Cosmic Microwave Background Theory

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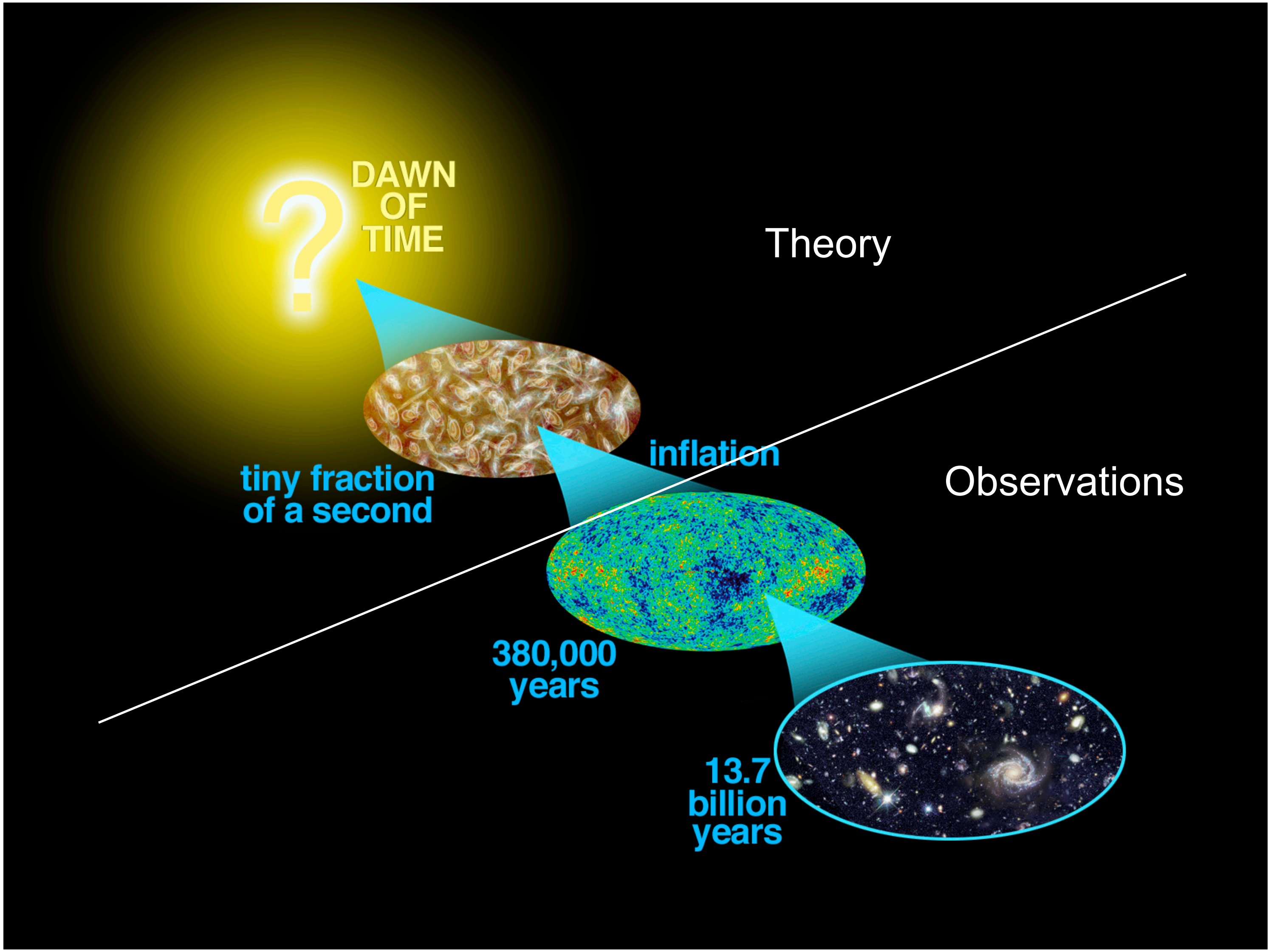
http://cosmologist.info

Outline

- Introduction and basic physics
- CMB temperature power spectrum and observables
- Parameter estimation
- Primordial perturbations
- CMB Polarization: E and B modes
- CMB lensing

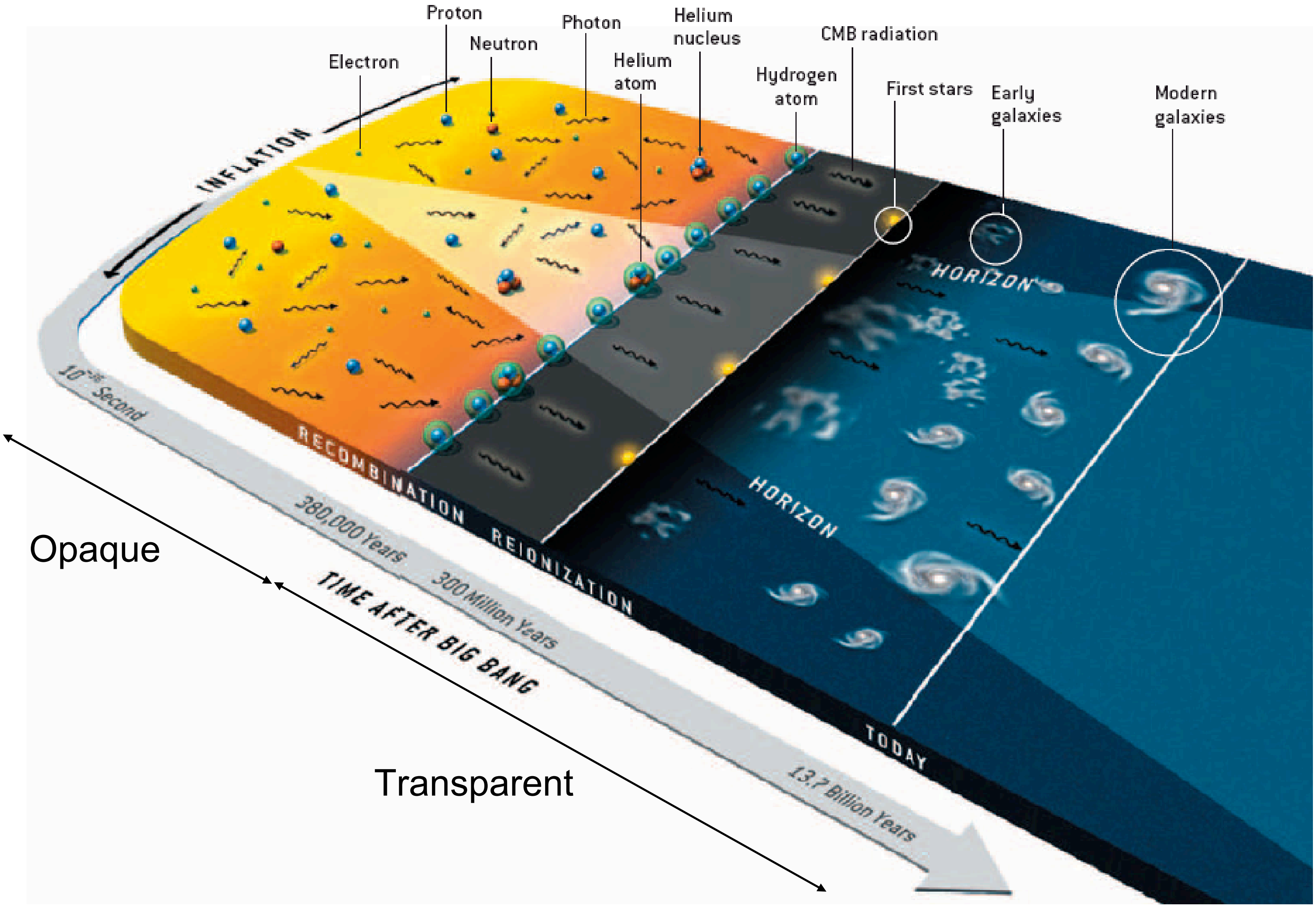
Not covered

Second order effects except lensing: SZ effect (clusters), OV, etc.
Mathematical details
CMB data analysis
etc..



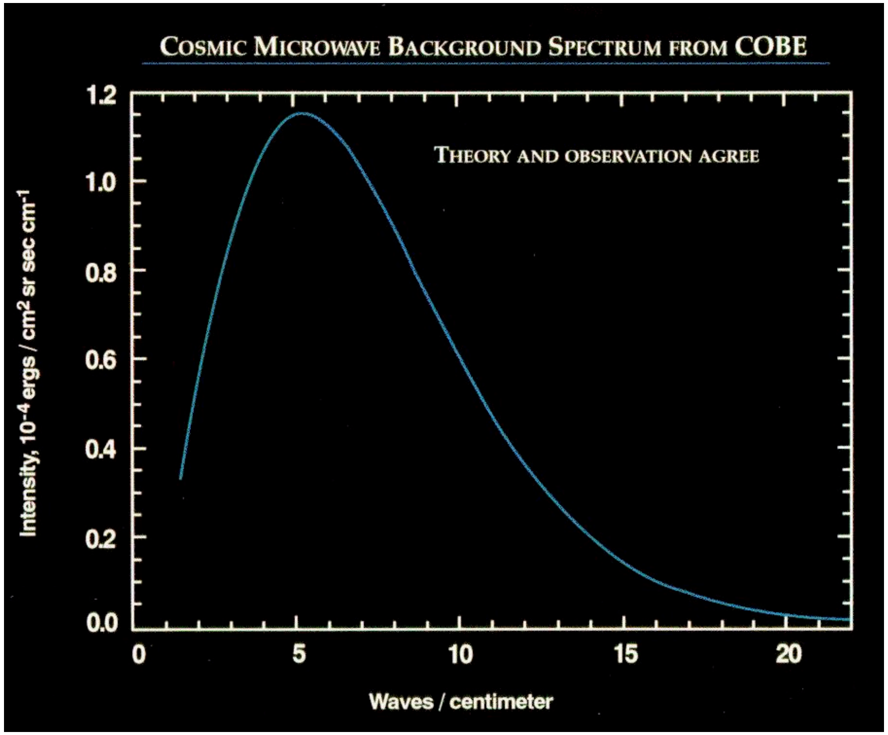
Source: NASA/WMAP Science Team

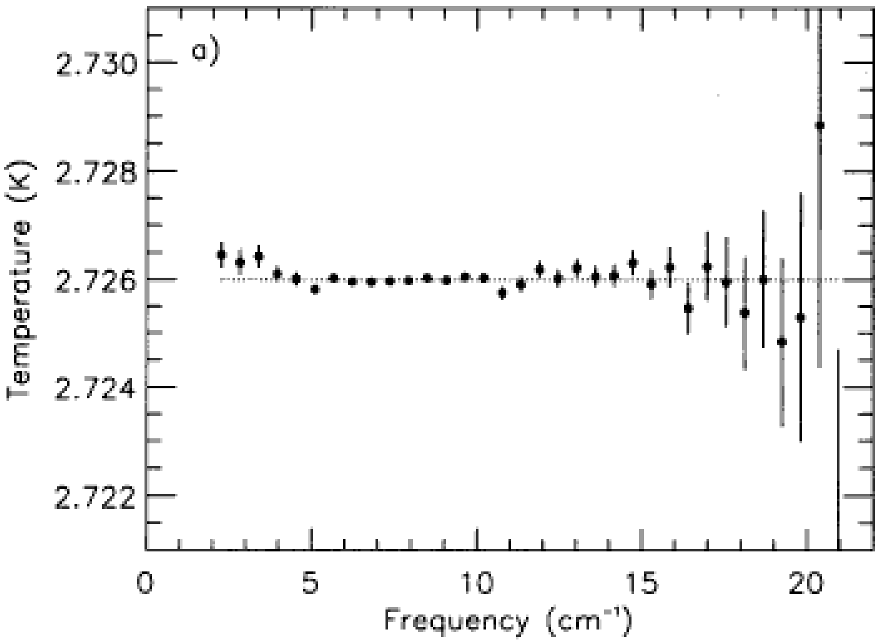
Evolution of the universe



Hu & White, Sci. Am., 290 44 (2004)

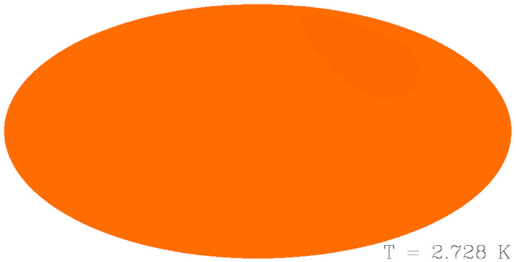
Black body spectrum observed by COBE



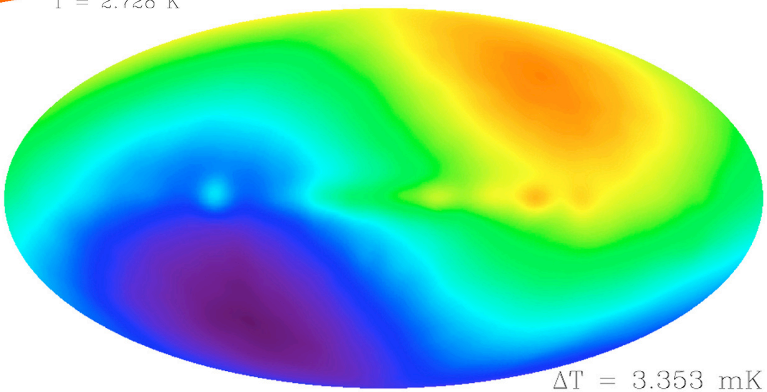


Residuals Mather et al 1994

- close to thermal equilibrium:
  temperature today of 2.726K (~ 3000K at z ~ 1000 because ν ~ (1+z))

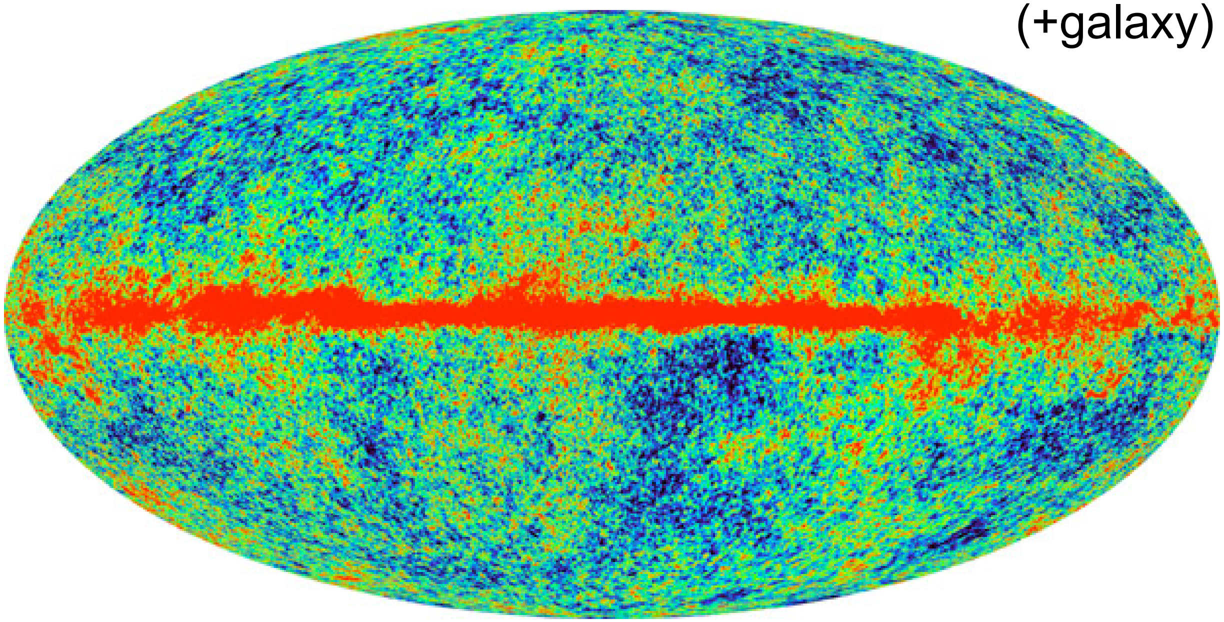


(almost) uniform 2.726K blackbody



Dipole (local motion)

O(10^{-5}) perturbations (+galaxy)

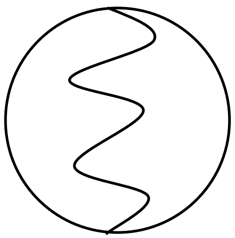


Observations: the microwave sky today

Source: NASA/WMAP Science Team

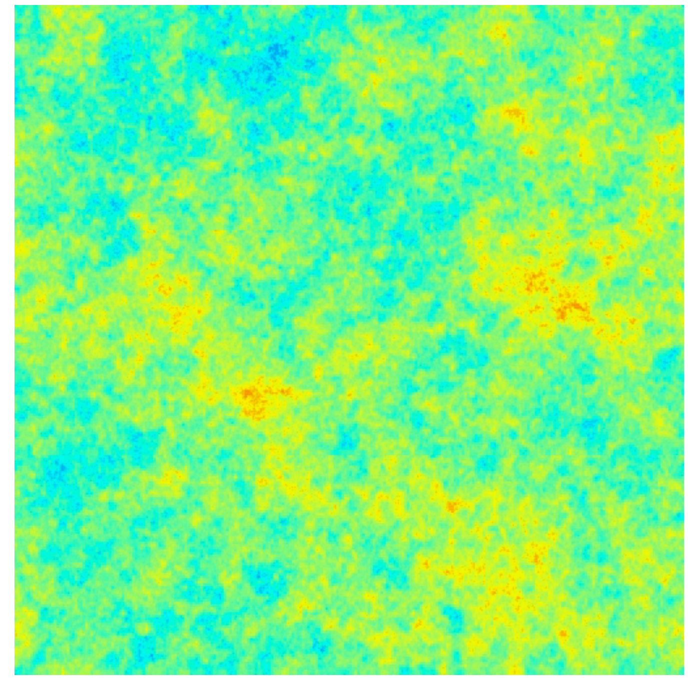
Can we predict the primordial perturbations?

