Tracking the Course of Mathematics Problems

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Abstract

The goal of this project is to improve our ability to track how students solve mathematical problems. This research will use eye tracking to make real-time inferences about what the student is thinking and fMRI imaging to make inferences about different styles of problem solving. This research is done in the context of both the ACT-R theory of human cognition, which allows us to produce computational models of cognition, and a series of cognitive tutors for mathematics education, which are based on the ACT-R theory. The ACT-R theory is a theory of how the cognitive system adaptively uses procedural and declarative knowledge to achieve its goals. The research will focus on the algebra tutor that is currently in use in high school and is being adapted for use in middle schools. The research will be concerned with the effect of different mathematical representations on problem solving and with different strategies for mathematical problem solving. There will be three lines of research. One, involving eye movements, will document the instructional opportunities associated with eye movements in the context of the cognitive tutors. It will particularly focus on the eye movements associated with competent use of graphical, tabular, and symbolic representations of functions. The second line of research, involving fMRI brain imaging, will study brain activation markers of the course of mathematical problem solving. It will particularly focus on distinguishing between students who use an informal, verbal form of reasoning with students who use a symbolic, visual form of reasoning. This line will also look at how we can merge information from imaging and eye scanning to make both methodologies more effective. The third line of research will study how one can use the information from fMRI scanning and eye tracking to produce more effective instruction. The three lines of research will converge on a culminating study that attempts to improve the effectiveness of the middle school tutor. It will first use fMRI imaging to identify the learning strategies of individual students and then collect real-time eye movement to guide instruction as students are learning. This will demonstrate how we can use some of the new emerging sensing technology to improve mathematics education.

Introduction

The basic premise of this proposal is that instruction is effective to the degree that it is sensitive to the individual student. Our focus will be on the acquisition of algebraic competence by middle-school and high-school students. Here, we are concerned with both understanding what a student is thinking while solving a particular problem and more generally with the style of problem-solving and learning that an individual student is bringing to the mathematical domain. The proposal brings together three threads in our past research history—cognitive tutors (e.g., Koedinger, Anderson, Hadley, & Mark, 1997), use of eye movement to track the fine-grained course of cognition (e.g., Salvucci & Anderson, 1998), and the use of functional magnetic resonance imaging (fMRI) to track the course of complex problem solving (e.g., Fincham, van Veen, Anderson, Stenger, Aizenstein & Carter, 2000). Our efforts in these three domains are unified by the ACT-R theory of human cognition (Anderson & Lebiere, 1998). We will be investigating the potential of eye movements and fMRI to develop better cognitive theory about the nature of the problem-solving and learning trajectories of individual students. We will also be investigating the potential of these methods as sensing devices to determine how particular students are solving problems and so more effectively individualize instruction.

In this proposal we will first review relevant background research on mathematics instruction, intelligent tutoring, use of eye movements and fMRI, and cognitive theory. Then we will describe the program of research. We will describe in some detail three initial experiments that will serve to test out the methodology for the subsequent research program. Then we will describe in more general terms the subsequent program of research and two related foci for the research. One foci will be to document two styles of reasoning that students bring to bear in mathematical problem solving. One style is an often brittle strategy of formal manipulation of visual symbols. The other style is an often limited strategy of informal verbal reasoning. Effective problem solving involves a flexible combination of these two strategies. The second, related foci will be to explore students' use of multiple representations for mathematical functions—verbal, graphical, symbolic, and tabular. At the end of the three-year project, we will perform a study to measure the achievement gain that we can achieve by responding to these differences in the mathematical thinking of individual students.

Background

Multiple Representations and Strategies in Mathematical Reasoning

The goal of this project is to find ways to better track the different representations and strategies that students use in reasoning about mathematical problems. There has been a substantial body of research looking at multiple representations in mathematical problem solving. A number of computer-based learning environments have been developed such as SimCalc (Roschelle, Kaput, & Stroup, 1998) that uses simulation technology to help students understand the mathematics of change, Function Probe (Confrey, 1992), which used graphs, equations, and tables to teach understanding of functions and Blocks World (Thompson, 1992) that helps children understand arithmetic by using Dienes blocks as well as the more conventional symbolic representations. There have been a variety of outcomes with use of external representation with negative evaluations (Tabachneck, Leonardo, & Simon, 1994, Yerushamy, 1991), positive evaluations (Ainsworth, Wood, & O'Malley, 1998, Cox & Brna, 1995, Thompson, 1992), and mixed (e.g., Schoenfeld, Smith, & Arcavi, 1993).

As Ainsworth (1999) notes there are three functions associated with multiple representations. One is that certain types of inferences can be more easily performed with one sort of representation than with another. Many inferences that would require a great deal of deliberation in a symbolic representation can be performed much more directly on more external, visual representations (Kaput, 1992, Scaife, & Rogers, 1996). A second function is that one representation can constrain the interpretation of a second representation. The third function is that relating multiple representations can result in a deeper understanding of the mathematical domain. As Kaput (1989) writes, "the cognitive linking of representations creates a whole that is more than the sum of its parts...It enables us to "see" complex ideas in a new way and apply them more effectively."

A major goal of this project is to have students use different representations to reason about functions in solving mathematical problems. Much of our research here is guided by work showing that students come to the classroom with informal ways of solving problems which are often more effective than the symbolic methods (Kirshner, 1989; Koedinger & McClaren, 1997, Koedinger & Nathan, submitted). Students' initial methods of treating algebraic expressions are often driven by the visual form of these expressions and not by their informal verbal understanding, nor indeed by any connection with other representations of functions like graphical (Koedinger & Alibali, 1999). However, their use of algebraic representations becomes more flexible when it is connected to these other representations. Thus, it becomes critical to develop methods for "sensing" which representations a particular student is bringing to bear on a problem and for responding to this diagnosis. We intend to develop such techniques.

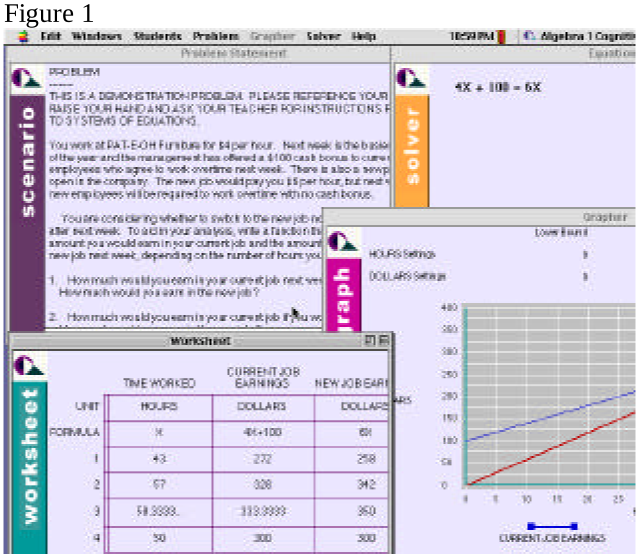
More generally our research will enrich our understanding of how students process different representations and of their different problem-solving strategies. This knowledge will not only inform tutor design but this understanding of student thinking can be transferred into improved teacher development (e.g., Carpenter & Fennema, 1992; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989).

