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Effect of Gyro-viscosity on Rayleigh-Taylor Instability of a Plasma

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ABSTRACT

The effect of finite ion Larmor radius on the development of Rayleigh-Taylor instability of a plasma bounded by a vacuum is studied. The macroscopic equations of motions are used where the finite ion Larmor radius effect is incorporated through off-diagonal terms in the pressure tensor in the momentum fluid equations. The effect of the finite Larmor radius stabilization of the interchange mode is demonstrated. For disturbances propagating along the magnetic field it is found that the inclusion of gyro-viscosity has the effect of reducing the instability of plasma vacuum interface.

I. INTRODUCTION

The importance of the effect of the finite Larmor radius (FLR) of the ions on plasma instabilities has been of considerable interest in the recent years. The effect of FLR on the gravitational instability of a plasma was considered by Rosenbluth et al[1] using Vlasov equation and the same results were recovered later by Roberts and Taylor[2] using macroscopic equations for a two dimensional plasma. Recently Kennel and Greene[3] have investigated the FLR effects using the appropriate scalings of the Vlasov equation and derived a set of asymptotic equations. Among several interesting results, they obtain the FLR stabilization of the interchange mode.

This paper considers the stability of an incompressible plasma bounded by a vacuum and supported against gravity by a static magnetic field. We use here the macroscopic moment equations where the effect of finite ion Larmor radius is included through off-diagonal terms in the pressure tensor in the momentum fluid equations. The boundary conditions relevant to the problem at hand are derived and the dispersion relation is obtained. The interchange mode stabilization is demonstrated. When the disturbance is propagating along the magnetic field, the dispersion relation is analyzed on the assumption that gyroviscosity is small, i.e. $N\Theta k^2/\rho n \omega_c \ll 1$, where N is the particle density, Θ the plasma temperature in energy units, k and n are the wave number and frequency of the disturbance respectively, ρ the density and ω_c the ion-gyrofrequency. It is found that the inclusion of gyro-viscosity has the effect of reducing the instability of plasma vacuum boundary.

II. FORMULATION OF THE PROBLEM.

We now consider a situation where an infinitely conducting plasma occupies the half space $0 < y < \infty$ and is supported against gravity by a uniform magnetic field which we shall take along the z-axis of a system of cartesian co-ordinates (x, y, z). We shall assume that the medium is incompressible and infinitely conducting. The equations basic to our discussion are the equations of motion

$$\rho \frac{d\vec{v}}{dt} = - \vec{\nabla} \pi + \frac{1}{4\pi} (\vec{B} \cdot \vec{\nabla}) \vec{B} + \rho \vec{g} - \vec{\nabla} \cdot \vec{P}_M \qquad (1)$$

and the generalized Ohm's law

$$\vec{E} + \frac{1}{c} \vec{v} \times \vec{B} - \frac{1}{4\pi Ne} (\vec{\nabla} \times \vec{B}) \times \vec{B} + \frac{1}{Ne} \vec{\nabla} p_e = 0 \qquad (2)$$

together with

$$\vec{\nabla} \cdot \vec{v} = 0 \quad \text{and} \quad \vec{\nabla} \cdot \vec{B} = 0, \qquad (3)$$

where $\pi = p + B^2/8\pi$, $\vec{g} = (0, -g, 0)$ is the constant gravitational field, N is the electron number density, p_e is the electron pressure, p is the total fluid pressure, ρ is the constant density, $\vec{v}$ is the fluid velocity, $\vec{E}$ the electric field, $\vec{B}$ the magnetic field and c the velocity of light. The electric and the magnetic fields satisfy Maxwell's equations in which we shall neglect the displacement current. The anisotropic part of the stress tensor, P_M, arises because of the finite Larmor radius effects. If the magnetic field is along the z-axis, its various components are given by[4]

$$P_{xx} = -\rho\nu A_{xy}, \quad P_{yy} = \rho\nu A_{xy}, \quad P_{zz} = 0, \tag{4}$$

$$P_{zx} = P_{xz} = -2\rho\nu A_{yz}, \tag{5}$$

$$P_{zy} = P_{yz} = 2\rho\nu A_{zx}, \tag{6}$$

$$P_{xy} = P_{yz} = \frac{1}{2}\rho\nu (A_{xx} - A_{yy}), \tag{7}$$

where

$$\vec{A} = \frac{1}{2}[\vec{\nabla}\vec{v} + (\vec{\nabla}\vec{v})^{tr}] - \frac{1}{3}\vec{\nabla}\cdot\vec{v}\,\vec{I}, \tag{8}$$

