What is hot in superconductivity

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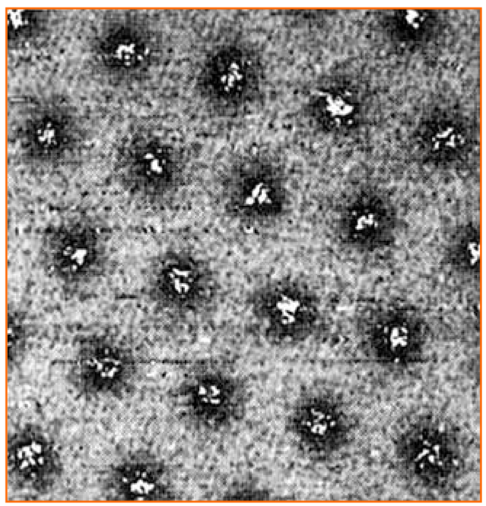
МЕЖДУНАРОДНАЯ ЗИМНЯЯ ШКОЛА ПО ФИЗИКЕ ПОЛУПРОВОДНИКОВ
ФТИ им. А.Ф.Иоффе Российской Академии наук

Physics of vortex matter

I. Pinning and creep
II. Melting and localization



L.B. Ioffe
M. V. Feigelman
D. Geshkenbein
A. Koshelev
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M.C. Marchetti
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S. Scheidl
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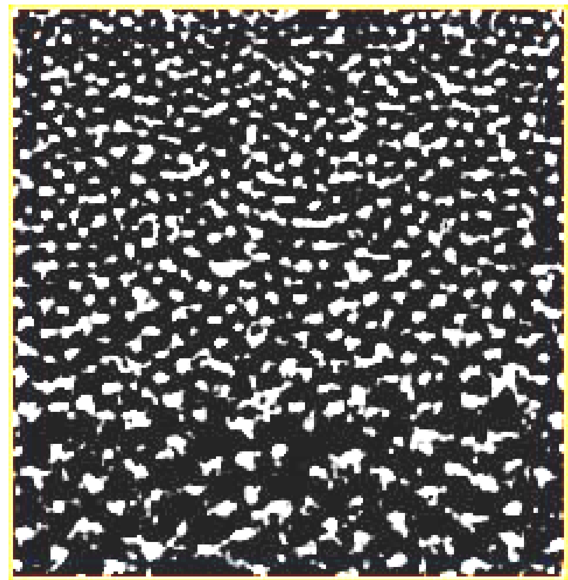
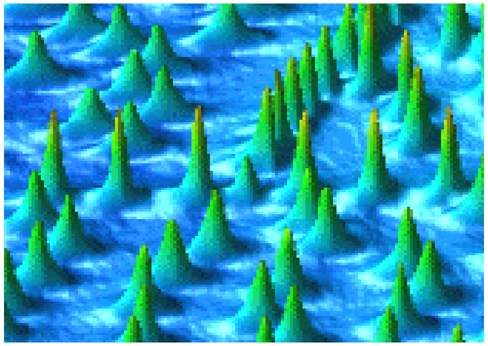


Part I Outline

1. **Elastic string in point disorder**
2. **Critical currents and creep**
3. **Quantum mechanical mapping**
4. **Creep through columnar defects**

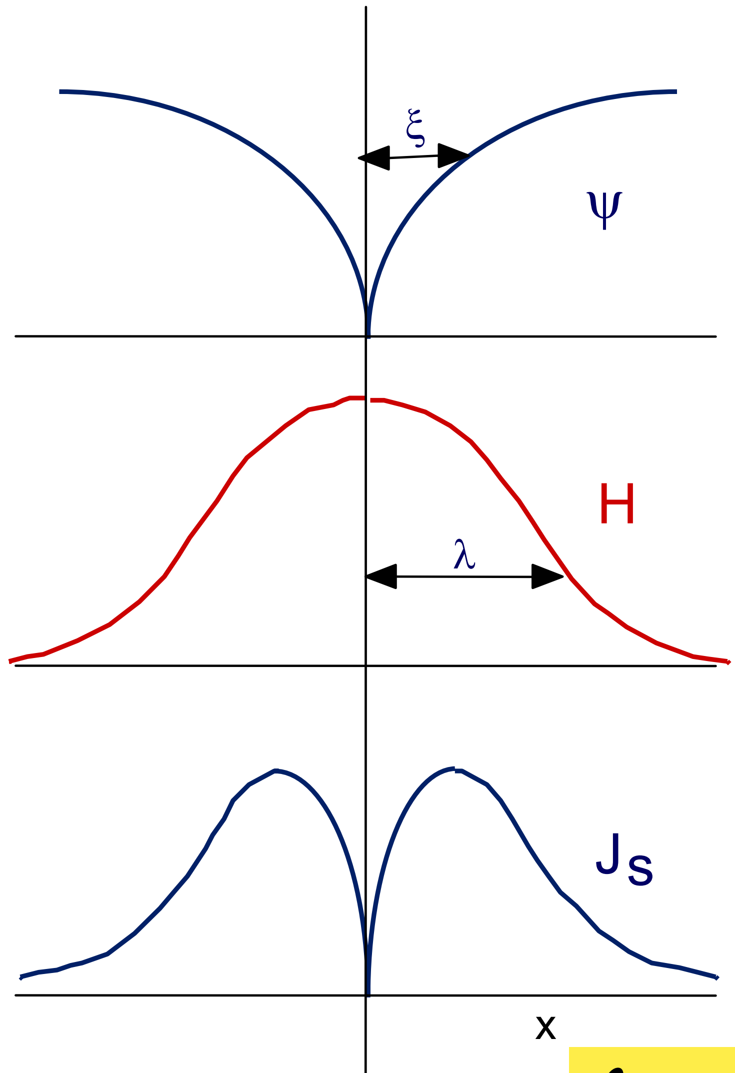


- On n'est pas sérieux, quand on a dix-sept ans
  Et qu'on a des tilleuls verts sur la promenade.

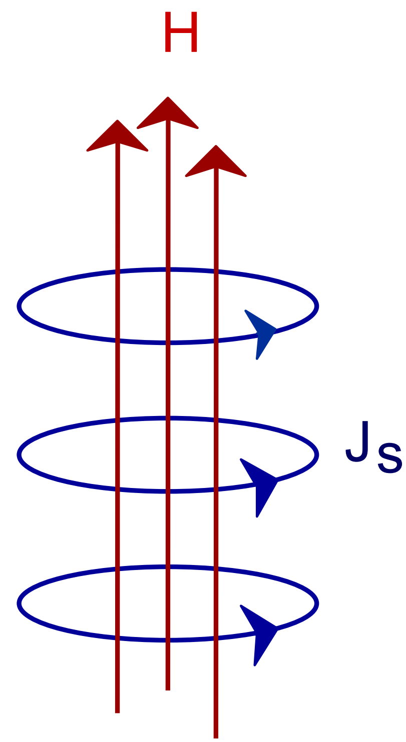
One isn't serious when one is seventeen,
And when there are green lime-trees on one's promenade.

Arthur Rimbaud

Abrikosov Vortex



$$\lambda \gg \xi$$



quantum of flux
hc/2e
20.7 Oe-μm ^{2}

Vortex line tension

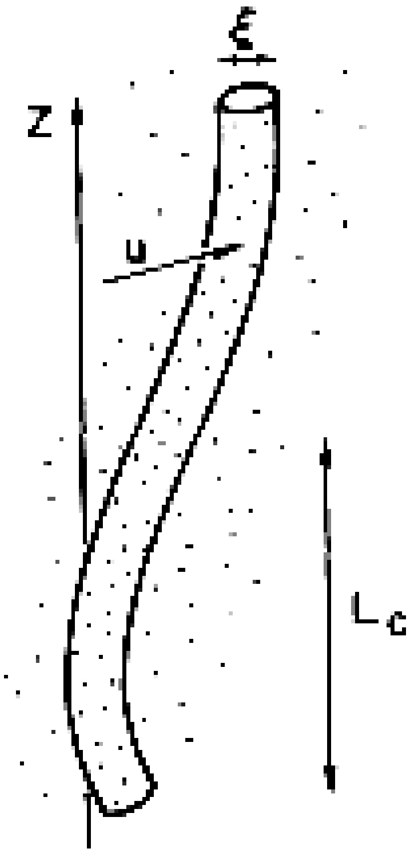
In order to bend vortex one has to distort distribution of circular currents→ linear tension can be ascribed to vortices.

$$e_l = \varepsilon_0 \ln\frac{\lambda}{\xi} = \frac{\Phi_0}{4\pi} H_{c1} \qquad \varepsilon_0 = \left(\frac{\Phi_0}{4\pi\lambda}\right)^2$$

line tension ε_l is equal to the line energy e_l

$$\mathcal{F} = \sum_i \int dz \frac{\varepsilon_l}{2}\left(\frac{\partial \mathbf{u}_i}{\partial z}\right)^2 + \sum_{i,j}\int dz\, dz'\, v[\mathbf{u}_i(z) - \mathbf{u}_j(z')]$$

Disorder



Single vortex line pinned by the collective action of many weak pointlike pinning centers. Only fluctuations in the pin density are able to pin the vortex. In order to accommodate optimally to the pinning potential, the vortex line deforms by ξ (the minimal transverse length scale the vortex core is able to resolve equals the scale of the pinning potential) on a longitudinal length scale L_c, the collective pinning length.

Exemplary system: elastic medium in random environment

Models a wealth of physical systems and phenomena:

$$\mathcal{H} = \int d^D x \left[\frac{C}{2} \left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}} \right)^2 + V(\mathbf{x}, \mathbf{u}) - \mathbf{F} \cdot \mathbf{u} \right]$$

Dislocations in crystals

CDW and SDW

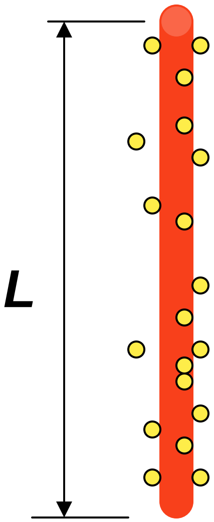
Domain walls

Interacting electrons

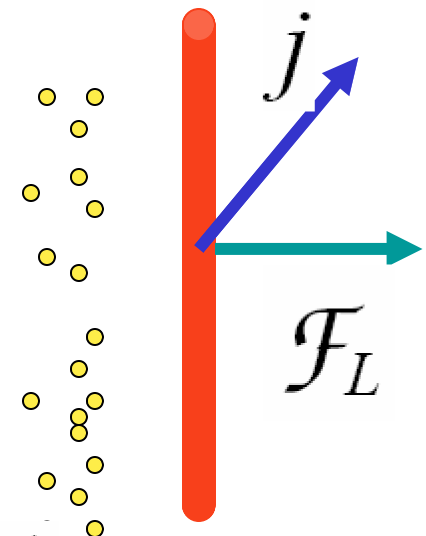
Wigner crystals on disordered substrates

Spin- and other glasses…

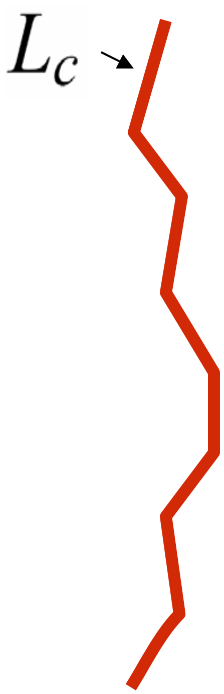
Collective pinning



$$\mathcal{F}_{pin} \simeq (f_{pin}^2 n_i \xi^2 L)^{1/2}$$



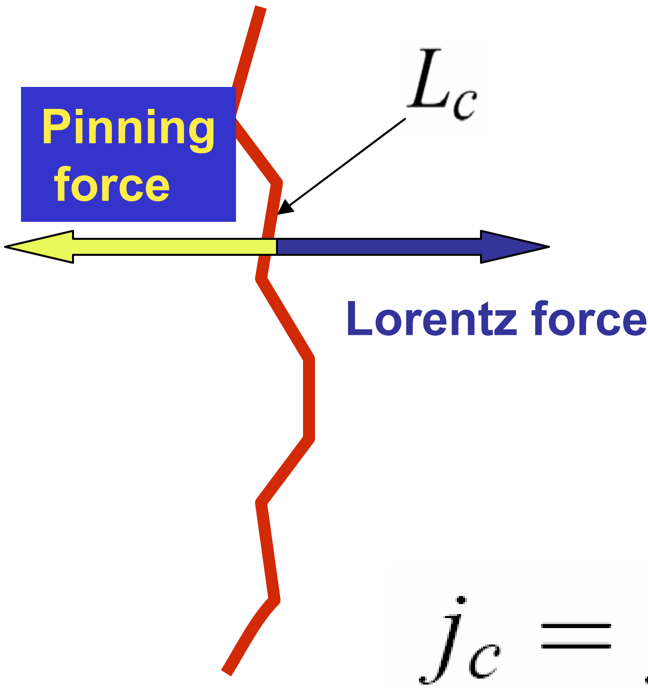
$$\mathcal{F}_L \simeq j\Phi_0 L/c$$



$$\mathcal{E}_{el}(L_c) \simeq \varepsilon_\circ \xi^2/L_c \sim \mathcal{E}_{pin}(L_c)$$

$$\mathcal{F}_{pin} \sim L/L_c$$

Critical current is defined by the relation:



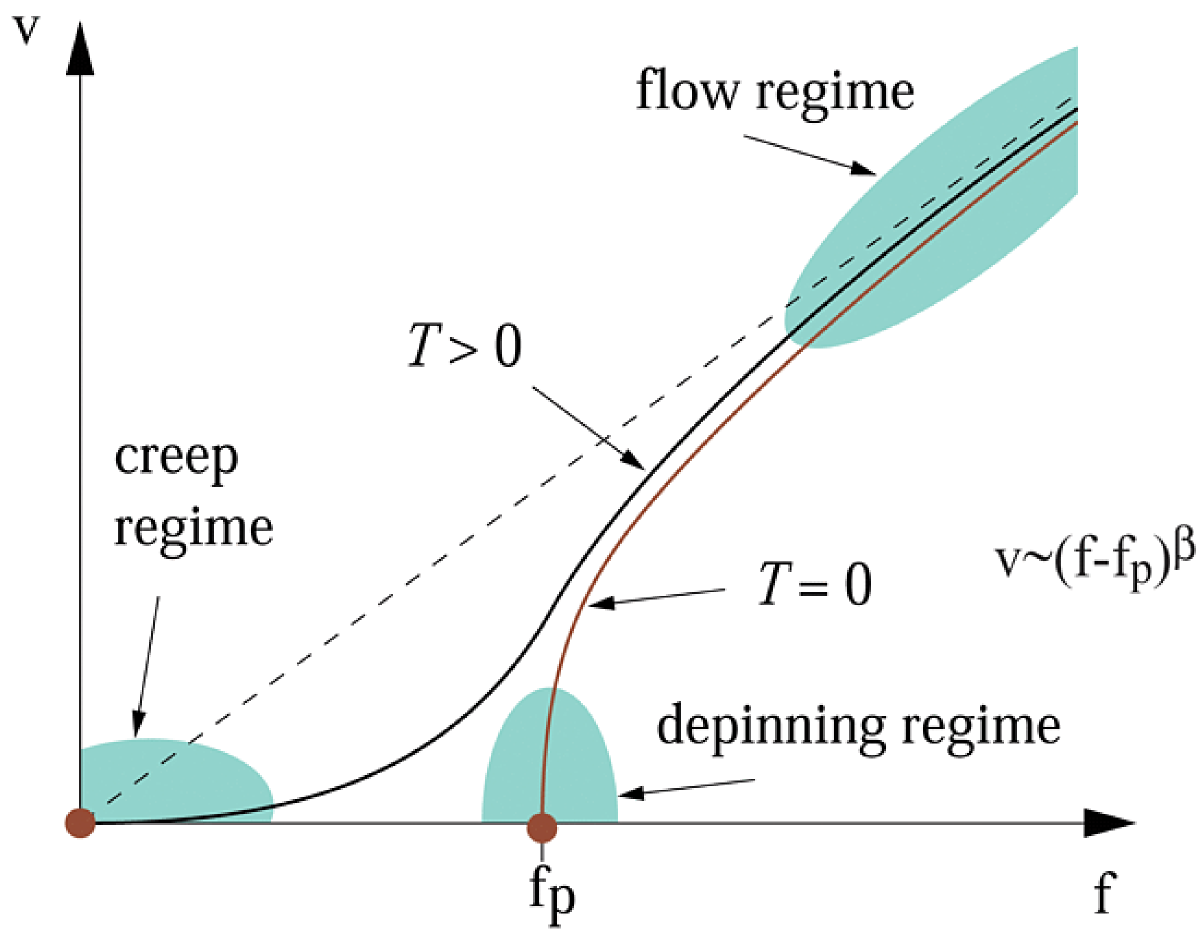
$$\mathcal{F}_{pin}(L_c) = \mathcal{F}_L(L_c)$$

and reads:

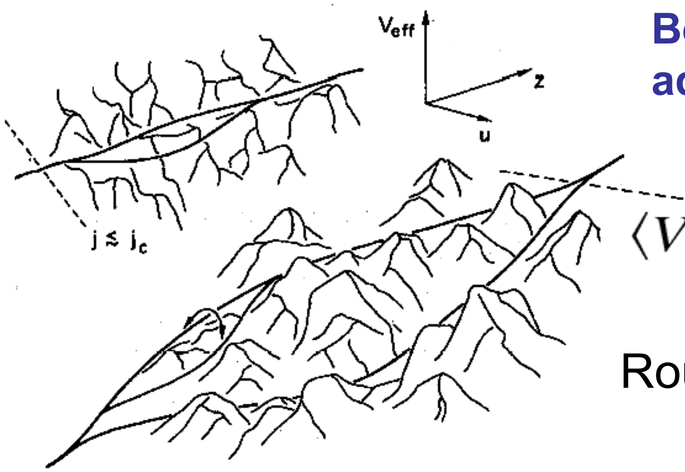
$$j_c = j_\circ(\xi\gamma/\varepsilon_\circ^2)^{2/3},\ \gamma \simeq n_i \xi^2 f_{pin}^2$$

TEMPERATURE BEHAVIOR?