- Maybe..



Quantum Mechanics
"waves in a box" calculation
vacuum state, etc…

Inflation
make $>10^{30}$ times bigger



After inflation
Huge size, amplitude ~ 10^{-5}

Perturbation evolution – what we actually observe

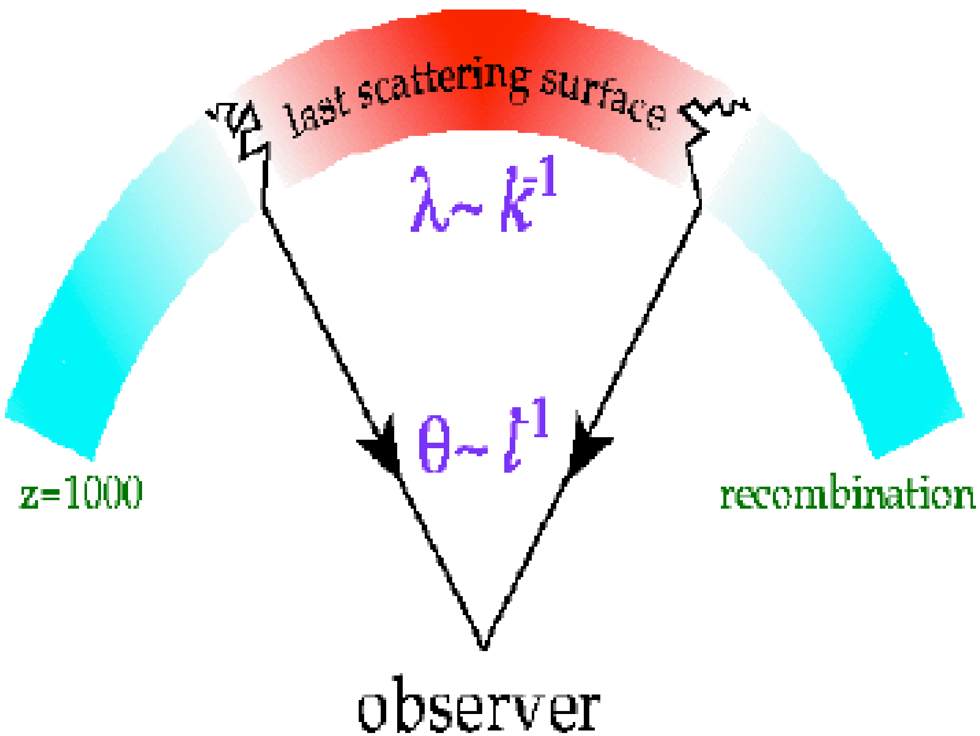
CMB monopole source till 380 000 yrs (last scattering), linear in conformal time

scale invariant primordial adiabatic scalar spectrum

photon/baryon plasma + dark matter, neutrinos

Characteristic scales: sound wave travel distance; diffusion damping length

Observed ΔT
as function of angle on the sky



Calculation of theoretical perturbation evolution

Perturbations O(10^{-5})

⇨ Simple linearized equations are very accurate (except small scales)

Can use real or Fourier space

Fourier modes evolve independently: simple to calculate accurately

Physics Ingredients

• Thomson scattering (non-relativistic electron-photon scattering)
  - tightly coupled before recombination: 'tight-coupling' approximation
  (baryons follow electrons because of very strong e-m coupling)
• Background recombination physics (Saha/full multi-level calculation)
• Linearized General Relativity
• Boltzmann equation (how angular distribution function evolves with scattering)

CMB power spectrum C_l

- Theory: Linear physics + Gaussian primordial fluctuations

$$a_{lm} = \int d\Omega \;\Delta T\; Y_{lm}^*$$

Theory prediction $C_l = \langle |a_{lm}|^2 \rangle$

- variance (average over all possible sky realizations)
- statistical isotropy implies independent of *m*

Initial conditions + cosmological parameters —(linearized GR + Boltzmann equations)→ C_l

CMBFAST: cmbfast.org
CAMB: camb.info
CMBEASY: cmbeasy.org
COSMICS, etc..

Sources of CMB anisotropy

Sachs Wolfe:
Potential wells at last scattering cause redshifting as photons climb out

Photon density perturbations:
Over-densities of photons look hotter

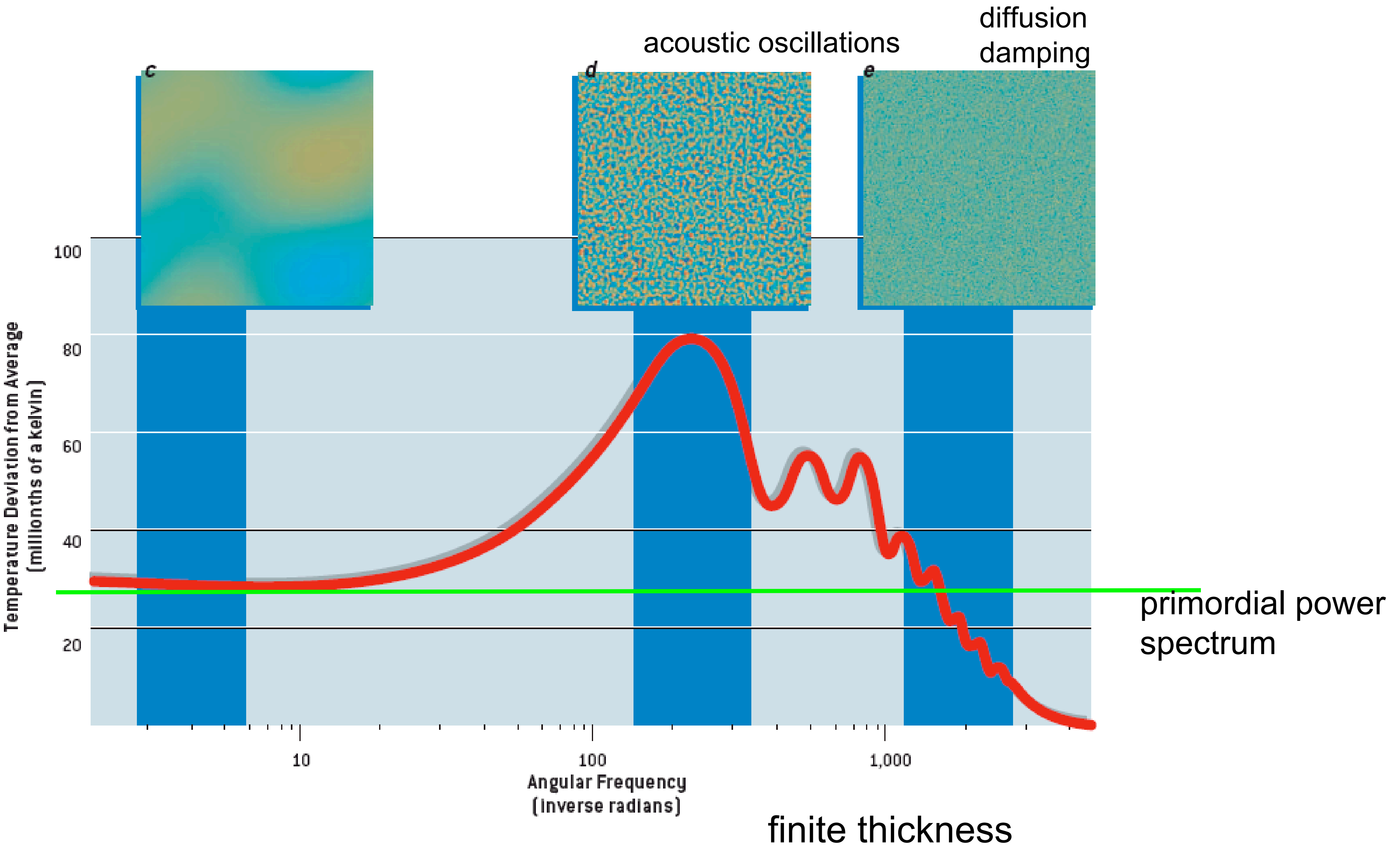
Doppler:
Velocity of photon/baryons at last scattering gives Doppler shift

Integrated Sachs Wolfe:
Evolution of potential along photon line of sight:
net red- or blue-shift as photon climbs in an out of varying potential wells

Others:
Photon quadupole/polarization at last scattering, second-order effects, etc.

CMB temperature power spectrum

Primordial perturbations + later physics



Hu & White, Sci. Am., 290 44 (2004)

Why C_l oscillations?

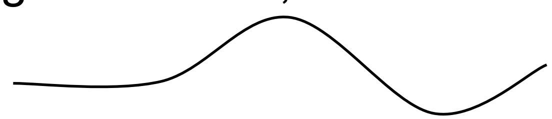
Think in k-space: modes of different size

- Co-moving Poisson equation: $(k/a)^2\, \Phi = \kappa\, \delta\rho / 2$
  - potentials approx constant on super-horizon scales
  - radiation domination $\rho \sim 1/a^4$

  à $\delta\rho/\rho \sim k^2\, a^2\, \Phi$

  à since Φ ~ constant, super-horizon density perturbations grow $\sim a^2$

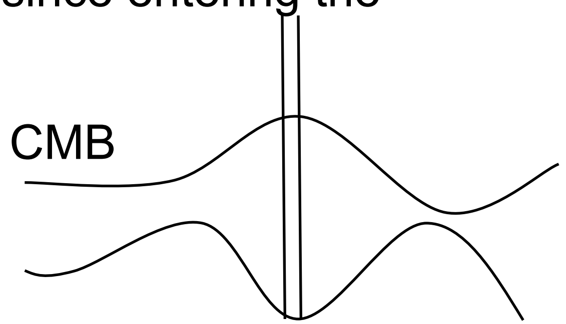
- After entering horizon pressure important: perturbation growth slows, then bounces back



  à series of acoustic oscillations (sound speed $\sim c/\sqrt{3}$)

- CMB anisotropy (mostly) from a surface at fixed redshift: phase of oscillation at time of last scattering depends on time since entering the horizon

  à k-dependent oscillation amplitude in the observed CMB



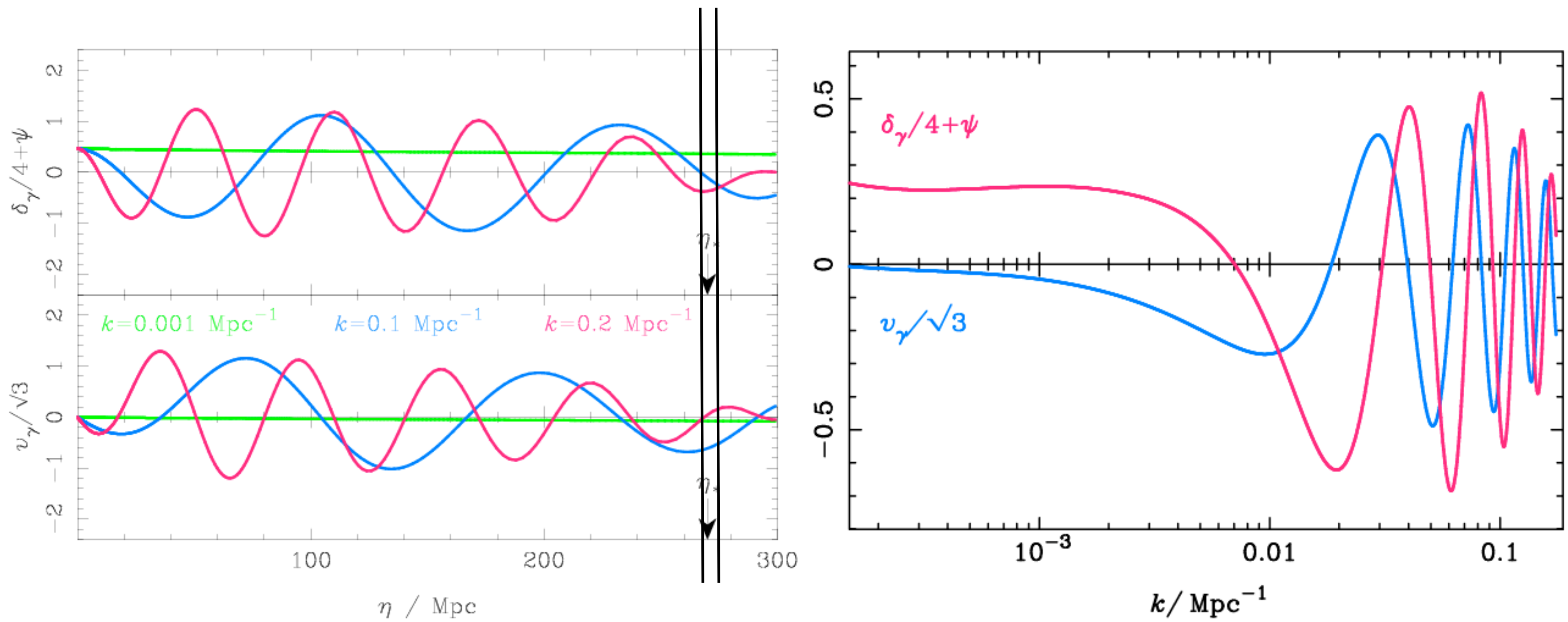
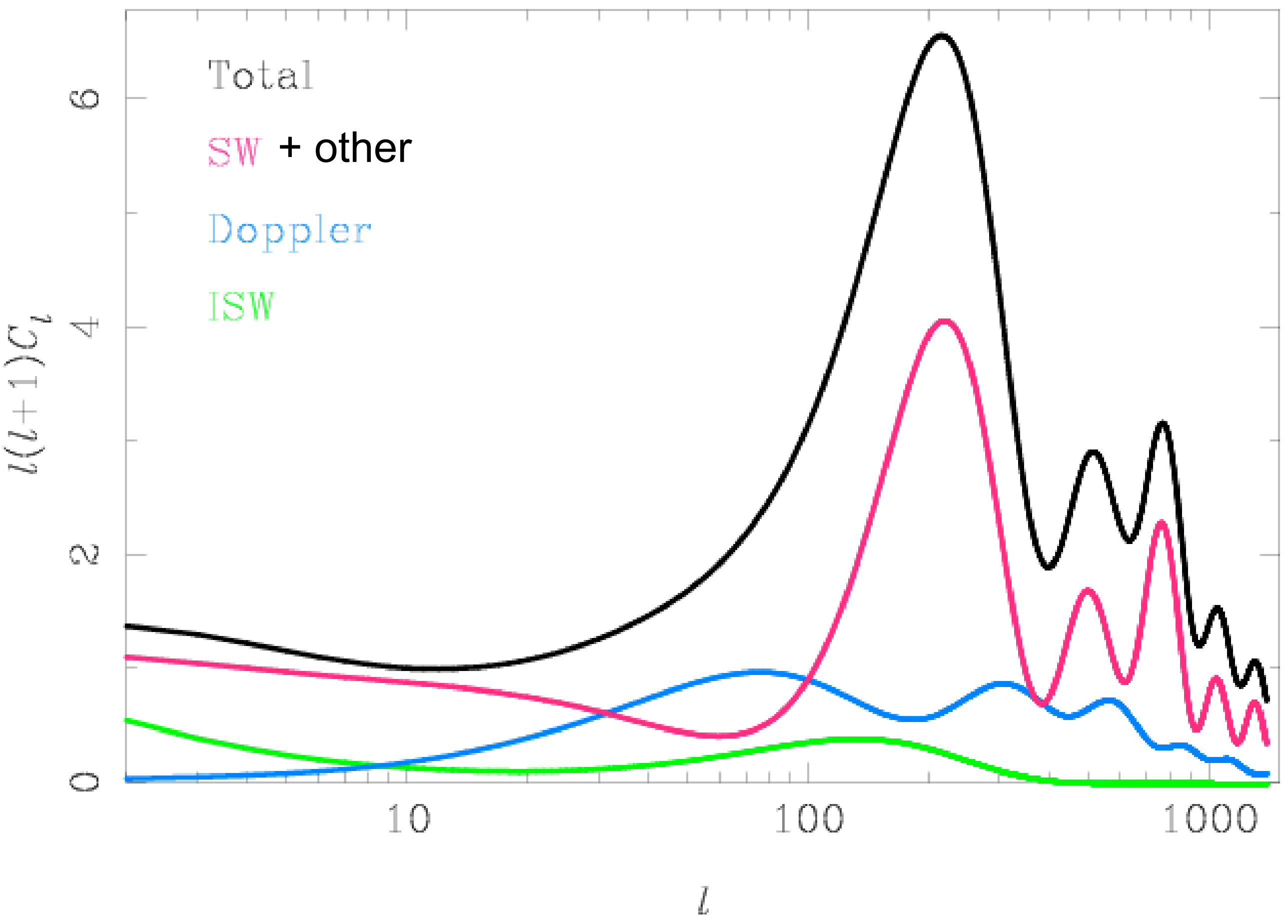


Fig. 3. Evolution of the combination $\delta_\gamma/4 + \psi$ (top left) and the photon velocity v_γ (bottom left) which determine the temperature anisotropies produced at last scattering (denoted by the arrow at η_*). Three modes are shown with wavenumbers $k = 0.001$, 0.1 and 0.2 Mpc^{-1}, and the initial conditions are adiabatic. The fluctuations at the time of last scattering are shown as a function of linear scale in the right-hand plot.

