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REDUCTION METHOD WITH SYSTEM ANALYSIS FOR MULTIOBJECTIVE OPTIMIZATION-BASED DESIGN

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ABSTRACT

We present an approach for reducing the number of variables and constraints, which is combined with System Analysis Equations (SAE), for multiobjective optimization-based design. In order to develop a simplified analysis model, the SAE is computed outside an optimization loop and then approximated for use by an optimizer. Two examples are presented to demonstrate the approach.

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1 Introduction

There are mainly two classes of methods for handling optimization-based design of *complex* engineering systems. In the context of this paper, a complex system entails two or more of the following features: (i) has computationally costly SAE, (ii) has a large number of variables and nonlinear constraints, (iii) is multiobjective, and (iv) is decomposable into a number of subsystems which *hierarchically* or *nonhierarchically* interact with one another.

The first class of methods are mostly applied to handle features (i) and (ii) (Vanderplaats, 1984, where further references can be found), and more recently (iii) (see, for example, Zhou and Tits, 1993a). In majority of the methods in this class, the system analysis is performed outside an optimization loop; often the system analysis (which is a part of an outside loop, see Figure 2 for an example) is approximated for use in the optimization loop (the inside loop, see Figure 2) in an attempt to reduce the number of costly and detailed analyses. The number of variables can also be reduced by *design variable linking* (Vanderplaats, 1984), this is usually done by problem-dependent assumptions (e.g., symmetry in structural design). Finally, the number of constraints can be reduced by employing only a significant subset of constraints (the *active set*) in the optimization loop (Zhou and Tits, 1993b; Gill et al., 1982).

The second class of methods are almost entirely applied to handle single-objective problems with the features (i), (ii), and (iv) (see, for example, Sobieski, 1992; Wu and Azarm, 1992; Zhao and Azarm, 1993, where further references can be found), with the exception of Haimes et al.(1990) method which is applicable to hierarchical multiobjective problems, and that of Azarm and Eschenauer (1993) which does not handle the SAE as formulated here. In majority of the methods in this class, the system analysis and/or optimization model can be decomposed into a number of submodels (subsystems or subproblems). The decomposition (which might be performed on the system analysis or optimization model or both) is called hierarchic, if it has an overall tree-type structure with two or more levels of subsystems. In hierarchic decomposition, horizontal interaction in between the subsystems located at the same level is not permissible. On the other hand, the decomposition is called nonhierarchic, if there is no restriction on the interaction which might exist in between the subsystems.

So far, none of the methods reported in the literature can simultaneously handle all of the above-mentioned features which are becoming increasingly common in the design of complex engineering systems. In an attempt towards removing this shortcoming, this paper presents an extension of a recent work by Azarm and Eschenauer (1993) whereby a multiobjective approach for reducing the number of variables and constraints is combined with the SAE which might be hierarchically or nonhierarchically coupled.

The remaining sections of this paper are organized as follows. We present, in Section 2, an overview of the definitions and formulation of the problem. This is followed by Section 3, whereby the solution approach is presented. The method is then demonstrated, as shown in Section 4, via two examples: a simple explosive actuated cylinder (to demonstrate the solution steps) and a fairly complex dual-wheel excavator. The paper is concluded in Section 5 with the final remarks. Parameters and variables for the formulation and examples are defined in the nomenclature of Section 8.

2 Formulation

As it was stated in Azarm and Eschenauer (1993), the overall multiobjective optimization problem is converted into a *minmax* form in that the collective objective is to minimize the maximum *loss* of all objectives (see also, Osyczka, 1984). Hence, the overall multiobjective optimization formulation is given as follows:

$$\begin{array}{ll} minmax & \tilde{f}(\tilde{z}) \\ s.t.: & \tilde{g}(\tilde{z}) \leq \tilde{0} \end{array} \tag{1}$$

where the objective and inequality constraint functions, $\tilde{f}$ and $\tilde{g}$ components, are smooth functions of design variables, $\tilde{z}$. The vector of design variables, $\tilde{z}$, is partitioned into three groups: $\tilde{x}$, $\tilde{X}_s$, and $\tilde{Y}$. $\tilde{x}$ represents an N-vector of *primary* variables, their impact on the design optimization is assumed to be global (e.g., variables that contribute to many design parts, disciplines, or nonphysical entities). $\tilde{X}_s$ represents an S-vector of *secondary* variables, their impact on the design is assumed to be local (e.g., variables that contribute to *detailed* dimensions or specifications of design parts, disciplines, or nonphysical entities). In general, as will be observed in the solution steps (Section 3), the primary variables are included into the solution process directly, while the secondary variables are considered indirectly (i.e., via *step-size* variables). $\tilde{Y}$ represents a vector of state variables obtainable as a solution of a set of simultaneous (coupled) equations which can be partitioned, for example, into $\tilde{Y}^a$, $\tilde{Y}^b$, $\tilde{Y}^c$, such that:

$$\tilde{Y}^a = \tilde{f}^a(\tilde{x}, \tilde{X}_s, \tilde{Y}^b, \tilde{Y}^c) \tag{2}$$
$$\tilde{Y}^b = \tilde{f}^b(\tilde{x}, \tilde{X}_s, \tilde{Y}^c, \tilde{Y}^a) \tag{3}$$
$$\tilde{Y}^c = \tilde{f}^c(\tilde{x}, \tilde{X}_s, \tilde{Y}^a, \tilde{Y}^b) \tag{4}$$

The above set of equations represents the SAE. Each equation, also referred to as Contributing Analysis (CA), may represent a particular engineering discipline or a distinct physical part (a subsystem) of a system, or both (Sobieski, 1990). Note that the division of SAE into CAs may: (i) correspond to physical boundaries present in the problem that suggest its separation into smaller parts, or (ii) be purely formal as in dividing a set of equations into several subsets. These two ways of the SAE partitioning are referred to as physical and nonphysical in the paper.