Cognitive Tutors

Our work on cognitive tutors developed out of a long tradition of intelligent tutoring systems that began with efforts (e.g., Carbonell, 1970, Stevens & Collins, 1977, Brown, Burton, & DeKleer, 1982, Wenger, 1987) to increase the flexibility of computer-aided instruction with artificial intelligence techniques. While these early efforts helped establish the technology, tutors became effective and were used in the real world when they were infused with a cognitive theory (e.g., Anderson, Corbett, Koedinger, & Pelletier, 1995, Lesgold, Lajoie, Bunzo, & Eggan, 1982, Van Lehn, 1996). These cognitive theories enabled informed inferences about cognitive state and allowed the tutors to adapt their instruction to the particular student.

The research in this proposal will be based on the PAT algebra tutor (Koedinger, Anderson, Hadley, & Mark, 1997), which is used in the ninth grade to help students learn to relate graphical, symbolic, verbal, and tabular representations of mathematical functions. It is part of a full year curriculum. Students spend about two days a week working with the tutor and spend other time doing other projects. This tutor is part of a larger curriculum which covers grades 9 to 11 and has recently been designated an "exemplary" program by the Department of Education. The computer tutor is used two days a week in a full curriculum in which students also work on large scale projects. Evaluations of the curriculum have shown that it improves student achievement by as much as a letter grade or a standard deviation. Carnegie Mellon has created a company, Carnegie Learning, that is

Figure 1



responsible for marketing the tutor. Thus, our research will make contact with what is happening in classrooms today. New curriculum development is extending our work down to middle schools. This is particularly appropriate to the current proposal since use of alternative representations and mathematical reasoning strategies is important in the "pre-algebra" work that occupies much of middle school.

Figure 1 shows a typical screen image from the algebra tutor. Students interact with this tutor, solving problems that involve inter-relating verbal, tabular, graphical, and symbolic representations of functions. Underlying the tutor is a cognitive model that can solve this class of problems. As students solve the problem, the model is solving the problem in the background trying to infer what the student is thinking about solving the problem. This model is used to individualize instruction to where the student is in the problem solving and individualize the curriculum for that student. The cognitive model is implemented in an earlier version of the ACT architecture (Anderson, 1983) which solved problems in a large grain size of tens of seconds.

While the PAT algebra tutor has had proven successes we have also identified ways it can be improved. In particular, it can misdiagnose students and consequently fails to deliver appropriate instruction. With respect to misdiagnosis and relevant to this proposal, it fails to be sensitive to the different ways in which different students solve problems. Koedinger and McClaren (1997) have shown that some students solve these problems in an informal verbal manner while other students solve these problems by reference to the algebraic expressions. Our tutor could be improved if we could identify students using the two strategies and adapt the instruction appropriately.

There are other ways that instruction can be inappropriate to where the student is cognitively. Particularly relevant to the current proposal is the problem of misreference. The tutor may think the student is currently processing one part of the problem while the student is in fact processing a different part of the problem. Such problems of misreference become particularly grievous as the problems become more complex. Given the current emphasis on instruction in complex problem-solving situations, issues of misreference are very significant. While problems of misreference are frequent in computer-based instruction they are not at all unique to computer-based dialog. Particularly important is the recent research of Herb Clark at Stanford University showing the critical role of shared visual field to communication and that people in dialog monitor what each other are looking out. This points to the critical role that eye tracking can have in enhancing tutorial dialogs.

Using Eye Movements to Track Cognition

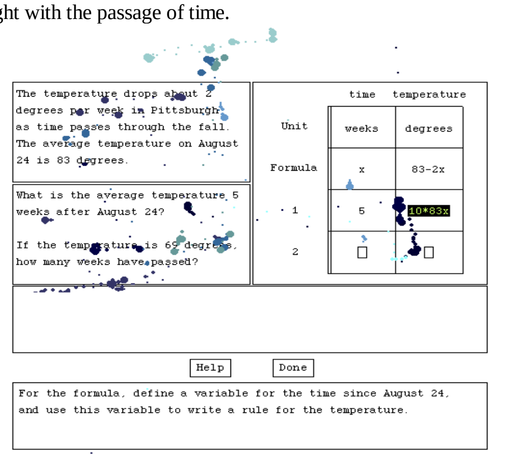
The wealth of eye-movement studies in past decades has provided solid evidence that eye movements can reveal a great deal about underlying cognitive processes (Just & Carpenter, 1984; Rayner, 1995). Researchers have successfully utilized eye-movement data in a variety of contexts, including menu selection (Aaltonen, Hyrskykari, & Räihä, 1998; Byrne et al., 1999), reading (Just & Carpenter, 1980, 1984; McConkie & Rayner, 1976; O'Regan, 1981; Rayner & Morris, 1990), gaze-based interfaces (Hutchinson et al., 1989; Jacob, 1995; Salvucci, 1999b; Stampe & Reingold, 1995; Zhai, Morimoto, & Ihde, 1999), driving (McDowell & Rockwell, 1978), image scanning (Noton & Stark, 1971), television viewing (Flagg, 1978), mathematics (Hegarty, Mayer, & Green, 1992; Suppes, 1990), analogy (Salvucci & Anderson, in press), and mental rotation (Carpenter & Just, 1978).

Eye movements are extremely informative as a data source for analysis of human cognition. Perhaps the most important reason for their usefulness is that eye movements indicate, generally speaking, the focus of visual attention. Although visual attention can move independently from eye-gaze location, attention always precedes eye movements: attention shifts to the location of the next fixation before the eye executes a saccade to that location (Hoffman, 1998). Thus, at the very least, eye movements provide clues to where the visual attention focuses at approximately the same time. The usefulness of eye movement data is further bolstered by the fact that eye-movement data collection (i.e. eye tracking) is generally non-intrusive and does not affect behavior. Eye-movement studies with both tracking and non-tracking conditions for the same algebraic task have shown no significant differences in performance between conditions (e.g., Salvucci, Anderson, & Douglass, submitted).