Challinor: astro-ph/0403344

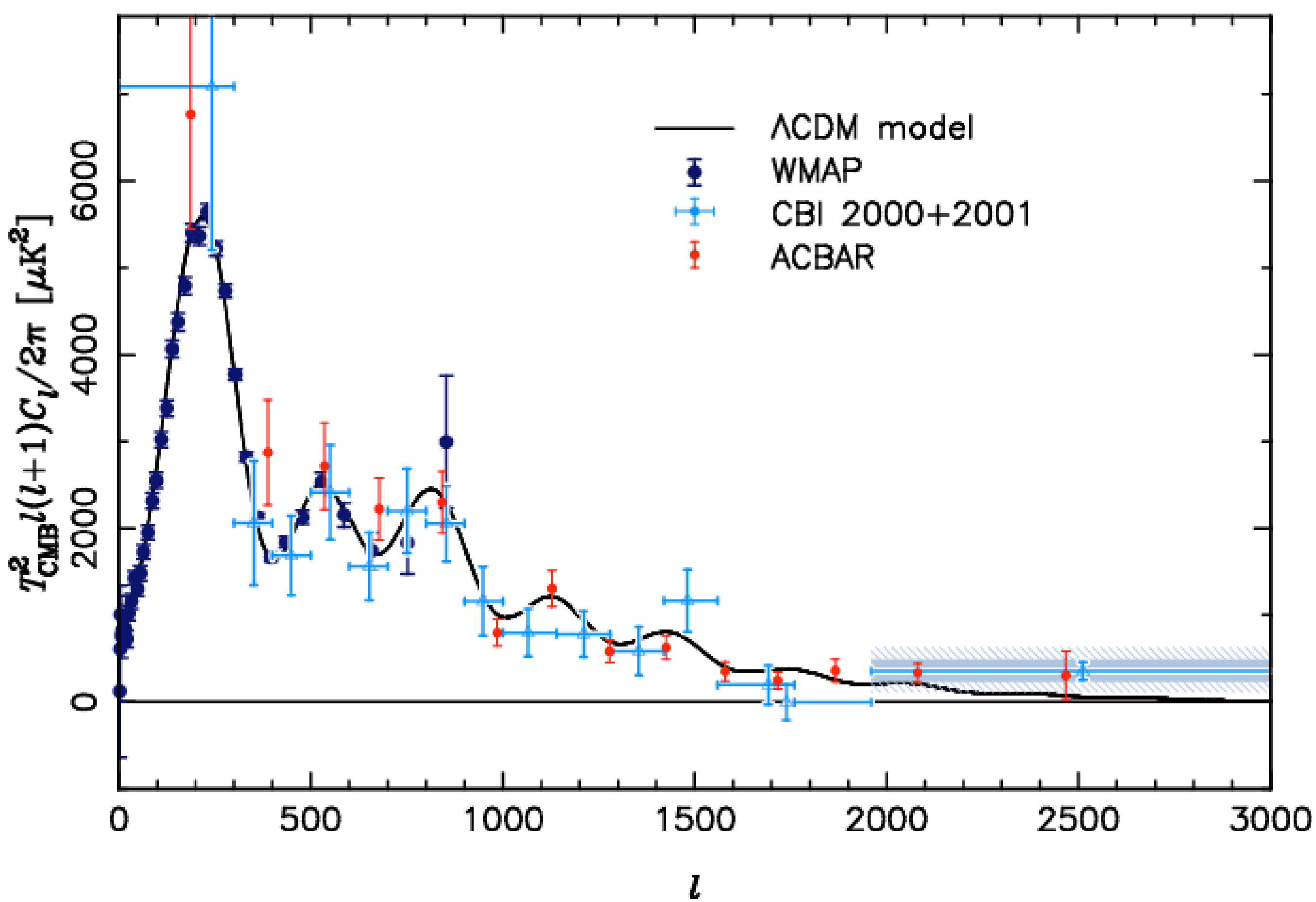
Contributions to temperature C_l



Challinor: astro-ph/0403344

Anisotropy observations

Current WMAP + other CMB data



Redhead et al: astro-ph/0402359

What can we learn from the CMB?

- **Initial conditions**
  What types of perturbations, power spectra, distribution function (Gaussian?);
  => learn about inflation or alternatives.
  (distribution of ΔT; power as function of scale; polarization and correlation)

- **What and how much stuff**
  Matter densities ($_b$, $_{cdm}$);; neutrino mass
  (details of peak shapes, amount of small scale damping)

- **Geometry and topology**
  global curvature $_K$ of universe; topology
  (angular size of perturbations; repeated patterns in the sky)

- **Evolution**
  Expansion rate as function of time; reionization
  - Hubble constant H_0 ; dark energy evolution w = pressure/density
  (angular size of perturbations; *l* < 50 large scale power; polarizationr)

- **Astrophysics**
  S-Z effect (clusters), foregrounds, etc.

• Cosmic Variance: only one sky

Use estimator for variance: $C_l^{obs} = \frac{1}{2l+1}\sum_m |a_{lm}|^2$

Assume a_{lm} gaussian:

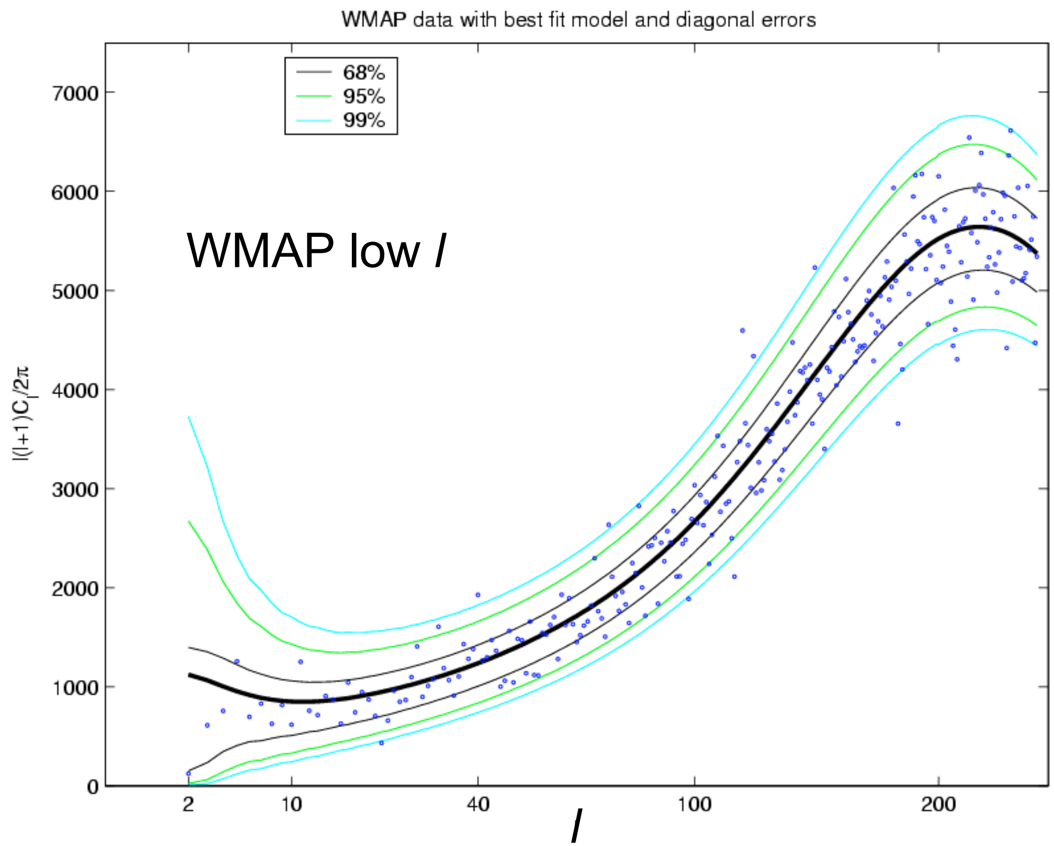
$C_l^{obs} \sim \chi^2$ with $2l+1$ d.o.f.

"Cosmic Variance"

$$\left\langle |\Delta C_l^{obs}|^2 \right\rangle \approx \frac{2C_l^2}{2l+1}$$

$P(C_l \mid C_l^{obs})$

- inverse gamma distribution (+ noise, sky cut, etc).



Cosmic variance gives fundamental limit on how much we can learn from CMB

Parameter Estimation

- Can compute P({Я} | data) = P(C_l({Я}) | c_l^{obs})

- Often want marginalized constraints. e.g.

$$<\theta_1> = \int \theta_1 P(\theta_1 \theta_2 \theta_3 \ldots \theta_n \mid data) d\theta_1 d\theta_2 .. d\theta_n$$

- BUT: Large *n* integrals very hard to compute!

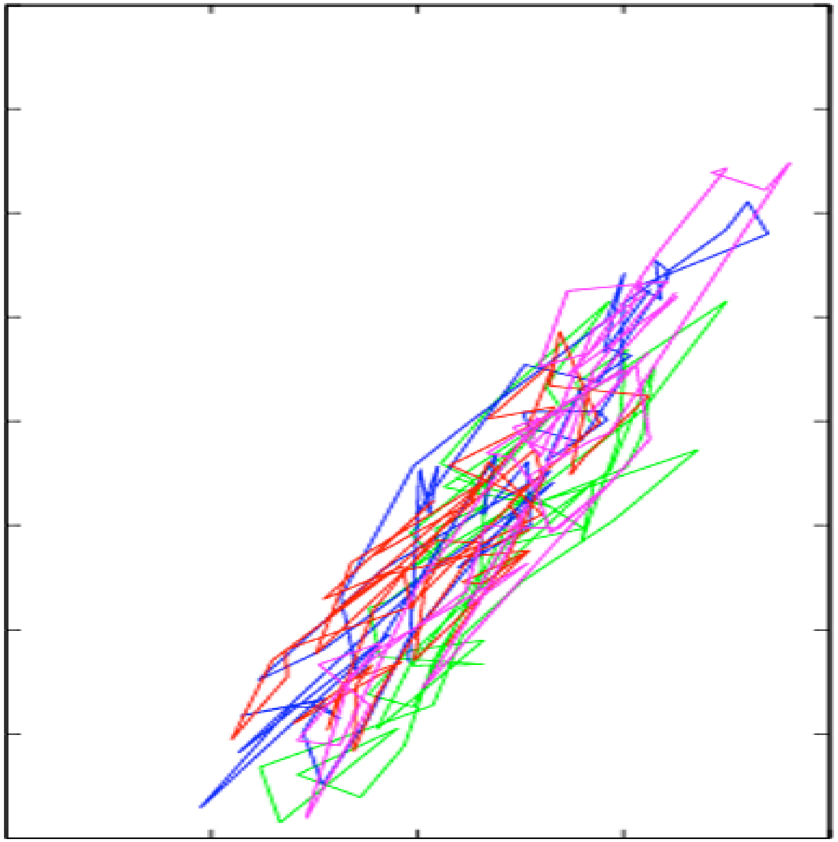
- If we instead sample from P({Я} | data) then it is easy:

$$<\theta_1> \approx \frac{1}{N} \sum_i \theta_1^{(i)}$$

Can easily learn everything we need from set of samples

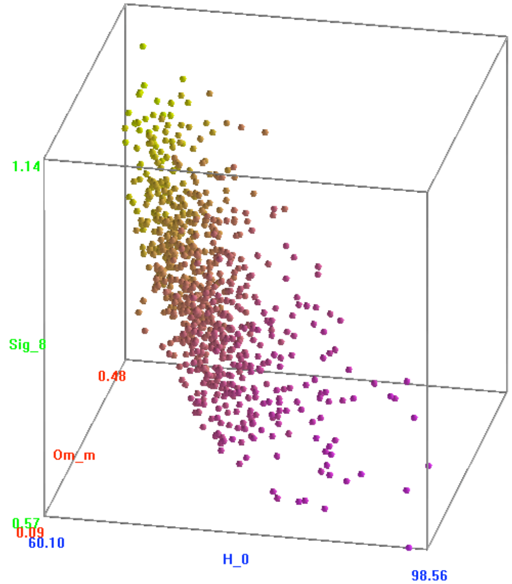
Markov Chain Monte Carlo sampling

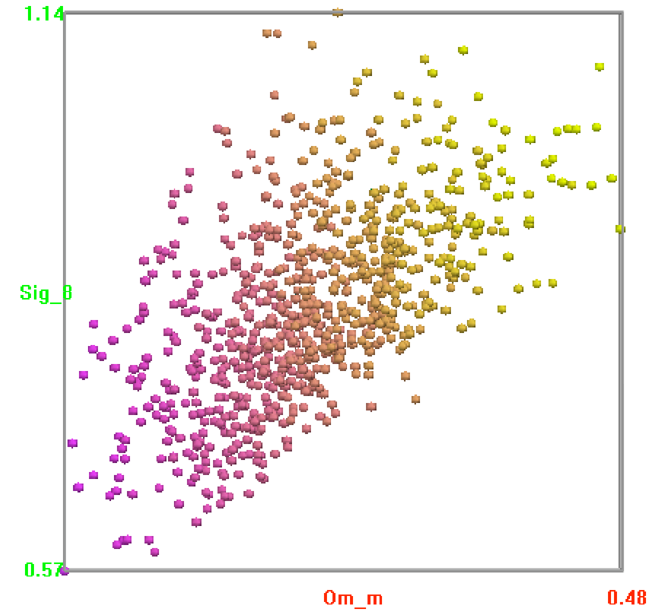
- Metropolis-Hastings algorithm
- Number density of samples proportional to probability density
- At its best scales linearly with number of parameters
  (as opposed to exponentially for brute integration)

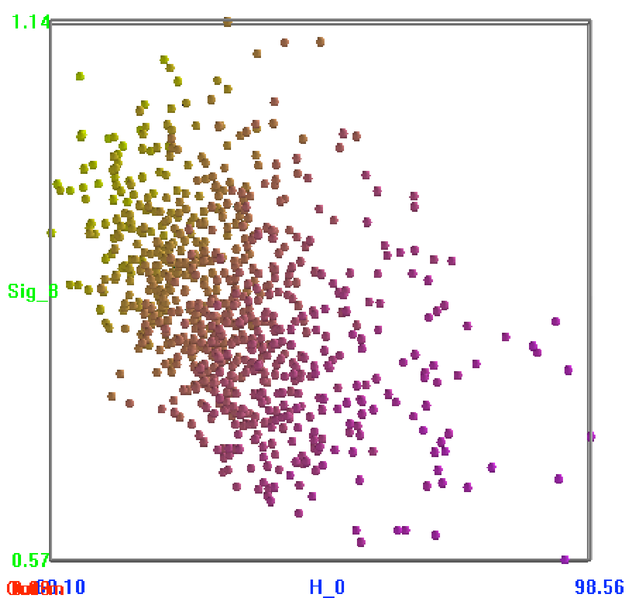


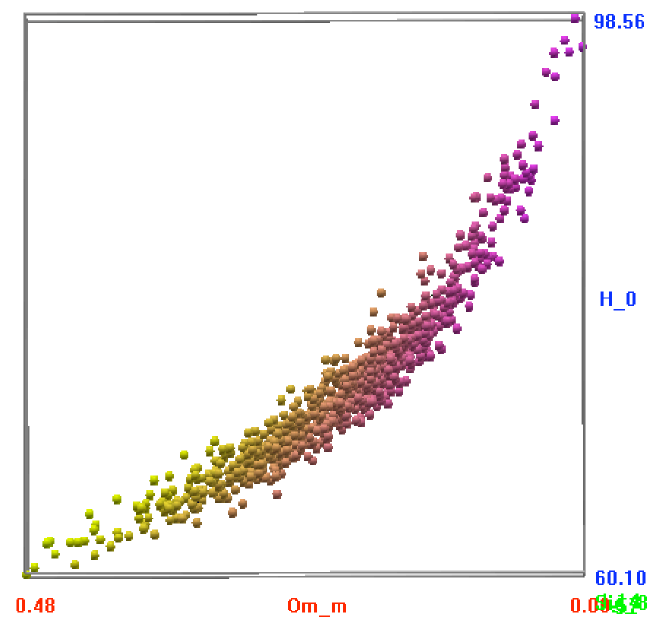
Now standard method for parameter estimation. Public CosmoMC code available at http://cosmologist.info/cosmomc (Lewis, Bridle: astro-ph/0205436)

