Dynamics of Vortex Structures

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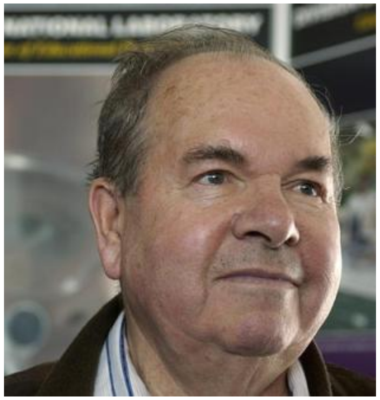
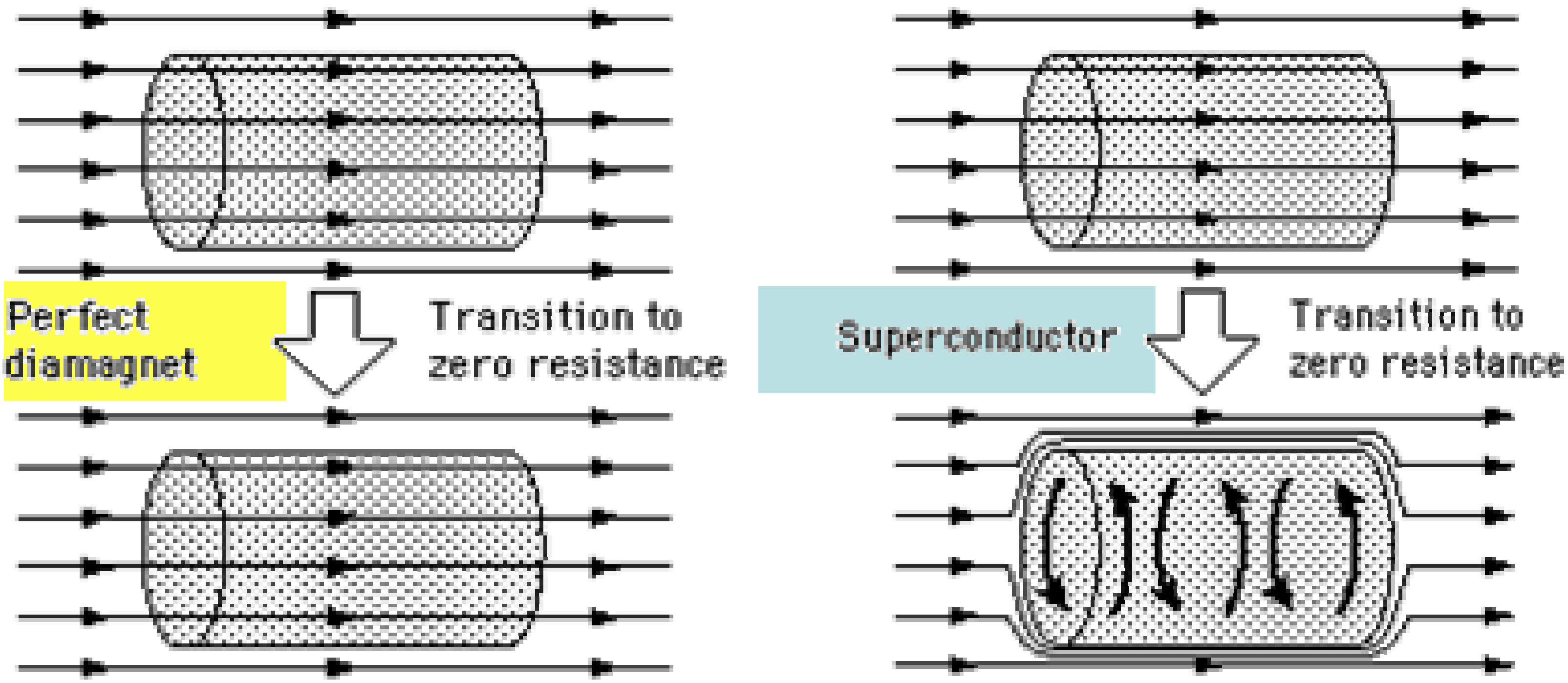