Moreover, eye movement technology has now reached the point where it can be used with computer applications to provide fine-grained temporal tracking of user attention (Ryder, Weiland, Stokes, & Barba, 1999). Cameras can be placed external to the student, capture the student face, locate the eyes, and track gaze with accuracy greater than 1 degree of visual angle. (Interestingly, humans can monitor the gaze of another person they are speaking to looking with a root mean square error of about 2 to 4 degrees, Gale, 1998.) Ryder et al. describe a system that has been used by the Navy to deliver instruction in an anti-air warfare combat information center. Eye movements promise to be another input modality to the human computer at least as effective as speech and requiring no real training or cooperativeness by the user. A number of projects (e.g. Salvucci, 1999) have begun exploring the relative advantages and disadvantages of eye movements over mouse movements for pointing on the screen. In the domain of tutoring, eye movements promise to enable more effective diagnosis of student intentions and more effective communication from the tutor.

Figure 2

Trace of eye movement across problem as the student tries to fill in the highlighted cell—the fixation points go from dark to light with the passage of time.



Kevin Gluck (1999) has just finished an experiment in our laboratory assessing the potential of eye movements to inform tutoring. This serves as the background for our proposed research on eye movements. He decided that for his initial research he wanted to use a screen display that was simpler than what is used in the PAT algebra tutor but still maintained its essential features. Figure 2 shows the screen display of a half-completed problem with student eye movements superimposed over the problem. The student's task is to fill in the column labels and units, enter a variable and an expression written in terms of that variable, and then answer whatever result-unknown and start-unknown questions are present. The key aspect of the problem is the expression 12 + 45x, which the student must enter into the formula row. The real goal of the lesson is to teach the student how to create such expressions and how to use them to solve problems. While we want to study student eye movements with the full PAT screen, the development of this display actually serves multiple research purposes. For instance, this is a version that would be more practical to use in an fMRI scanner.

Gluck studied 18 students of a wide range of achievement levels, from grades 7 to 10, solving 16 problems over a four-day period. He established that one can reliably track students of this age and he identified a number of ways in which eye movements can help inform instruction. This proposal is too brief to enumerate his results but he found many examples where eye movements disambiguated students' intentions, could avoid miscommunication between student and tutor, and indicate cases where students had failed to read critical aspects of the problem and instruction. As just one example, Figure 2 illustrates an interesting case where we find students not solving the problem by the expected means. This student, typical of many, is calculating an answer without using the expression (83-2x) he has just created. As illustrated in the eye movement trace, such students do not fixate the expression while solving the problem but spend a lot of time rereading the problem statement. These eye movements are typical of a student who is reasoning about the problems in an informal, verbal way. One of our goals in this research is to be able to identify such students and instruct them differently than students who are using the expressions.

Using fMRI to Track Problem Solving and Learning

This section serves as fulfilling the requirement for a report on results from prior NSF support. This is a report on the grant SBR-98S73465 (10/01/98—09/30/01, $793,584) entitled "Computational Models on Coordinated Neuroimaging of Learning and Cognitive Function." This grant has 7 principal investigators who include John Anderson and Cam Carter. While many other projects are progressing under this grant, we will describe our research results. They serve as the basis of our proposed fMRI research. However we will begin this section with a more general review of fMRI research.

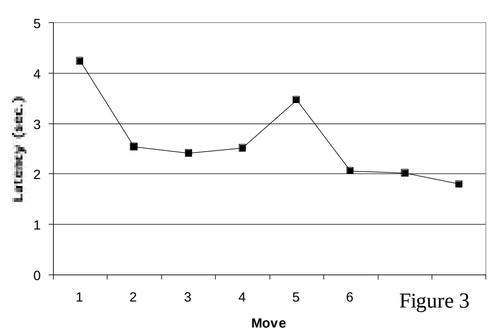


Figure 3

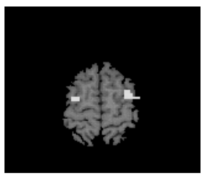
The technology of fMRI does not allow the temporal resolution of eye movements because of the slowness of the fMRI BOLD (blood oxygen level dependent) response. However, fMRI technology does allow one to track problem solving that is taking place over the time span of roughly 10 seconds to 5 minutes which is the time scale of much of algebraic problem solving. It also allows one to track learning that is taking place over many hours. Because of its ability to track the different brain regions involved, it allows one to identify the different cognitive components involved in solving a task and the changes in the composition of these components with learning. These are things one cannot do with eye movement technology. So it is really quite well situated to deepen our understanding of the learning of mathematical competence.

Recent work using a cognitive neuroscience approach to investigating mathematical competence in humans has emphasized the existence of two complementary cognitive systems related to these processes (Dahaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). This work has focused on arithmetic and has shown that for "exact" arithmetic computations (e.g., performance of a single operation such as multiplication or addition to produce a finite result) a language-based system is used. This would seem to parallel the language-based strategies used early in the learning of algebra. This system transfers poorly to another language in bilingual subjects and does not generalize well to novel instances. Arithmetic for approximate solutions (e.g. when the result of an operation must be compared numerically with approximate solutions) seems to rely on non-linguistic representations of magnitude which is language independent, transfers readily to other languages in bilinguals, and generalizes readily to novel instances. This may be analogous to the non-verbal symbolic coding associated with advanced learning of algebraic problem solving.

Initial studies using fMRI provide strong support for the above distinctions during arithmetic performance and provide the basis for predictions related to the use of the algebra tutor in the current proposal (Dahaene et al. 1999). Performing arithmetic problems requiring exact solutions activated a left sided inferior frontal-based neural network which overlapped with the network previously associated with verbal production and phonological rehearsal mechanisms during working memory performance (Smith, Jonides, Marshuetz, & Koeppe, 1998; Fiez & Petersen 1998). In contrast, solving problems with approximate solutions activated a distinct parietal-based circuit overlapping with posterior brain regions previously associated with visual spatial performance (Dahaene et al 1999, Corbetta, Shulman, Meizin, & Petersen, 1995). Interestingly, Reichle, Carpenter, and Just (in press) report a similar contrast between a verbal strategy and a visual strategy for sentence-picture verification that involves similar brain regions as in the Dahaene, et al. study. The same problem could be solved using either strategy and involved different neural areas when it did. We predict that we will see this sort of strategy-dependence for students solving the same algebra problem.