As it was stated before, presumably the system analysis is performed outside where the objective and constraint functions (optimization formulation) are evaluated. Furthermore, it is assumed that in the optimization formulation, the number of variables and constraint functions (and perhaps the number of objective functions as well) during the solution process has to be reduced. The reduction in the number of variables is achieved through the secondary variables, $\tilde{X}_s$. This has recently been developed for a multiobjective case by Azarm and Eschenauer (1993). Briefly, it involves partitioning of secondary variables, $\tilde{X}_s$, into several groups: $\tilde{X}_{s_i}$, $i = 1, \cdots, I$. Each $\tilde{X}_{s_i}$ represents a S_i-vector of secondary variables in an optimization subsystem i (which might be an engineering discipline, a physical part, or a nonphysical entity). Note that: $S = \sum_{i=1}^{I} S_i$; the total number of secondary variables is equal to the sum of the number of secondary variables in each optimization subsystem: $i = 1, \cdots, I$. The vector of secondary variables $\tilde{X}_{s_i}$ is replaced by a scalar *step-size* variable s_i as follows:

$$\tilde{X}_{s_i}^{new} = \tilde{X}_{s_i}^{old} + s_i \, \tilde{d}_i \tag{5}$$

where $\tilde{d}_i$ (as given in Section 3) is a descent direction in the subspace of active constraints which is obtained from the optimization subsystem post-optimality sensitivity analysis. In general, the optimization subsystems (or system) can be different from the analysis subsystems (here we use the terms: *optimization subsystem* and *analysis subsystem* to distinguish them from each other).

In order to reduce the number of constraints, the cumulative function (or the KS function) introduced by Kreisselmeier and Steinhauser (1979), which was also used by Sobieski et al. (1985), is utilized here. As an example, g_{ks} represents a cumulative function of $g_1, \cdots, g_J$:

$$g_{ks} = \frac{1}{\rho} ln \left(\sum_{j=1}^{J} exp\,(\rho g_j) \right) \tag{6}$$

where ρ is a user controlled coefficient. Sobieski et al.(1988) were first to develop a technique, based on the KS function, to convert a multiobjective optimization problem into a single objective form. The KS function can also be applied to convert a multiobjective optimization problem formulated in a *minmax* form into a single objective form. Figure 1 shows an example of this whereby f_{ks} and g_{ks} are the KS functions for the objectives and constraints, respectively. Note that, as shown in the Figure 1, depending on the value of ρ there might be a gap in between the minimum of f_{ks} and the $minmax(f_1, f_2, f_3)$, points B and A, respectively. These two points however will coincide when $\rho \rightarrow \infty$. Also, note that the choice of the KS function as a means to reduce the problem to a single objective form implies that the objective that has the steepest slope after normalization controls the optimization.

Finally, the SAE can be approximated in the optimization formulation in which the number of variables and constraints are reduced as well, i.e., the optimization problem of (1)

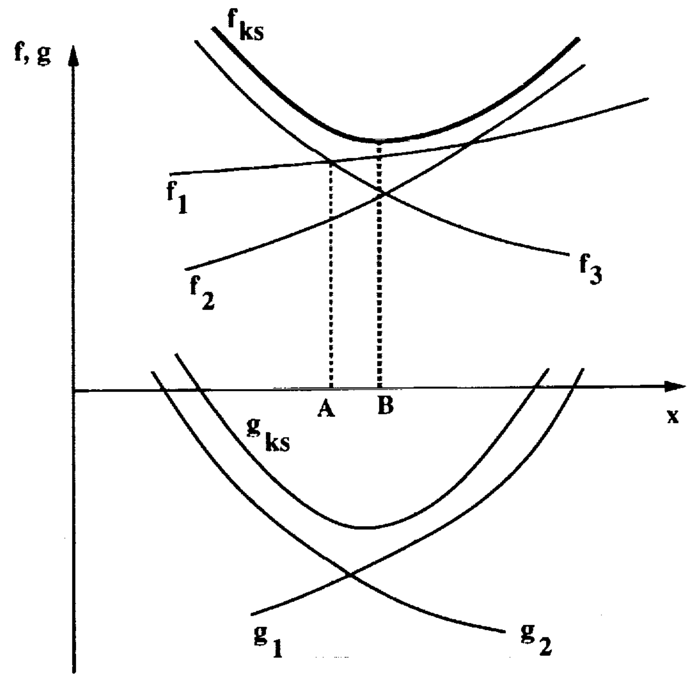

Figure 1: KS Function for Objectives and Constraints

can be rewritten as follows ($\tilde{x}$ and s are varied while $\tilde{\tilde{Y}}$ fixed):

$$\begin{array}{ll} minmax & \tilde{f}\left(\tilde{x}, s_1, \cdots, s_I, \tilde{\tilde{Y}}\right) \\ s.t.: & g_{ks}\left(\tilde{x}, s_1, \cdots, s_I, \tilde{\tilde{Y}}\right) \leq 0 \end{array} \tag{7}$$

where $\tilde{\tilde{Y}} \in [\tilde{\tilde{Y}}_l^c, \tilde{\tilde{Y}}_u^c]$, $\tilde{\tilde{Y}}$ represents a linear approximation of the $\tilde{Y}$ which is computed by the SAE, $\tilde{\tilde{Y}}_l^c$ and $\tilde{\tilde{Y}}_u^c$ represent lower and upper move limits on the current value of $\tilde{Y}$. The advantage of (7), when compared with (1), is that the number of constraints is reduced to one and the number of variables is reduced from (N+S+number of state variables) in (1) to $(N+I)$ in (7). As an example, in a large-scale structural optimization problem, the reduction in the number of variables and constraints can (among other reasons) substantially reduce the computer storage for an otherwise very large jacobian matrix needed by the optimizer.

3 Solution Steps

The solution steps are summarized as follows (see also the flow-chart in Figure 2):

- Step (0): Identify primary, secondary, step-size and state variables. Initialize with $\tilde{x} = \tilde{x}^0$, $\tilde{X}_{s_i} = \tilde{X}^0_{s_i}$, $s_i = 0$ and set k=0,

- Step (1): Solve system analysis equations for $\tilde{Y}$, and perform system sensitivity analysis (GSE as described by Sobieski, 1990) to obtain $\partial\tilde{Y}/\partial\tilde{x}$ and $\partial\tilde{Y}/\partial\tilde{X}_s$.