Outline

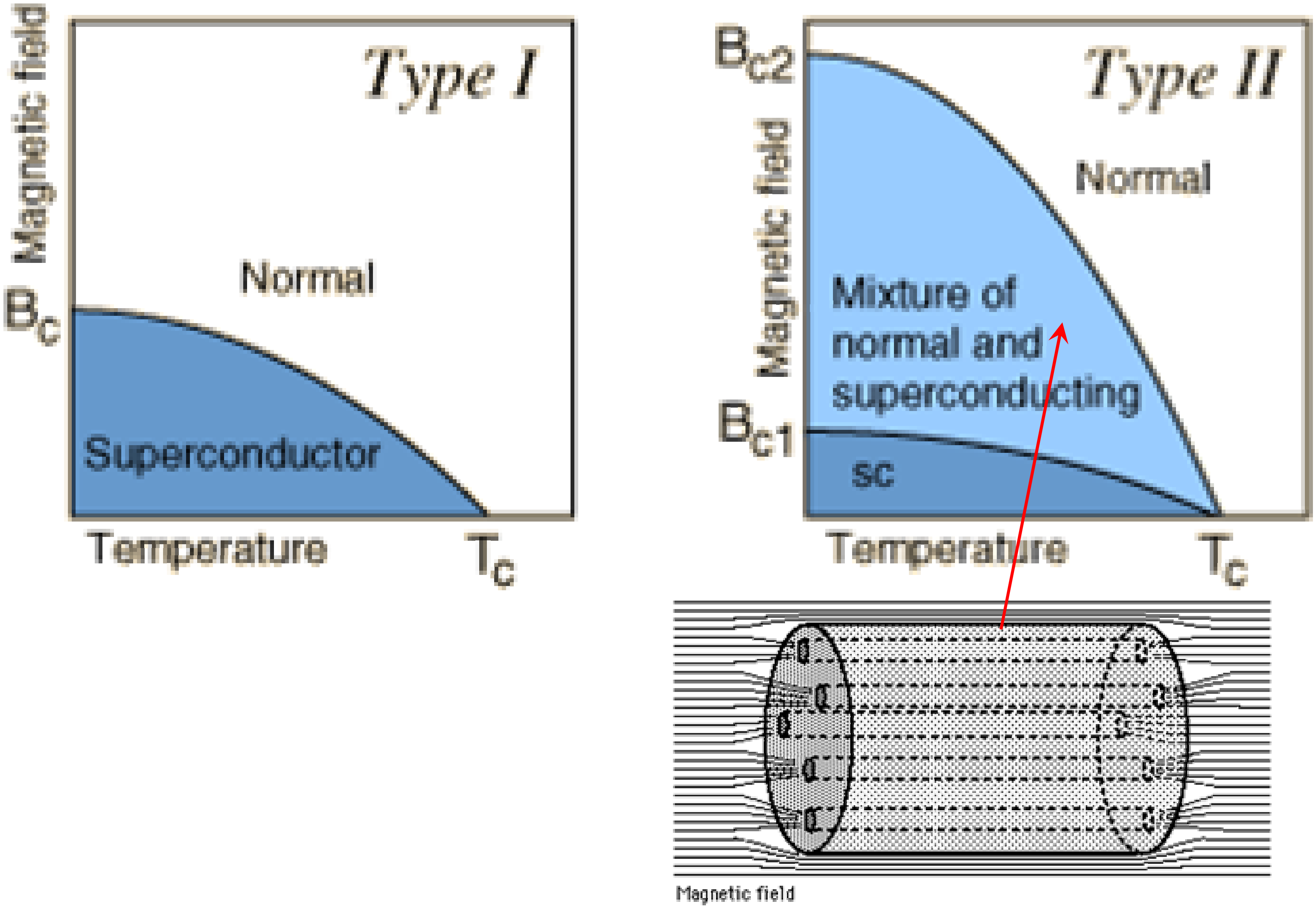
1. **Superconductors and vortices**
2. **Vortices and point disorder**
3. **Critical currents and creep**
4. **Quantum mechanical mapping and Bose glass**
5. **Competing localization**