and $\vec{I}$ is the unit dyadic. The coefficient ν occurring in the above relations is given by $\nu = N\Theta/\rho\omega_c$, where $\Theta = kT$ and $\omega_c = eB/Mc$ is the ion gyrofrequency. It is clear from the foregoing equations that these admit the static stationary solution

$$\vec{\nabla}\pi = \vec{\nabla}p = \rho\vec{g}. \tag{9}$$

The condition of pressure balance in equilibrium at the interface then becomes

$$p + \frac{B_p^2}{8\pi} = \frac{B_v^2}{8\pi}, \tag{10}$$

where B_p and B_v denote the strength of the magnetic fields in the plasma and in the vacuum respectively.

For small departures from the state of equilibrium, let the various quantities be denoted by $\vec{v}$, $B_p + \vec{b}$, $p + \delta p$. The equations governing these are readily obtained from Eqs. (1) - (4) together with Maxwell's equation $\partial \vec{B}/\partial t = -c\vec{\nabla} \times \vec{E}$. We thus obtain

$$\frac{\partial \vec{v}}{\partial t} = -\vec{\nabla}\pi_1 + \frac{1}{4\pi\rho}(\vec{B}_p \cdot \vec{\nabla})\vec{b} - \frac{1}{\rho}\vec{\nabla} \cdot P_M, \tag{11}$$

and

$$\frac{\partial \vec{b}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}_p) - c_1 \vec{\nabla} \times [(\vec{\nabla} \times \vec{b}) \times \vec{B}_p], \tag{12}$$

where $\pi_1 = \delta p/\rho + \vec{B}_p \cdot \vec{b}/4\pi\rho$ and $c_1 = c/4\pi Ne = B_p/4\pi\rho\omega_c$.

We now assume the (t,z) dependence of the various quantities to be of the form $f(x, y, z, t) = f(x, y) \exp(nt + ik_z z)$. Equations (11) and (12) then reduce to

$$n\vec{v} = -\vec{\nabla}\pi_1 - \frac{1}{\rho}\vec{\nabla} \cdot P_M + \frac{ik_z B_p}{4\pi\rho}\vec{b} \tag{13}$$

and

$$n\vec{b} = ik_z B_p \vec{v} + ik_z B_p c_1 \vec{\nabla} \times \vec{b}. \tag{14}$$

The gradient term in Eq. (13) can be eliminated by taking the curl of this equation. Assuming that the temperature variations in the initial state are negligible, we find after some straight forward reductions that

$$n\vec{\nabla} \times \vec{v} = ik_z \frac{\nu}{2}(\vec{\nabla} \times \vec{\nabla} \times \vec{v} - 3k_z^2 \vec{v}) + \frac{ikB_p}{4\pi\rho}\vec{\nabla} \times \vec{b}. \tag{15}$$

We can then eliminate the velocity field from Eq. (15) by using Eq. (14) and obtain the equation for perturbation in the magnetic field $\vec{b}$:

$$a_3 k_z^{-3} \operatorname{curl}^3 \vec{b} + a_2 k_z^{-2} \operatorname{curl}^2 \vec{b} + a_1 k_z^{-1} \operatorname{curl} \vec{b} + a_0 \vec{b} = 0, \tag{16}$$

where we have put

$$a_0 = -\frac{3}{2} \frac{n\omega_\nu}{\Omega^2}, \tag{17}$$

$$a_1 = i \left(1 + \frac{n^2}{\Omega^2} - \frac{3}{2} \frac{\omega_\nu}{\omega_c}\right) \tag{18}$$

$$a_2 = -\frac{n}{\omega_c} + \frac{1}{2} \frac{n\omega_\nu}{\Omega^2}, \tag{19}$$

$$a_3 = \frac{1}{2}\, i\, \frac{\omega_\nu}{\omega_i}, \tag{20}$$

and

$$\omega_\nu = \nu k_z^2, \quad \Omega^2 = k_z^2 A^2, \quad A^2 = \frac{B_p^2}{4\pi\rho}. \tag{21}$$