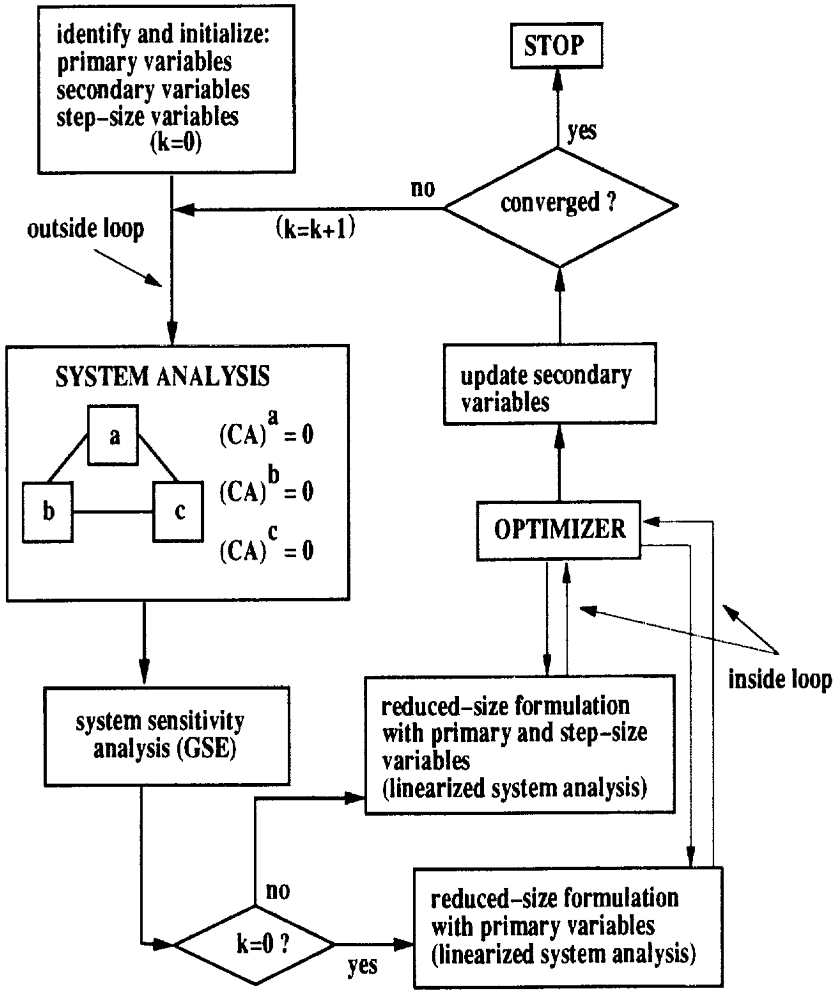

Figure 2: Flow-Chart of the Solution Steps

- Step (2): if k=0 then solve (8) where $\tilde{x}$ is varied while $\tilde{\tilde{Y}}$ is fixed:

$$\begin{aligned} &minmax \quad \tilde{f}(\tilde{x}, \tilde{\tilde{Y}}) \\ &s.t.: \qquad g_{ks}(\tilde{x}, \tilde{\tilde{Y}}) \leq 0 \end{aligned} \tag{8}$$

  otherwise (i.e., if k $\geq$ 1), solve (9) where $\tilde{x}$ and s are varied while $\tilde{\tilde{Y}}$ fixed:

$$\begin{aligned} &minmax \quad \tilde{f}(\tilde{x}, s_1, \cdots, s_I, \tilde{\tilde{Y}}) \\ &s.t.: \qquad g_{ks}(\tilde{x}, s_1, \cdots, s_I, \tilde{\tilde{Y}}) \leq 0 \end{aligned} \tag{9}$$

  where in (8) and (9), there are move limits on $\tilde{\tilde{Y}}$, i.e., $\tilde{\tilde{Y}} \in [\tilde{\tilde{Y}}^c_l, \tilde{\tilde{Y}}^c_u]$.

- Step (3): As a part of post optimality sensitivity analysis of (8) or (9), compute:

$$\tilde{d}_i = -(\frac{\partial \tilde{f}}{\partial \tilde{X}_{s_i}} \tilde{\omega} + \frac{\partial g_{ks}}{\partial \tilde{X}_{s_i}} \lambda) \tag{10}$$

and find:

$$\tilde{X}_{s_i}^{new} = \tilde{X}_{s_i}^{old} + s_i\ \tilde{d}_i \tag{11}$$

- Step (4): Set k=k+1, repeat steps (1)-(3) until convergence is achieved.

In the above-mentioned solution steps, $\tilde{\omega}$ and λ are Lagrange multipliers corresponding to the objective and constraint functions of (8) or (9). Furthermore, $\tilde{\tilde{Y}}$ is obtained as follows:

$$\tilde{\tilde{Y}} = \tilde{\tilde{Y}}^c + \sum_i \{(\partial\tilde{Y}/\partial\tilde{x}_i)\Delta x_i\} + \sum_i \{(\partial\tilde{Y}/\partial\tilde{X}_{s_i})(\partial\tilde{X}_{s_i}/\partial s_i)\Delta s_i\} \tag{12}$$

where $\tilde{\tilde{Y}}^c$ is the *current* value of $\tilde{Y}$. Also:

$$(\partial(\tilde{f} \text{ or } g_{ks})/\partial\tilde{x}) = (\partial(\tilde{f} \text{ or } g_{ks})/\partial\tilde{x})_{\text{local}} + \sum_{i=a,b,c} (\partial(\tilde{f} \text{ or } g_{ks})/\partial\tilde{Y}^i)(\partial\tilde{Y}^i/\partial\tilde{x}) \tag{13}$$

$$(\partial(\tilde{f} \text{ or } g_{ks})/\partial s_i) = \sum_{i=a,b,c} \{(\partial\tilde{Y}^i/\partial\tilde{X}_s)(\partial(\tilde{f} \text{ or } g_{ks})/\partial\tilde{Y}^i)\}(\partial\tilde{X}_s/\partial s_i) \tag{14}$$

The minimax problems of (8) and (9) are solved by the subroutine FSQP (Zhou and Tits, 1991). FSQP (Feasible Sequential Quadratic Programming) is a set of Fortran subroutines which implements algorithms which are described and analyzed by Panier and Tits (1993), Bonnans et al. (1992), and Zhou and Tits (1993a).