Superconductors

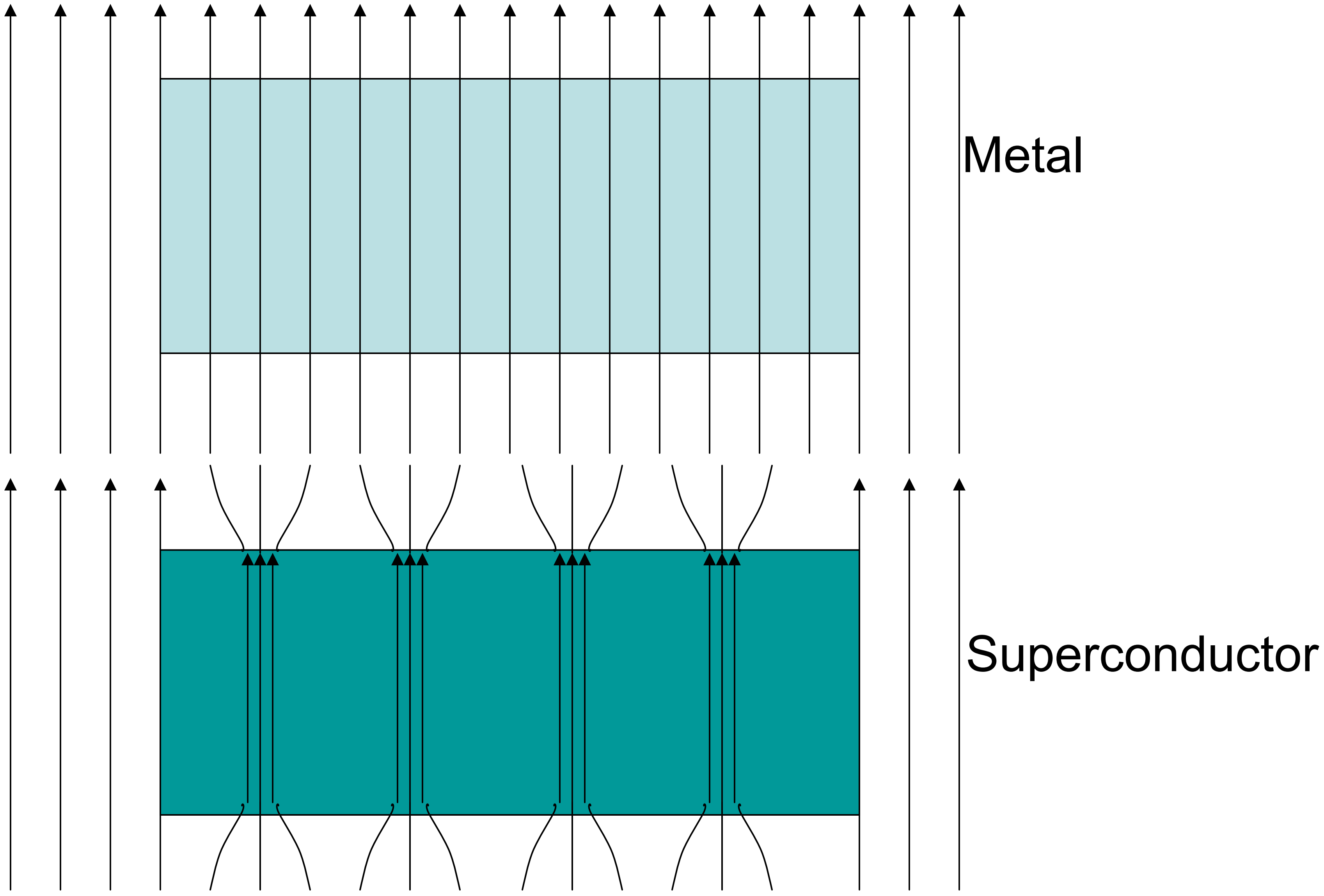
The Meissner Effect