Samples in 6D parameter space









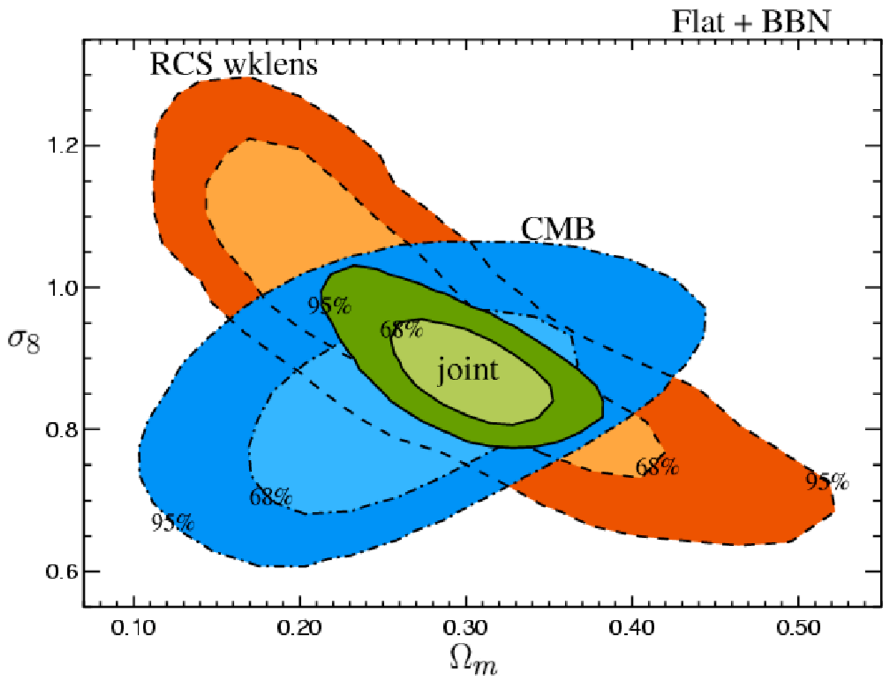
CMB data alone

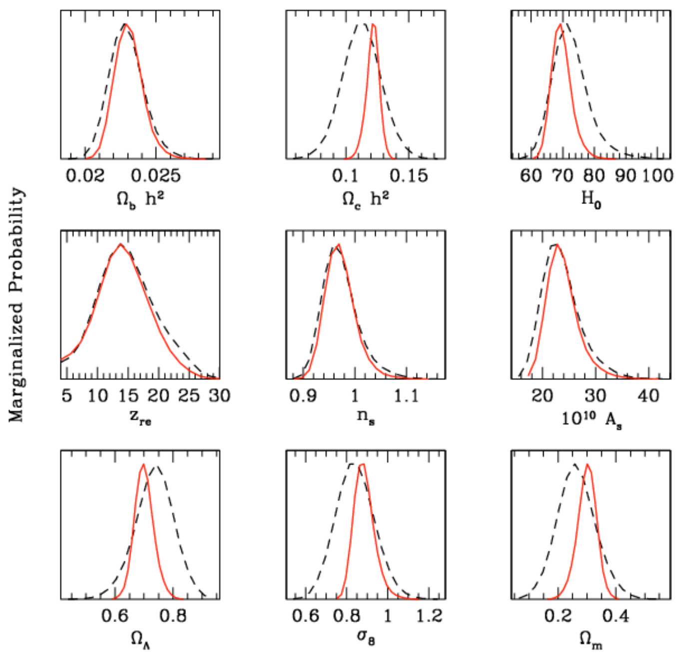
color = optical depth

Plot number density of samples as function of parameters

Often better constraint by combining with other data

e.g. CMB+galaxy lensing +BBN prior

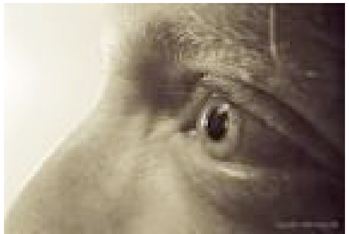




Contaldi, Hoekstra, Lewis: astro-ph/0302435

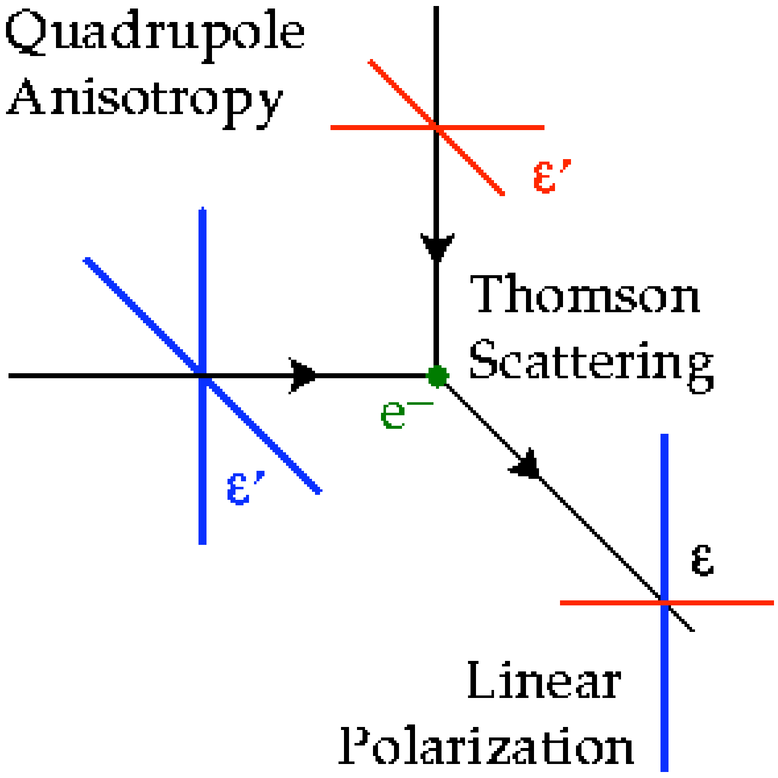
Thomson Scattering Polarization

W Hu



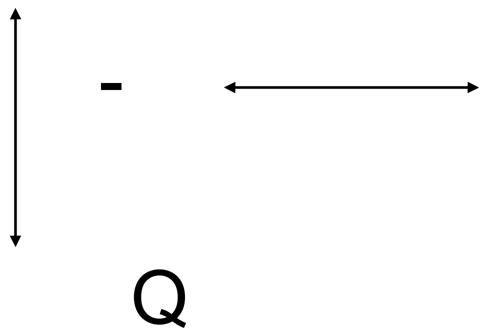
CMB Polarization

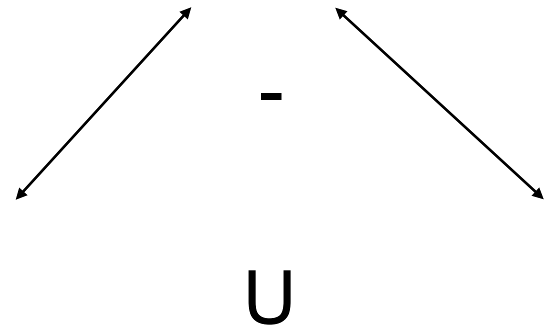
Generated during last scattering (and reionization) by Thomson scattering of anisotropic photon distribution



Hu astro-ph/9706147

Polarization: Stokes' Parameters



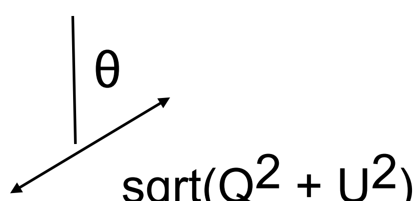


Q → -Q, U → -U under 90 degree rotation

Q → U, U → -Q under 45 degree rotation

⇒ Spin-2 field Q + i U
or Rank 2 trace free symmetric tensor

$$P = \begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}$$



$\theta = \frac{1}{2} \tan^{-1} U/Q$

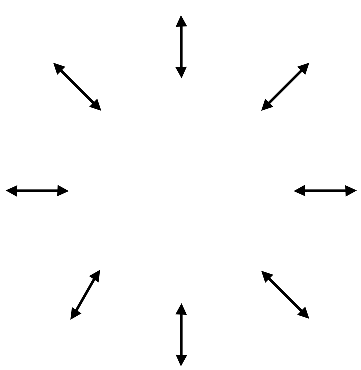
E and B polarization

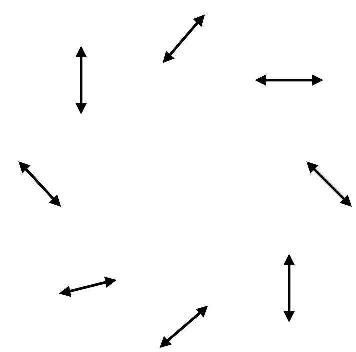
$$\mathcal{P}_{ab} = \nabla_{(a}\nabla_{b)} P_E - \epsilon^{c}{}_{(a}\nabla_{b)}\nabla_c P_B$$

“gradient” modes
E polarization

“curl” modes
B polarization

e.g.





E and B harmonics

- Expand scalar P_E and P_B in spherical harmonics
- Expand P_{ab} in tensor spherical harmonics

$$\mathcal{P}_{ab} = \frac{1}{\sqrt{2}} \sum_{lm} \left(E_{lm} \, Y^{G}_{(lm)ab} + B_{lm} \, Y^{C}_{(lm)ab} \right)$$

$$E_{lm} = \sqrt{2} \int_{4\pi} \mathrm{d}S \, Y^{G\,ab*}_{(lm)} \mathcal{P}_{ab} \qquad B_{lm} = \sqrt{2} \int_{4\pi} \mathrm{d}S \, Y^{C\,ab*}_{(lm)} \mathcal{P}_{ab}$$

Harmonics are orthogonal over the full sky:

E/B decomposition is exact and lossless on the full sky

Zaldarriaga, Seljak: astro-ph/9609170
Kamionkowski, Kosowsky, Stebbins: astro-ph/9611125

Primordial Perturbations

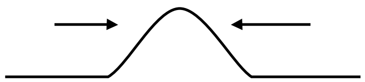
fluid at redshift < 10^9

- Photons
- Nearly massless neutrinos
  Free-streaming (no scattering) after neutrino decoupling at z ~ 10^9
- Baryons + electrons
  tightly coupled to photons by Thomson scattering
- Dark Matter
  Assume cold. Coupled only via gravity.
- Dark energy
  probably negligible early on

Perturbations O(10^{-5})

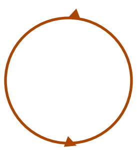
- Linear evolution
- Fourier k mode evolves independently
- Scalar, vector, tensor modes evolve independently
- Various linearly independent solutions

Scalar modes: Density perturbations, potential flows



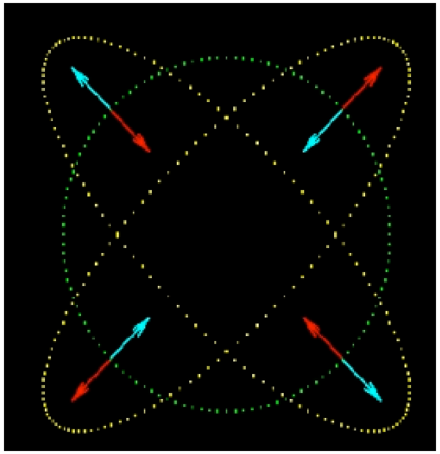
$\delta\rho, \nabla\delta\rho, etc$

Vector modes: Vortical perturbations



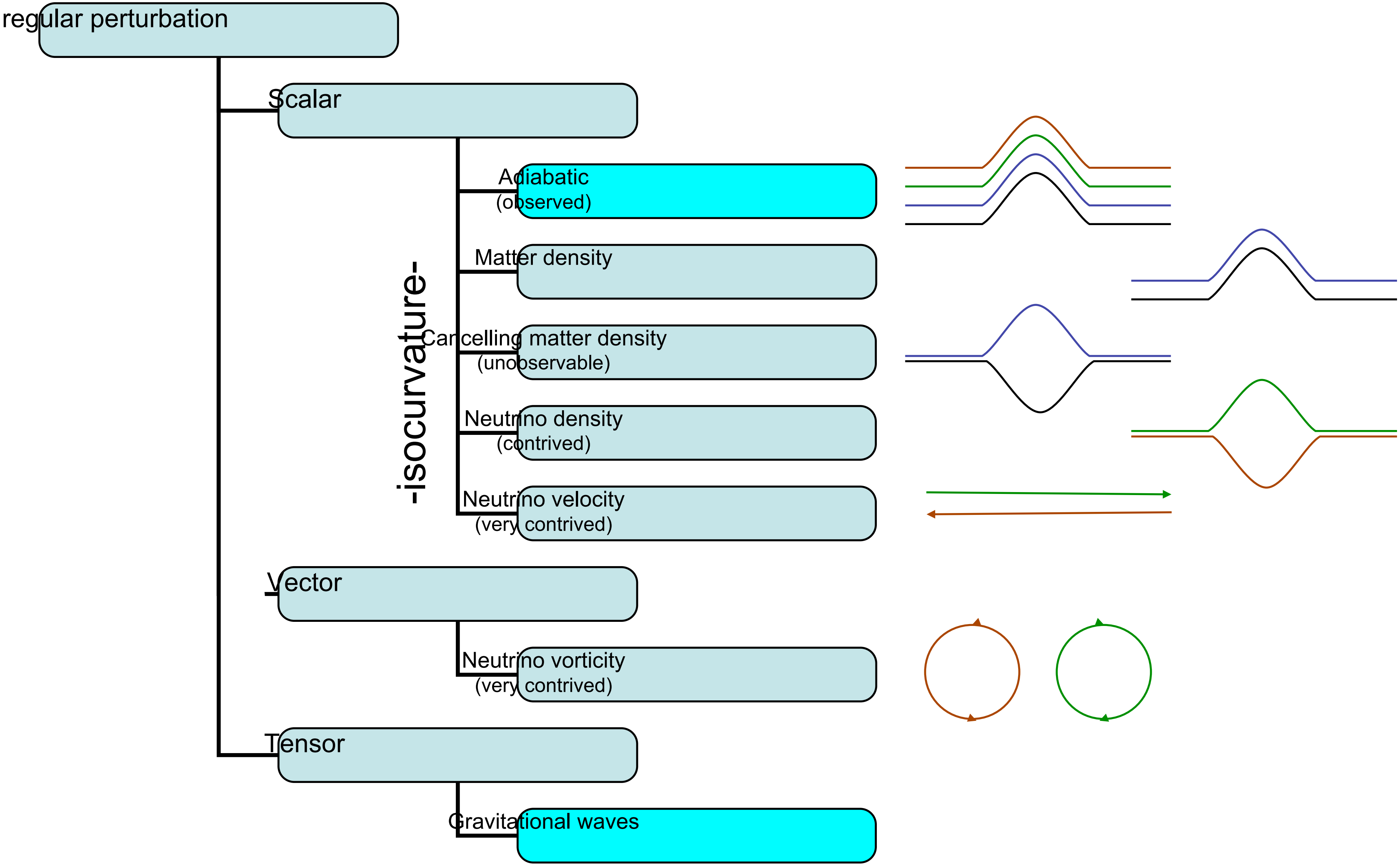
velocities, *v* $(\nabla \bullet v = 0)$

Tensor modes: Anisotropic space distortions – gravitational waves



http://www.astro.cf.ac.uk/schools/6thFC2002/GravWaves/sld009.htm

General regular linear primordial perturbation



+ irregular modes, neutrino n-pole modes, n-Tensor modes Rebhan and Schwarz: gr-qc/9403032
+ other possible components, e.g. defects, magnetic fields, exotic stuff…

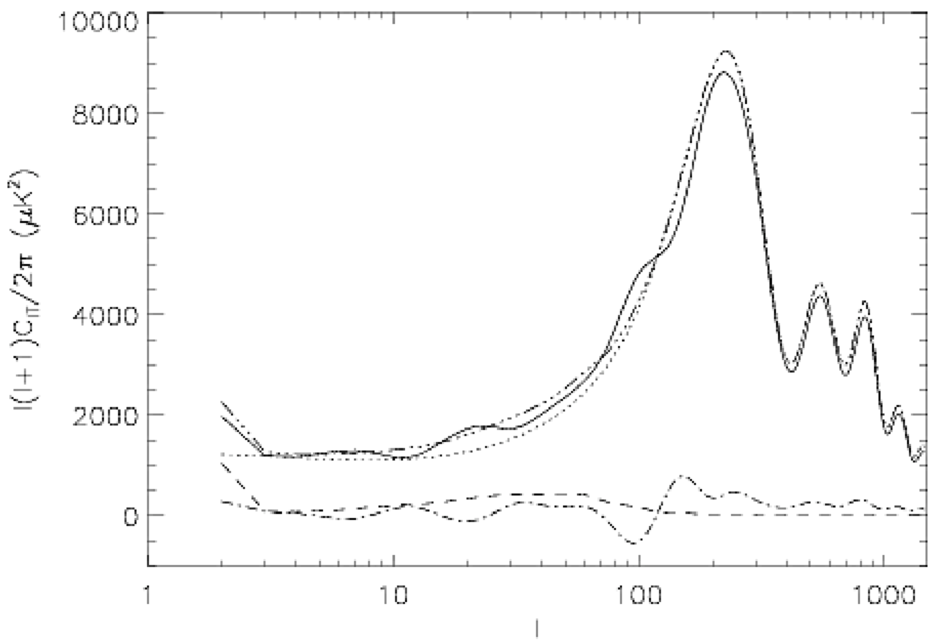
Irregular (decaying) modes

- Generally ~ a^{-1}, a^{-2} or $a^{-1/2}$
- E.g. decaying vector modes unobservable at late times unless ridiculously large early on