Three regions of vortex motion:



T. Nattermann and S. Scheidl, Vortex glass phases in type II superconductors, Advances in Physics, 2000, vol. 49, No. 5, 607, 704

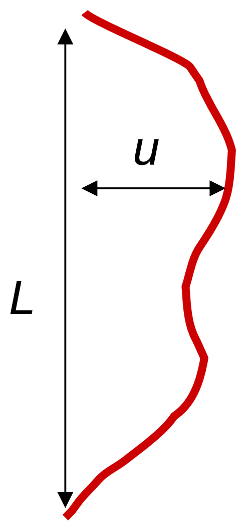


Being placed in disordered medium elastic object adjusts itself to rugged potential relief

$$\langle V(\mathbf{x},\mathbf{u})V(\mathbf{x}',\mathbf{u}')\rangle = \Delta^2\delta^D(\mathbf{x}-\mathbf{x}')f(|\mathbf{u}-\mathbf{u}'|/\xi)$$

Roughness: $w(L) = \langle[\mathbf{u}(\mathbf{x}+\mathbf{L}) - \mathbf{u}(\mathbf{x})]^2\rangle^{1/2}$

On the intermediate scales where $u < a_0$



$$w \sim \xi\left(\frac{L}{L_c}\right)^{\varsigma}$$

$\zeta < 1$ is the roughness exponent

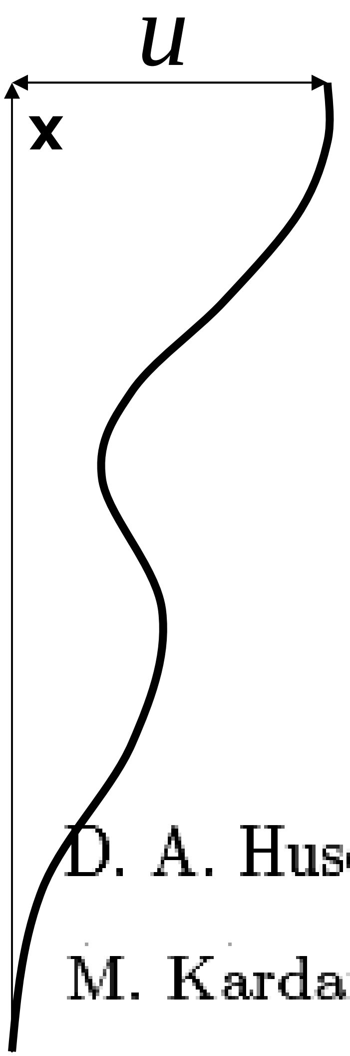
$$E_{barrier} = E_p\left(\frac{L}{L_c}\right)^{\chi} f\left(\frac{x-x'}{w(L)}\right)$$

$$E_p = a_0\Delta L_c^{D-2}$$

$$L_c = (C\xi^2/\Delta)^{2/(4-D)}$$

$$\chi = D - 2 + 2\zeta$$

the free energy of the string ε
as the function of the position of its right end (x,u)



$$\frac{\partial \varepsilon}{\partial x} = -\frac{1}{2C}\left(\frac{\partial \varepsilon}{\partial u}\right)^2 - \frac{T}{2C}\frac{\partial^2 \varepsilon}{\partial u^2} + V(x,u)$$

$$\varepsilon \sim x^{1/3}$$

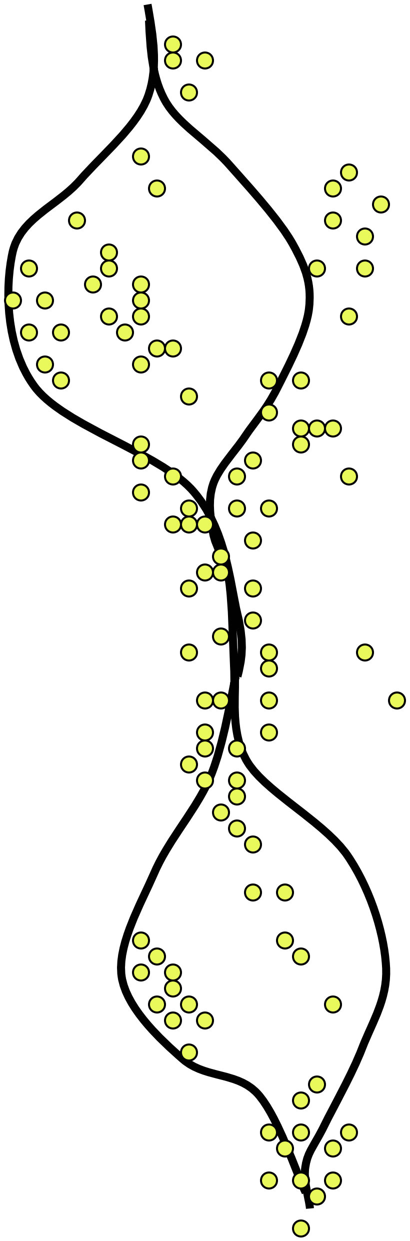
$$u \sim x^{2/3}$$

D. A. Huse and C. L. Henley, Phys. Rev. Lett. **54**, 2708 (1985)

M. Kardar and D. R. Nelson, Phys. Rev. Lett. **55**, 1157 (1985)

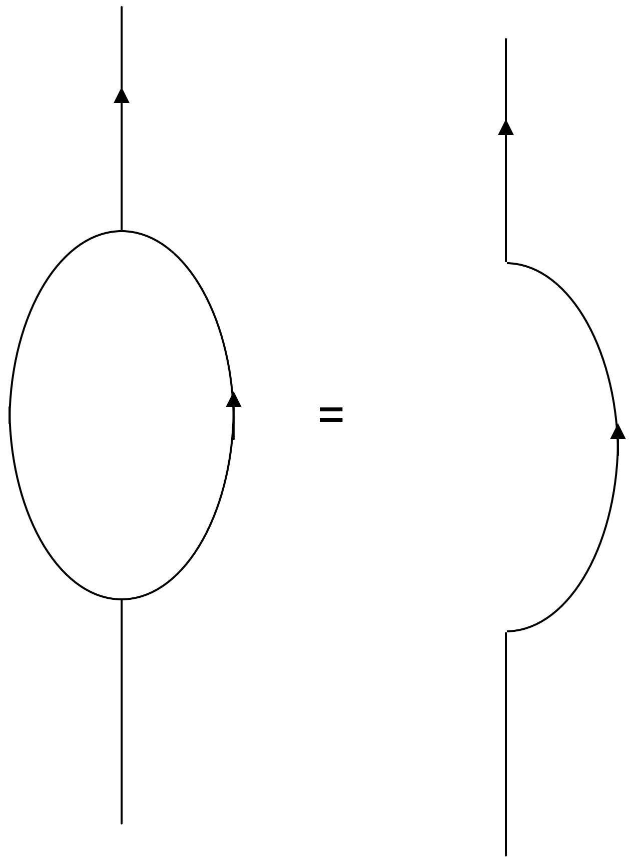
L. B. Ioffe and V. M. Vinokur, J. Phys. C **20**, 6149 (1987)

Thermally activated vortex dynamics in random media



Activated vortex motion:

Creation a vortex loop



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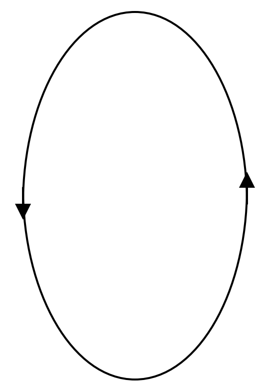
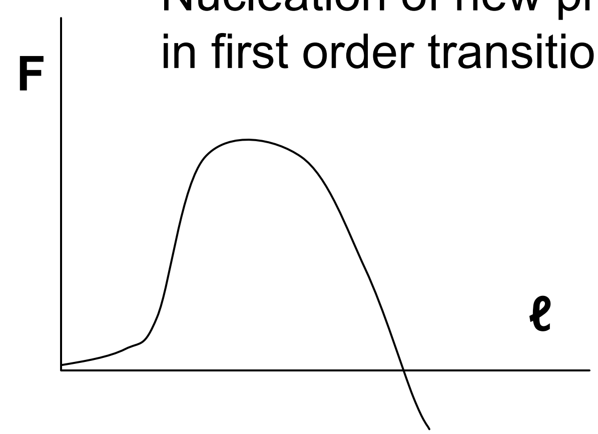


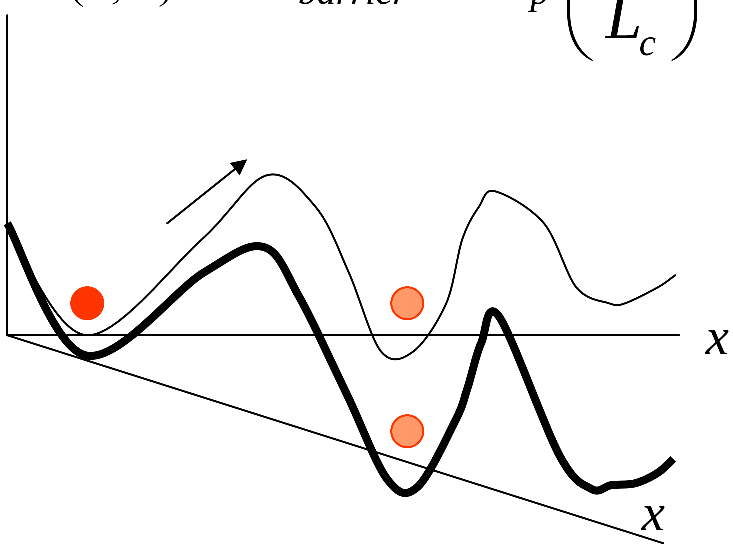
Nucleation of new phase
in first order transitions



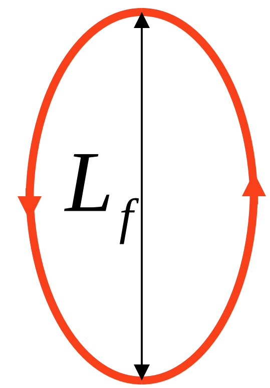
Driving force *f* :

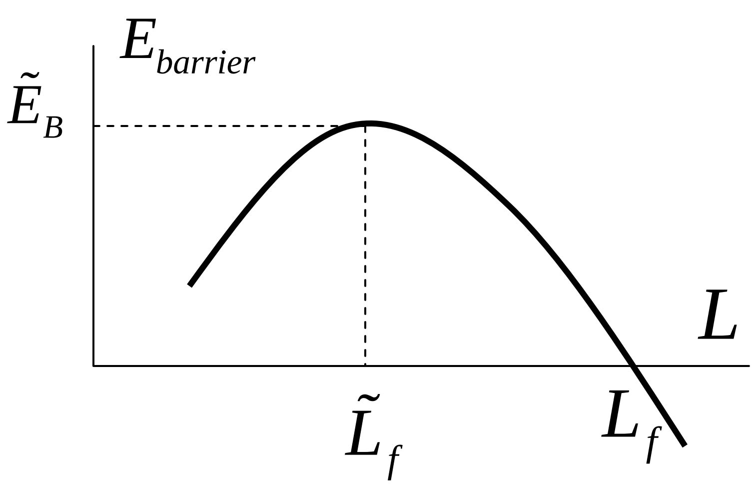
$$E^{(0)}_{barrier} = E_p \left(\frac{L}{L_c}\right)^{\chi}$$

$$E_{barrier} \simeq E_p \left(\frac{L}{L_c}\right)^{\chi} - w(L)\, f L^D = E_p \left(\frac{L}{L_c}\right)^{\chi} \left[1 - \left(\frac{L}{L_f}\right)^{2-\zeta}\right]$$



$$L_f = L_c \left(\frac{f_p}{f}\right)^{1/(2-\varsigma)}$$





Meaning of L_f :

At distances $L > L_f$

pinning is not effective

$$E_{barrier}(L_f) \equiv \tilde{E}_B = E_p \left(\frac{f_p}{f}\right)^{\mu}$$

Linear response

Experiments show that if a *force* $\vec{F}$ is imposed to a system, its response is a *current* $\vec{j}$ vanishing as the force vanishes. Thus for $\vec{F}$ small

$$\vec{j} \; = \; L \cdot \vec{F} + O(\vec{F}^2) \, ,$$

$$X_a = -\frac{\partial S}{\partial x_a}$$

$$\frac{\partial x_a}{\partial t} = -\sum_b \gamma_{ab} X_b$$

Here L is a matrix of *transport cœfficients*.

Examples:

1. FOURIER's law: a temperature gradient produces a heat current $\vec{j}_{heat} = -\lambda \vec{\nabla} T$.
2. OHM's law: a potential gradient (electric field) produces an electric current $\vec{j}_{el} = -\sigma \vec{\nabla} V$.
3. FICK's law: a density gradient produce a flow of matter $\vec{j}_{matter} = -\kappa \vec{\nabla} \rho$.