III. THE METHOD OF SOLUTION.

The solution of Equation (16) can be written as

$$\vec{b} = \sum_{i=1}^{3} \vec{b}^i, \tag{22}$$

where $\vec{b}^i$ is governed by the equation

$$\vec{\nabla} \times \vec{b}^i = \alpha_i \vec{b}^i \tag{23}$$

and α_i's are solutions of the cubic equation

$$a_3 k_z^{-3} \alpha_i^3 + a_2 k_z^{-2} \alpha_i^2 + a_1 k_z^{-1} \alpha_i + a_0 = 0. \tag{24}$$

To solve Equation (23) we further assume that the x-dependence of the perturbation is of the form $i k_x x$. We then obtain for the solution valid in the upper half plane ($y > 0$):

$$b_x^i = C_i p_i e^{-\lambda_i y}, \tag{25}$$

$$b_y^i = i C_i q_i e^{-\lambda_i y}, \tag{26}$$

and

$$b_z^i = C_i e^{-\lambda_i y}, \tag{27}$$

where

$$\lambda_i^2 = k^2 - \alpha_i^2, \quad k^2 = k_x^2 + k_z^2,$$
$$p_i = \frac{k_x k_z + \alpha_i \lambda_i}{k_z^2 - \alpha_i^2}, \quad q_i = \frac{k_x \alpha_i + k_z \lambda_i}{k_z^2 - \alpha_i^2} \tag{28}$$

and C_i are the three integration constants to be determined by the boundary conditions. Having obtained $\vec{b}$ the perturbation in the velocity field can be obtained from Equation (14) to be

$$\vec{v} = \sum_{i=1}^{3} \vec{v}_i = \frac{-i}{k_z B_p} \sum_{i=1}^{3} \left[n + \frac{i \Omega^2 \alpha_i}{k_z \omega_c} \right] \vec{b}^i. \tag{29}$$

The perturbation in the total pressure π_1 can be obtained by taking the z-component of Eq. (13) and making use of the foregoing results. We thus find

$$\pi_1 = \sum_{i=1}^{3} \pi_i = \frac{1}{k_z^2 B_p} \sum_{i=1}^{3} \left[\Omega^2 + \left(n + \frac{i \Omega^2 \alpha_i}{k_z \omega_i} \right) (n - i \nu k_z \alpha_i) \right] \vec{b}_z^i . \tag{30}$$

Finally the perturbation in the vacuum magnetic field, $\vec{b}^{(0)}$ is the solution of equations $\vec{\nabla} \times \vec{b}^{(0)} = 0$ and $\vec{\nabla} \cdot \vec{b}^{(0)} = 0$; it is given by

$$\vec{b}^{(0)} = [\vec{e}_x i k_x + \vec{e}_y k + \vec{e}_z i k_z] C_4 e^{ky} \quad (y < 0), \tag{31}$$

where C_4 is another integration constant to be determined by the boundary condition.

IV. THE BOUNDARY CONDITIONS

The dispersion relation can now be obtained by using the boundary conditions appropriate to the problem. These are: (1) At the interface $y = 0$, the normal component of the velocity must be compatible with the assumed form of the deformed boundary, i.e.,

$$v = n \epsilon e^{i(k_x x + k_z z + n t)}, \tag{32}$$

where ϵ is a constant.

(2) The normal component of the magnetic field must be continuous. The linearized form of this condition is

$$\Delta[\vec{b} \cdot \vec{N}_0 + \vec{B} \cdot \overline{\delta N}] = 0. \tag{33}$$

where $\Delta[X]$ denotes the jump in the quantity X at the interface. Here $\vec{N}_0$ is the unit normal to the undisplaced interface and $\overline{\delta N}$ denotes its displacement. For the problem at hand

$$\vec{N}_0 = \vec{e}_y \text{ and } \overrightarrow{\delta N} = -\epsilon(\vec{e}_x\, ik_x + \vec{e}_z\, ik_z). \tag{34}$$

The remaining conditions are obtained by integrating the equation of motion (1) across the interface. We thus obtain

$$\vec{N}\,\Delta(\pi) = \vec{N} \cdot \Delta(\vec{B}\vec{B} - \mathbf{P}). \tag{35}$$

(3) If we multiply Eq. (35) scalarly with $\vec{N}$ we obtain the boundary condition for the continuity of the normal component of the stress:

$$\Delta(\pi) = \vec{N} \cdot \Delta(\vec{N} \cdot \vec{\mathbf{P}}). \tag{36}$$

The linearized form of this equation leads to the condition

$$\pi_1^p - \epsilon\rho g + P_{yy} = \pi_1^v = \frac{B_v}{4\pi}\, b_z^{(0)}, \quad (y = 0), \tag{37}$$

where the superscripts 'p' and 'v' on π_1 refer to the plasma and the vacuum quantities and in writing Eq. (37) we have made use of the relation $\partial\pi_0^p/\partial y = -\rho g$