Adiabatic decay ~ $a^{-1/2}$ after neutrino decoupling.

possibly observable if generated around or after neutrino decoupling

Otherwise have to be very large (non-linear?) at early times



Amendola, Finelli: astro-ph/0411273

CMB Polarization Signals

- E polarization from scalar, vector and tensor modes
- B polarization only from vector and tensor modes (curl grad = 0) + non-linear scalars

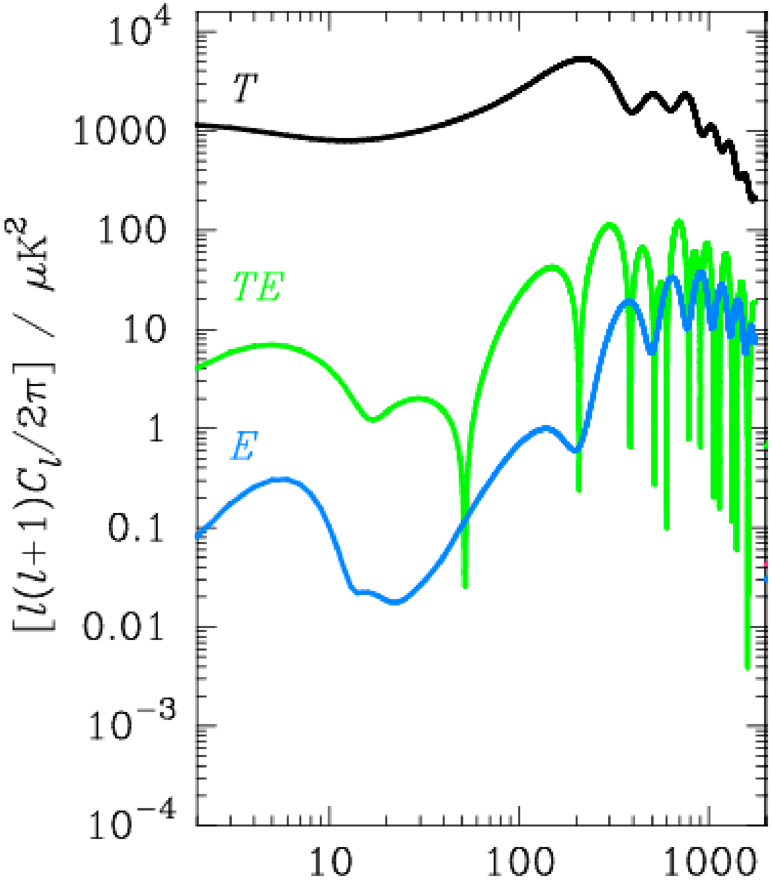
Average over possible realizations (statistically isotropic):

$$\langle E^*_{l'm'} E_{lm} \rangle = \delta_{l'l} \delta_{m'm} C_l^{EE} \qquad \langle B^*_{l'm'} B_{lm} \rangle = \delta_{l'l} \delta_{m'm} C_l^{BB}$$

Parity symmetric ensemble: $\langle E^*_{l'm'} B_{lm} \rangle = 0$

Power spectra contain all the useful information if the field is Gaussian

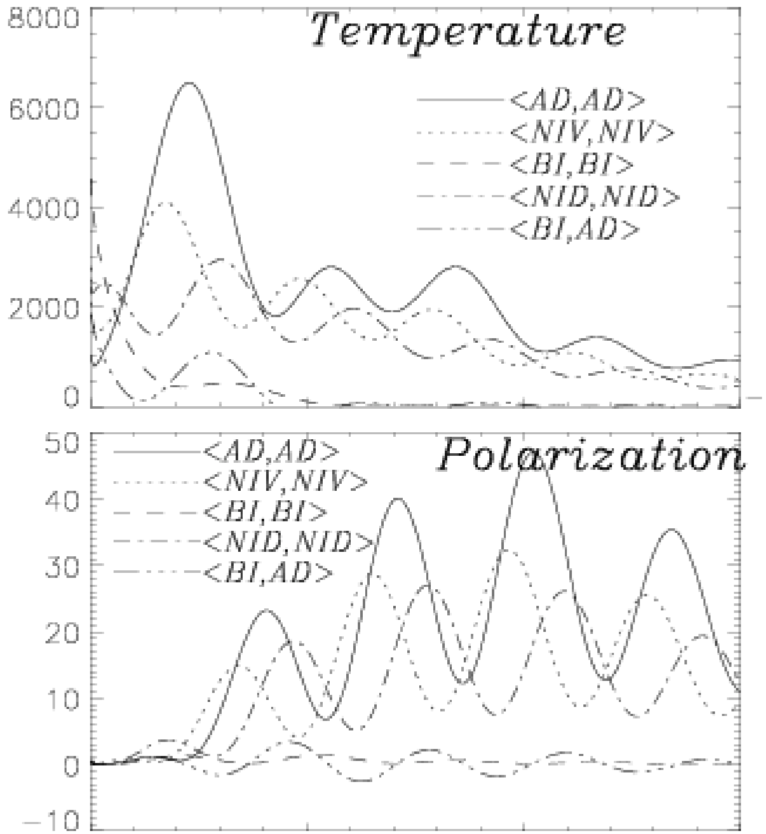
Scalar adiabatic mode



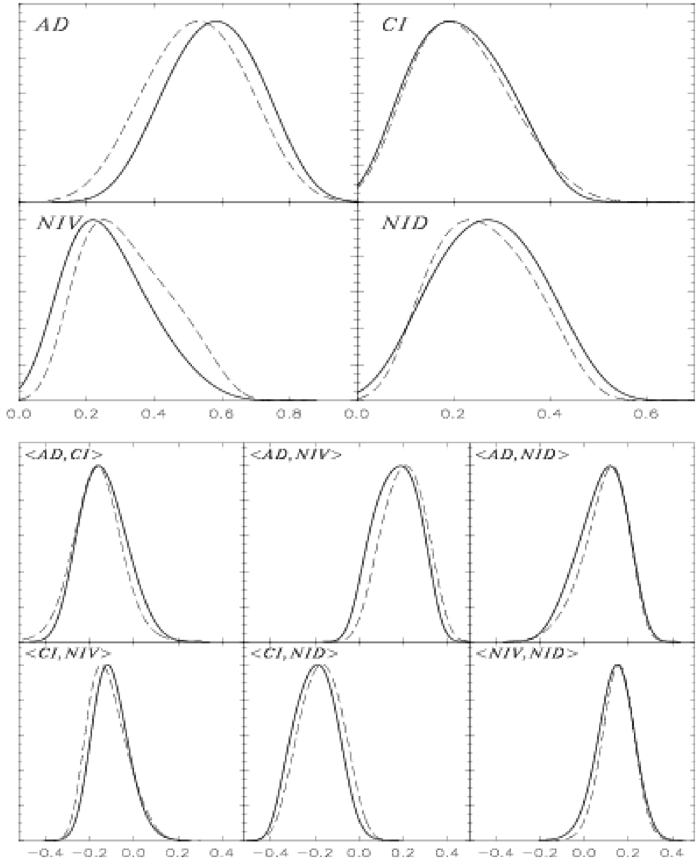
E polarization only

correlation to temperature T-E

General isocurvature models



- General mixtures currently poorly constrained



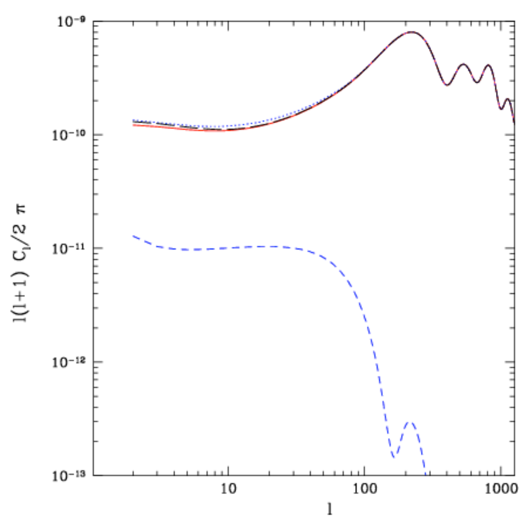
Bucher et al: astro-ph/0401417

Primordial Gravitational Waves
(tensor modes)

- Well motivated by some inflationary models
  - Amplitude measures inflaton potential at horizon crossing
  - distinguish models of inflation

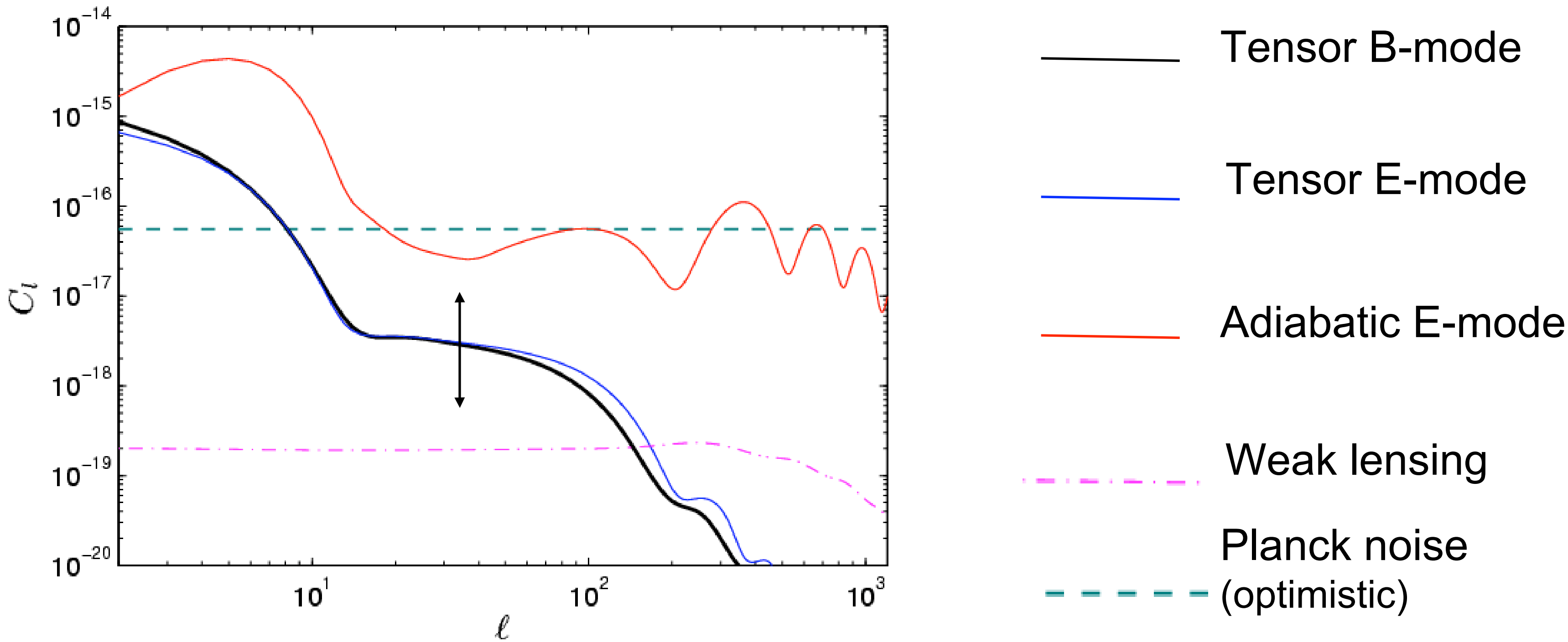
- Observation would rule out other models
  - ekpyrotic scenario predicts exponentially small amplitude
  - small also in many models of inflation, esp. two field e.g. curvaton

- Weakly constrained from CMB temperature anisotropy
  - cosmic variance limited to 10%
  - degenerate with other parameters (tilt, reionization, etc)



Look at CMB polarization:
'B-mode' smoking gun

CMB polarization from primordial gravitational waves (tensors)



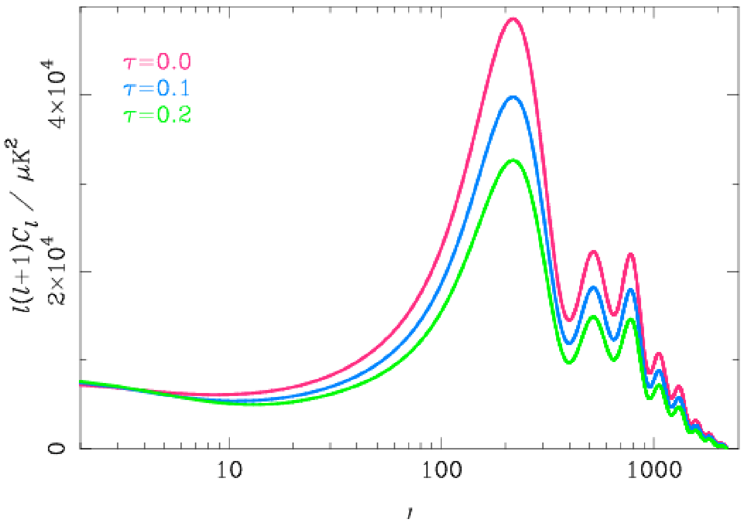
- Amplitude of tensors unknown
- Clear signal from B modes – there are none from scalar modes
- Tensor B is always small compared to adiabatic E

Seljak, Zaldarriaga: astro-ph/9609169

Reionization

Ionization since z ~ 6-20 scatters CMB photons

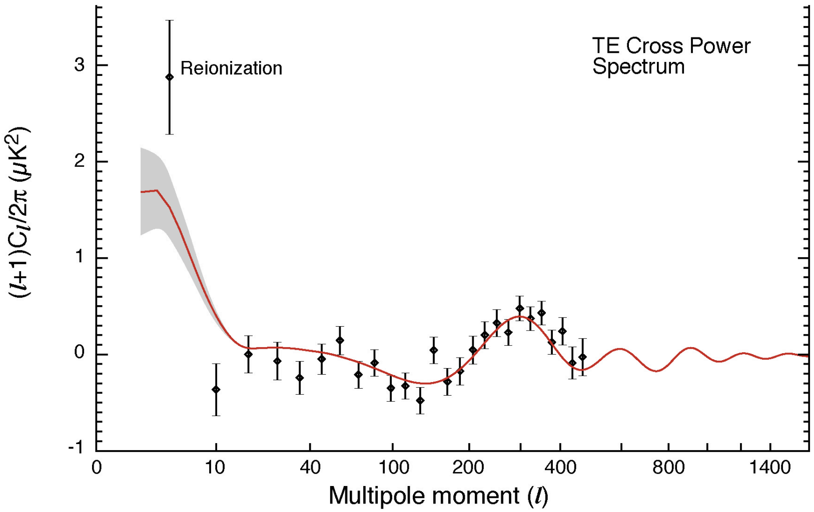
Temperature signal similar to tensors



Quadrupole at reionization implies large scale polarization signal

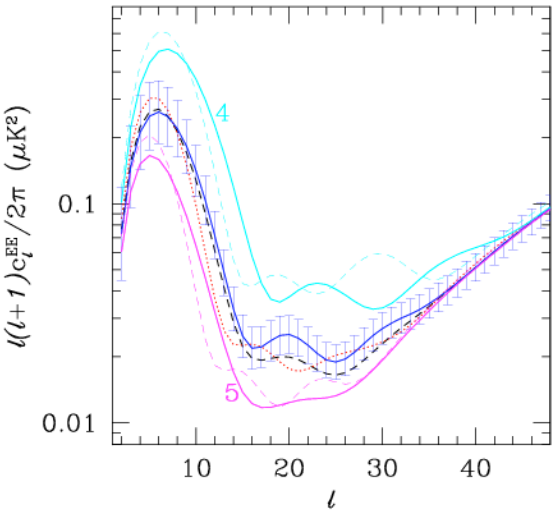
⇒

Measure optical depth with WMAP T-E correlation

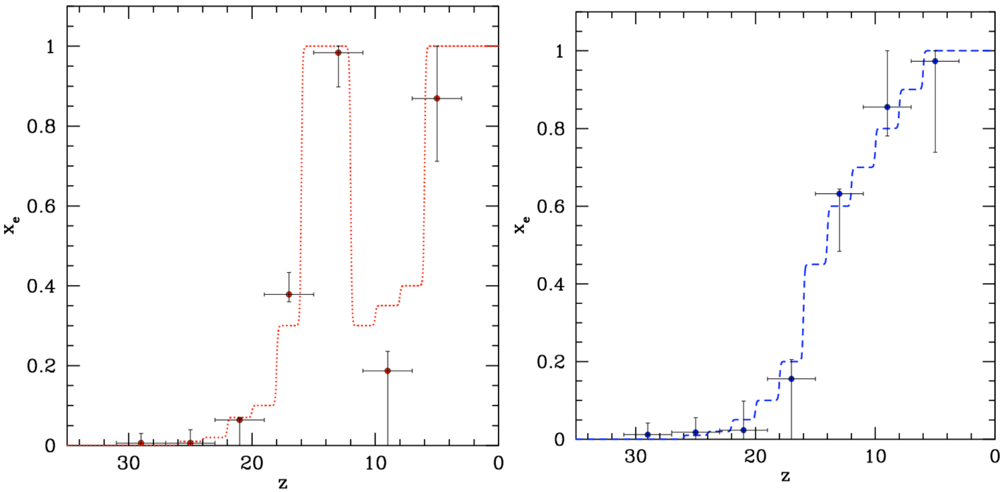


Cosmic variance limited data – resolve structure in EE power spectrum

(Weakly) constrain ionization history



Holder et al: astro-ph/0302404



Weller, Lewis, Battye (in prep)

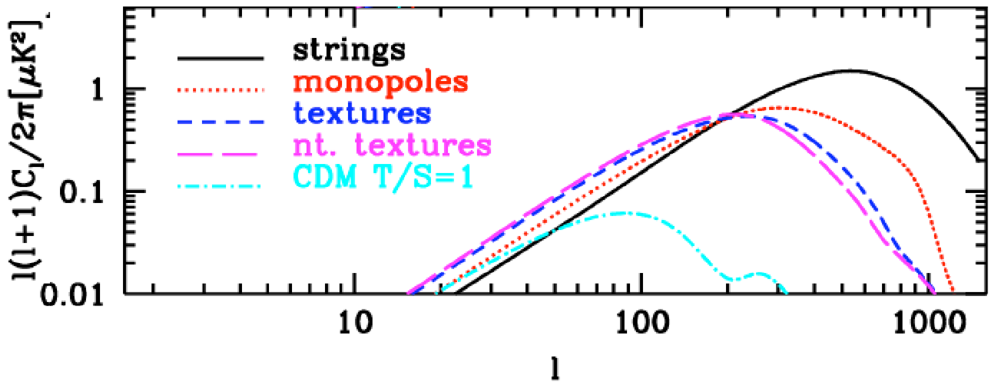
Other B-modes?