Thermally activated dynamics of random systems:

Governed by the wide distribution of barriers

time to overcome barrier E:

$$\tau \simeq \tau_0 \exp\left(\frac{E}{T}\right)$$

Basic law of relaxation in random (glassy) systems:

$$E \simeq T \ln(\tau/\tau_0)$$

Consequences of general law of relaxation:

$$\tilde{E}_B = E_p \left(\frac{f_p}{f} \right)^{\mu} \simeq T \cdot \ln(\tau / \tau_0)$$

CREEP

Motion under Constant force:

$$v \sim \tau^{-1}$$

$$v \sim \exp\left[-\frac{E_p}{T} \left(\frac{f_p(T)}{f} \right)^{\mu} \right]$$

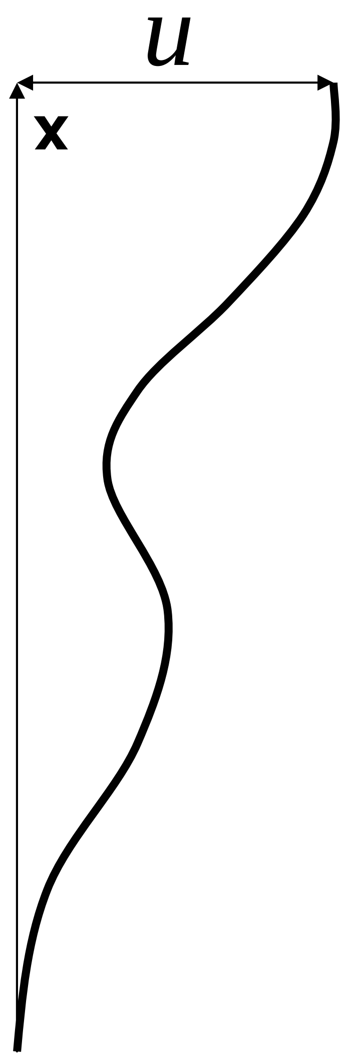
$$\mu = \frac{\chi}{2-\varsigma}$$

Current (magnetization relaxation:

$$f = \frac{n_v \Phi_0}{c} J \Longrightarrow J(t) \sim \frac{J_0}{\left[\ln(t/\tau_0) \right]^{1/\mu}}$$

O Snail,
Climb Fuji slope, slowly, slowly
Up to the top…
Matsuo Bashō

the free energy of the string ε
as the function of the position of its right end (x,u)



$$\frac{\partial \varepsilon}{\partial x} = -\frac{1}{2C}\left(\frac{\partial \varepsilon}{\partial u}\right)^2 - \frac{T}{2C}\frac{\partial^2 \varepsilon}{\partial u^2} + V(x,u)$$

$\varepsilon \to v, \quad x \to t$ Burgers equation for turbulence

$$\varepsilon \sim x^{1/3} \quad \to \quad v \sim t^{1/3}$$

Experiment

Unambiguous proof: divergent barriers $E \propto J^{-\mu}$

PHYSICAL REVIEW B VOLUME 42, NUMBER 4 1 AUGUST 1990

Dependence of flux-creep activation energy upon current density in grain-aligned $YBa_2Cu_3O_{7-x}$

M. P. Maley and J. O. Willis
Los Alamos National Laboratory, Los Alamos, New Mexico 87545

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Department of Metallurgical Engineering and Materials Science, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213
(Received 2 April 1990; revised manuscript received 21 May 1990)

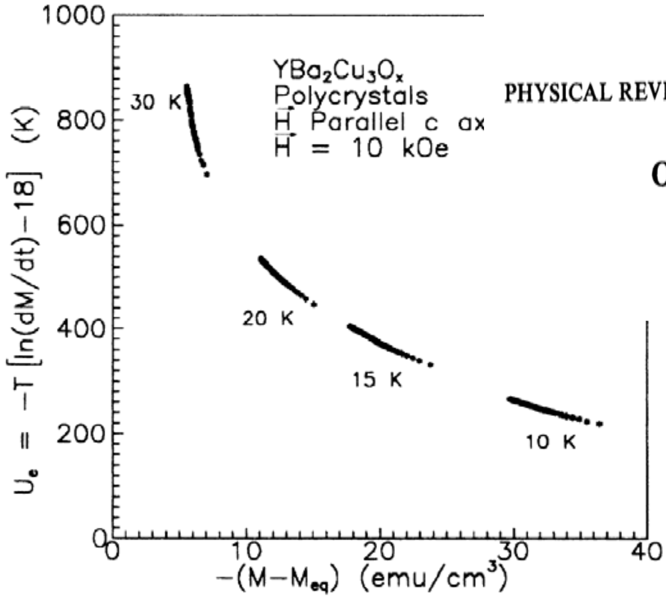

FIG. 4. (a) Plots of nonthermally cycled data: $T \ln(dM/dt)$ vs $M-M_{eq}$ for temperatures 10, 15, 20, and 30 K. (b) Plots of the same data shown in (a) with 18 T added to each data set where T is the temperature. As discussed in the text, this represents U_e vs $M-M_{eq}$.

RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 47, NUMBER 9 1 MARCH 1993-I

Observation of flux creep through columnar defects in $YBa_2Cu_3O_7$ crystals

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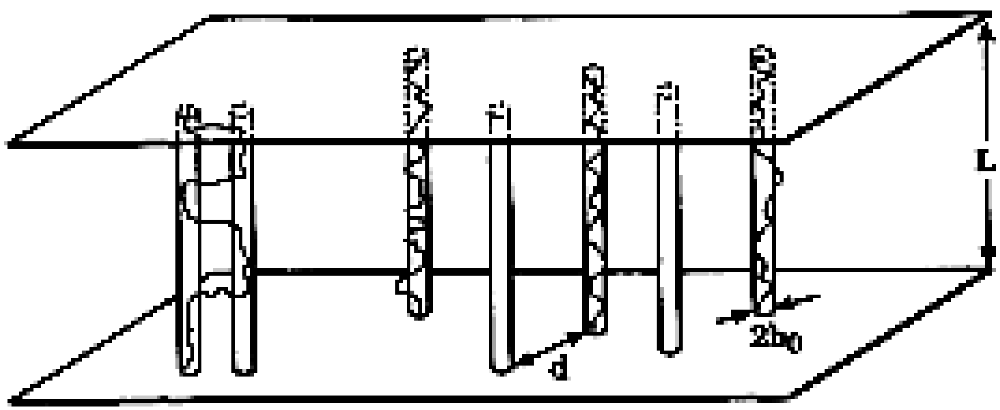
et al

BOSE GLASS:
VORTICES + COLUMNAR DEFECTS

13 062 DAVID R. NELSON AND V. M. VINOKUR 48

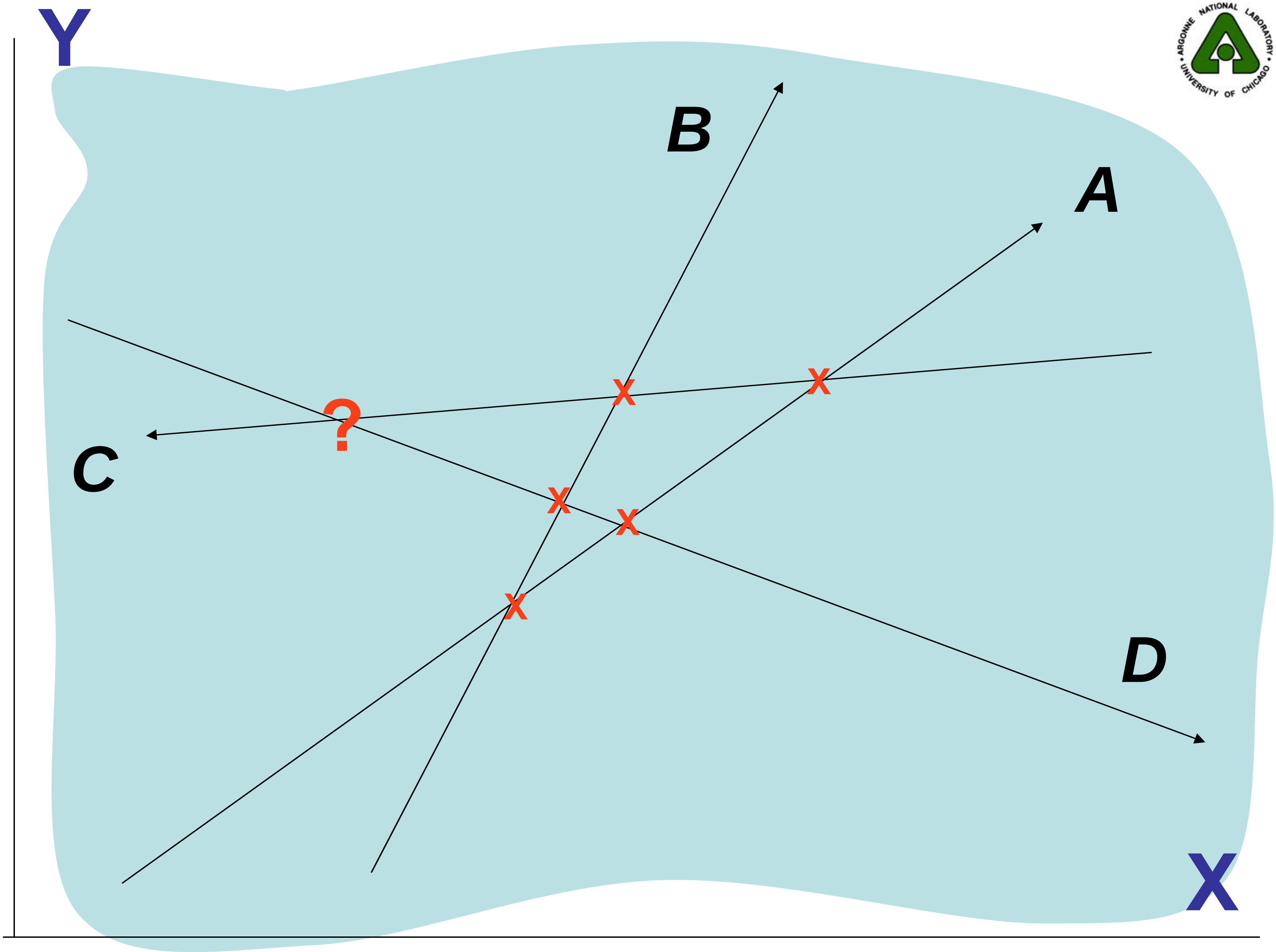


A PROBLEM:

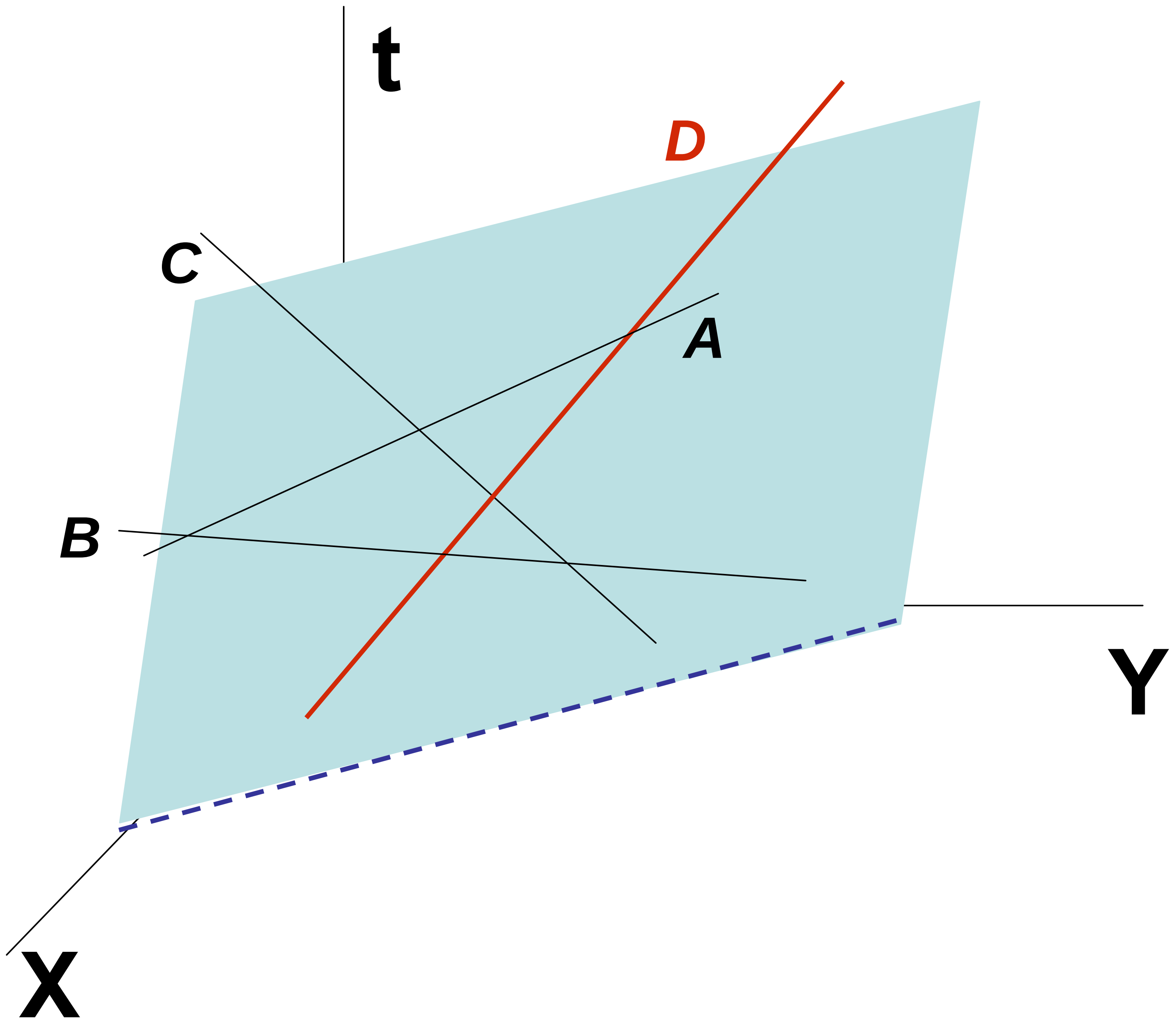
Four ships,

sail over the sea. The paths are straight lines and all the velocities are different. Ship *A* 'collided' with ships *B*, *C*, and *D; B* collided with *C* and *D* (triple collisons excluded).

Prove that *C* collided (or will collide) with *D*



Y
B
A
C
?
X
X
X
X
X
D
X


t
D
C
A
B
Y
X

Quantum mechanical mapping

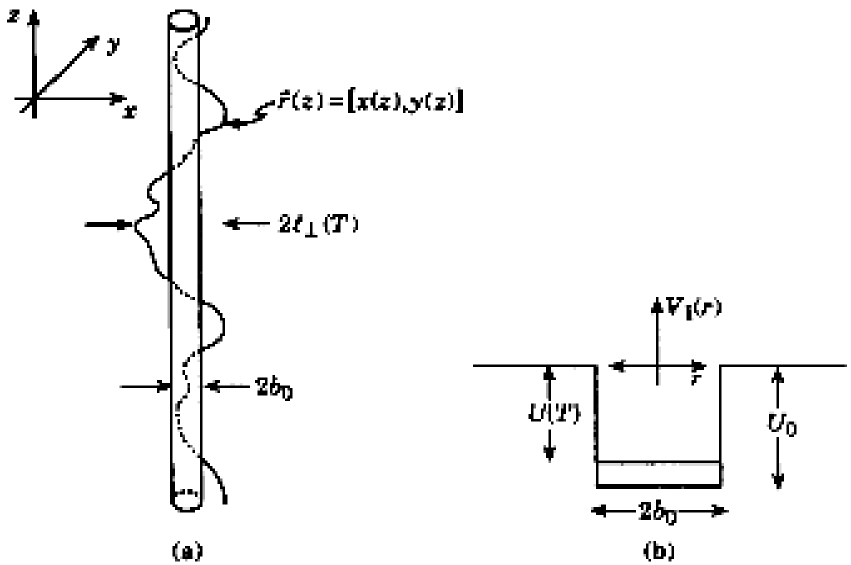
Vortex trajectories can be mapped onto world lines of the 2D Bosons

$$F_N = \int dz \left[\frac{\varepsilon_1}{2} \sum_i \left(\frac{dr_i}{dz} \right)^2 + U(r_i) + \sum_{i<j} V(r_{ij}) \right]$$

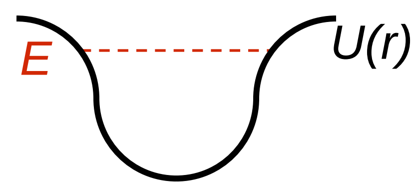
TABLE I. Boson analogy applied to vortex transport.