We (Carter, Fincham, van Veen, and Anderson) have been studying the time course of solution of the Tower of Hanoi problem (Simon, 1975) which has been modeled in ACT-R (Anderson & Lebiere, 1998). This is a puzzle which requires tens of moves to solve and usually takes a few minutes to solve. It has typically been used to study the setting and manipulation of goals. Certain moves in the solution of the problem are associated with these goal manipulations. Figure 3 shows the latency profile associated with making a sequence of 8 moves. Note that there are two peaks in the latency profile associated with moves 1 and 5. These are points where the student must engage in planning. Figure 4 identifies three regions of interest and Figure 5 displays their activation profile over the same 8 moves as Figure 3. Two regions of interest are left and right premotor cortex (Figure 4a). Unlike motor cortex, where we only get activation in left hemisphere (controlling the right hand that is making the moves), this area shows bilateral activation and is related to the planning process. Figure 5 shows the activation pattern of this area across the 8 moves. Each move takes 8 seconds followed by 8 seconds for the BOLD response to return to base-line. The BOLD response rises for each move but shows its largest rise associated with moves 1 and 5 where planning is involved. The correlation between the peaks in the BOLD response and the latency profile is .95. A very different pattern appears bilaterally in the anterior cingulate region (Figure 4b) which shows a decrease in activation as we move away from the goal states (1 and 8). This data dramatically demonstrates our ability to track problem transformation and goal processing. This is quite relevant to the current proposal where we will want to track similar problem transformation and goal processing in algebra.

(a) Slice 4 (approximately 60 mm above AC-PC line at slice 19) showing significant (subgoal count x scan) differential premotor activity:



(b) Slice 16 (approximately 12 mm above AC-PC line at slice 19) showing significant (subgoal count x scan) differential anterior cingulate activity:

Figure 4

The example above illustrates how we have developed the fMRI methodology to track problem solving. We have also been working on tracking the development of competence in a domain across problems. For instance, in another project under the NSF grant Schneider, Fincham, and Anderson have performed an fMRI study of the learning of rules. These were artificial rules for the setting of sports events. The processes involved in learning these rules have been subject to a substantial series of studies in the experimental literature starting with Anderson and Fincham (1994) and have recently been successfully modeled in ACT-R by Taatgen (1999). Here we were interested in what happens to these rules as people gain expertise in using them. Based on Taatgen's model we expected to see early heavy involvement of regions involved in goal manipulation (dorsolateral prefrontal cortex, anterior cingulate) and later heavy involvement of regions involved in episodic memories for past solutions (e.g. hippocampus). Our preliminary research is consistent with these identifications.

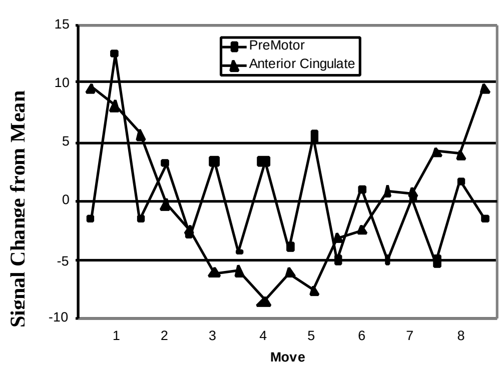


Figure 5

The examples of our research described above have involved adult participants, but in recent research Carter has been working with middle-school children. In an ongoing study, which uses comparable methods to those proposed in this application, Carter has studied 10 healthy young children in the age group (10-14) and obtained good quality data from all but one participant, whose data had to be discarded because of excessive head movement. This study involves a guessing game in which participants can either win or lose small amounts of money, and is being used to examine reward-related brain activity in children at high risk for depression compared to healthy matched controls. Robust activation in subcortical and frontal cortical regions previously shown to be responsive to reward in adults has been observed in the healthy children. Figure 6 shows reward-related activation in the ventral striatum in these children. Robust reward-related activation is seen which peaks after participants are informed of their reward and is sustained until the beginning of the next trial. This is an example of how fMRI can be used for diagnosis—a goal in our project although we will be concerned with diagnosing different mathematical strategies.

The ACT-R Theory

The development of our cognitive tutors has been guided by earlier version of the ACT-R theory (Anderson, 1983, 1993). The current version of the theory (Anderson & Lebiere, 1998) attempts to address cognition at a much finer grain size and Anderson and Byrne (1998) developed ACT-R/PM that addresses issues associated with the integration of perception and action. Salvucci (2000) has developed a rigorous extension of ACT-R/PM to deal with eye movements. Our recent explorations into fMRI have been guided by an effort to map the ACT-R theory onto brain imaging. The theory has not stood still with these extensions and the research proposed here will result in further development of the theory. Ultimately, we intend that the theory will result in a new generation of cognitive tutors that are able to be more sensitive to mental states of students.

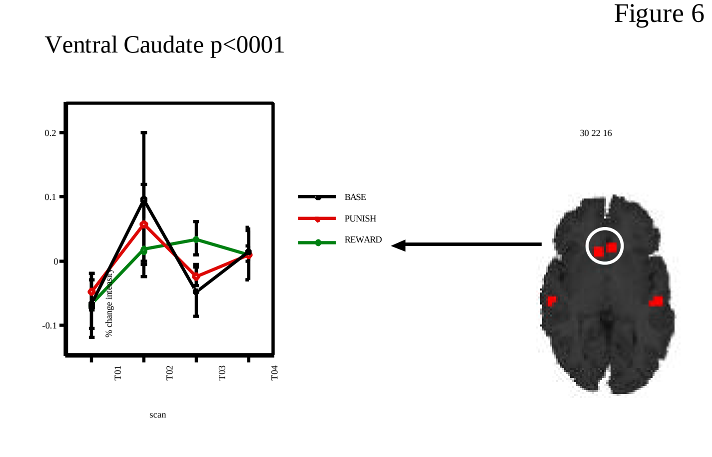


Figure 6

A number of features of the current ACT-R theory make it particularly appropriate for educational applications. It is a theory that models cognition in fine temporal grain size and so is appropriate for using information like eye movements or latency of processing for cognitive diagnosis. It also assumes that there is both a symbolic level and a subsymbolic level to cognition. The symbolic level represents the knowledge the student has but continuous, activation-like processes at the subsymbolic level enable us to represent the varying degrees of strength of that knowledge. Thus, we are not constrained to represent students in terms of simple overlay models. The various features of the ACT-R theory have made it popular with the research community concerned with training and human performance. As such it has received substantial use within the cognitive psychology with a user community of over 100 researchers around the world. There are summer schools offered on ACT-R both in Europe and America and an annual Summer workshop that attracts over 50 researchers. At last year's Cognitive Science meetings about 10% of the papers involved ACT-R or ACT-R/PM (Byrne & Anderson, 1998) and 50% of the papers at the International Conference on Cognitive Modeling involve these models. Thus, our efforts to make ACT-R contact instruction will leverage a substantial research base.

Proposed Research

In our background section we described a number of studies that have done much to establish the viability of the proposed research plan. In this section we will begin by describing in some detail three further studies that extend the research to the proposed integration of tutoring, eye movements, and fMRI. These studies will occupy the first 6 months of the proposal and will establish the basis for subsequent research. Our description of these studies will give some of the specifics of our methodology that we will be using throughout the project. These three studies also exemplify the three goals of our research. The first study uses eye movements to trace how students solve problems and to identify individual differences. The second study will use this information to make our cognitive tutors more sensitive to individual students. The third study uses fMRI to trace student problem solving and identify differences among students at different stages of learning. It is not feasible to use brain imaging in real-time instructional situations. However, we can use the diagnostic information we get from the brain imaging research to inform our cognitive tutors.