- Topological defects

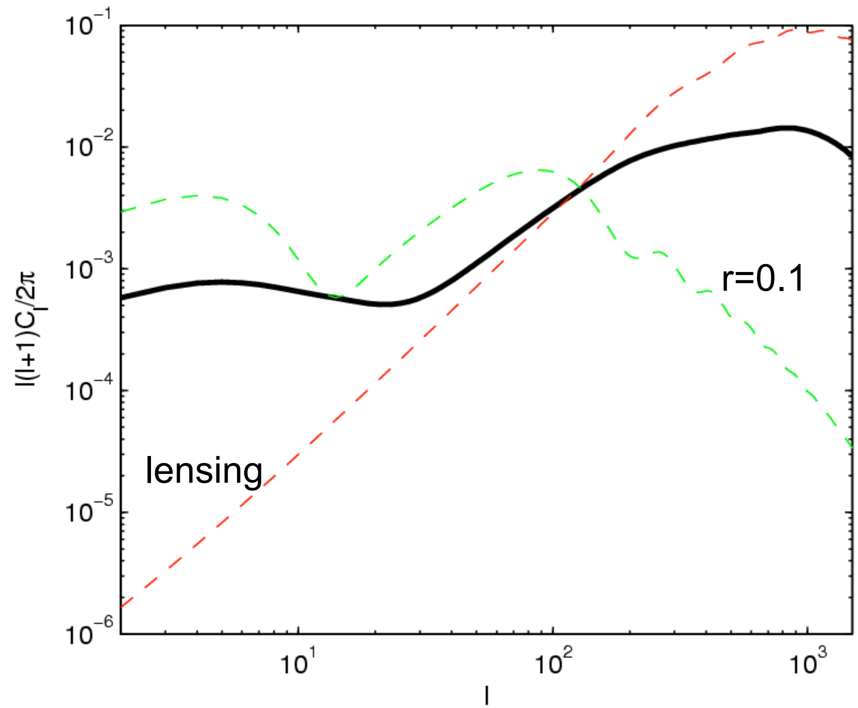
Seljak, Pen, Turok: astro-ph/9704231

Non-Gaussian signals

global defects:



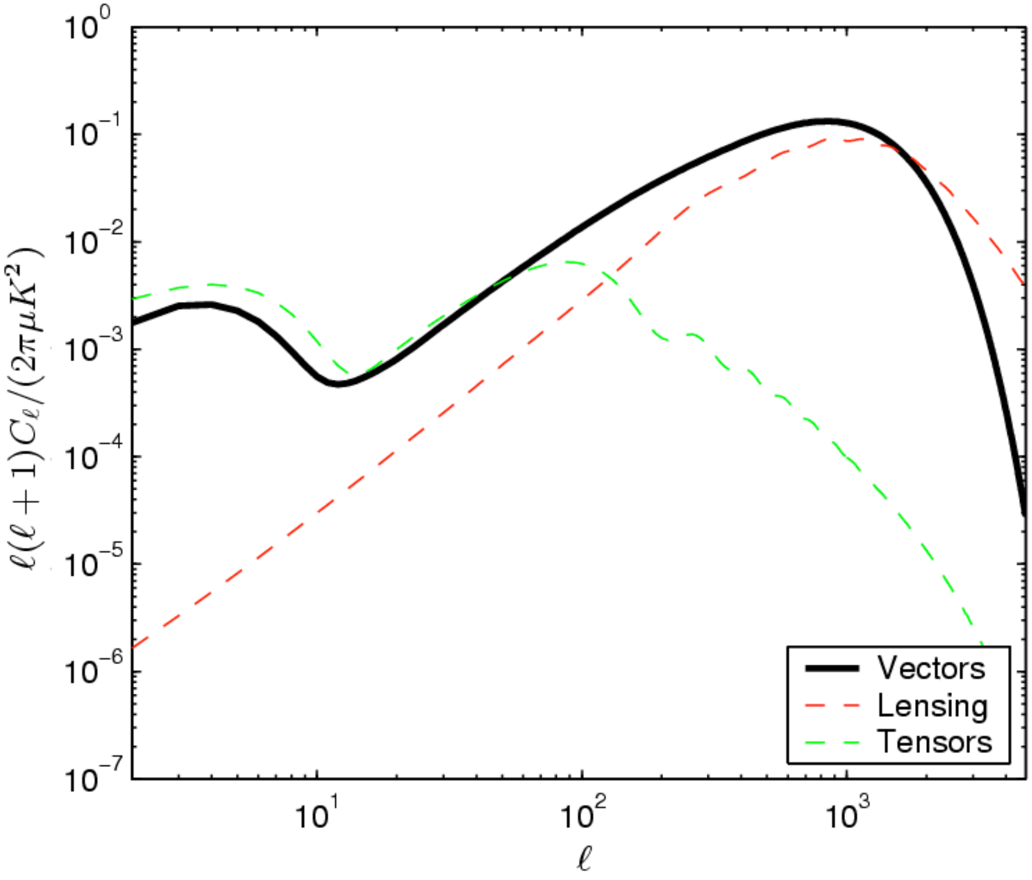
10% local strings from brane inflation:



Pogosian, Tye, Wasserman, Wyman: hep-th/0304188

- Regular vector mode: ‘neutrino vorticity mode’

- logical possibility but unmotivated (contrived). Spectrum unknown.

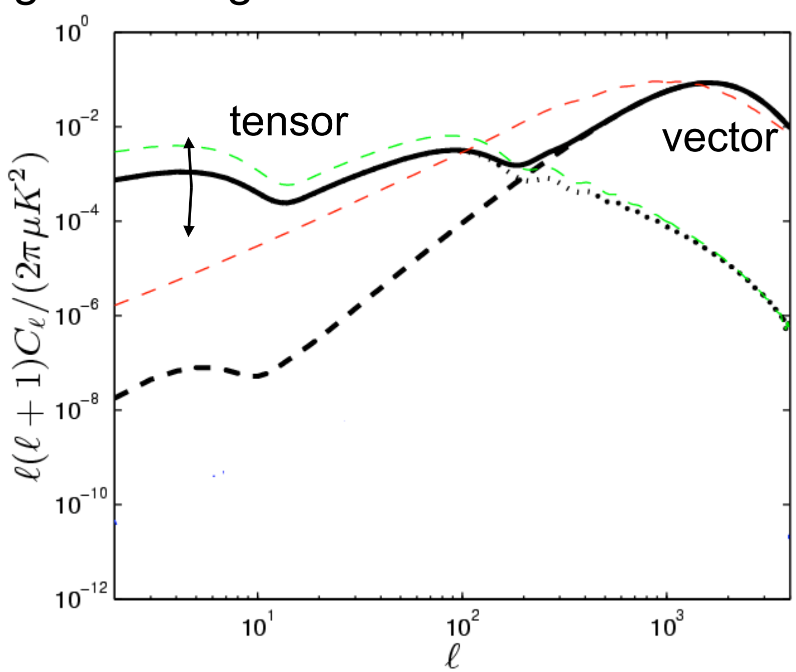


Similar to gravitational wave spectrum on large scales: distinctive small scale

Lewis: astro-ph/0403583

- Primordial magnetic fields
  - not well motivated theoretically, though know magnetic fields exist
  - contribution from sourced gravity waves (tensors) and vorticity (vectors)

e.g. Inhomogeneous field B = 3x10^{-9} G, spectral index n = -2.9



Tensor amplitude uncertain.

Non-Gaussian signal.

Check on galaxy/cluster evolution models.

Lewis, astro-ph/0406096.
Subramanian, Seshadri, Barrow, astro-ph/0303014

the initial properties of the magnetic field. (c) Concerning studies of generation of cosmic microwave background (CMBR) anisotropies due to primordial magnetic fields of $B \sim 10^{-9}$ Gauss on $\gtrsim 10\,$Mpc scales, such fields are not only impossible to generate in early causal magnetogenesis scenarios but also seemingly ruled out by distortions of the CMBR spectrum due to magnetic field dissipation on smaller scales and the overproduction of cluster magnetic fields. (d) The most promising detection

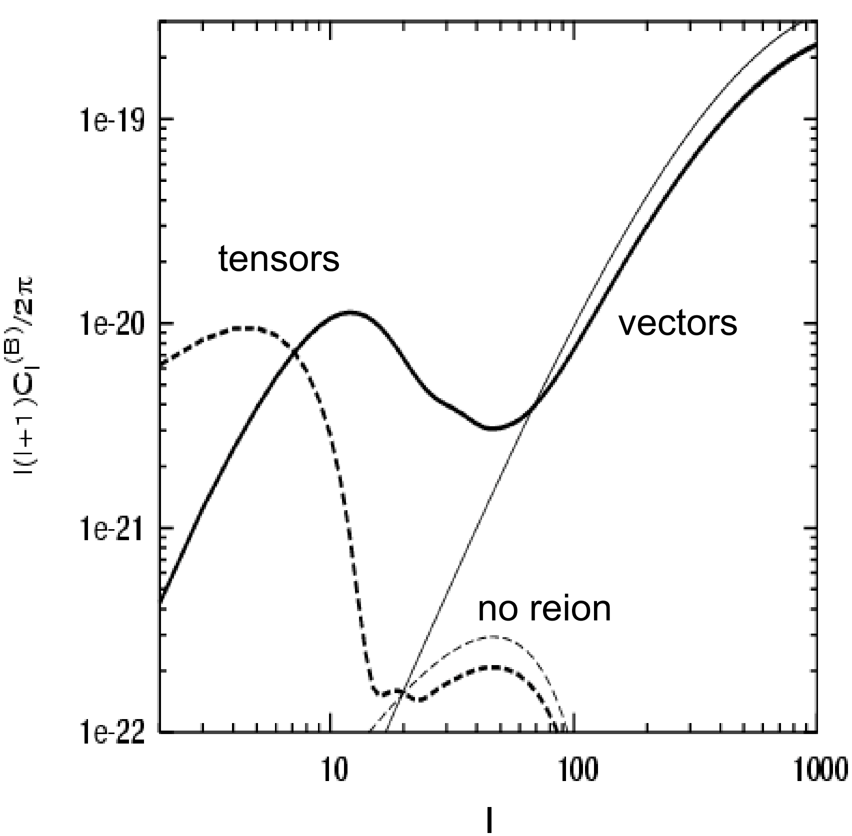
Banerjee and Jedamzik: astro-ph/0410032

- Also Faraday rotation B-modes at low frequencies

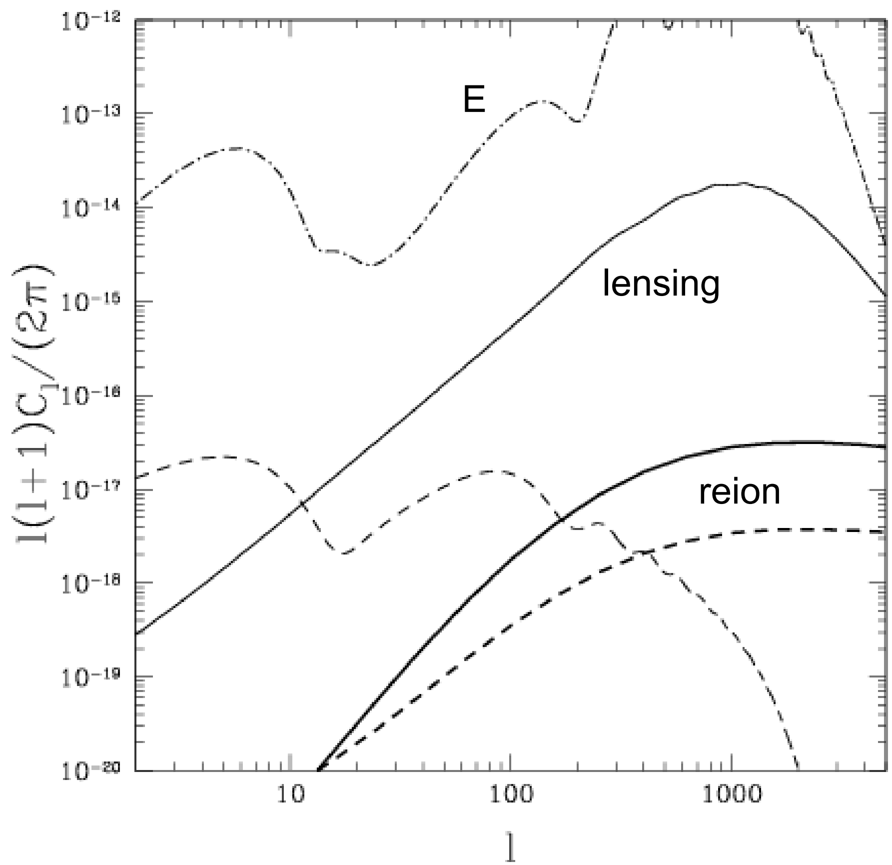
Kosowsky, Loeb: astro-ph/9601055, Scoccola, Harari, Mollerach: astro-ph/0405396

- Small second order effects, e.g.