Charged bosons	Mass	$\hbar$	$\beta\hbar$	Pair potential	Charge	Electric field	Current
Superconducting Vortices	$\tilde{\varepsilon}_1$	T	L	$2\varepsilon_0 K_0(r/\lambda_{ab})$	ϕ_0	$\frac{\hat{\mathbf{z}}\times\mathbf{J}}{c}$	$\mathcal{E}$

Temperature dependence of the critical current from the elementary quantum mechanics



Schematic of a flux line interacting with a columnar pin. (a) The line is confined to a tube of radius $l_\perp(T)$. (b) Cylindrical square well potential which models the binding of the line to the pin. The binding potential is reduced from U_0 to $U(T)$ by thermal fluctuations.



$$|E| \sim \exp[-(\hbar^2/m) \mid \int_0^\infty dr\, r U(r) \mid^{-1}]$$

$$\hbar \to T, \quad m \to \varepsilon_l$$

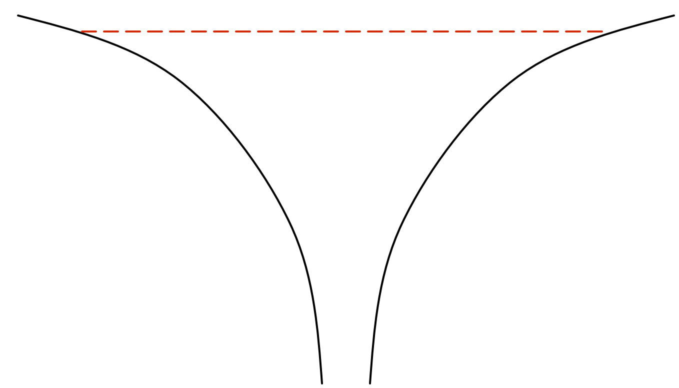
$$j_c(T) \sim j_c(0) \exp[-(T/T^*)^2] \qquad T^* = b_0 \sqrt{U_0 \varepsilon_l}$$

A problem

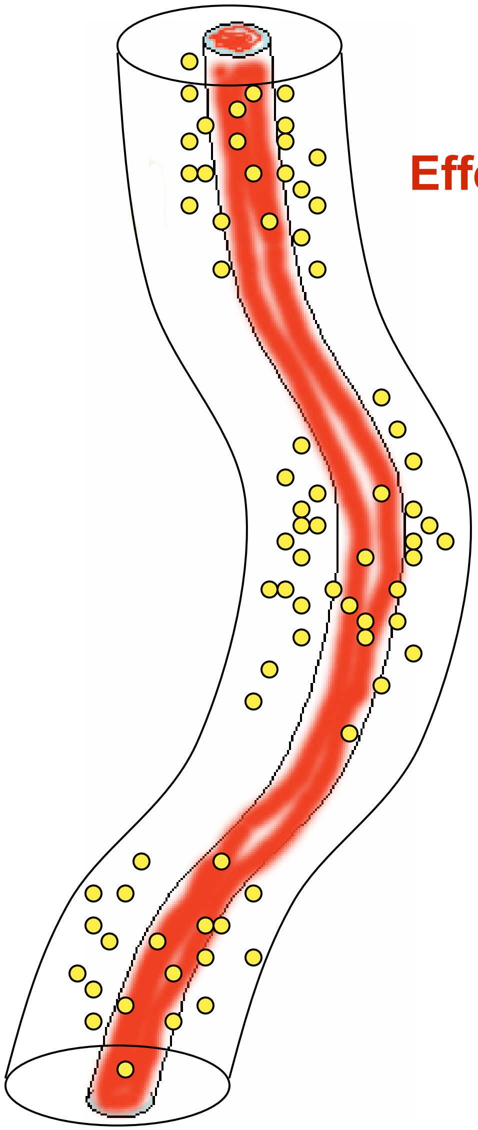
- Let us consider two dimensional potential well

$$U(r) = -\frac{\alpha}{r^2}$$

Find first bound state energy level



Temperature dependence of the point defects-induced critical current



Effective potential well

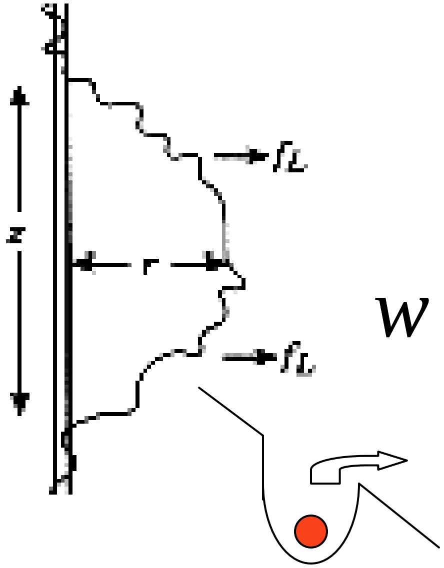
$$|E| \sim \exp\left(-\frac{\hbar^2}{m\xi^2 |U_{eff}|}\right)$$

$$j_c \sim \exp\left(-\frac{T^2}{\varepsilon_\ell \xi^2 |\tilde{U}_{eff}|}\right)$$

$$T \simeq \sqrt{\gamma L} \qquad \tilde{U}_{eff} \equiv \frac{\sqrt{\gamma L}}{L} = \frac{\sqrt{\gamma}}{\sqrt{L}} = \frac{\gamma}{T}$$

$$j_c \sim \exp\left(-\left(\frac{T}{T_{depin}}\right)^3\right)$$

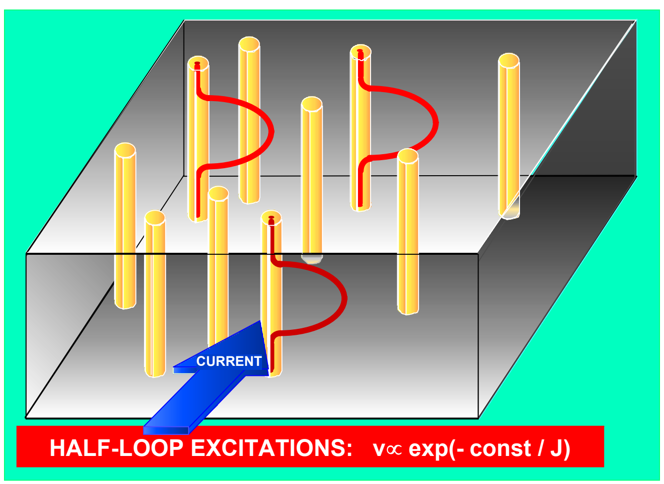
$$T_{depin} = \left(\varepsilon_\ell \xi^2 \gamma\right)^{1/3}$$



$$w \sim \exp\left(-\frac{E_k}{T}\frac{J_1}{J} \right) \equiv \exp\left(-\frac{4\sqrt{2}}{3}\frac{c\varepsilon_l^{1/2}U_0^{3/2}}{\Phi_0 J} \right)$$

TABLE I. Boson analogy applied to vortex transport.

Charged bosons	Mass	$\hbar$	$\beta\hbar$	Pair potential	Charge	Electric field	Current
Superconducting Vortices	$\tilde{\varepsilon}_1$	T	L	$2\varepsilon_0 K_0(r/\lambda_{ab})$	ϕ_0	$\frac{\hat{z}\times \mathbf{J}}{c}$	$\mathcal{E}$



$$w \simeq \exp\left(-\frac{4\sqrt{2}}{3}\frac{m^{1/2}\left|U\right|^{3/2}}{\hbar\left|e\right|E} \right)$$

Variable range vortex hopping

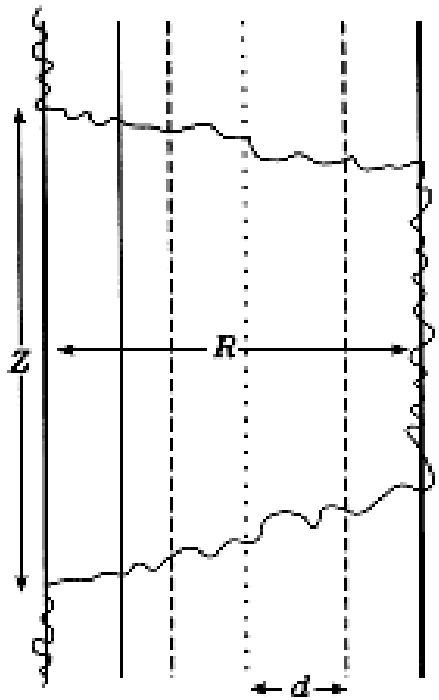


FIG. 15. Double-superkink configuration required for variable-range hopping. The "tongue" of vortex line seeks out a compatible low-energy pin so that the line can spread.

$$\rho \sim \exp\left[-\frac{E_k}{T}\left(\frac{J_0}{J}\right)^{1/3}\right]$$

BOSE GLASS DYNAMICS

PHYSICAL REVIEW B VOLUME 51, NUMBER 6 1 FEBRUARY 1995-II

Experimental evidence for Bose-glass behavior in $Bi_2Sr_2CaCu_2O_8$ crystals with columnar defects

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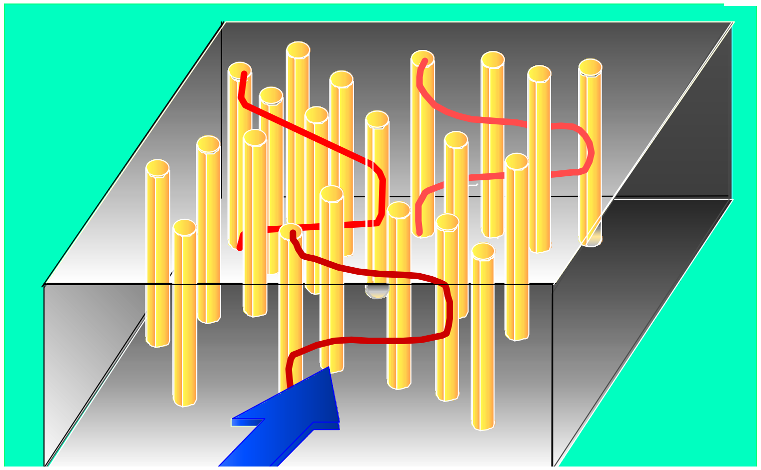
et al



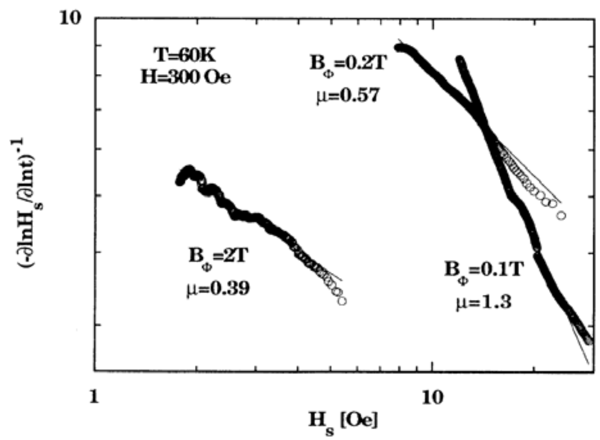


FIG. 4. Energy barrier for flux creep ($1/S$) vs stray field H_s recorded at 300 Oe and 60 K on BSCCO:2212 samples irradiated to various doses.

VOLUME 78, NUMBER 16 PHYSICAL REVIEW LETTERS 21 APRIL 1

Superfast Vortex Creep in $YBa_2Cu_3O_{7-\delta}$ Crystals with Columnar Defects: Evidence for Variable-Range Vortex Hopping

J. R. Thompson,[1] L. Krusin-Elbaum,[2] L. Civale,[3] G. Blatter,[4] and C. Feild[2]

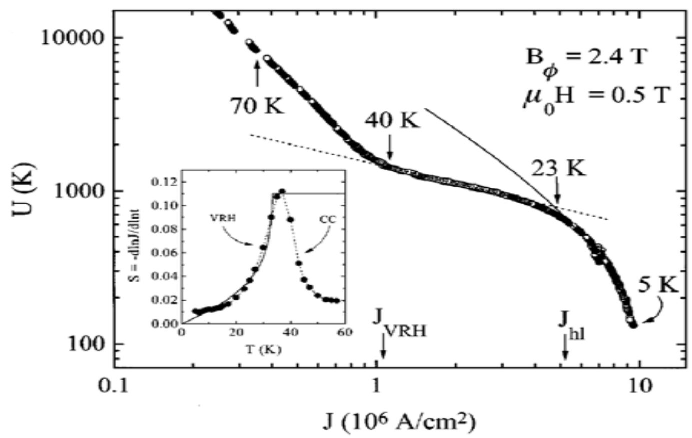
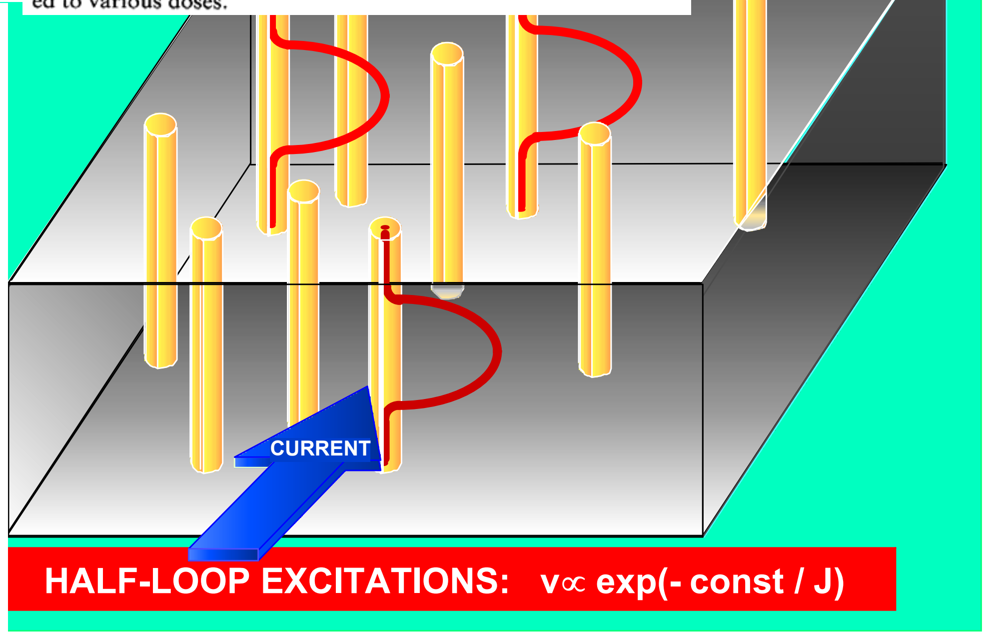


FIG. 4. $U(J)$ for the $B_\Phi = 2.4$ T crystal in a 0.5 T magnetic field. The solid line is the fit to the full glassy expression for $U(J)$ (see text) with $\mu \simeq 1$. The slope $\mu \sim 1/3$ (dashed line) fits the data well between 23 and 40 K. Crossover currents are indicated by the arrows. Inset: Fit to variable-range hopping [Eq. (3)] with $\mu = 1/3$ (see text) is shown as the solid line. The decreased rate on the high-T side of the peak is due to slower creep in the collective regime.



P. Chauve, T. Giamarchi, and P. Le Doussal, Europhys.Lett., 44,110 (1998)
L. Radzikhovsky, 1998 March Meeting of APS, talk E37 8

developed RG approach describing the whole range of velocities.

Creep is a general phenomenon

VOLUME 80, NUMBER 4 PHYSICAL REVIEW LETTERS 26 JANUARY 1998

Domain Wall Creep in an Ising Ultrathin Magnetic Film

S. Lemerle,[1] J. Ferré,[1] C. Chappert,[2] V. Mathet,[2] T. Giamarchi,[1] and P. Le Doussal[3]

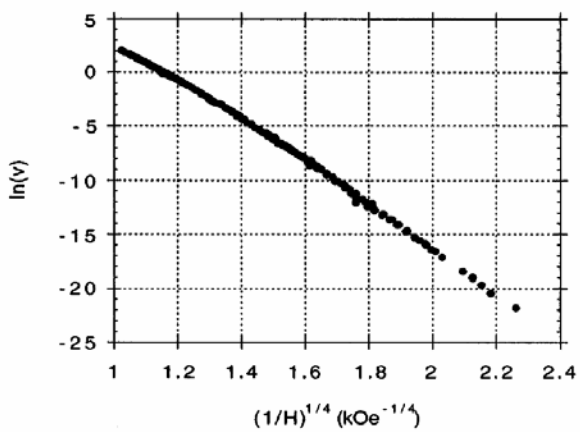


FIG. 3. Natural logarithm of MDW velocity as a function of $(1/H)^{1/4}$ (room temperature, $H \leq 955$ Oe).

VOLUME 89, NUMBER 9 PHYSICAL REVIEW LETTERS 26 AUGUST 2002

Domain Wall Creep in Epitaxial Ferroelectric $Pb(Zr_{0.2}Ti_{0.8})O_3$ Thin Films

T. Tybell,[1,2] P. Paruch,[1] T. Giamarchi,[3] and J.-M. Triscone[1]

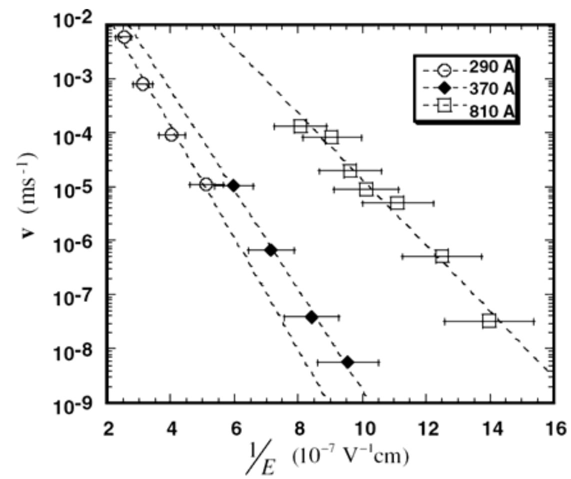


FIG. 3. Domain wall speed as a function of the inverse applied electric field for 290, 370, and 810 Å thick samples. The data fit well to $v \sim \exp[-\frac{R}{k_B T}(\frac{E_0}{E})^{\mu}]$ with $\mu = 1$, characteristic of a creep process.

Other random systems?

All of them?