While the first three studies will yield interesting results in themselves they will also serve to establish the methodological groundwork for pursuing major themes for the next 2.5 years of research. One theme is concerned with the use of different representations of functions and the other theme is concerned with different problem-solving and learning strategies. The ultimate products of this research will be: (1) a better theory of these dimensions of individual differences and (2) cognitive tutors that are more sensitive to these differences. In the last 6 months of the proposal, we will perform an evaluation to test our success.

Our initial studies will be with the ninth grade PAT algebra tutor but we will switch to middle school algebra as the middle-school curriculum develops. We will be working with students in the range from sixth to the ninth grade. Thus, the focus of the project will be on children from approximately 11 to 15 years of age. We have had very successful experiences working with such children both in eye-tracking studies (Gluck, 1999) and in fMRI and have developed good procedures for involving parents. We have our tutors in a number of Pittsburgh area high schools (Langley H. S., Carrick H. S.) and middle schools (Greenway M. S., Chartiers Valley M. S., North Hills M. S.) and so have access to an ample pool of children to study. In addition, we are always fielding requests from local parents to have their children involved in our tutor studies. Children and parents find the involvement with the technology exciting and the instructional benefits are considerable.

Study 1: Eye Movements and the Actual PAT Tutor

The tutor that Gluck studied simplified the PAT interface and was only concerned with the first unit in the curriculum. It was developed when our eye movement software only allowed automatic analysis of eye movement data when the application (in this case the tutor) inserted the eye movement data into its record of the event stream. This forced us to simplify the software to maintain accuracy of recording. We now have software that can take eye movement data from one machine and problem state data from another machine and interleave the two. Thus, we can work with the actual PAT software and have students process the full screen. This will give us information closer to the actual classroom situation. This study is intended to inform us on the issues involved in using eye movements with the actual tutor and the true potential of such eye movements.

The curriculum unit that we will look at involves graphing and is the second unit in the ninth grade algebra curriculum. It involves a complex screen with multiple regions—a graph, a table, a problem statement, questions, instructional text and so creates multiple opportunities for the misreference problems we described earlier. An interesting example of a reference problem involves determining which point from a table a student is trying to plot on a graph. We should be able to disambiguate this by which cells are fixated in the table.

We can get students prepared for this curriculum by recruiting them from the classroom after the prerequisite Unit 1 has been mastered. Thus, students will be comfortable with the general tutor interface. Students generally spend about 5 days with the tutor in the classroom to cover this unit. We anticipate that they will be able to cover the unit in the laboratory during a week of after-school sessions. There are 25 problems, each taking about 10 minutes, associated with this unit. The exact number of problems that a student solves depends on what if any difficulties they manifest. One thing we would like to assess is the degree to which eye movements can inform the actual selection of additional problems.

The graphing facilities involve a number of opportunities to assess students' facility with the graphical representation of a function. For instance, in plotting points the grapher provides tracer lines to help identify x and y coordinates. Eye movements should help us assess student understanding of these. Once students have graphed a function they can use it to find values to enter in the table but we have been uncertain how often students go from function graph to table entries. Again eye movements will allow us to assess this.

The basic eye movement configuration involved in the data collection involves a configuration of three computers. There is computer that is delivering the instruction, in this case the PAT tutor. There is the computer attached to the eye-tracking equipment (in this case, a system from Eyelink) that is involved in tracking the eye movements and calculating the gaze position. There is a third machine that collates state information from the tutor machine and gaze position information from the Eyelink machine and organizes this into a data file that facilitates later analysis. Over the past few years we have developed methods for the rapid analysis of the large data files and for replaying fragments of eye movement protocols. The Eyelink system can either be head-mounted or involve a remote camera. The head-mounted system offers greater precision and we have one but we are proposing to acquire a remote camera system to have a less intrusive system and a system that is like the kind of system that might come with a computer in the next decade.

A major goal of this project will be to assess the issues associated with using eye movements with real classroom software rather than the mock-up that Gluck used. The displays are much richer and there will be increased demand for high calibration of the eye tracker. Based on this study, we will follow on with an instructional intervention using the PAT tutor like the one we describe next (Study 2) using Gluck's mock up.

Study 2: Using Eye Movements to Guide Instructional Interventions

The goal of this study is to show that the tutor can be made more sensitive to individual differences and more effective by incorporating information from eye movements. Gluck's dissertation showed that there are a number of points where we can diagnose a student's mental state more accurately using eye movement data. The goal of this study is to demonstrate that this more accurate diagnosis can result in additional instructional opportunities. An instructional opportunity in this study will be an association of a trigger condition defined on eye movements with some instructional maneuver. Our trigger conditions will all be defined as a pattern of fixations over the screen. The instructional maneuver can either involve a spoken message or a change to what the tutor delivers visually or both. In the case of spoken messages, they will be brief based on research indicating a low tolerance for long instructional messages (Laddaga, Levine, & Suppes, 1984).

To evaluate the effectiveness of the tutor, we will compare it to the identical system without those instructional opportunities. We will use the same three measures as Gluck used—errors made with the tutor, time in the tutor, and pretest-to-posttest gain.

Below are the instructional opportunities that we identified for the tutor.

1. Some students, particularly when they are working on the expression, fall into a superficial scanning of the problem statement in which they simply hunt out the numbers and try them in linear expressions. We will tabulate amount of time reading in the vicinity of the numbers versus the amount of time reading the rest of the problem statement. Should the ratio of these numbers be substantially out of proportion and should the student make an error, we will suggest they more carefully read the text of the problem statement.

2. Students often lack the metacognitive skills to use the help that tutors provide (Aleven & Koedinger, 2000, Woods & Woods, 2000). Students often get stuck but do not seek help. We have contemplated using long latencies without progress as a sign that we should volunteer help. However, we have not been able to do this because such long latencies may simply indicate the student is not working on the problem. Off-task examples in the classroom include the student talking to another student, the teacher, or perhaps daydreaming. While one might contemplate intervention in such "off-task" states, we do not regard this as an appropriate role for our tutor. On the other hand, if the student is stuck and is earnestly trying to solve the problem it seems appropriate to intervene with help. Eye movements now allow us to judge whether the student is scanning the screen trying to solve the problem. Gluck has shown that in such cases if students spend abnormally long periods of time they are likely to make errors. We propose to volunteer help if there is a long-latency on-task period.