4 Examples

Two examples are presented here to demonstrate the method developed in the paper. Both examples are selected from the literature where they are formulated as a single-objective problem. They are revised here to form multiobjective examples.

4.1 Example 1: Explosive Actuated Cylinder

This example is constructed from a well-known single-objective optimization problem, a minimum length design (f_1) of an explosive actuated cylinder (Papalambros and Wilde, 1979). The constraints for this example express the specifications for: kinetic energy (g_1), wall stress (g_2), and geometry constraints (g_3-g_5). Its formulation is revised here by taking out one of its constraints (piston force) in the original formulation and including it as an additional objective (f_2) in the problem. As shown below, both objectives have been scaled so that they are of the same order of magnitude:

$$\begin{aligned} &minmax\quad \{f_1, f_2\} \\ &\qquad\qquad f_1 = (z_1 + z_2)/L_{max} \end{aligned}$$

$$f_2 = [(1000\pi/4)z_4 z_5^2]/F_{max}$$

$$s.t.: \tag{15}$$

$$g_1: \quad (W_{min}(1-\gamma))/[1000 z_4 v_1^{\gamma}(v_2^{(1-\gamma)} - v_1^{(1-\gamma)})] - 1 \leq 0$$

$$v_1 = v_c + (\pi/4)z_1 z_5^2$$

$$v_2 = v_1 + (\pi/4)z_2 z_5^2$$

$$g_2: \quad \bar{\sigma}_e N/\bar{\sigma}_Y - 1 \leq 0$$

$$\bar{\sigma}_e = (\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)^{1/2}$$

$$\sigma_1 = z_4(z_3^2 + z_5^2)/(z_3^2 - z_5^2)$$

$$\sigma_2 = -z_4$$

$$g_3: \quad z_3/D_{max} - 1 \leq 0$$

$$g_4: \quad (z_1 + z_2)/L_{max} - 1 \leq 0$$

$$g_5: \quad z_5 < z_3$$

$$z_i > 0 \quad i = 1, \cdots, 5$$

For convenience, the nomenclature for this example is given in the appendix (see also Papalambros and Wilde, 1979, for further details).

4.1.1 Solution

Following the solution steps in Section 3, we assume: $\tilde{x} = (z_1,z_4)^t$, $\tilde{X}_s = (z_2,z_3,z_5)^t$, $\tilde{Y} = (Y^a,Y^b)^t = (v_1,v_2)^t$. We then initialize with $\tilde{x}^0 = (0.4,0.34)^t$, $\tilde{X}_s^0 = (1.2,0.82,0.67)^t$, s^0=0, and k=0. Next, as in step (1) of the solution steps, we select and partition the SAE into two nonphysical analysis subsystems: subsystem a and subsystem b (Figure 3 shows the interaction between these two subsystems). We then form the global sensitivity equations

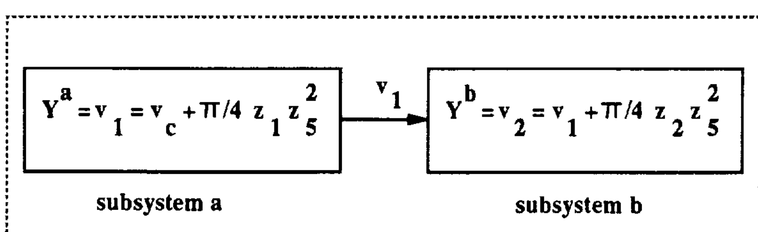

Figure 3: Analysis Subsystems for Example 1

(Sobieski, 1990):

$$\begin{pmatrix} 1 & -\partial v_1/\partial v_2 \\ -\partial v_2/\partial v_1 & 1 \end{pmatrix} \begin{pmatrix} \partial v_1/\partial z_k \\ \partial v_2/\partial z_k \end{pmatrix} = \begin{pmatrix} \partial f^a/\partial z_k \\ \partial f^b/\partial z_k \end{pmatrix} \qquad k = 1,2,5 \tag{16}$$

where:

$$f^a = v_c + (\pi/4)z_1 z_5^2 \tag{17}$$

$\tilde{f}$ and $\tilde{z}$	Test Cases				
	a	b	c	d	e
f_1	0.721	0.722	0.723	0.724	0.724
f_2	0.721	0.722	0.723	0.724	0.724
z_1	0	0	0	0	0
z_2	1.44	1.44	1.44	1.44	1.45
z_3	1	1	0.97	0.99	0.89
z_4	23.4	23.4	23.04	23.04	22.5
z_5	0.166	0.166	0.167	0.167	0.169

Key:
a: original double-objective problem
b: original double-objective problem was reduced to single-objective by KS function
c: original multi-constraint problem was reduced to single-constraint by KS function
d: no. of variables and constraints are reduced
e: no. of variables and constraints are reduced with linearization on g_1

Table 1: Summary of Results for Example 1

$$f^b = v_1 + (\pi/4)z_2 z_5^2 \tag{18}$$

As an example, GSE for z_5 will be as follows:

$$\begin{pmatrix} +1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \partial v_1/\partial z_5 \\ \partial v_2/\partial z_5 \end{pmatrix} = \begin{pmatrix} (\pi/2)z_1 z_5 \\ (\pi/2)z_2 z_5 \end{pmatrix} \tag{19}$$

With k=0 at step (2), we formulate (8) with ρ=150, and solve for updated primary variables: $\tilde{x}=(0.75,2.3)^t$. We also obtain (as a by-product of FSQP): $(\omega_1,\omega_2)=(0,1)$ and λ=0.2. Next, we go to step (3), whereby (10) we compute d_1, d_2, d_3 for X_s^1, X_s^2, X_s^3, respectively. We then express the secondary variables via (11) as a function of the step-size variable s (new values for the step-size s and primary variables $\tilde{x}$ are then obtained via (9) for k $\geq$ 1). We now go to step (4) and set k=1 and repeat steps (1)-(3). This iterative process continues until there is no further improvement in the value of objective functions. A summary of results are given in Table 1. As shown in the table, the results from the solution steps (test case e) compare well with those obtained when the original problem is solved (test case a), or when the number of objectives are reduced to one (test case b), or when the number of constraints are reduced to one (test case c), or when the number of variables (including the step-size variable s) are reduced to three and the number of constraints to one (test case d).