II. Vortex lattice melting

1. Lindemann criterion

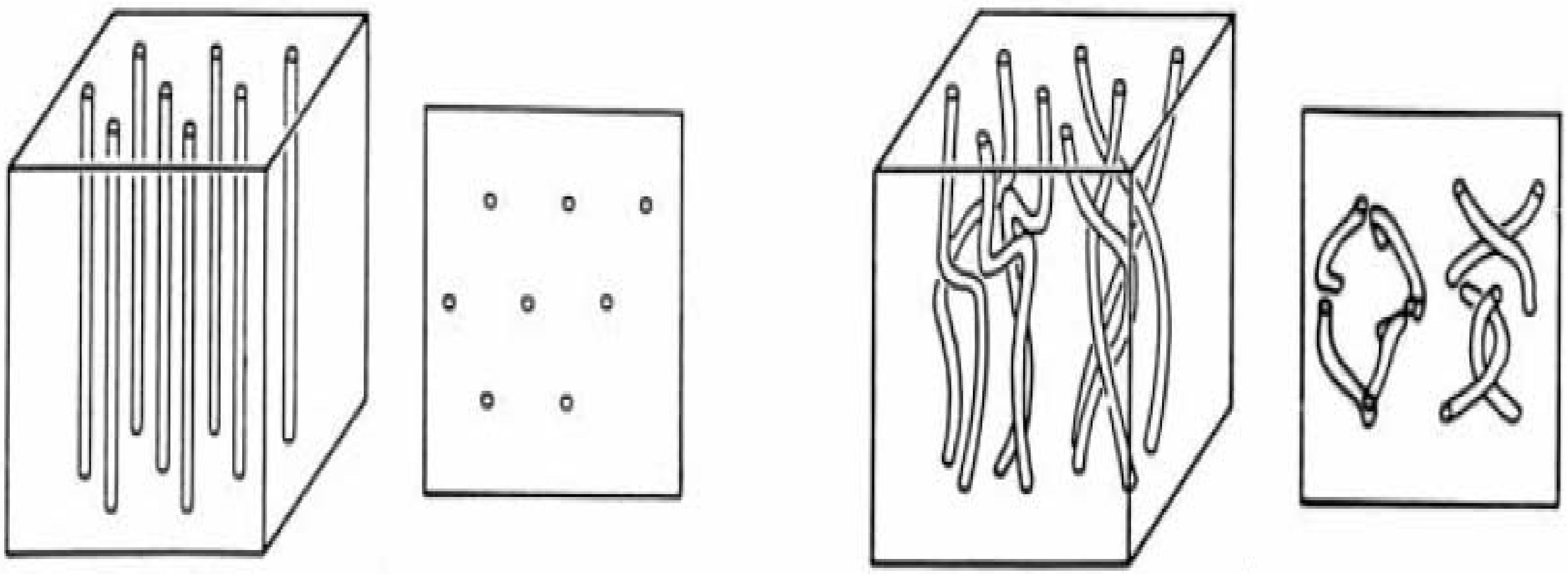
2. Role of disorder

3. Disorder-induced melting

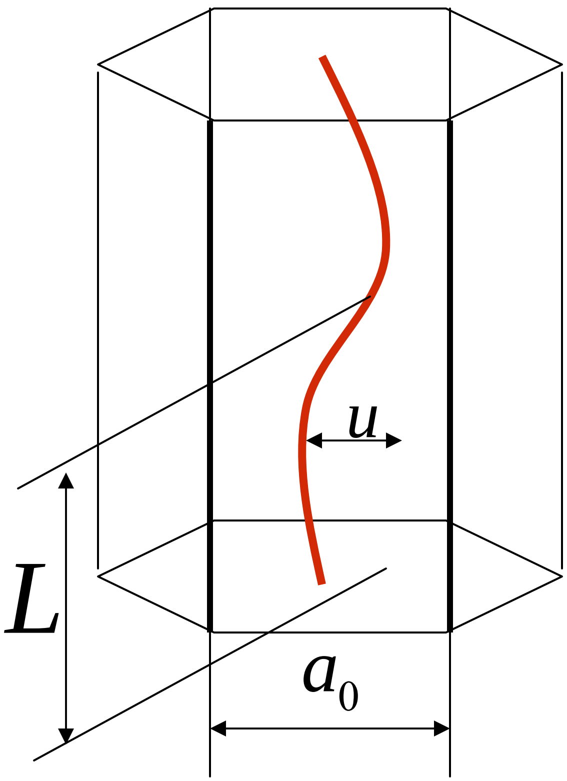
4. Dynamic melting

5. Columnar defects:
   - Upward shift
   - Bose glass phase
   - Localization in vortex liquid
   - Delocalization-induced melting

Vortex lattice melting



Vortex lattice (1957)   Vortex liquid (1988)



Mean-field (cage) approach

$C_{66}u^2L$ - elastic energy to deform vortex in a cage

T - energy of thermal fluctuations

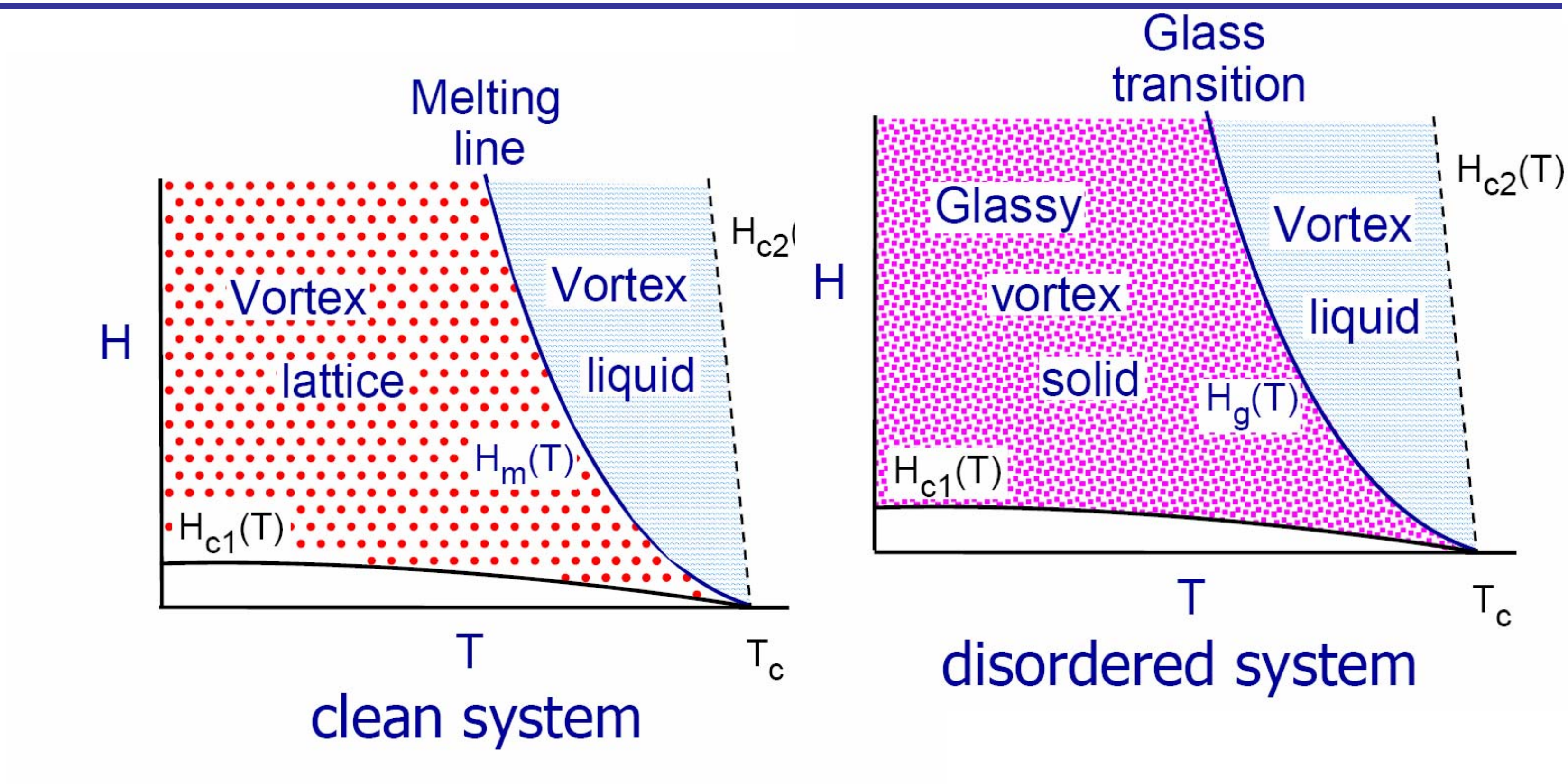
$$C_{66}u^2L \simeq \varepsilon_\ell \frac{u^2}{L}, \quad C_{66} \approx \varepsilon_\ell / a_0^2 \Longrightarrow L \approx a_0$$

Energy balance: $C_{66}u^2a_0 \simeq T$

Melting condition: $C_{66}a_0^3 \simeq T \Longrightarrow u \simeq c_L\, a_0$

$$c_L \approx 0.2 \div 0.3$$

melting and disorder



Weak disorder does not influence melting much since disorder becomes relevant on the scales of the order L_c, and melting characteristic distances are a_0 , and

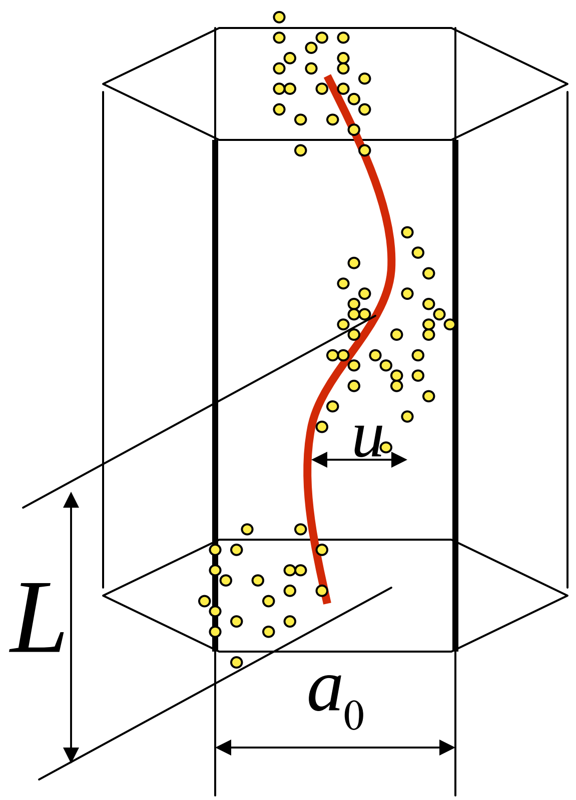
$$L_c \gg a_0$$

?

disorder-induced melting

$$L_c \gg a_0$$

$$a_0 \approx \frac{\Phi_0}{H}$$



Let us estimate contribution of disorder:

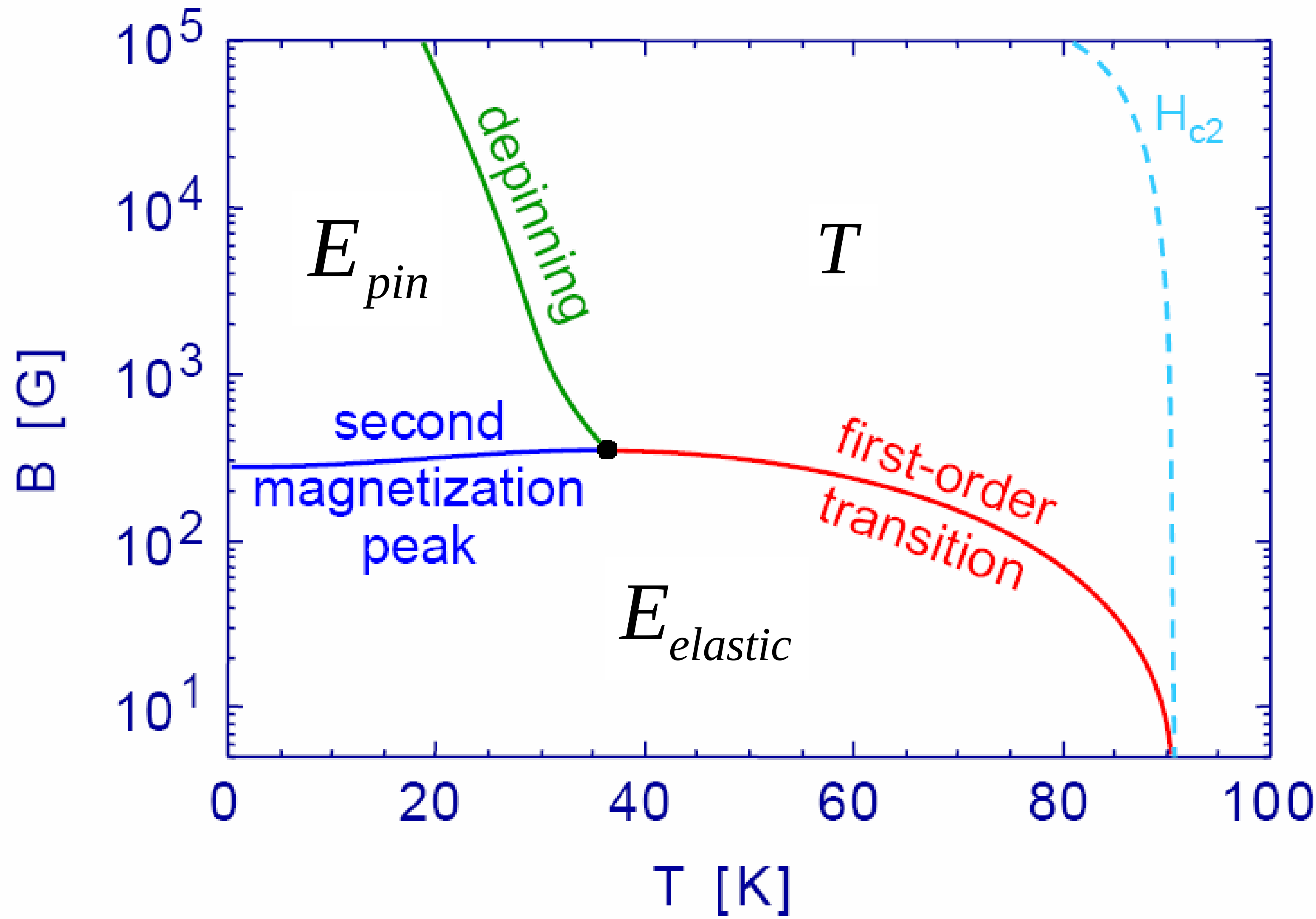
$$E_{pin} \simeq \sqrt{\gamma L} \sim \sqrt{\gamma a_0} \qquad \propto \frac{1}{\sqrt[4]{\sqrt{H}}}$$

At the same time:

$$E_{elastic} \simeq C_{66} a_0^3 \approx \varepsilon_\ell a_0 \qquad \propto \frac{1}{\sqrt{H}}$$

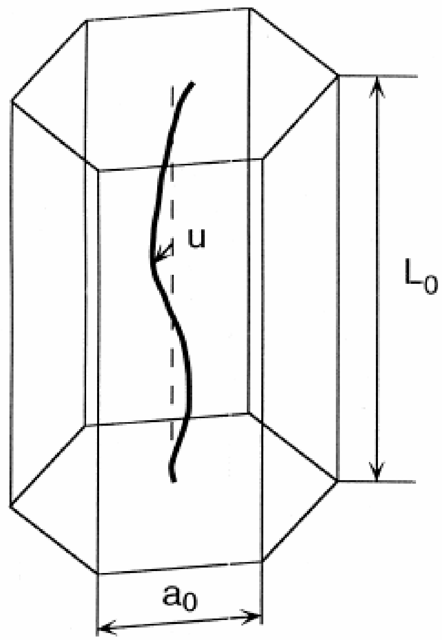
This means that at some field the transition form elasticity dominated behavior (low fields) to disorder dominated state (high fields) takes place!