3. Our tutor delivers a sequence of more specific help messages ending with one that basically tells the student what to do. There are some students who quickly click through the help messages until they get to the bottom-out help message. We can now suggest that they read each message before going onto the next. In addition we can suggest to students that they read the problem statement before they ask for any help.

4. When students make errors, the tutor typically delivers a "bug message" intended to explain what was wrong with the answer. These bug messages are distinct from the help messages discussed in (2) and (3) above. Gluck found that students fail to read over 40% of these bug messages. In many cases this is because students know what they did wrong and can self correct without reading the message. However, Gluck found that students were unlikely to self correct if more than 20 seconds pass without an action and without reading the bug message. We propose to ask them to read the bug message if a long period of time passes without their entering the answer.

5. Gluck identified in the first unit one frequent case of misreference between student and tutor. Students would sometimes indicate that they wanted to enter a value in one column of the table but actually enter a value appropriate for the other column. In such cases students typically fixate the wrong source of the information. For instance, if they enter a given where a result should go they will spend time reading the question that contains the given rather than fixating the problem statement or expression that would help them calculate the result. The opposite fixation pattern occurs when students enter a result where the given should go. We propose to warn them about the potential confusion if they spend too much time fixating the wrong source of information.

6. As illustrated in Figure 2, sometimes students' eye movements indicated they are not using algebraic expressions. Some of these students are highly error prone. We propose to identify students who do not fixate the expression and are making errors in cells where the expression could be used. We will highlight the expression and present instruction on how to use these expressions.

In all of these examples, the basic character of the tutor interaction will not change. The tutor will still deliver suggestions to help the student learn. The suggestions will just be more appropriate. While the student will obviously know their eye movements are being monitored, we hope it will not be apparent which messages are informed by eye movements. Rather we hope they will be less aware of messages that should have been informed by eye movements. A simple example of this is that at a number of points the tutor currently asks the student to read the problem statement just after the student has read the problem statement. This is a message we would like to avoid.

Study 3. Using fMRI to Track Algebra Learning

We would like to use fMRI to track the solving of algebra problems and the acquisition of algebraic competence. In the first fMRI study we will use it to track students solving algebraic equations using the symbolic manipulation tool in the algebra tutor. We are focusing on this piece of the tutor because it involves a minimum of eye movements and we will transform the interface to further minimize eye movements. We are working on methodologies to subtract out eye movement artifacts (different scanning protocols, omitting scans on which eye movements occur, avoid scanning of the orbital regions) but we did not want this to be a significant complication in the first study. In later years we will move onto other aspects of the algebra curriculum, particularly those concerned with relating different algebraic representations.

The equation-solving tutor has a successful mode in which students do not enter new equations but just choose from a menu of operators for transforming equations. This is particularly appropriate for the scanner because it avoids dealing with issues of complex response modalities. It focuses students on appreciating the critical aspects of equation structure and legal transformations and obviates many of the time-consuming aspects in doing the bookkeeping involved in rewriting equations. Our achievement results have been good with this version of the equation-solving tutor. The typical problem involves presenting a student with an equation like

$$6y + 4y - 14 = 26$$

and presenting them with a menu of operations that includes adding to both sides, subtracting from both sides, dividing both sides, multiplying both sides, collecting like terms, and distributing. Whatever operation they select they then have to parameterize the operation. This parameterization can also be selected from a menu. Once the operation is selected and parameterized the transformation is applied. So if the student chooses to collect y terms on the left the equation would become

$$10y - 14 = 26$$

The tutor can follow any legal sequence and provides strategic advice. A problem can involve 1 to 5 or more of these transformations. These problems have the advantage that students can evolve considerably in their competency over the couple of hours they can be in the scanner.

The transformation cycle is appropriate for an event-related methodology like what we have used with the Tower of Hanoi problems. We will place a 12-second period between problems to allow the fMRI BOLD signal to return to baseline from the previous problems. Then the student will be presented with an equation and will perform a sequence of transformations on the problem. Each transformation will result in a change of the equation. We can plot the fMRI signal for each transformation either stimulus-aligned (which means anchored at the beginning of the new display of the equation) or response-aligned (which means anchored with the menu selection that terminated that display).

The natural manipulations are problem difficulty and practice. Problem difficulty can be manipulated by means of the size of the problem. We have found that students have greater difficulty applying the same transformation (say, adding to both sides) if the problem involves more symbols (i.e., $10y-14 = 26$ is more difficult than $y - 14 = 26$) or if it involves more complex numbers like decimals (i.e., $y-1.4 = 4.2$ is more difficult than $y - 4 = 2$) even though it is the same menu selections and the tutor actually performs the calculations. Finding which regions respond differently to problem difficulty is a way of identifying brain regions involved in the task. Our expectation is that problem difficulty will activate different brain regions for practiced students rather than novices. As in our Tower of Hanoi task we would expect involvement of the frontal cortex early in practice. On the other hand, our hypothesis is that for skilled students this will involve parietal regions associated with visual reasoning. This hypothesis is relevant to our more general interest in the difference between formal-symbolic and informal-verbal methods of reasoning about algebraic problems.

Research Themes: Different Representations and Different Strategies

We have just described the three studies that will occupy the first six months of the project. They will serve to establish the methodology for the later studies. We do not have the space to describe in detail the subsequent studies but Table 1 indicates the basic research plan. There will be three tracks of research—one looking at eye movements, one assessing instructional interventions, and the third focused on fMRI studies. These will continue the lines of research in the first three studies. There are a number of 6-month studies proposed in Table 1. At the end of the research program we will converge on an instructional study assessing what we have learned. The exact studies need to be tentative and open to being revised in response to earlier results. However, the themes that will unify the research will be the themes of tracking individual differences in mathematical representations and problem-solving strategies. Our tutors present students with verbal, tabular, graphical, and symbolic representations of functions and require students to use all these representations in a problem. The strongest achievement gain displayed by our students is in their ability to relate the multiple representations. Nonetheless, there is still much room for improvement.

Year	Eye Movement	Instructional Interventions	fMRI
0.5	Study 1	Study 2	Study 3
1.0	Graph Interpretation	PAT Tutor	Verbal Symbolic
1.5	Equation & Expressions	Representation Competence	Different competencies.
2.0	Spreadsheets	Strategic Competence	Determinate/Indeterminate
2.5	Middle School Tutor	Middle School Tutor	Eye Movement Coordination
3.0	← Informing Instruction by Selection by fMRI Middle School and eye movement →		

Table 1: Studies for Each Half Year of Project

Eye Movement Studies (first column in Table 1)

With respect to eye movements we would like to have the ability to more accurately diagnose the source of student difficulties with the various representations. Therefore, we will engage in a program of research that identifies the eye movements associated with competent use of these representations and those associated with particular difficulties that students have with a representation. Basically, we will be concerned with four representations illustrated by:

Verbal: Concert tickets cost 45 dollars a pieces. A friend offers to stand in line and buy a number of tickets if you pay him 12 dollars to do so.