Second order vectors and tensors:
Mollerach, Harari, Matarrese: astro-ph/0310711



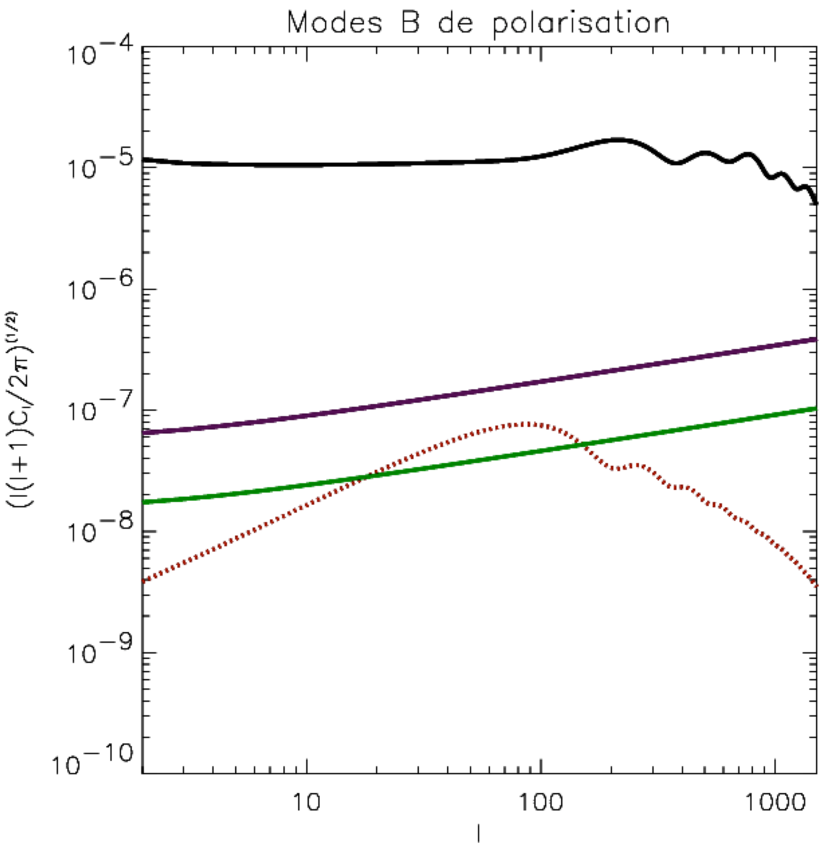
Inhomogeneous reionization
Santon, Cooray, Haiman, Knox, Ma: astro-ph/0305471; Hu: astro-ph/9907103



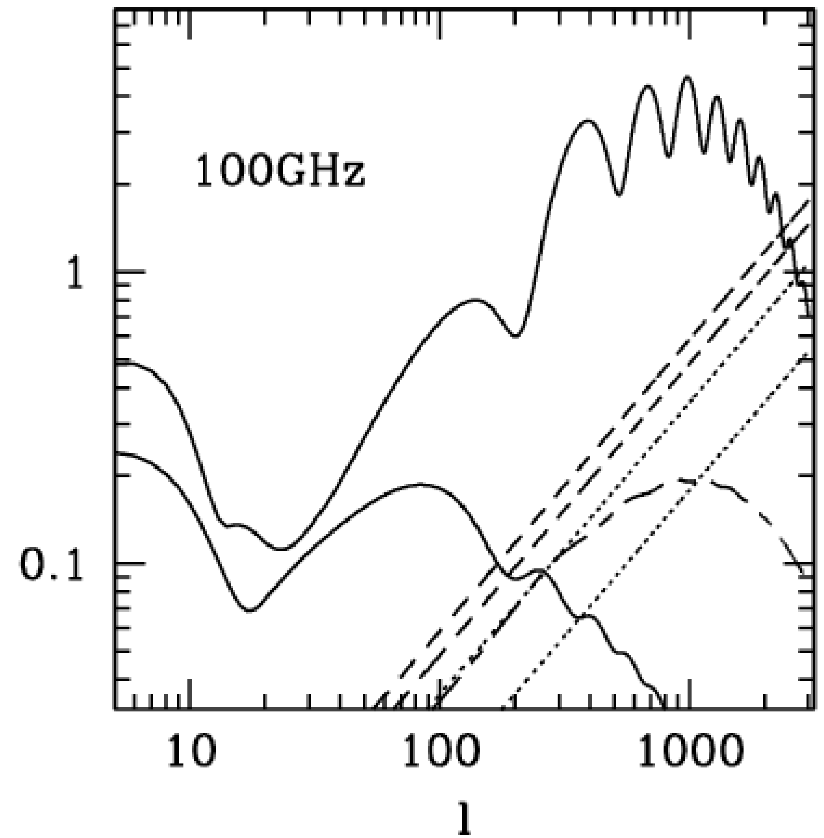
non-Gaussian

- Systematics and foregrounds, e.g.

Galactic dust (143 and 217 GHz):
Lazarian, Prunet: astro-ph/0111214



Extragalactic radio sources:
Tucci et al: astro-ph/0307073



B modes potentially a good diagnostic of foreground subtraction problems or systematics

Partial sky E/B separation problem

$$\mathcal{P}_{ab} = \nabla_{\langle a}\nabla_{b\rangle} P_E - \epsilon^c{}_{(a}\nabla_{b)}\nabla_c P_B$$

Pure E: $\nabla^a\nabla^b\mathcal{P}_{ab} = (\nabla^2 + 2)\nabla^2 P_E$

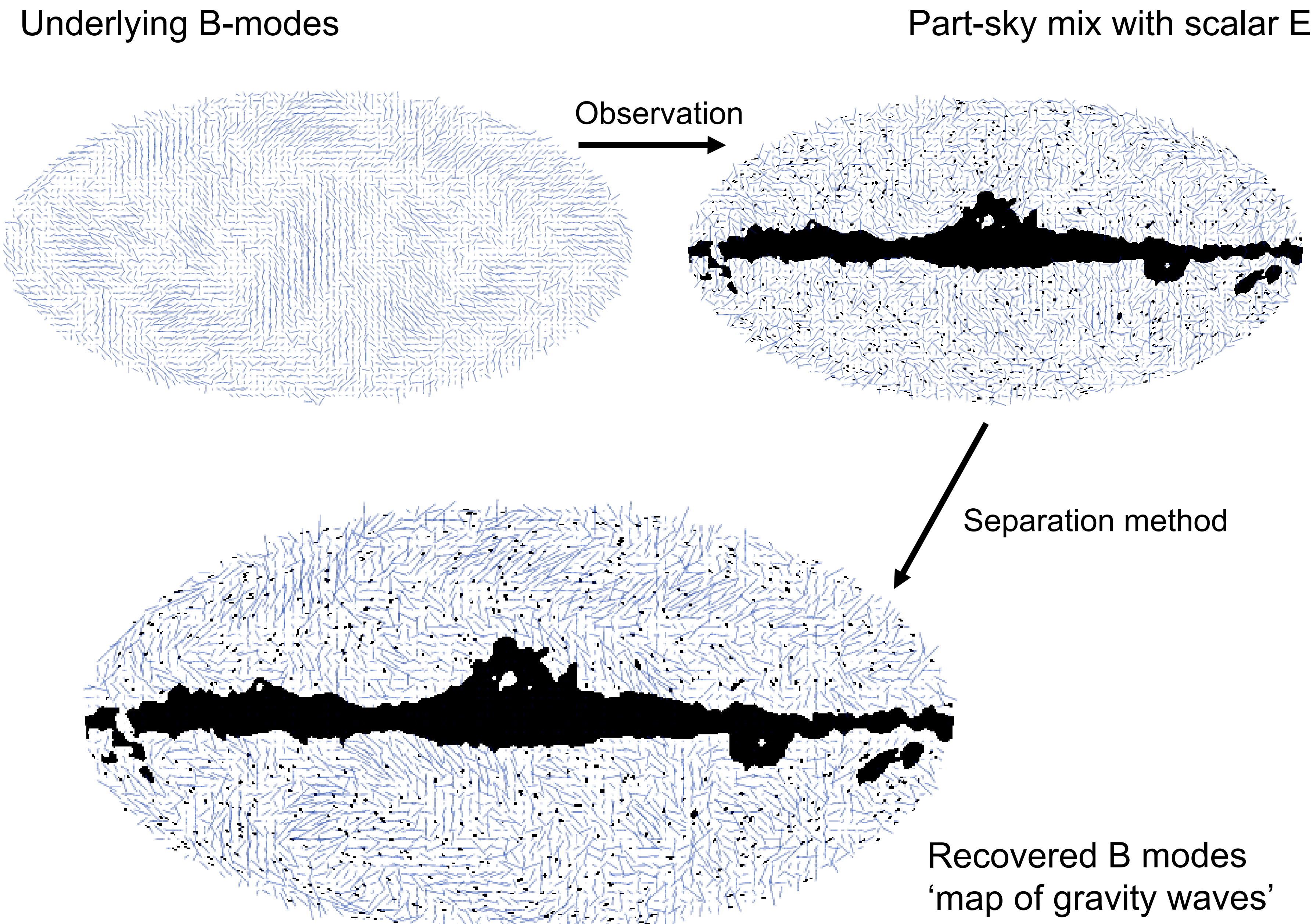
Pure B: $\epsilon^b{}_c\nabla^c\nabla^a\mathcal{P}_{ab} = (\nabla^2 + 2)\nabla^2 P_B$

Inversion non-trivial with boundaries

Likely important as reionization signal same scale as galactic cut

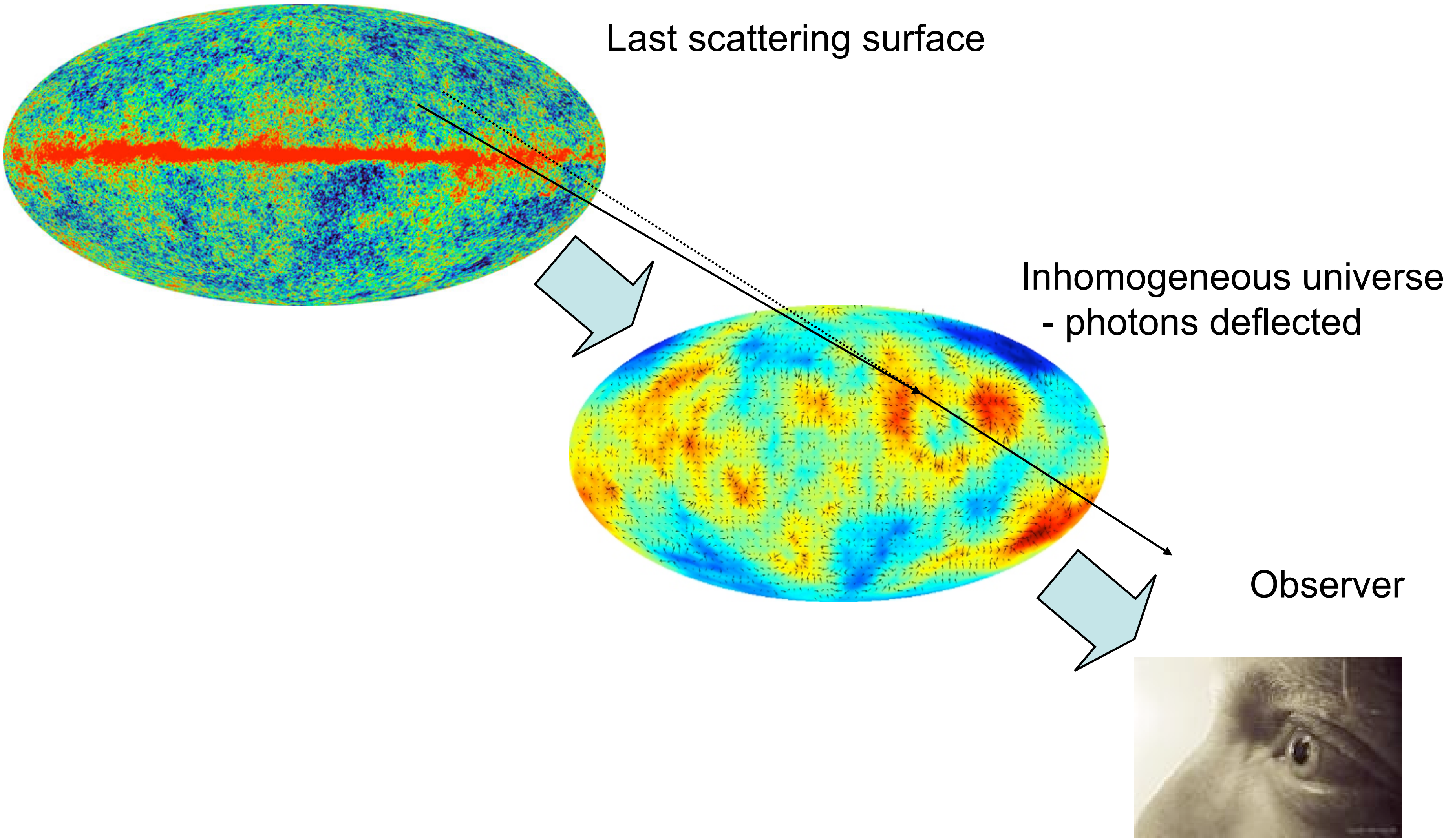
→ Use set of E/B/mixed harmonics that are orthogonal and complete over the observed section of the sphere.
Project onto the `pure' B modes to extract B.

(Nearly) pure B modes do exist   Lewis, Challinor, Turok astro-ph/0106536



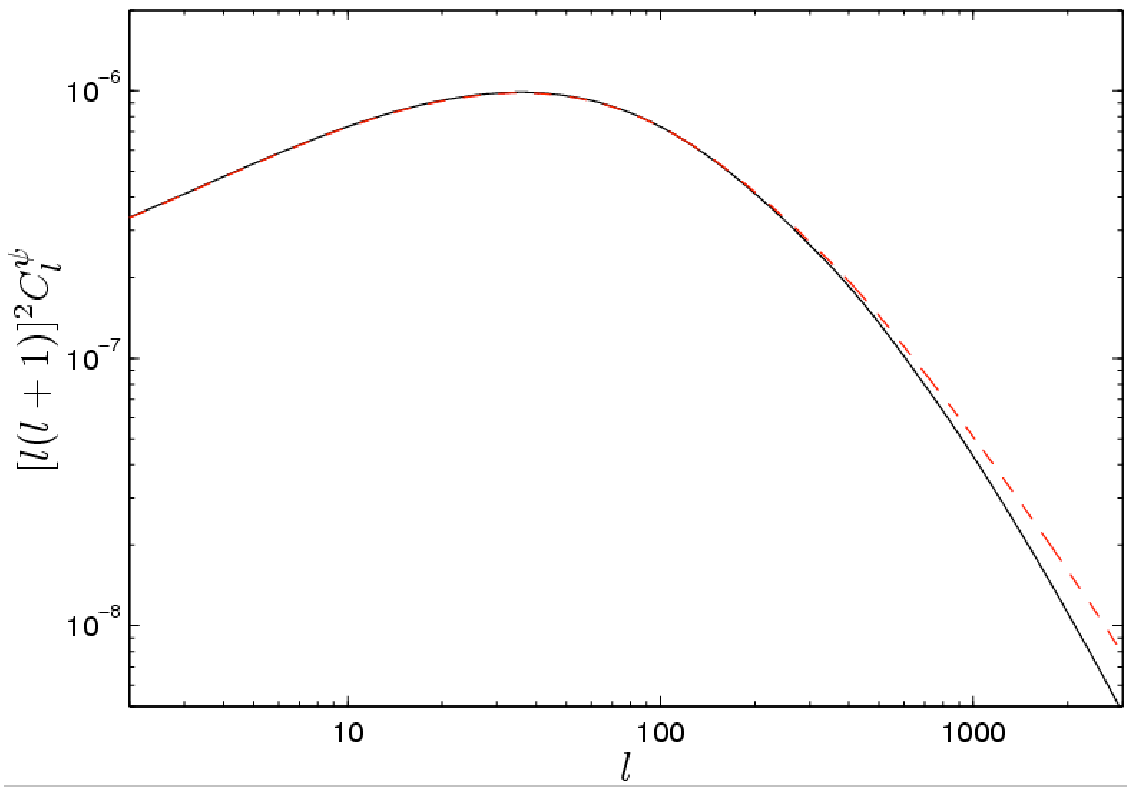
Lewis: astro-ph/0305545

Weak lensing of the CMB



Lensing Potential

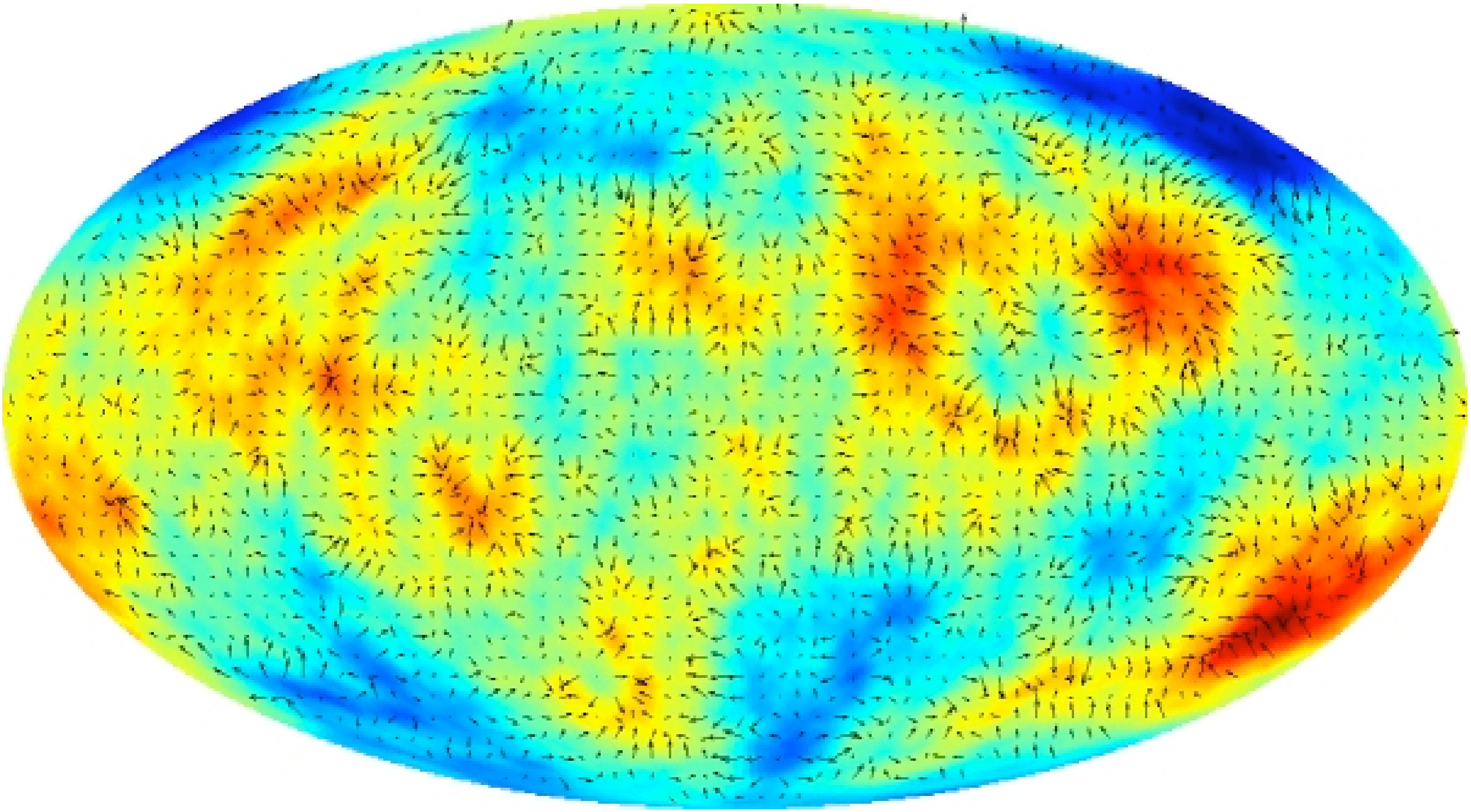
$$\bar{X}(\mathbf{n}) = X(\mathbf{n}') = X(\ \mathbf{n} + \nabla\psi(\mathbf{n})\ )$$



Deflections O(10^{-3}), but coherent on degree scales à important!