Vortex matter phase diagram in $Bi_2Sr_2CaCu_2O_8$

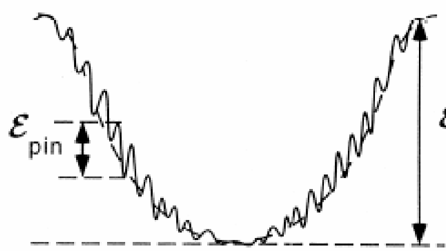


Formation of the amorphous phase

V. Vinokur et al. / Physica C 295 (1998) 209–217



(a)



(b)

Dislocation mediated melting

Disorder free part of free energy in terms of dislocation density:

$$f(\rho)=2\rho\left(E_c-T\frac{1}{2a_z}\ln\left(1+\frac{2\pi T a_z}{\epsilon_D a^2}\right)\right)$$
$$+2\rho\frac{Kb^2}{4\pi}0.3\ln\left(\frac{1}{b(T)\,a^2\rho}\right)+\rho^3\frac{\pi^2}{3}\frac{T^2a^2}{\epsilon_D}$$

Disorder-related part:

$$f(\rho)\approx e_D(\rho)-\begin{cases}\text{BrG:} & 2A_{BrG}E_c\rho\\ \text{RM:} & 2A_{RM}\dfrac{E_c}{a^2}(\rho a^2)^{\frac{13}{15}}\left(\dfrac{a}{R_a}\right)^{\frac{4}{15}}\end{cases}$$

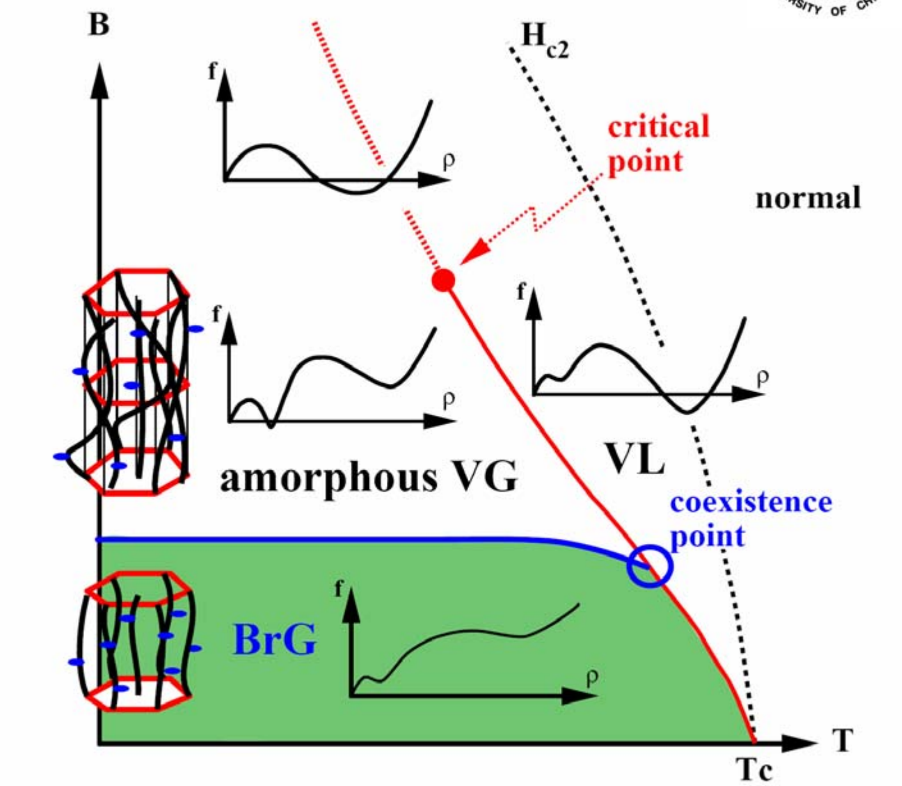


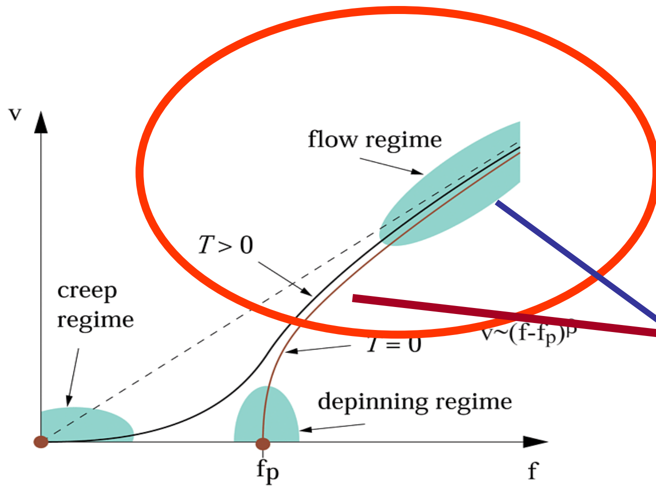
FIG. 1. Schematic phase diagram of YBCO. Insets show typical free energy densities f of a dislocation ensemble as function of the dislocation density ρ.

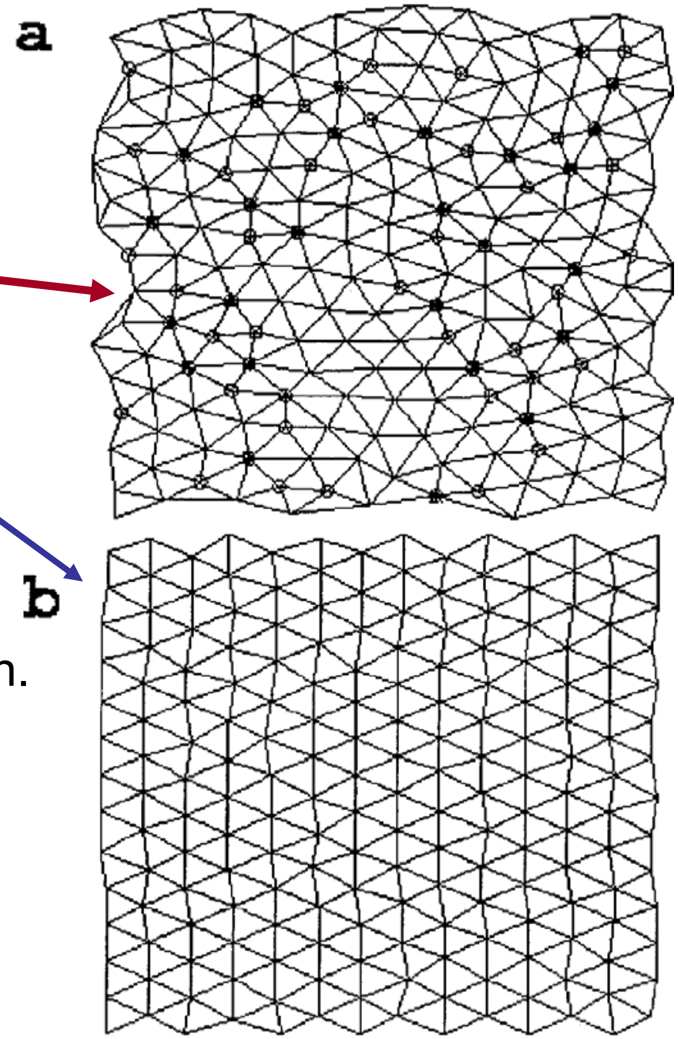
$$e_D(\rho)=2\rho(E_c+(Kb^2/4\pi)\ln(1/a\rho^{1/2}))$$

A local minimum in the free-energy density at $\rho \simeq R_a^{-2}$

VL local minimum: $\rho \simeq 0.2a_0^{-2}$

Dynamic melting





Driven lattice hits defects.
These random collisions cause positional fluctuations of vortices – resembling effect of temperature.

Effect of fluctuations increases as lattice slows down.
Thus lattice can melt.

Shaking temperature:

$$T_{\rm sh} = \frac{1}{4\eta}\sum_{\alpha}\int d\mathbf{r}\int dt\, \aleph_{\alpha\alpha}(\mathbf{r},t) = \frac{1}{4\sqrt{2\pi}}\frac{n_v\gamma_U}{F_{\rm ext}r_p^3}$$

$$\aleph_{\alpha\alpha'}(\mathbf{r},t) = \langle F_{p\alpha}(0,0)F_{p\alpha'}(\mathbf{r},t)\rangle$$

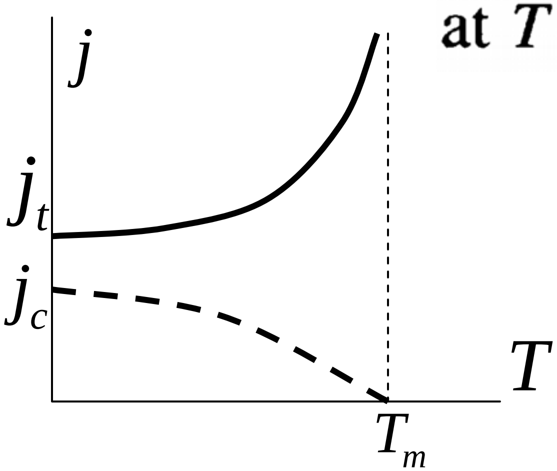
Dynamic phase transition line

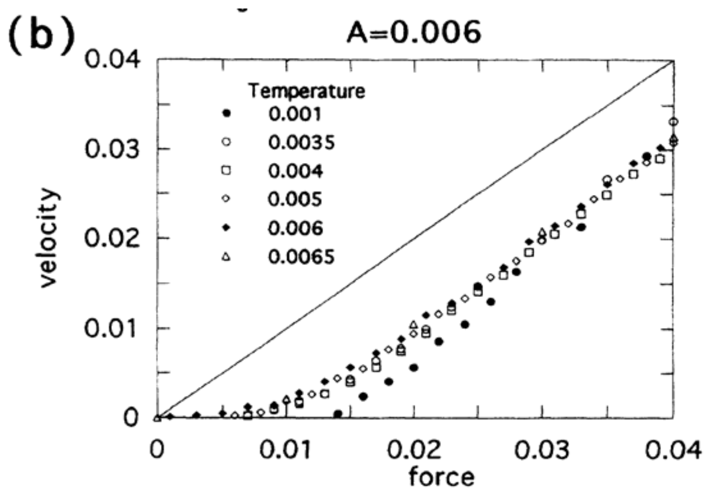
$$T_{\text{eff}} = T + T_{\text{sh}}$$

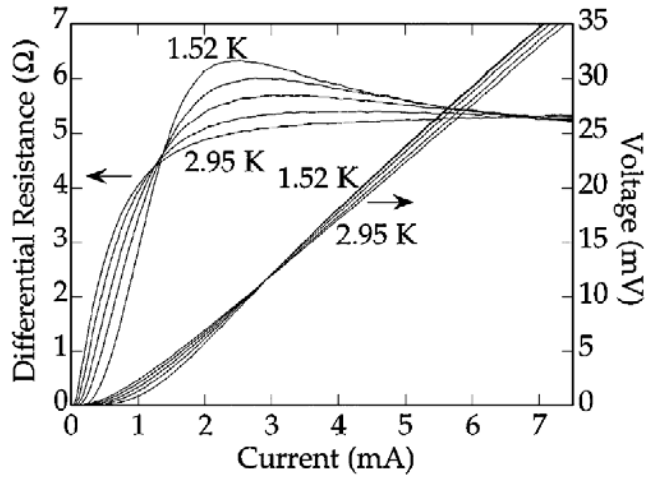
$$T_{\text{eff}}(F_{\text{ext}} = F_t) = T_m, \text{ where } T_m = 0.62 C_{66} a^2 / 4\pi$$

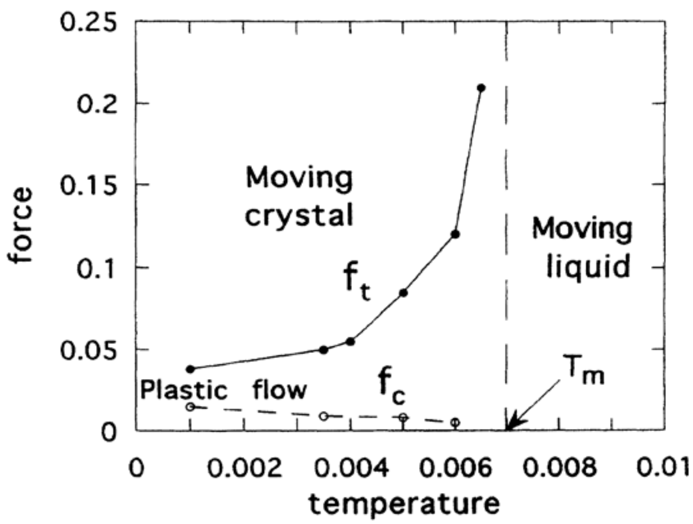
At $T = 0$ ⟹ $$F_t = \frac{n_v \gamma_U}{4\sqrt{2\pi} T_m r_p^3}$$

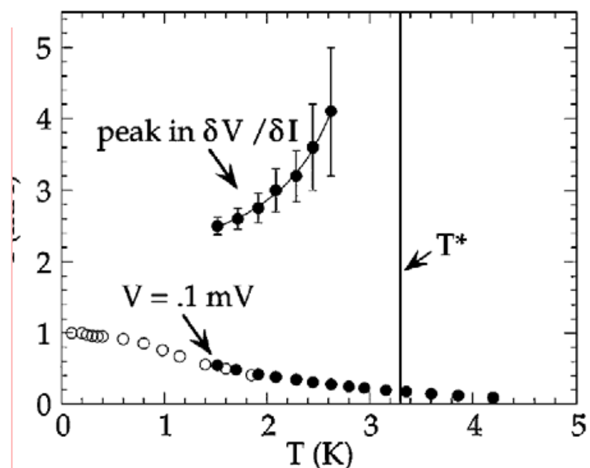
at $T \to T_m$ ⟹ $$F_t = \frac{n_v \gamma_U}{4\sqrt{2\pi}(T_m - T) r_p^3}$$









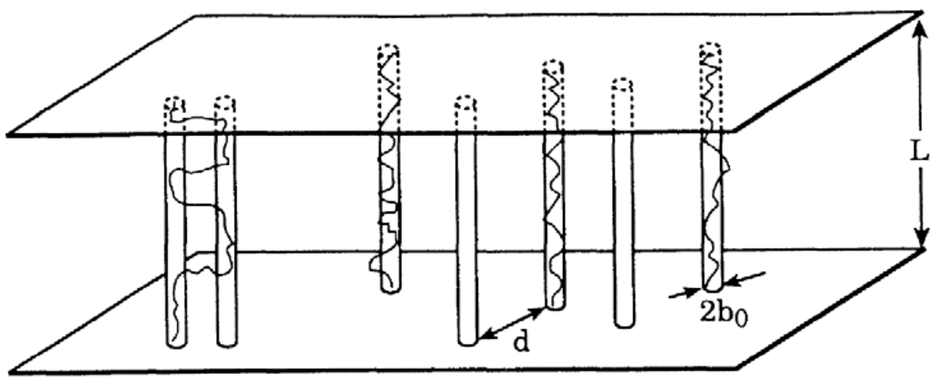


VOLUME 76, NUMBER 21 PHYSICAL REVIEW LETTERS 20 MAY 1996

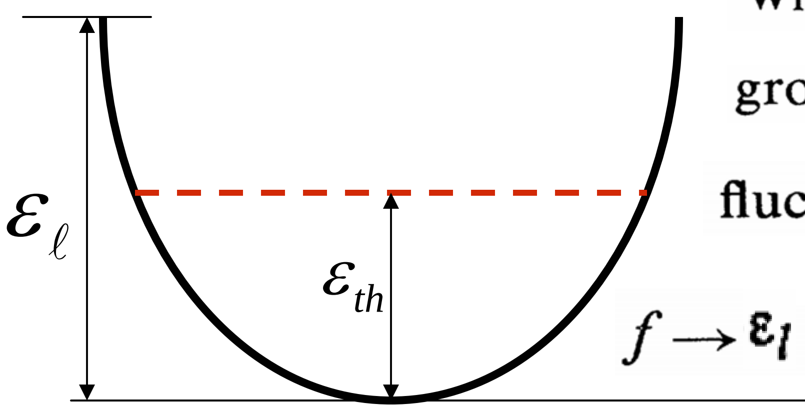
Vortex Dynamics in Two-Dimensional Amorphous $Mo_{77}Ge_{23}$ Films

M. C. Hellerqvist, D. Ephron, W. R. White,* M. R. Beasley, and A. Kapitulnik
Department of Applied Physics, Stanford University, Stanford, California 94305
(Received 29 August 1995)

Bose glass transition and vortex localization



Energy Lindemann criterion



Within a harmonic approximation $[V(R)=fR^2/2]$

ground-state energy: $\hbar\omega/2 \qquad \omega=\sqrt{f/m}$

fluctuation amplitude $\langle R^2\rangle \simeq \hbar\omega/f$

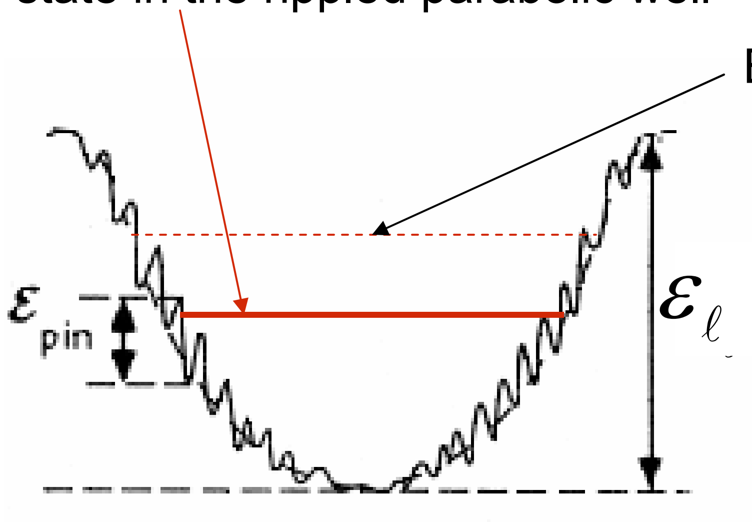
$f \to \varepsilon_l/a_0^2, \quad m \to \varepsilon_l, \quad \hbar \to T, \quad \omega \to 1/a_0$

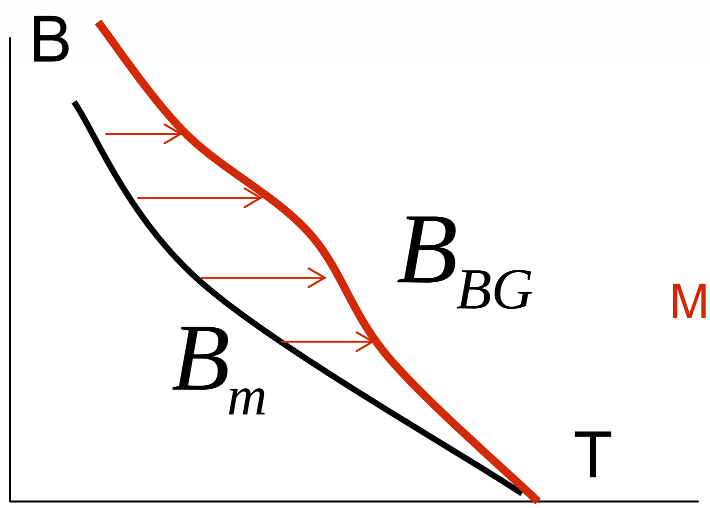
$$\varepsilon_{th} \simeq \frac{T}{a_0} \qquad \qquad \langle u_{th}^2\rangle \simeq \frac{Ta_0}{\varepsilon_\ell}$$

Lindemann criterion: $\langle u_{th}^2\rangle = c_L^2 a_0^2 \Longrightarrow T_m = c_L^2 \varepsilon_\ell a_0$

Energy criterion: $\varepsilon_{th}(T_m) = c_L^2 \varepsilon_\ell \Longrightarrow T_m = c_L^2 \varepsilon_\ell a_0$

Shift of the melting line due to columnar defects





In the first order with respect to disorder the correction to the energy is zero (after averaging with respect to disorder). Thus the first correction appears in the second order of perturbation theory. But the second order correction is always negative. Therefore, the bound state in the rippled well is more deep.