4.2 Example 2: Dual-Wheel Excavator

This example (Figures 4 and 5) is constructed from a single-objective optimization problem (Wilson, 1992). It involves a minimum weight design of a 120-inch threaded hub-and-shaft assembly for a dual-wheel excavator. There are 9 variables and 25 constraints in this example. The constraints in the example express specifications on: hub stress (g_1, g_2), stress on shaft at various critical positions (g_3-g_{18}), and other *practical* constraints (g_{19}-g_{25}). Here, the original single-objective problem (sum of the weights for the hubs and shaft) has been converted into two objectives: weight of the shaft (f_1), weight of the hub (f_2), with both objectives scaled to be of the same order of magnitude:

$$
\begin{array}{ll}
minmax & \{f_1, f_2\} \\
 & f_1 = (\rho_S \pi/4)\{Z_{SP} z_3^2 + Z_{SH} z_4^2 + 2 Z_B D_B^2 + 2 z_8 z_2^2 \\
 & +2 z_7 z_1^2 + 2 Z_c z_5^2 + 2 z_9 z_6^2\}/3300 \\
 & f_2 = (\rho_S \pi/4)\{[z_7((z_1+a)^2 - z_1^2)] + [(z_7 + B_2)/2(D_{11}^2 \\
 & -(z_1+a)^2)] + [(B_1+B_2)/2(D_{10}^2 - D_{11}^2)\}/5000 \\
s.t.: & \\
g_1(z_1, z_7): & \sigma_{ah}/\sigma_y + \sigma_{mh}/\sigma_y - 1/f_s \leq 0 \\
g_2(z_1, z_7): & \sigma_{ah}/\sigma_e + \sigma_{mh}/\sigma_{ut} - 1/f_s \leq 0 \\
g_3(z_3, z_7, z_8): & \sigma_{ab}/\sigma_y + \sigma_{mb}/\sigma_y - 1/f_s \leq 0 \\
g_4(z_3, z_7, z_8): & \sigma_{ab}/\sigma_e + \sigma_{mb}/\sigma_{ut} - 1/f_s \leq 0 \\
g_5(z_1, z_7): & \sigma_{au}/\sigma_y + \sigma_{mu}/\sigma_y - 1/f_s \leq 0 \\
g_6(z_1, z_7): & \sigma_{au}/\sigma_e + \sigma_{mu}/\sigma_{ut} - 1/f_s \leq 0 \\
g_7(z_1, z_7): & \sigma_{athd}/\sigma_y + \sigma_{mthd}/\sigma_y - 1/f_s \leq 0 \\
g_8(z_1, z_7): & \sigma_{athd}/\sigma_e + \sigma_{mthd}/\sigma_{ut} - 1/f_s \leq 0 \\
g_9(z_1): & \sigma_{axth}/\sigma_y + \sigma_{mxth}/\sigma_y - 1/f_s \leq 0 \\
g_{10}(z_1): & \sigma_{axth}/\sigma_e + \sigma_{mxth}/\sigma_{ut} - 1/f_s \leq 0 \\
g_{11}(z_1, z_5, z_6): & \sigma_{aend}/\sigma_y + \sigma_{mend}/\sigma_y - 1/f_s \leq 0 \\
g_{12}(z_1, z_5, z_6): & \sigma_{aend}/\sigma_e + \sigma_{mend}/\sigma_{ut} - 1/f_s \leq 0 \\
g_{13}(z_1, z_5): & \sigma_{aent}/\sigma_y + \sigma_{ment}/\sigma_y - 1/f_s \leq 0 \\
g_{14}(z_1, z_5): & \sigma_{aent}/\sigma_e + \sigma_{ment}/\sigma_{ut} - 1/f_s \leq 0 \\
g_{15}(z_1, z_2): & \sigma_{arev}/\sigma_y + \sigma_{mrev}/\sigma_y - 1/f_s \leq 0 \\
g_{16}(z_1, z_2): & \sigma_{arev}/\sigma_e + \sigma_{mrev}/\sigma_{ut} - 1/f_s \leq 0 \\
g_{17}(z_1, z_5, z_9): & \sigma_{aes}/\sigma_y + \sigma_{mes}/\sigma_y - 1/f_s \leq 0 \\
g_{18}(z_1, z_5, z_9): & \sigma_{aes}/\sigma_e + \sigma_{mes}/\sigma_{ut} - 1/f_s \leq 0
\end{array}
\tag{20}
$$