Symbolic: $12 + 45x$

Graphic: A graph of the same function on x-y coordinates.

Tabular: A set of pairs of values satisfying the function.

There are a number of ways that one can look at student ability to access these various function representations. These include answer result-unknown questions like "What is the value of the expression when x =3", start-unknown questions like "How many tickets would you have bought if you paid your friend $147" and questions where one has to decide if one function form corresponds to another functional form. We believe that students who have deficits with one of these representations will show these deficits in the way that they visually scan these representations in answering questions. We would like to document these distinctive eye movements so that we can use them as basis for diagnosing student misunderstandings in the tutors. For instance,

1. Consider a student who does not appreciate order of precedence in $3 + 2x$. Such a student, when asked to evaluate the expression for $x = 2$, would read the expression in a left to right manner rather than first fixating on the 2x.

2. Consider a student who is confused about x and y in the graph. When asked to evaluate the function $y = 3 + 2x$ for $x = 5$. Such a student might scan to the (1, 5) point on the graph.

3. Consider a student who does not appreciate how the verbal statement maps onto the graphical expression. Such a student might fail to check that the intercept on the graph is 12.

In Table 1, we propose three studies on three topics which will have special units developed in the middle school curriculum—graph interpretation, equations and expressions, and spreadsheets. The graph interpretation unit is concerned with teaching students to use multiple-function graphical displays to answer questions. Here we will be concerned with detecting evidence for things like x-y confusions, failure to use intercepts, and inability to use function intersections. The equation and expression unit will be concerned with symbol manipulation. We will be looking for evidence for misparsing and for differential processing of visually salient and non-salient rules (e.g. Kirshner & Awtry's, submitted, contrast between $2(x-y) = 2x-2y$ versus $(x+y)(x-y) = x^2 - y^2$). The spreadsheet lesson will look at sixth grader's ability to use spreadsheets to find answers to questions like when one phone plan is cheaper than another. We will be looking for patterns such as appropriate comparison of rows in the spreadsheet.

fMRI Research (third column in Table 1)

While eye movement research is well suited to assess differences in how external representations are processed, fMRI research is well suited for studying differences in how internal processes are brought to bear on the same external representation. In particular, we would like to focus on the differences between informal, verbal representations of functions and symbolic, visual representations of functions. We would assume the verbal representation will involve ventral frontal cortex (BA 44, 45) and the graphical representation will involve parietal activation in solving problems that are formally equivalent. It is less transparent to us what areas will be involved in processing of equations but we have a hypothesis: We believe that at initial levels of competence students tend to process equations and expressions as spatial patterns of graphical marks but that at high levels of competence algebraic expressions become another linguistic system for students and so we should see additional verbal regions of involvement. More generally, it would be valuable if we could document for any of these representations that increased competency was associated with different regions of activity. It would provide a new way to assess the different patterns of competency of different students.

The study in Table 1 labeled "Verbal Symbolic" will use the following paradigm to compare verbal versus symbolic problem solving in a fMRI study. We would either show students an expression like

$$4x + 11$$

or a verbal statement like

multiply a number by 4 and then add 11

and give students time to commit this statement to memory. At encoding, we would expect to see differential activation of visual versus verbal areas depending on which statement they read. However, the critical issue is to show that, after the information is removed, they reason about it differently. So, after reading either statement, they would then be presented with a digit that they are either to substitute for the number (variable) or find the value of the number (variable) which would produce that result. Generalizing from the work reviewed of Dahaene, et al. (1999) we would expect to see different regions activating to this same stimulus (frontal versus parietal) activated reflecting the different solution methods.

The study in Table 1 labeled "Different Competences" would compare three types of students. We have reviewed evidence that early in the development of algebraic competence some students tend to rely exclusively on verbal representations and informal reasoning strategies while other students tend to rely exclusively on symbolic representations and various rules symbol manipulation. Students who are using a symbol manipulation strategy often show a brittleness for their use of the strategy. While they can solve familiar problems like

1a. $3x + 5 = 32$

they are unable to extend that strategy to solve formally similar problems like

1b. $(x - 5)/3 = 9$

Other students cannot solve either of these but have no difficulty when faced with verbal restatements of these problems like

2a. Starting with some number, if I multiply it by 3 and add 5, I get 32.
What number did I start with?

2b. Starting with some number, if I subtract 5 and then divide by 3, I get 9.
What number did I start with?

Third there are students who can solve all forms of the statements and equations and who can translate between them. We propose that these students activate both verbal and symbolic-visual areas. Moreover, it should be possible to improve students' performance on the difficult equations by instructing them to translate an equation like 1b into a verbal statement like 2b. Students trained with such a strategy should show activation of both verbal and the symbolic-visual areas like those students who are competent in both representations without training.

The fMRI study labeled "Determinate/Indeterminate" will look at the differences between solution of determinate and indeterminate problems. An example of a determinate problem would be to find a combination of dimes and quarters who's sum is 20 and show value if \$3.20. An example of an indeterminate problem is to find some combination of dimes and quarters who's sum is 20 and whose value is between 3 and 4 dollars. In his dissertation Nouyvanisvong (1999) found that determinate problems tended to evoke symbolic solutions while indeterminate problems evoked a guess and check procedure. This is analogous to the difference between exact and approximate arithmetic problems found by Dehaene, et al. (1999).

The fMRI study labeled "Eye Movement Coordination" is intended to study how we can use the eye movement equipment that the fMRI center will be installing. We are particularly interested in whether more effective visual parsing strategies are associated with less activations of the parietal cortex.

Instructional Interventions (second column in Table 1)

There are a series of studies listed where we would like to evolve our ability to adapt instruction to student differences. Some of this adaptation will be informed by eye tracking data but also it will be informed by other information including fMRI assessments. The study label "PAT Tutor" will attempt to extend the same methods in Study 2 to the full tutor. The study labeled "Representation Competence" will focus on problems students have with graphs and equations with the goal of getting students to effectively relate the two. The study "Strategic Competence" will attempt to develop different instruction for students who are relying on informal verbal reasoning and for students who are relying on visual-symbolic with the goal to getting them to inter-related the two. The last year of instructional intervention will involve a two-pass effort surrounding the middle-school tutor which will be deployed by then. The first formative study will try to extend the work done earlier with the high-school PAT tutor to use eye movements to gain instructional opportunities. The final study will first use a fMRI assessment of students reasoning patterns and then inform the instruction by this information as well as eye movements. In is not feasible to have fMRI inform real-time instruction but we can use it for diagnosis of student differences before instruction.