Melting line shifts upwards

Bose glass melting vs. delocalization process

Formation of the Bose glass phase is equivalent to localization of 2D quantum particles in the random field of point defects. Melting into a liquid phase corresponds to delocalization effect

Then little Gerda shed burning tears; and they … thawed the lumps of ice, and consumed the splinters of the Bose-glass…

Hans Christian Andersen, The Snow Queen

Disordered Bose systems: long history of study…

… … … … … …

Dilute interacting Bose gas in a random potential:

Kerson Huang and Hsin-Fei Meng, Phys. Rev. Lett. **69**, 644 (1992)

S. Giorgini, L. Pitaevskii, S. Stringari, Phys. Rev. B **49**, 12938 (1994)

A.V. Lopatin and V.M. Vinokur, Phys. Rev. Lett. 88, 235503 (2002)

– finite temperatures

– described quasiparticles dissipation and depletion of superfluidity at zero temperature

•The described depletion of n_s reflects only scattering of zero energy quasiparticles by random potential, but possible contribution from the quasiparticle bound states was overlooked

• Some quasiparticles leave condensate and get localized: this suggests the existence of the intermediate state where superfluid and localized components present simultaneously

→ **This poses the next**

Question:

Will even *arbitrarily weak* disorder localize the part of the condensate –
***or* localization effects *vanish* if disorder is too weak and/or the boson interactions are sufficiently strong?**

General problem: localization in interacting (Bose) systems

Quantum mechanical mapping

Vortex trajectories can be mapped onto world lines of the 2D Bosons

$$F_N = \int dz \left[\frac{\varepsilon_1}{2} \sum_i \left(\frac{dr_i}{dz} \right)^2 + U(r_i) + \sum_{i<j} V(r_{ij}) \right]$$

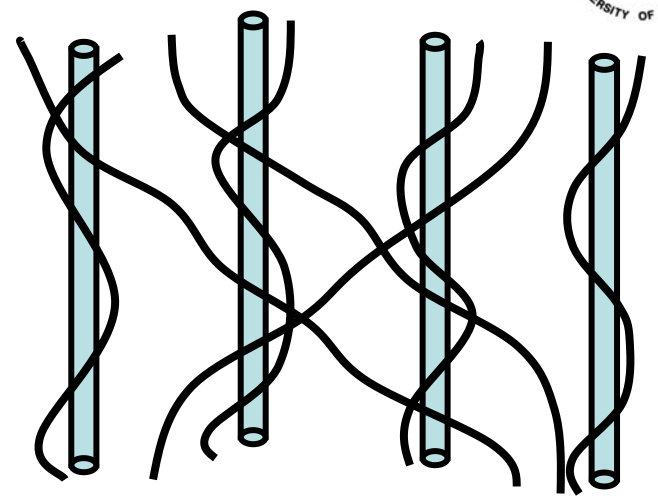
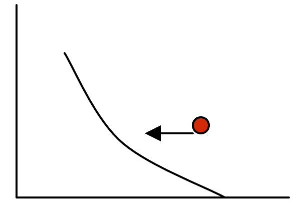


TABLE I. Boson analogy applied to vortex transport.

Charged bosons	Mass	$\hbar$	$\beta\hbar$	Pair potential	Charge	Electric field	Current
Superconducting Vortices	$\tilde{\varepsilon}_1$	T	L	$2\varepsilon_0 K_0(r/\lambda_{ab})$	ϕ_0	$\frac{\hat{\mathbf{z}}\times\mathbf{J}}{c}$	$\mathscr{E}$

Bose glass transition:



1. Low field: go from the superfluid - vortex liquid side

c_{44} is related to the superfluid density of bosons. $c_{44} = \left(B^2 / 4\pi\right)\left[1 + (4\pi n_s \lambda\ )^{-1}\right]$

Superfluid density: $n_s = n - n_n$ where n_n is the normal density.

Disorder-induced depletion of superfluid density ⟷ Stiffening of tilt modulus

$$n_s = n - 4n_2/3$$

$$n_2 = \frac{\kappa}{4\pi}\frac{a^2 m^{3/2}}{\sqrt{\mu}}$$

$$c_{44} \simeq (B^2/4\pi)[1 + (4\pi\lambda^2 n_s)^{-1}]$$

$$\frac{1}{c_{44}^{vR}} = \frac{1}{c_{44}^{v}} - \frac{T^4 n \Delta_1}{(c_{44}^{v})^2 \epsilon_1}\int \frac{d^2q\, q^4}{\varepsilon^4(q)}$$

Kerson Khuang and Hsin-Fei Meng, PRL 69, 644 (1992)
S. Giorgini, L. Pitaevskii, S. Stringari, PR B 49, 12938 (1994)
A. Lopatin and V. Vinokur, PRL 88, 235503 (2002)

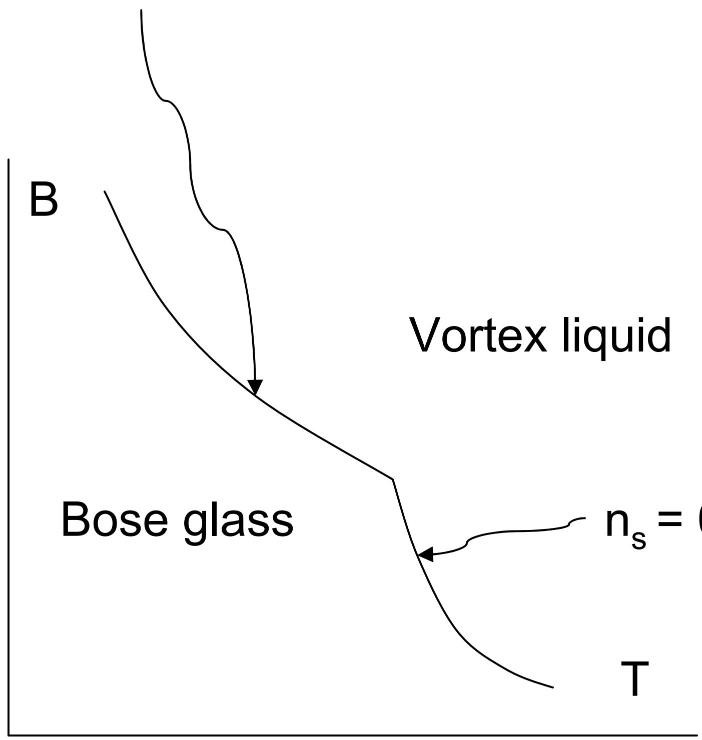
T. Hwa, P. Le Doussal, D. Nelson, V.V., PRL 71, 3545 (1993)

Bose glass transition takes place at *$n_s = 0$*

2. Higher fields: vortex-vortex interactions dominate.

Melting is determined by balance between the elastic and thermal energies:

$$\varepsilon_{th}(T_m) \approx c_L^2 \varepsilon_0 \quad \Longrightarrow \quad T_{BG} = T_m(1 + corrections)$$



Re-examine liquid/superfluid state

Disorder-induced depletion of superfluid density ⟷ Stiffening of tilt modulus

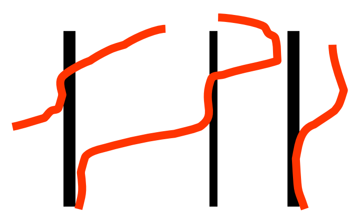
$$n_s = n - 4n_2/3$$

$$n_2 = \frac{\kappa}{4\pi}\frac{a^2 m^{3/2}}{\sqrt{\mu}}$$

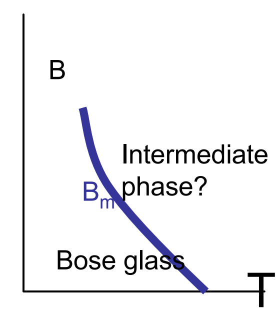
$$c_{44} \simeq (B^2/4\pi)[1 + (4\pi\lambda^2 n_s)^{-1}]$$

$$\frac{1}{c_{44}^{vR}} = \frac{1}{c_{44}^{v}} - \frac{T^4 n \Delta_1}{(c_{44}^{v})^2 \epsilon_1}\int \frac{d^2 q\, q^4}{\varepsilon^4(q)}$$

Any disorder reduces n_s. ⟷ May be interpreted as the fact that some fraction of the vortices is localized (partially pinned)



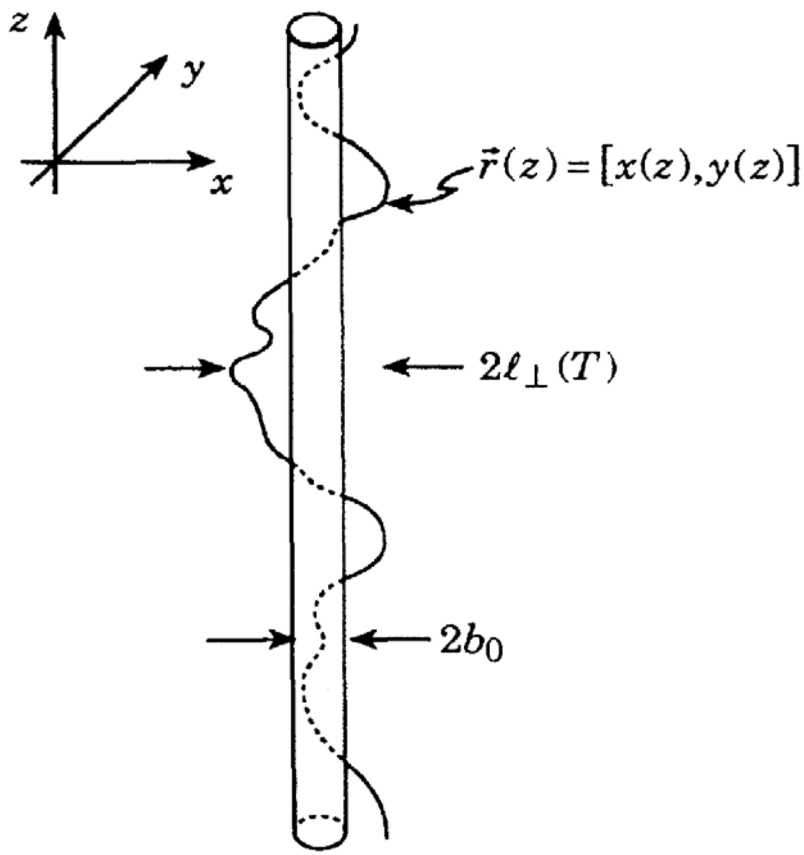
Intermediate vortex state liquid+pinned?

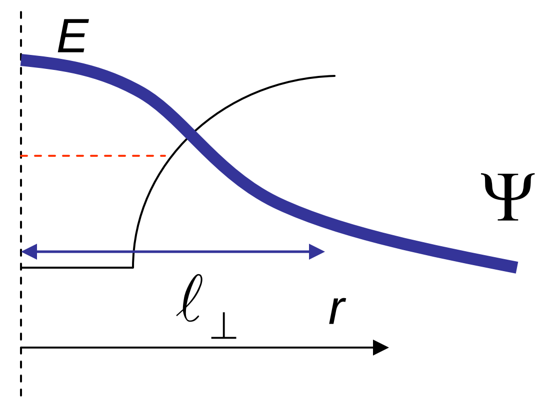


What happens to pinned vortices as we raise temperature?

(A. Andreev, I. Lifshitz, ’68)

Re-examine pinning: pinning by a single defect

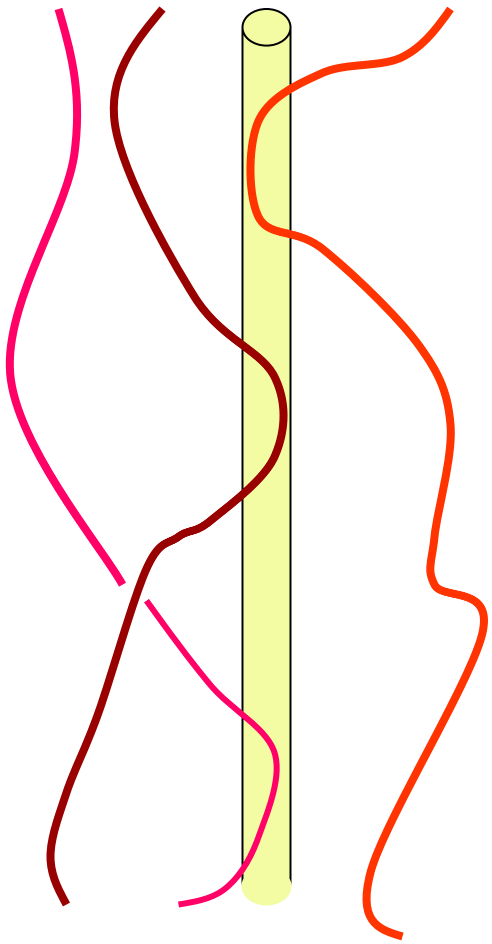




$$E_b \sim J_c \propto \exp\left(-\frac{T}{T^*}\right)$$

There are always bound states in 2D: one vortex is always pinned by one Columnar defect.

What happens if there are many interacting vortices (take high enough temperatures where binding is exponentially weak)?



Naïve picture:
Vortices wander freely and screen each other out from columnar defect.
Thus, if the defect potential is not sufficiently strong, vortices may get depinned.

Quantitative analysis:

Localization and delocalization of vortices on/from the columnar defects.

There are two contributions to the normal density: $n_n = n_{loc} + n_{del}$

(a) Contribution from localized states n_{loc}.