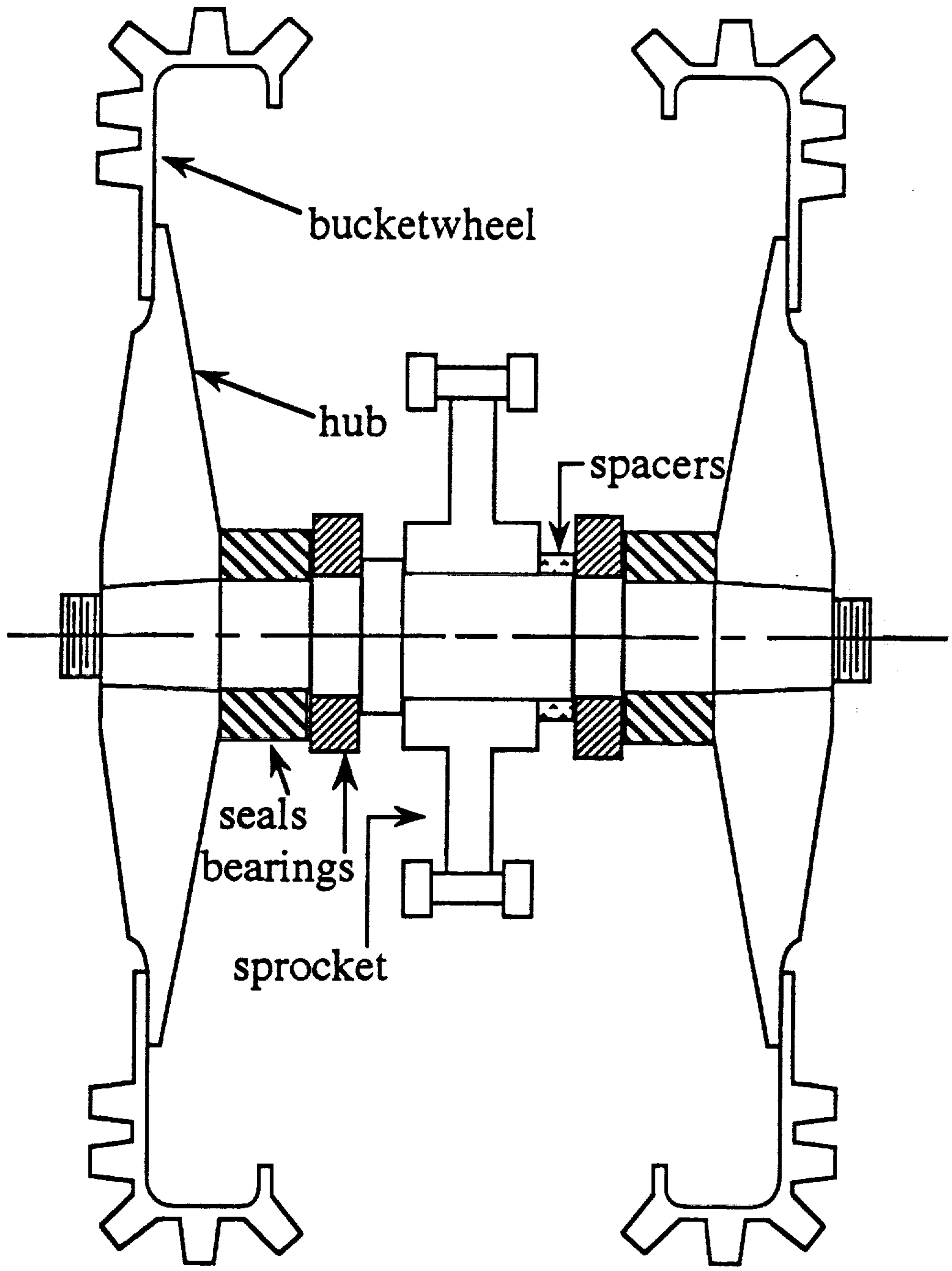

Figure 4: A Dual-Wheel Excavator

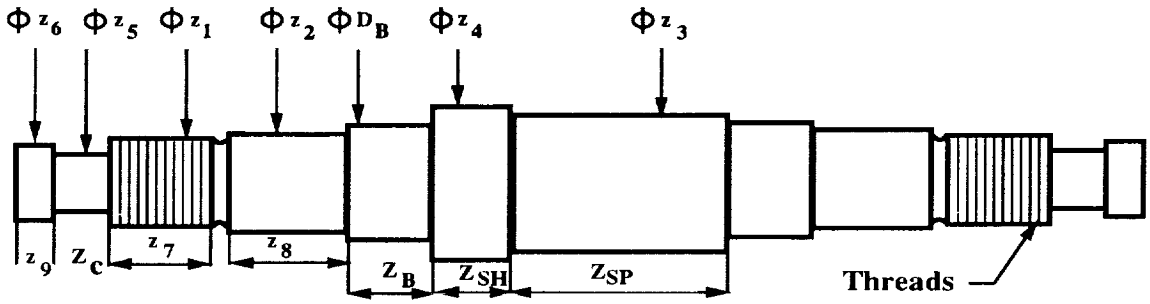

Figure 5: Shaft of the Dual-Wheel Excavator

$g_{19}(z_8)$:	$0.127N_S - z_8 \leq 0$
$g_{20}(z_2, z_3)$:	$z_2 - 0.95D_B \leq 0$
$g_{21}(z_2, z_3)$:	$0.8D_B - z_2 \leq 0$
$g_{22}(z_3, z_4)$:	$S_{hldr} + D_B/2 - z_4/2 \leq 0$
$g_{23}(z_5, z_6)$:	$z_5 + 0.0165 - z_6 \leq 0$
$g_{24}(z_1, z_2)$:	$z_1 + 0.0267 - z_2 \leq 0$
$g_{25}(z_1, z_5)$:	$0.25z_1 - z_5 \leq 0$

For convenience, the nomenclature for this example is given in the appendix (see also Wilson, 1992, for further details).

4.2.1 Solution

Following the solution steps in Section 3, we assume: $\tilde{x}=(z_1,\ z_7)^t$, $\tilde{X}_{s_1}=(z_2,\ z_3,\ z_4,\ z_8)^t$, $\tilde{X}_{s_2}=(z_5,\ z_6,\ z_9)^t$. Note that the secondary variables are grouped (optimization subsystems) according to the physical makeup of the excavator; $\tilde{X}_{s_1}$ represents shaft dimensions in between the two hubs while $\tilde{X}_{s_2}$ represents those outside the two hubs (Figures 4 and 5). State variables $\tilde{Y}$, or the SAE (Figure 6), for this example are established so that they only contribute to the hub stress constraints (g_1 and g_2). The hub is modeled (Wilson, 1992) as a