While the specifics of this final assessment remain to be determined, we would like to use as large a fraction of the middle-school curriculum as possible. There will be essentially three conditions—students learning the curriculum with the standard tutor in classroom, students working with the standard tutor in the laboratory, and students working with the enhanced tutor in the laboratory. The students working the standard tutor in the laboratory will have all the same treatments (fMRI study, eye tracking) as the students with the enhanced tutor. They will just not receive instruction informed by this information. The comparison between the students in classroom versus laboratory with the standard tutor will allow us to assess the Hawthorne effects associated with the experimental treatment. The comparison between the laboratory students with standard versus enhanced tutor will give a pretty pure measure of the effect of instruction contingent on the information we get from advanced sensing techniques. If successful, this study would serve as a basis for introducing eye tracking into the actual classroom.

Significance

This would be the first research on algebraic competence to associate use of different problem representations and different problem-solving strategies with different eye movements and different patterns of brain activation. There would be five more or less immediate and important consequences of this research program. First, technology is now becoming available for such advanced "sensing" of students and this research will provide a good assessment of the potential of this technology. Second, for the graduate students and post-docs involved in this project, this would be unique and valuable cross-disciplinary training. Third, it would drive the ACT-R theory of knowledge representation. The critical feature for the theory development is that we would have converging behavioral and neural evidence for different processes involved in solving formally equivalent problems. Fourth, it would enable us to diagnose the different strengths and strategies that different students have at different stages in the development of their mathematical competence. Fifth, it would enable us to make our tutors more sensitive to individual students. To repeat the theme of the introduction, our goal is to make instruction sensitive to the cognition of the individual student.

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Summary: Relevant to Quandrants I and II

The goal of this project is to improve our ability to track how students solve mathematical problems. This research will use eye tracking to make real-time inferences about what the student is thinking and fMRI imaging to make inferences about different styles of problem solving. This research is done in the context of both the ACT-R theory of human cognition, which allows us to produce computational models of cognition, and a series of cognitive tutors for mathematics education, which are based on the ACT-R theory. The research will focus on the algebra tutor that is currently in use in high school and is being adapted for use in middle schools. The research will be concerned with the effect of different mathematical representations on problem solving and with different strategies for mathematical problem solving. There will be three lines of research. One, involving eye movements, will document the instructional opportunities associated with eye movements in the context of the cognitive tutors. It will particularly focus on the eye movements associated with competent use of graphical, tabular, and symbolic representations of functions. The second line of research, involving fMRI brain imaging, will study brain activation markers of the course of mathematical problem solving. It will particularly focus on distinguishing between students who use an informal, verbal form of reasoning with students who use a symbolic, visual form of reasoning. The third line of research will study how one can use the information from fMRI scanning and eye tracking to produce more effective instruction. The three lines of research will converge on a culminating study that attempts to improve the effectiveness of the middle school tutor. It will first use fMRI imaging to classify students and then collect real-time eye movement to guide instruction as students are learning.

ROLE: Tracking the Course of Mathematics Problems

Revised Plan of Work

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In addition to the work described in our initial proposal, we will engage in the following activities:

1. An additional post-doctoral researcher will be hired to investigate the relationship between eye movement and mathematical problem solving at the neural and cognitive levels. The person for this position will have to master technical issues in using eye movement research, technical issues in imaging research, and be very familiar with visual attention. We would be satisfied finding a post doc who knew visual attention and had mastered the one of the technical areas. His or her training would involve mastery of the other technical area and development of a research program that traverses these content areas.

This research will take advantage of a new ASL infrared eye-tracker that is being put into the scanner. Two additional directions of research will be pursued. The first is to use eye movements to classify problem-solving strategies on a problem-by-problem basis. As we noted in the proposal there are different eye movements associated with the informal verbal strategy and the formal symbolic strategy. It is the case that some students will switch from problem to problem the strategy they use. If we did fMRI studies of such students and did not know what strategy they were using on a problem, the averaged results would be severely compromised by noise. On the other hand, if we can classify their strategy on the basis of their eye movements, we may be able to sort problems according to methods. Not only would this clean up our imaging data, but it would also serve as an important converging test of our strategy classification. Moreover, this ability to use eye movements to sort trials according to strategy will be generally important as researchers move to imaging studies of more complex tasks.

The second direction is to look at the implicit attentional learning that is part of the development of competence in many domains, and to connect that line of research to the body of work on attention that has emerged from neural and eye-movement-based studies. As noted in the proposal, Kirshner has claimed that a significant component of symbolic competence in algebra is the ability to correctly parse the visual form of algebraic expressions. In some of our eye movement research we have shown that a significant component of the development of skill in domains like air-traffic control is learning which regions of a complex display to look at for relevant information. Learners are not aware that they are improving their scanning patterns, but such learning is critical to their having enough time to do the task in the time-pressured situations. Such issues of visual attention become important as algebra students have to parse complex tables, graphs, and geometric displays. It seems likely that learning how to scan these displays is a critical part of the development of their mathematical competence. There is a high degree of overlap between brain regions involved in eye movement control (frontal eye fields, dorsolateral prefrontal and parietal cortex) and those involved in visual spatial attention. Therefore, we would like to pursue the issue of whether activation in these regions reflects the fluency that students are displaying in their scanning of mathematical problems.

2. We will develop a consulting relationship with Stanislas Dehaene, with whom we will organize a conference at NSF. At the beginning of the first year we will bring Dr. Dehaene to Pittsburgh to consult with us on our research directions and to give talks in conjunction with the Center for the Neural Basis of Cognition (CNBC -- a joint organization between Pitt and CMU). As we noted he might also profit by exposure to our event-related imaging techniques. At the beginning of the second year we propose to visit Dehaene at his lab. Then at the beginning of the third year we propose hold a conference, jointly organized with Dehaene at NSF, reporting on our work and that of others on the relationship between mathematics learning and brain research. Funds for the conference itself are not provided for in the present award, and may be negotiated at a later date in the form of a supplement.

In addition, specific aspects of our the fMRI work that we have proposed are clarified as follows:

The predictions relate to our hypothesis that the initial development of algebra problem solving skills will engage a language-based circuit involving left inferior frontal cortex, and that with increasing skill a more efficient and generalizable visual spatial symbolic mode of processing will be utilized with corresponding activation of a parietal-based circuit. Within both modes of processing we predict increasing activation correlating with increased problem difficulty. Based upon our previous work related to executive functions in working memory and planning and problem solving we also predict for both parietal and frontal regions that for a given level of difficulty increased activation will be associated with better problem solving performance in individual differences analyses. Finally, an open question is whether with the development of advanced skill activity in parietal regions will increase or decrease. Although speculative, we predict, based upon work of Raichle and others, that with advanced skill processing in the symbolic mode activation in parietal cortex will actually decrease, reflecting more efficient processing in that brain region.