The other two conditions are obtained by taking the vector product of Eq. (35) with $\vec{N}$. This leads to the conditions for the continuity of the tangential stress:

$$(\vec{N} \cdot \vec{B})\, N \times \Delta(\vec{B}) = \vec{N} \times \vec{\Delta}\, (\vec{N} \cdot \vec{\mathbf{P}}) \tag{38}$$

The linearized form of Eq. (38) leads to the conditions

$$P_{xy} = 0 \qquad (\text{at } y = 0) \tag{39}$$

and

$$\Delta\,[B_p(b_y + B_p\, \delta N_z)] = P_{zy} \quad (y = 0). \tag{40}$$

On making use of the foregoing boundary conditions we obtain the following characteristic value equations:

$$\sum_i (n + \beta\alpha_i)\, q_i\, C_i = n\epsilon k_z B_p, \tag{41}$$

$$\sum_i q_i\, C_i = -\, i k C_4 + k_z \epsilon\, (B_p - B_v), \tag{42}$$

$$\sum_i (n + \beta\alpha_i)\, (k_x p_i + \lambda_i q_i) = 0, \tag{43}$$

$$\sum_i \nu(n + \beta\alpha_i)\,(k_x + k_z p_i)\, C_i = \frac{k_z B_p}{\rho}\, (B_p - B_v)(k C_4 - i\, k_z \epsilon B_v), \tag{44}$$

$$\sum_i \left[\Omega^2 + (n + \beta\alpha_i)(n - i\nu k_z \alpha_i) + \frac{i\nu}{2} k_z \beta_i (k_x q_i + \lambda_i p_i)\right] C_i$$

$$= k_z^2 B_p \left[\epsilon g + \frac{i k_z B_v}{4\pi\rho} C_4\right], \tag{45}$$

where we have put $\beta = i\Omega^2/k_z\omega_c$. Equations (41)-(45) are five equations involving the five unknown constants C_i, $(i = 1, 2, 3, 4)$ and ϵ. The dispersion relation is obtained by setting the determinant of their coefficients equal to zero.

V. THE INTERCHANGE MODE.

In view of the complexity of the general case we first consider the simple case of the interchange mode. We observe that in this case $k_z = 0$, $k_x \neq 0$ and Equations (13) and (14) reduce to

$$n\vec{v} = -\vec{\nabla}\pi_1 - \frac{1}{\rho}\vec{\nabla}\cdot\vec{\mathbf{P}}_M, \tag{46}$$

and

$$n\vec{b} = 0. \tag{47}$$

Equation (47) gives $\vec{b} = 0$, since $n \neq 0$, while Eq. (46) leads to

$$n\vec{v} = -\nabla\pi_1 + \frac{\nu}{2\rho}\left[\vec{e}_x\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)v - \vec{e}_y\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)u\right], \tag{48}$$

where we put $\vec{v} = (\vec{e}_x u + \vec{e}_y v + \vec{e}_z w)$. The z-component of Eq. (48) requires that $w = 0$. The other two equations together with the divergence condition yield the solutions

$$u = -\, i\, C\, e^{-k_x y + i k_x x + n t}, \tag{49}$$

$$v = C\, e^{-k_x y + i k_x x + n t}, \tag{50}$$

and

$$\pi_1 = \frac{nC}{k_x}\, e^{-k_x y + i k_x x + n t}. \tag{51}$$

The constant C is determined by the requirement that at $y = 0$, $v = n\epsilon$ and we get

$$C = n\,\epsilon. \tag{52}$$

The boundary condition (37) now reduces to

$$\pi_1 + \frac{1}{2}\,\nu \left(i\, k_x\, v + \frac{\partial u}{\partial y} \right) = \epsilon\, g \text{ at } y = 0. \tag{53}$$

On using the foregoing results in this equation we obtain the dispersion relation

$$n^2 = g\, k_x - i\, n\, \nu\, k_x^2. \tag{54}$$

This leads to the result

$$n = \pm\, (g\, k_x)^{1/2} \left(1 - \frac{\nu^2\, k_x^3}{4g} \right)^{1/2} - \frac{i\, \nu k_x^2}{2}. \tag{55}$$

We may observe here that when $\nu \to 0$, $n = \pm\, (g\, k_x)^{1/2}$ which agrees with the classical result that the presence of a magnetic field has no effect on the interchange mode. However, when $\nu \neq 0$, we find from relation (55) that (a) the instability sets in as overstability and (b) the growth rate of instability is reduced due to the gyro-viscosity; and this effect is large for short wavelengths. In fact we can see from Eq. (55) that all wave numbers with k greater than a certain critical k_* are stabilized by the gyro-viscosity, where k_* is given by

$$k_* = \left(\frac{4g}{\nu^2}\right)^{1/3}, \tag{56}$$

and all wave numbers $k < k_*$ are unstable.