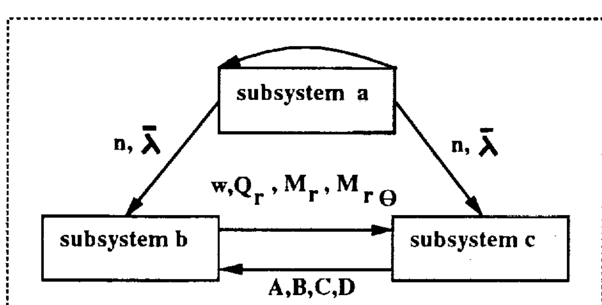

Figure 6: Analysis Subsystems for Example 2

ring plate of linearly varying thickness subject to concentrated transverse load and bending moment at its outer edge. The SAE is partitioned into three nonphysical analysis subsystems: subsystem a ($\tilde{Y}^a$), subsystem b ($\tilde{Y}^b$), and subsystem c ($\tilde{Y}^c$). Note that, the analysis subsystems as shown in the Figure 6 are different from the optimization subsystems. The SAE for subsystem a represents:

$$\text{subsystem a:} \quad \begin{cases} Y_1^a = n = f_1^a(z_1, z_7) \\ Y_2^b = \bar{\lambda} = f_2^a(n) \end{cases} \tag{21}$$

where n is the hub's thickness constant, and $\bar{\lambda}$ is an eigenvalue computed iteratively (as shown by an arched arrow on subsystem a in Figure 6) by the following characteristic equation. The characteristic equation is obtained from partial differential equations established for the hub based on the excavation loading conditions and using small-deflection plate theory (Conway, 1958; Wilson, 1992):

$$\begin{aligned} &\bar{\lambda}^4 - 2\bar{\lambda}^3(n+2) + \bar{\lambda}^2[(n+2)^2 + n(1-\nu) - 2m^2] \\ &+\bar{\lambda}(n+2)[2m^2 - n(1-\nu)] + m^2[m^2 - 1 - (3+\nu n)(n+1)] = 0 \end{aligned} \tag{22}$$

$\bar{\lambda}$ is obtained for each m (only 11 terms are considered here, i.e., m=0 to m=10) of the deflection (w) equation in subsystem b. As shown below, subsystem b computes the deflection and loads (shear force, Q_r, and moments, $M_r, M_\theta, M_{r\theta}$) on the hub:

$$\text{subsystem b:} \quad \begin{cases} Y_1^b = w = \sum_{m=0}^{\infty}(Ar^{\bar{\lambda}_1} + Br^{\bar{\lambda}_2} + Cr^{\bar{\lambda}_3} + Dr^{\bar{\lambda}_4})\cos(m\theta) \\ Y_2^b = Q_r \ = f_1^b(w, z_1, z_7, n) \\ Y_3^b = M_r \ = f_2^b(w, z_1, z_7, n) \\ Y_4^b = M_\theta \ = f_3^b(w, z_1, z_7, n) \\ Y_5^b = M_{r\theta} = f_4^b(w, z_1, z_7, n) \end{cases} \tag{23}$$

where:

$$\begin{aligned} Q_r &= D(r)[\partial/\partial r]((\partial^2 w/\partial r^2) + (1/r)(\partial w/\partial r) + (1/r^2)(\partial^2 w/\partial \theta^2)) \\ M_r &= -D(r)[(\partial^2 w/\partial r^2) + \nu((1/r)(\partial w/\partial r) + (1/r^2)(\partial^2 w/\partial \theta^2))] \\ M_\theta &= -D(r)[(1/r)(\partial w/\partial r) + (1/r^2)(\partial^2 w/\theta^2) + \nu(\partial^2 w/\partial r^2)] \\ M_{r\theta} &= -(1-\nu)D(r)[(1/r)(\partial^2 w/\partial r\partial\theta)] \end{aligned} \tag{24}$$

and $D(r)$, the flexural rigidity, is a function of hub radius, r, and θ is an angular coordinate on the hub.

Finally, subsystem c represents the boundary conditions which can be used to find A, B, C, D coefficients for the above-mentioned deflection (w), Wilson (1992):

$$\text{subsystem c:} \quad \begin{cases} Y_1^c = A; \quad Y_2^c = B; \quad Y_3^c = C; \quad Y_4^c = D \\ w \qquad\qquad\quad = f_1^c(A, B, C, D, \bar{\lambda}) = 0 \\ \partial w/\partial r \qquad\ = f_2^c(A, B, C, D, \bar{\lambda}) = 0 \\ \sum(\text{forces}) \quad\ = f_3^c(A, B, C, D, \bar{\lambda}, n, Q_r, M_{r\theta}) = 0 \\ \sum(\text{moments}) = f_4^c(A, B, C, D, \bar{\lambda}, n, M_r) = 0 \end{cases} \tag{25}$$

Initially, we assumed: $\tilde{x}=(12.1,\ 8.9)^t$, $\tilde{X}_{s_1} = (14.4,\ 16.6,\ 19.1)^t$, $\tilde{X}_{s_2} = (9.2,\ 12,\ 1.8)^t$, s_1=0,

$\tilde{f}$ and $\tilde{z}$	Test Cases				
	a	b	c	d	e
f_1	0.80	0.79	0.80	0.81	0.81
f_2	0.80	0.80	0.80	0.81	0.81
z_1	12.8	12.7	12.7	11.9	12
z_2	14.3	14.4	14.3	13.6	13.9
z_3	15.2	15.2	15.4	15	14.8
z_4	17.6	17.5	17.8	18.6	18.2
z_5	7.5	6.9	7.7	9	9
z_6	10	8.9	9. 7	11.8	11.8
z_7	5.9	5.9	5.9	6	6
z_8	10	10	10	12	11.5
z_9	1.3	1.5	1.4	1.2	1.2

Key:
a: original double-objective problem
b: original double-objective problem was reduced to single-objective by KS function
c: original multi-constraint problem was reduced to single-constraint by KS function
d: no. of variables and constraints are reduced
e: no. of variables and constraints are reduced with linearization on g_1 and g_2

Table 2: Summary of Results for Example 2

s_2=0, and k=0. We then followed the solution steps in Section 3 to obtain the results which are summarized in Table 2. Finally, it should be stated that the minimax solution reported here (for case a of Table 2, the total hub-and-shaft assembly weight is 10,640 lb) is different from that reported by Wilson (1992) in which the problem is formulated in a single objective form (sum of the weights for the hubs and shaft was 10,211 lb). These two solutions are essentially two different Pareto solutions for the example as formulated here (Osyczka, 1984).