Lensing potential and deflection angles

LensPix sky simulation code: http://cosmologist.info/lenspix



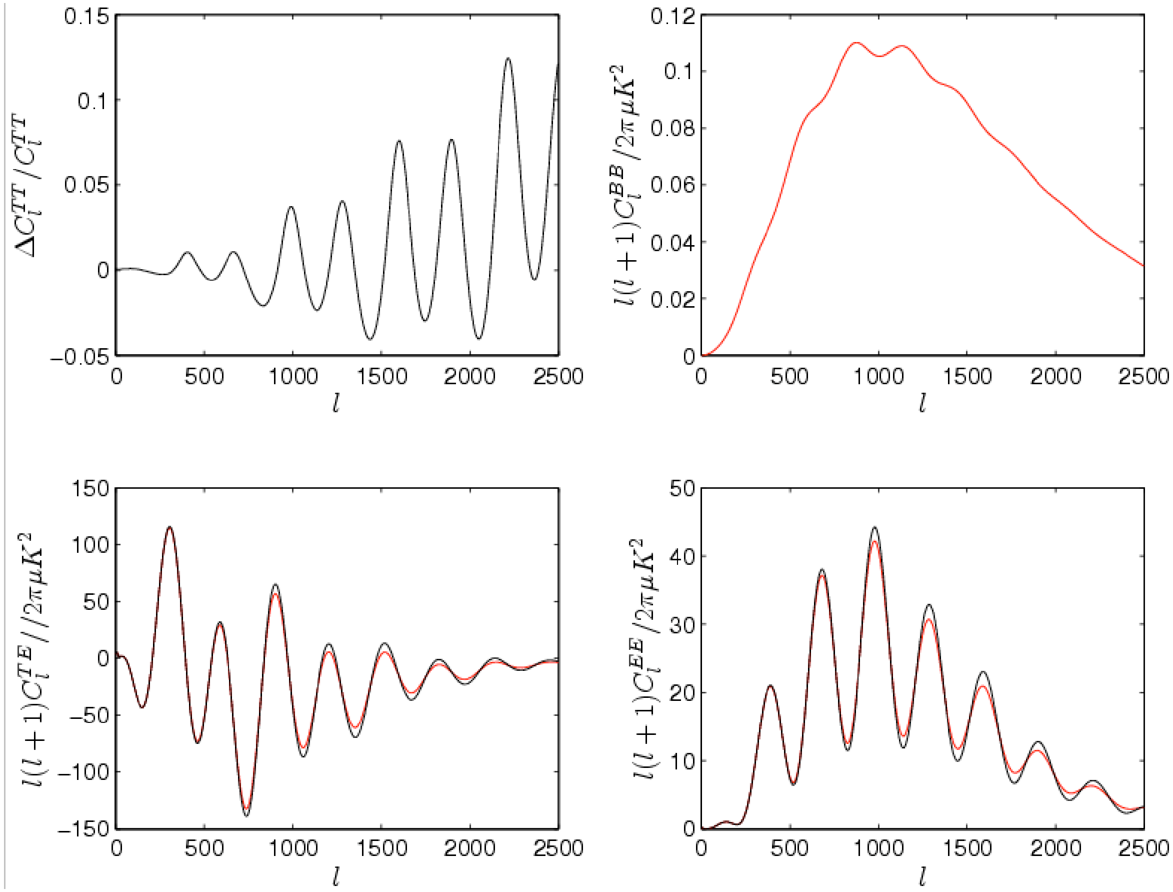
- Changes power spectra
- Makes distribution non-Gaussian

Lensed CMB power spectra

Few % on temperature

10% on TE/EE polarization

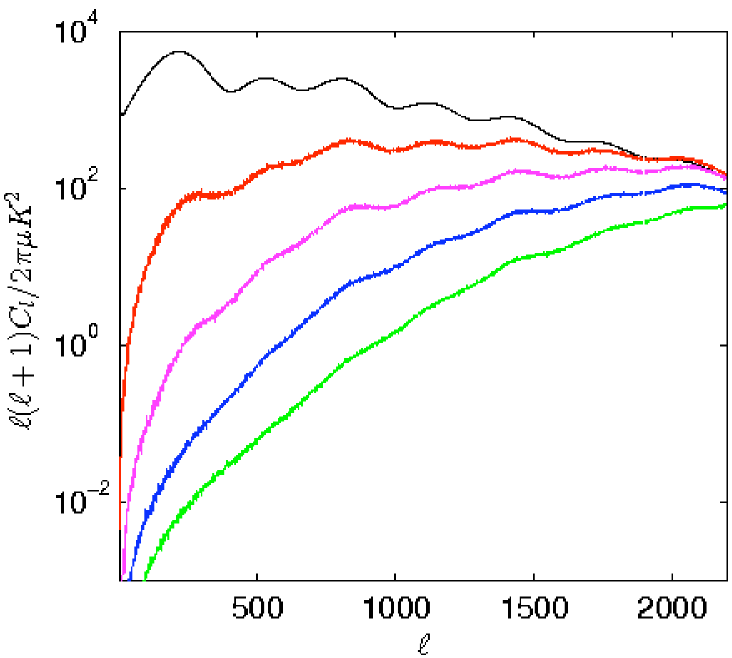
New lensed BB signal

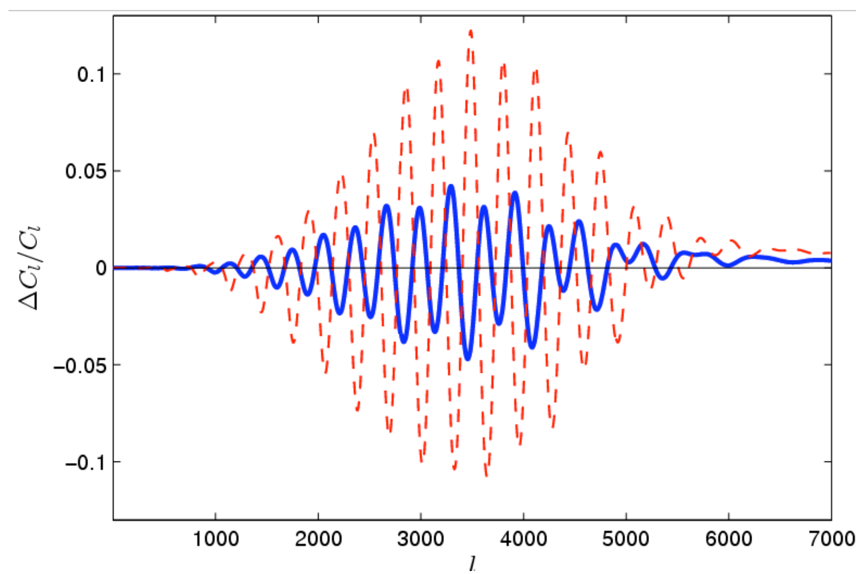


Series expansion in deflection angle?

$$\bar{X}(\mathbf{n}) = X(\mathbf{n}') = X(\ \mathbf{n} + \nabla\psi(\mathbf{n})\ )$$

$$\begin{aligned}\bar{T} = T \ &+\ \nabla_a T \nabla^a \psi + \tfrac{1}{2}\left\{\nabla^{\langle a}\psi\nabla^{b\rangle}\psi\nabla_{\langle a}\nabla_{b\rangle}T + \tfrac{1}{2}|\nabla\psi|^2\nabla^2 T\right\} \\ &+\ \tfrac{1}{3!}\left\{\nabla^{\langle a}\psi\nabla^{b}\psi\nabla^{c\rangle}\psi\nabla_{\langle a}\nabla_b\nabla_{c\rangle}T + \tfrac{1}{4}|\nabla\psi|^2\nabla\psi_a\nabla^a(3\nabla^2+2)T\right\} \\ &+\ \tfrac{1}{4!}\left\{\nabla^{\langle a}\psi\nabla^{b}\psi\nabla^{c}\psi\nabla^{d\rangle}\psi\nabla_{\langle a}\nabla_b\nabla_c\nabla_{d\rangle}T + \tfrac{1}{4}|\nabla\psi|^2\nabla^{\langle a}\psi\nabla^{b\rangle}\psi\nabla_{\langle a}\nabla_{b\rangle}(4\nabla^2+8)T + \tfrac{1}{8}|\nabla\psi|^4\nabla^2(3\nabla^2+2)T\right\} \\ &+\ \ldots\ldots\end{aligned}$$

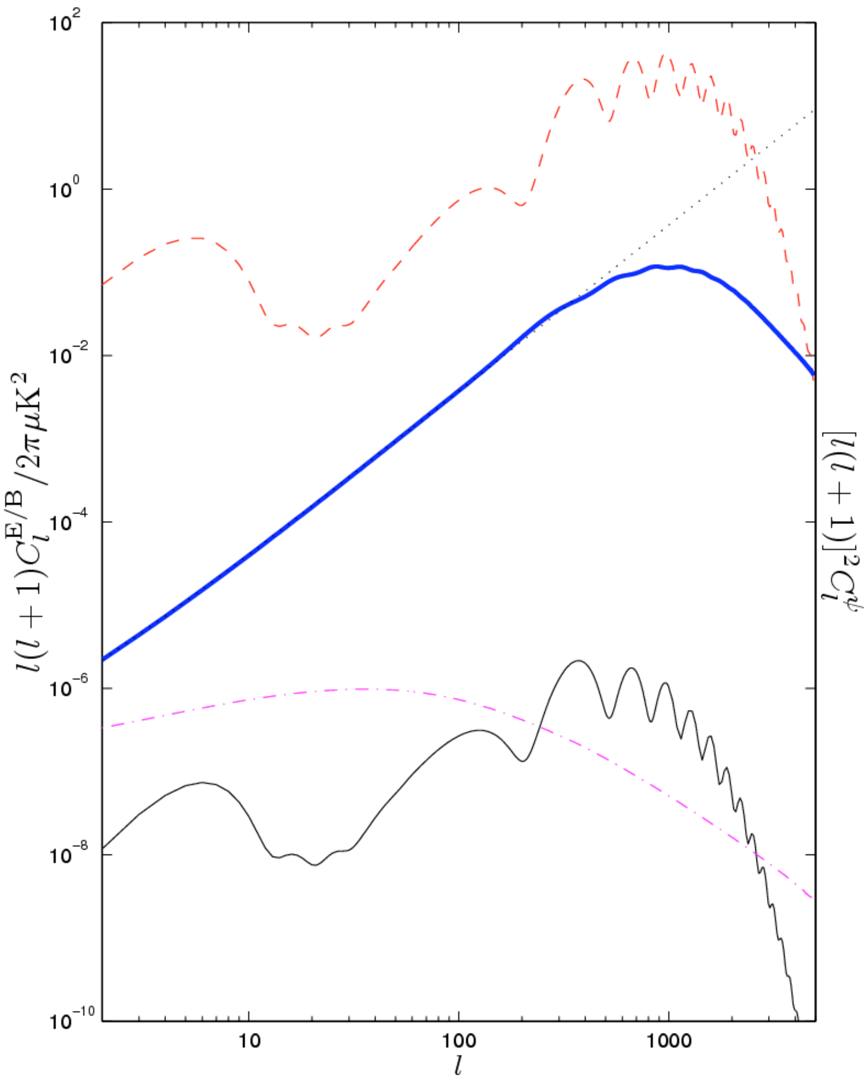




Series expansion only good on large and very small scales

Accurate calculation uses correlation functions: Seljak 96; Challinor, Lewis 2005

Lensing of CMB polarization



Lewis, Challinor review in prep

Nearly white BB spectrum on large scales

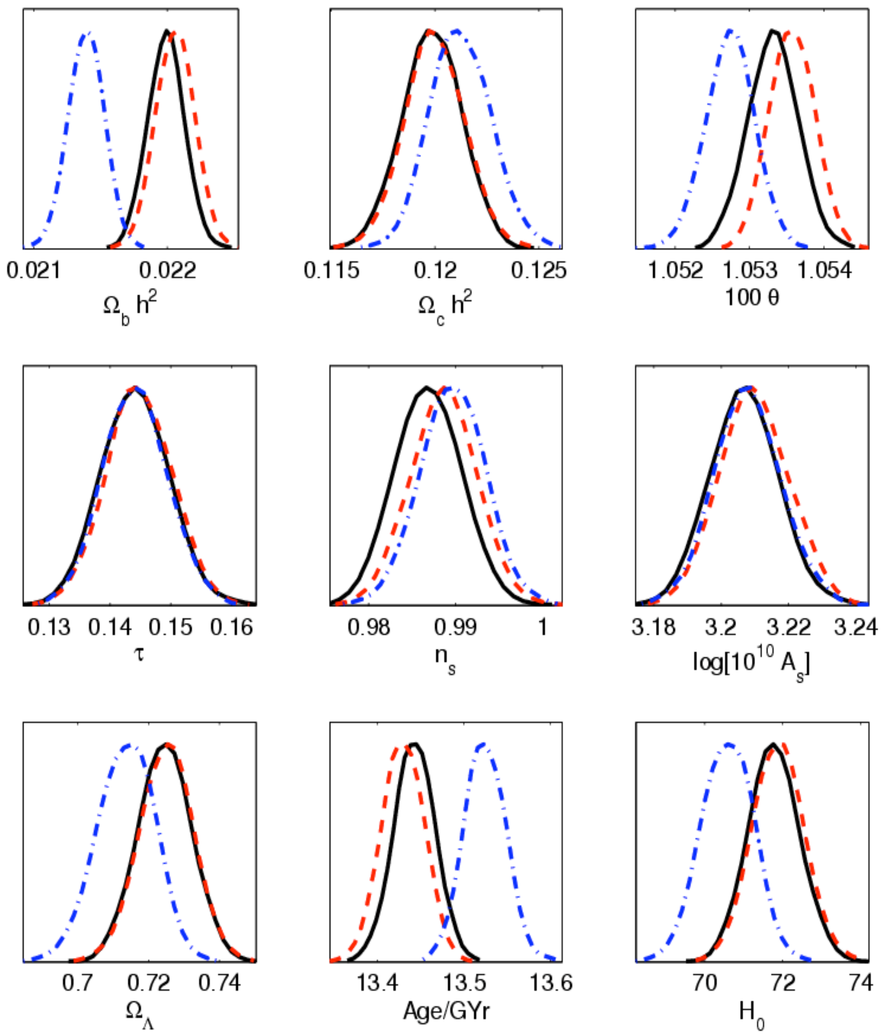
Potential confusion with tensor modes

Lensing effect can be largely subtracted if only scalar modes + lensing present, but approximate and complicated (especially posterior statistics).

Hirata, Seljak : astro-ph/0306354, Okamoto, Hu: astro-ph/0301031

Planck (2007+) parameter constraint simulation

(neglect non-Gaussianity of lensed field; BB noise dominated so no effect on parameters)



Important effect, but using lensed CMB power spectrum gets 'right' answer

Lewis 2005

Other non-linear effects

- Thermal Sunyaev-Zeldovich
  Inverse Compton scattering from hot gas: frequency dependent signal
- Kinetic Sunyaev-Zeldovich (kSZ)
  Doppler from bulk motion of clusters; patchy reionization; (almost) frequency independent signal
- Ostriker-Vishniac (OV)
  same as kSZ but for early linear bulk motion
- Rees-Sciama
  Integrated Sachs-Wolfe from evolving non-linear potentials: frequency independent
- General second order
  includes all of the above + more

Conclusions

- CMB contains lots of useful information!
  - primordial perturbations + well understood physics (cosmological parameters)

- Precision cosmology
  - constrain many cosmological parameters + primordial perturbations

- Currently no evidence for any deviations from standard near scale-invariant purely adiabatic primordial spectrum

- E-polarization and T-E measure optical depth, constrain reionization; constrain isocurvature modes

- Large scale B-mode polarization from primordial gravitational waves:
  - energy scale of inflation
  - rule out most ekpyrotic and pure curvaton/ inhomogeneous reheating models and others

- Small scale B-modes
  - Strong signal from any vector vorticity modes, strong magnetic fields, topological defects

- Weak lensing of CMB :
  - B-modes potentially confuse primordial signals
  - Important correction to theoretical linear result

- Foregrounds, systematics, etc, may make things much more complicated!

http://CosmoCoffee.info

arXiv paper discussion and comments

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