VI. THE CASE $k_x = 0$ AND $k_z \neq 0$.

We shall now drop the subscript z on k as it is no longer necessary. Equations (41)-(45) now lead to the dispersion relation:

$$\begin{vmatrix} (n+\beta\alpha_1) & (n+\beta\alpha_2) & (n+\beta\alpha_3) & 0 & 0 \\ Q_1 & Q_2 & Q_3 & 0 & 0 \\ (n+\beta\alpha_1)q_1 & (n+\beta\alpha_2)q_2 & (n+\beta\alpha_3)q_3 & 0 & n \\ q_1 & q_2 & q_3 & 1 & 1-\eta \\ P_1 & P_2 & P_3 & -\eta\Omega^2 & kg \end{vmatrix} = 0, \tag{57}$$

where

$$Q_i = \beta\alpha_i q_i - \frac{in\rho\nu}{B_p^2(1-\eta)}(n+\beta\alpha_i)P_i, \tag{58}$$

$$P_i = \Omega^2 + (n+\beta\alpha_i)\left(n - \frac{i\nu k}{2}\alpha_i\right) \tag{59}$$

and $\eta = B_v/B_p > 1$. In order to solve the dispersion relation (57), we now need the expressions for α_i's, the roots of Eq. (24). As it seems quite difficult to find the general solution of Eq. (24), we shall restrict ourselves to the case when the gyro-viscosity effects are small and we shall assume that $\omega_\nu/n \ll 1$. The roots of Eq. (24) are then determined by an iteration procedure and we find that correct to the first order terms in ω_ν/n, the roots are given by ($\omega_\nu = \nu k^2$):

$$\alpha_1 = \frac{ik\omega_c}{n}\left(1+\frac{n^2}{\Omega^2}\right) - \frac{ik\omega_\nu}{2n}\left[\frac{\omega_c^2}{n^2}\left(1+\frac{n^2}{\Omega^2}\right) + \frac{3}{(1+n^2/\Omega^2)}\right], \tag{60}$$

$$\alpha_2 = -\frac{3}{2}\frac{ink\omega_\nu}{n^2+\Omega^2} \tag{61}$$

and

$$\alpha_3 = -\frac{2ink}{\omega_\nu}\left(1+\frac{1}{2}\frac{\omega_\nu\omega_c}{n^2}\right). \tag{62}$$

The appearance of the term $(1/\omega_\nu)$ in Eq. (62) leads to no difficulties as we demand that in the limit $\nu \to 0$, the constant $C_3 \to 0$. With α's given by Eqs. (60) - (62) we can readily see that to the lowest significant order, we have

$$p_1 = \frac{i\omega_c}{n}\Gamma\sigma^2\left(1-\frac{1}{2}\delta X\right), \tag{63}$$

$$q_1 = \Gamma\left[1+\frac{\delta\omega_c^2}{2n^2}(1+2\Gamma^2)\right], \tag{64}$$

$$\lambda_1 = \frac{k}{\Gamma}\left[1-\frac{\delta\omega_c^2}{2n^2}(1+2\Gamma^2)\right] \tag{65}$$

$$\lambda_2 = k,\ q_2 = 1,\ p_2 = -\frac{3}{2}\,i\,\frac{\delta n\omega_c}{\Omega^2\sigma^2}\ ; \tag{66}$$

$$\lambda_3 = \frac{2nk}{\delta \omega_c}, \quad q_3 = -\frac{\delta}{2}\frac{\omega_c}{n}, \tag{67}$$

$$p_3 = -i\left(1 + \frac{1}{2}\delta\frac{\omega_c^2}{n^2}\right) \tag{68}$$

where

$$\Gamma = \left(1 + \frac{\omega_c^2}{n^2}\sigma^4\right)^{-1/2}, \quad \sigma^2 = 1 + \frac{n^2}{\Omega^2}, \quad \delta = \frac{\omega_\nu}{\omega_c} \tag{69}$$

and

$$X = 1 - \frac{\omega_c^2}{n^2} + \frac{3}{\sigma^4} - \frac{2\omega_c^2}{n^2}\,\frac{1}{1 + \frac{\omega_c^2}{n^2}\sigma^4}. \tag{70}$$