5 Concluding Remarks

In this paper we have discussed a method for combining a reduction technique with system analysis equations for multiobjective optimization problems. The main characteristics of the method presented are that: (i) the number of variables and constraints can be reduced, (ii) the SAE, which might be costly and hierarchically or nonhierarchically decomposed, is performed outside the optimization loop, and (iii) it is applicable to multiobjective optimization problems.

The method has been demonstrated by two examples: (i) a simple explosive actuated cylinder, and (ii) a fairly complex dual-wheel excavator. It has been shown that, for both examples, when the reduction measures (test cases (b)-(e) for both examples) are applied they can usually obtain a solution fairly close to that of the original problem (test case (a)). The small difference in the solution could be eliminated, for example, by increasing the number of variables, constraints, etc., to resolve a lack of sufficient *degrees-of-freedom.*

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8 Nomenclature

<u>Formulation:</u>

$(CA)^a$	= Contributing analysis equations in analysis subsystem a
$\tilde{d}_i$	= Descent direction in optimization subsystem i
$\tilde{f}$	= Vector of objective functions
$\tilde{f}_{ks}$	= Cumulative objective function
$\tilde{g}$	= Vector of inequality constraint functions
$\tilde{g}_{ks}$	= Cumulative constraint function
I	= Number of Optimization subsystems
k	= Iteration counter
N	= Number of elements in $tildex$
s_i	= Step-size variable in optimization subsystem i
S	= Number of elements in $\tilde{X}_s$
S_i	= Number of elements in $\tilde{X}_{s_i}$
$\tilde{x}$	= Vector of primary variables
$\tilde{X}_s$	= Vector of secondary variables
$\tilde{X}_{s_i}$	= Vector of secondary variables in optimization subsystem i
$\tilde{Y}$	= Vector of state variables
$\tilde{Y}^a$	= Vector of state variables in analysis subsystem a
$\tilde{\tilde{Y}}_l^c, \tilde{\tilde{Y}}_u^c$	= Lower and upper move limits on the current value of $\tilde{Y}$
$\tilde{\tilde{Y}}$	= Linear approximation to $\tilde{Y}$
$\tilde{z}$	= Vector of overall design variables
λ	= Lagrange multipliers for constraint functions

ρ	= User defined coefficient for KS function
ω	= Lagrange multipliers for objective functions

Example 1:

D_{max}	= Maximum allowable cylinder outside diameter, 1 in
F_{max}	= Maximum piston force, 700 lb
L_{max}	= Maximum cylinder total length, 2 in
N	= Safety factor, 3
v_1, v_2	= Initial, final volume of combustion, in^3
v_c	= Fixed chamber volume, 0.084 in^3
W_{min}	= Minimum kinetic energy, 600 lb-in
z_1	= Unswept cylinder length, in
z_2	= Working stroke of piston, in
z_3	= Outside diameter of cylinder, in
z_4	= Initial pressure of combustion, ksi
z_5	= Piston diameter, in
γ	= Ratio of specific heats, 1.2
$\bar{\sigma}_e$	= Equivalent stress, ksi
$\bar{\sigma}_Y$	= Yield strength, 125 ksi
σ_1, σ_2	= Principal stress, ksi

Example 2:

a	= Diameter constant for the hub, in
B_1, B_2	= Hub dimensions, in
D_{10}	= Hub dimension, in
D_{11}	= Hub dimension, in
D_B	= See Figure 5, in
f_s	= Safety factor
N_S	= Number of seals
S_{hldr}	= Radial height of the shoulder at the bearings, in
z_i	= Shaft dimension ($i = 1, \cdots, 9$), see Figure 5, in
Z_B	= See Figure 5, in
Z_c	= See Figure 5, in
Z_{SH}	= See Figure 5, in
Z_{SP}	= See Figure 5, in
$\bar{\lambda}$	= Eigenvalue
ν	= Poisson's ratio
ρ_S	= Density, lbm/in^3
σ_{ab}	= Alternating stress on shaft at bearings, ksi
σ_{aend}	= Alternating compressive stress on shaft at end connections, ksi
σ_{aent}	= Alternating tensile stress on shaft at end connections, ksi
σ_{aes}	= Alternating shear stress on shaft at end connections, ksi
σ_{ah}	= Alternating stress on hub, ksi
σ_{arev}	= Alternating stress on shaft due to reverse loading, ksi
σ_{athd}	= Alternating stress on ACME threads, ksi
σ_{au}	= Alternating stress on shaft at ACME thread undercut, ksi

σ_{axth}	= Alternating shear stress on ACME threads, ksi
σ_e	= Endurance limit, ksi
σ_{mb}	= Mean stress on shaft at bearings, ksi
σ_{mend}	= Mean compressive stress on shaft at end connections, ksi
σ_{ment}	= Mean tensile stress on shaft at end connections, ksi
σ_{mes}	= Mean shear stress on shaft at end connections, ksi
σ_{mh}	= Mean stress on hub, ksi
σ_{mrev}	= Mean stress on shaft due to reverse loading, ksi
σ_{mthd}	= Mean stress on ACME threads, ksi
σ_{mu}	= Mean stress on shaft at ACME thread undercut, ksi
σ_{mxth}	= Mean shear stress on ACME threads, ksi
σ_{ut}	= Ultimate tensile strength, ksi
σ_y	= Yield strength, ksi

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13. ABSTRACT *(Maximum 200 words)*

We present an approach for reducing the number of variables and constraints, which is combined with System Analysis Equations (SAE), for multiobjective optimization-based design. In order to develop a simplified analysis model, the SAE is computed outside an optimization loop and then approximated for use by an operator. Two examples are presented to demonstrate the approach.

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