(b) Contribution due to scattering of bosons (vortices) on the disorder potential n_{del}
(even smooth disorder suppresses the superfluid density)

The Task: Theory that includes both of these terms

Contributions to the normal density from localized and delocalized bosons

Hamiltonian of Bosons:

$$\hat{H} = \int d^2 r\, \psi^+ \left[p^2/2m - \mu + U(r) \right] \psi + \int d^2 r_1\, d^2 r_2\, \psi^+(r_1)\psi(r_1)\, V(r_{12})\, \psi^+(r_2)\psi(r_2)$$

$$U(r) = \sum_i u(r - r_i)$$

Low defect concentration:
Defects can be considered separately

$$u(r) \cong \varepsilon_0 \frac{r_0^2}{r^2 + 2\xi^2} \qquad \varepsilon_0 = \frac{\phi_0^2}{(4\pi\lambda)^2}$$

Single defect pinning energy: $E_1 \sim \dfrac{T^2}{\varepsilon_1 \xi^2} e^{-1/\sqrt{\beta}}$ $\qquad \beta = \varepsilon_0 \varepsilon_1 r_0^2 / 4T^2$

Localization length: $\ell_\perp \sim \dfrac{T}{|E_1|\varepsilon_1}$

Now we use the basis of the exact eigenstates of the *noninteracting* problem

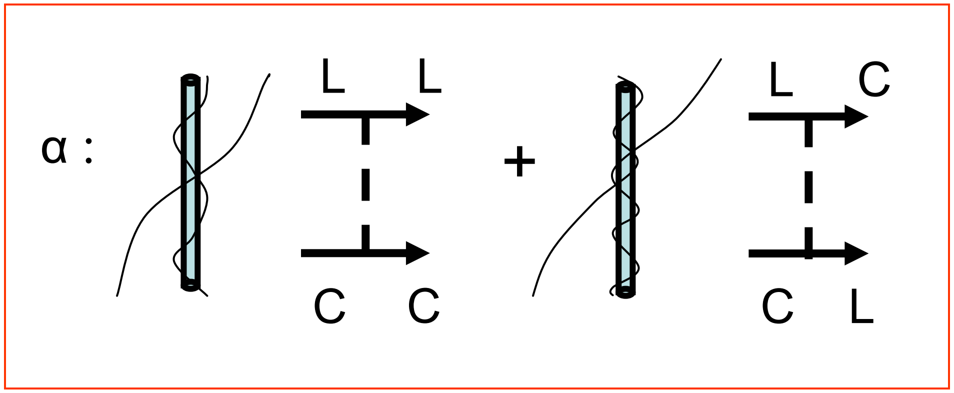
$$\psi = \varphi_0 \hat{b}_0 + \varphi_1 \hat{b}_1 + \sum_k \varphi_k \hat{b}_k$$

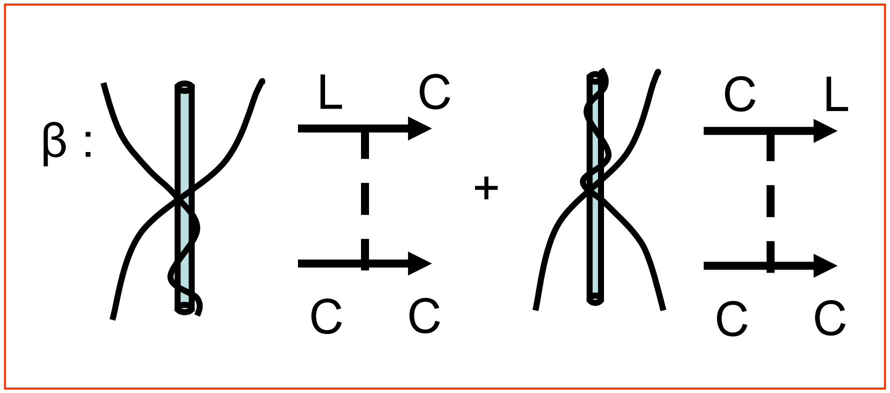
$\hat{b}_0$ – condensate, $\hat{b}_1$ – localized state, $\hat{b}_k$ – excitations

In case of strong vortex interaction double occupation is prohibited $\hat{b}^+ \hat{b}^+ = 0$

Effective model that describes occupation of the localized sates:

$$\hat{H}_{eff} = \hat{b}_1^+ (E_1 + \alpha - \mu) \hat{b}_1 + \beta (\hat{b}_1^+ + \hat{b}_1)$$





$$\psi = a_0 \left|0\right\rangle + a_1 \left|1\right\rangle \Longrightarrow E_\pm = \frac{1}{2}\left(E \pm \sqrt{E^2 + 4\alpha^2}\right)$$

$$E = E_1 + \alpha - \mu$$

Occupation of the lowest state: $n = a_1^2 = \dfrac{2\alpha^2}{4\alpha^2 + E^2 + E\sqrt{E^2 + 4\alpha^2}}$

$n \to 1$ when $E/\alpha \ll -1$ $\quad$ $n \to 0$ when $E/\alpha \gg 1$

At $E = 0$ localization-delocalization crossover occurs

$$E = E_1 + \frac{\mu}{v_0} \int d^2 r_1 d^2 r_2 \phi_1(r_1) v(r_1 - r_2) \phi_2(r_2)$$

$$v_0 = \int d^2 r v(r)$$

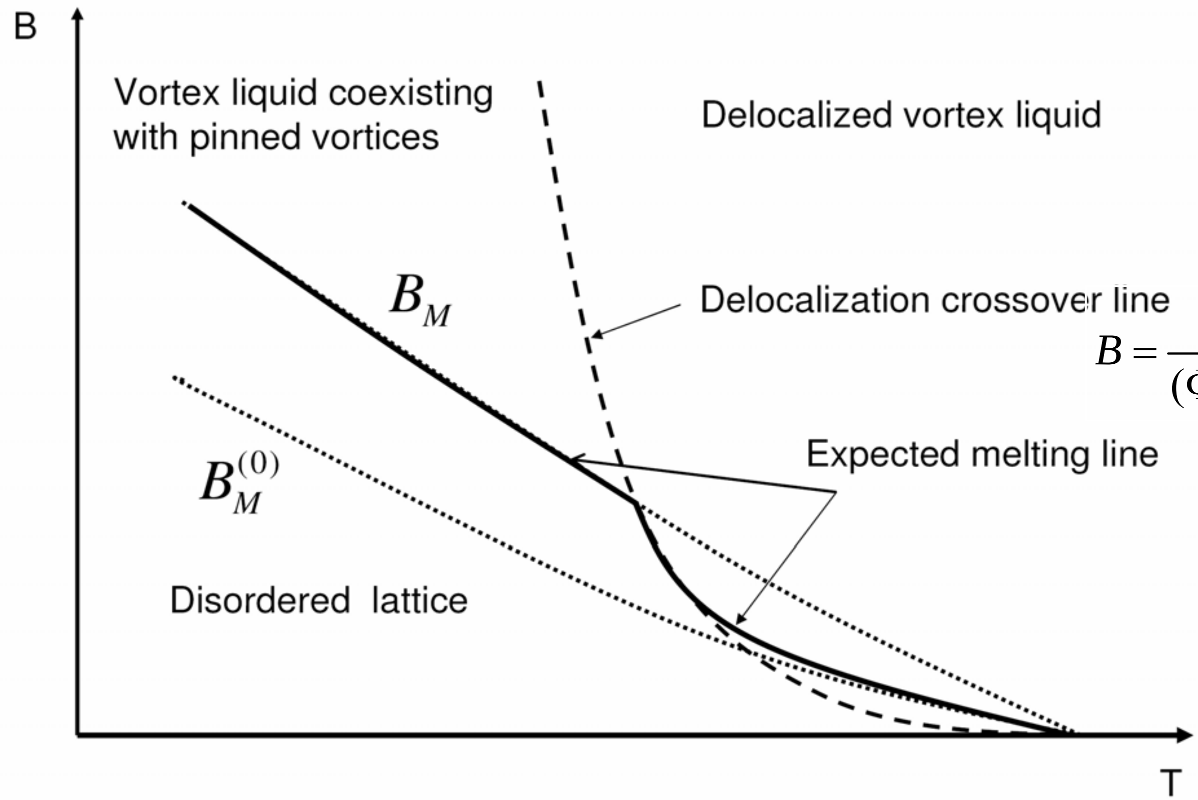
$$n_V \ell_\perp^2 \varepsilon_0 \ln(\lambda / \ell_\perp)$$

Localization-delocalization crossover:

$$n_V \equiv \frac{B}{\Phi_0} \simeq \frac{T^2}{\varepsilon_0 \varepsilon_1 \ell_\perp^4 \ln(\lambda / \ell_\perp)} \simeq \frac{\kappa^4 T^2}{(\Phi_0 / 4\pi)^4 \ln(\lambda / \ell_\perp)} \exp\left(-\frac{T}{T^*}\right)$$

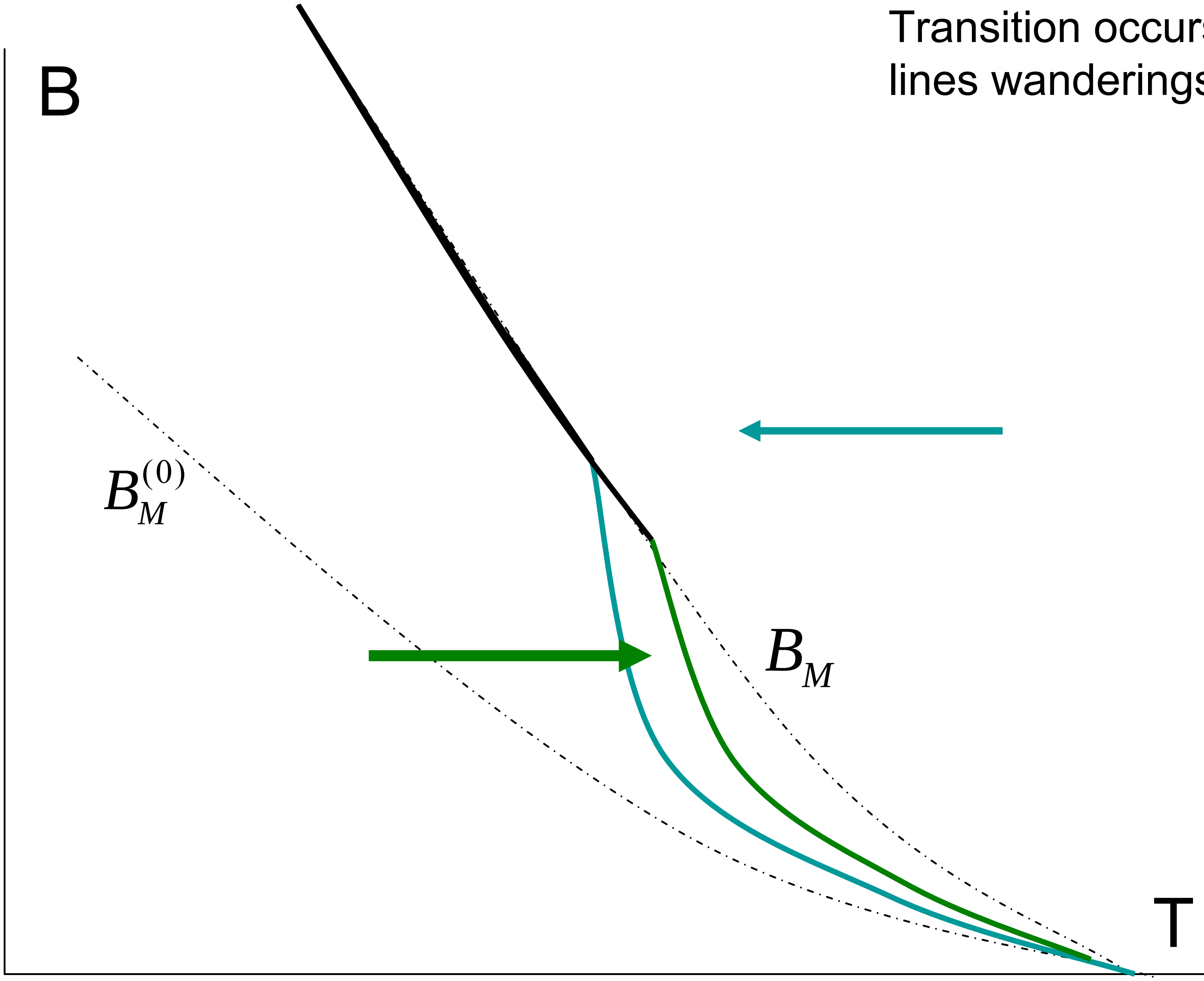
$$T^* = (1/8) r_0 \sqrt{\varepsilon_1 \varepsilon_0}$$

VORTEX PHASE DIAGRAM



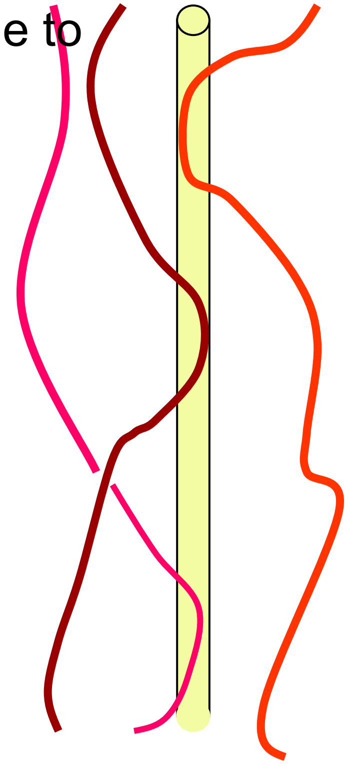
$$B = \frac{\Phi_0 \kappa^4 T^2}{(\Phi_0 / 4\pi)^4 \ln(\lambda / \ell_\perp)} \exp\left(-\frac{T}{T^*}\right)$$

Schematic phase diagram for the vortex system with columnar defects. The melting line under assumption of pinning is denoted by B_M, while the melting line of the pristine lattice is denoted by $B_M^{(0)}$.



FIRST ORDER TRANSITION

Transition occurs due to lines wanderings

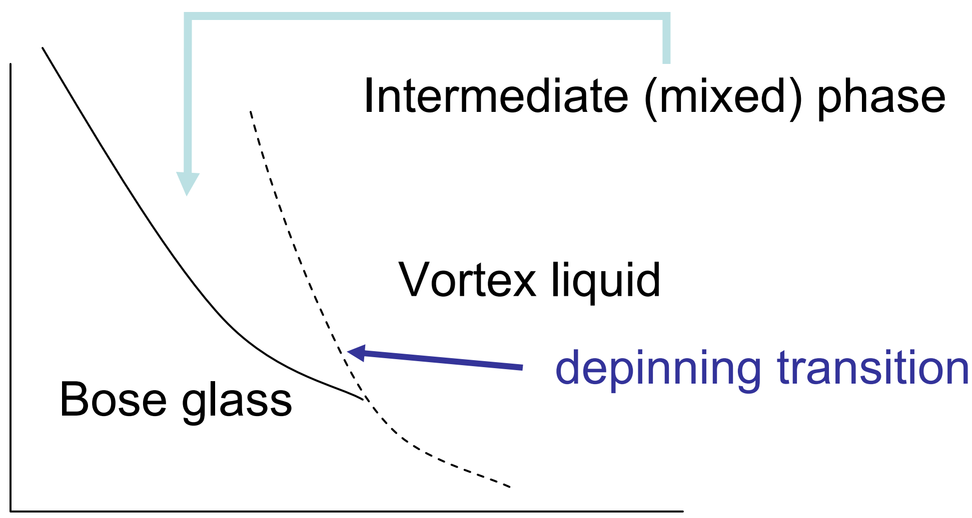


In a liquid vortices fluctuate much easier, thus we can expect the transition to shift to higher temperatures, if we go from the solid side!

- The exact position of the Bose glass transition depends on the filling parameter *n*, which is to be found self-consistently

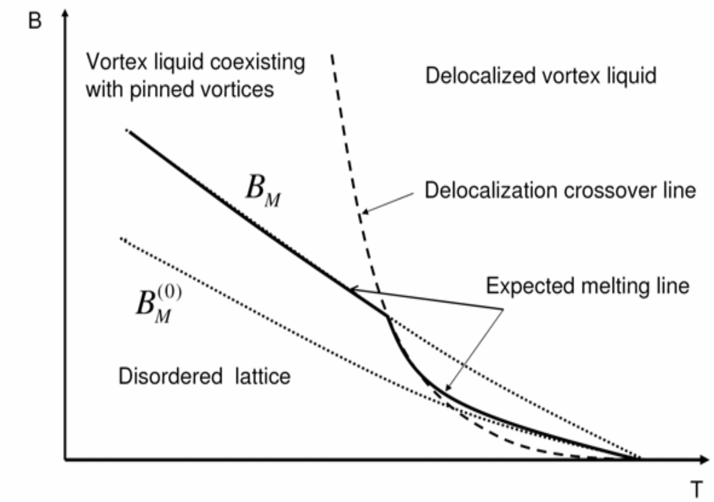
- Hybridization of the bound states with condensate: the number of trapped vortices is not exactly 1

- In the limit of low density (low hybridization) a localization-delocalization transition ($n=1 \rightarrow n=0$) may occur



$$v \sim \exp\left[-\frac{E_p}{T}\left(\frac{f_p(T)}{f}\right)^{\mu}\right]$$

$$\mu = \frac{\chi}{2-\varsigma}$$



Schematic phase diagram for the vortex system with columnar defects. The melting line under assumption of pinning is denoted by B_M, while the melting line of the pristine lattice is denoted by $B_M^{(0)}$.