The dispersion relation given by Eq. (57) can now be written as

$$\sum_{i=1}^{3} (n + \beta\alpha_i)\, K_i = 0, \tag{71}$$

where

$$K_i = \epsilon_{ijk}\, Q_j\, M_k, \tag{72}$$

$$M_i = (n + \beta\alpha_i)\, q_i\, [kg + \eta\Omega^2 (1 - \eta)] - n\, [q_i\, \eta\Omega^2 + P_i] \tag{73}$$

and ϵ_{ijk} is the unit tensor of third rank which is completely antisymmetric in all the indices. On using the relations (60) - (70) in equations (71) - (73) and carrying out all the expansions consistently to the lowest significant order in δ, we obtain after some straightforward reductions for the dispersion relation:

$$D_0 + \frac{\delta}{2}\left[\frac{\omega_c^2}{n^2} D_0 \sigma^2 + D_0 \left(2\Gamma^2 \frac{\omega_c^2}{n^2} - \frac{3}{\sigma^4} + \frac{\omega_c^2}{n^2} a\right)\right.$$

$$+ \frac{3}{\sigma^2}(N - n^2) - \frac{3}{\sigma^4}(N + \eta n^2) \tag{74}$$

$$\left. + \frac{\omega_c}{n}\frac{b}{\sigma^2}(N + \eta n^2) - \frac{\omega_c}{n} N - \frac{\omega_c}{n}\frac{b}{\Gamma}\frac{D_0}{\sigma^2}\right] = 0,$$

where

$$D_0 = kg - n^2 - \Omega^2 (1 + \eta^2),$$

$$N = kg + \eta\Omega^2 (1 - \eta^2), \tag{75}$$

$$a = \frac{n^2}{2\pi\Omega^2 (1 - \eta)} \quad \text{and} \quad b = 1 - a.$$

First of all we observe that in the limiting case when $\delta \to 0$, i.e., the absence of gyro-viscosity, Eq. (74) reduces to the classical result

$$D_0 = kg - n_0^2 - \Omega^2 (1 + \eta^2) = 0, \tag{76}$$

where we put a subscript on n to denote its value in the limit $\delta \to 0$. In order to obtain the first order correction to the dispersion relation, we substitute n_0 given by Eq. (76) into all the terms containing δ in Eq. (74) and obtain the result

$$n^2 = kg - \Omega^2 (1 + \eta^2) - \frac{\omega_\nu}{2} n_0 f,$$

$$= n_0^2 \left(1 - \frac{\omega_\nu f}{2n_0}\right) \tag{77}$$

where

$$f = 1 - \frac{(\eta + 1)}{2\pi(\eta - 1)}$$

Note that f is positive only when η is larger than $\eta_c = (2\pi + 1)/(2\pi - 1) \approx 1.4$.

If we consider a situation where $\eta > \eta_c$ and which is unstable in the absence of ω_ν i.e., $n_0 > 0$, then for this unstable mode in the presence of ω_ν, $n_0^2 > n^2 > 0$ i.e., the instability is reduced. If on the other hand we consider a situation which is stable in the absence of ω_ν i.e., $n_0^2 < 0$, then the inclusion of gyro-effects in the equations of motion leads to damping of the oscillations.

VII. CONCLUSIONS

We find that for the interchange mode with the inclusion of gyro-viscosity (a) the instability sets in as overstability and (b) the growth rate of instability is reduced. For disturbances propagating along the magnetic field, for a high β-plasma ($\eta > \eta_c$) the instability is reduced due to the inclusion of gyro-viscosity.

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REFERENCES

1. M. N. Rosenbluth, N. A. Krall and N. Rostoker, Nuclear Fusion Supplement Pt. 1, 143, (1962).

2. K. V. Roberts and J. B. Taylor, Phys. Rev. Letters 8, 197 (1962).

3. C. F. Kennel and J. M. Greene, Ann. of Phys. 38, 63 (1966).

4. I. B. Bernstein and S. K. Trehan, Nuclear Fusion 1, 3 (1960).