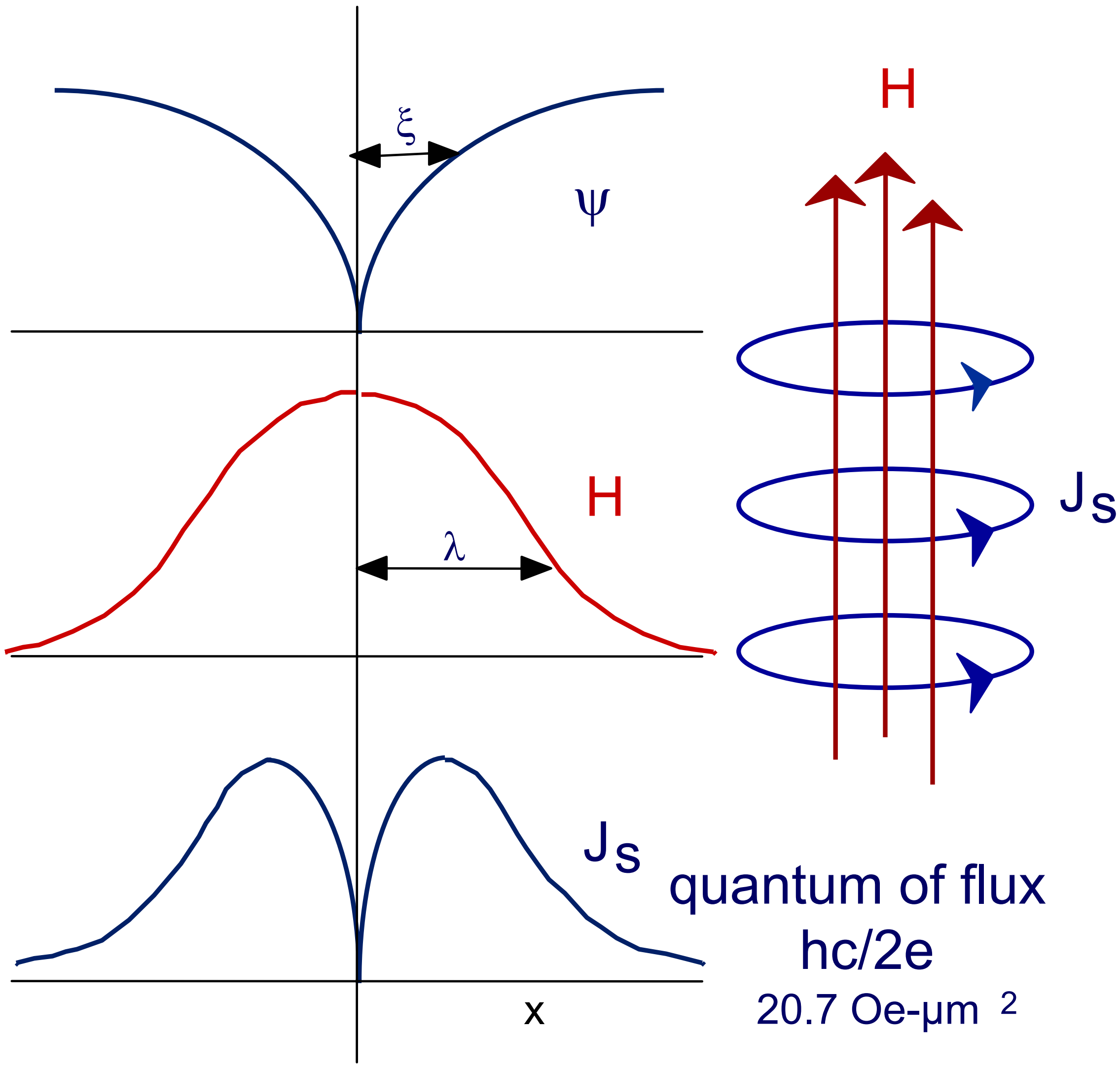
TYPE I AND TYPE II SUPERCONDUCTORS



VORTEX FORMATION

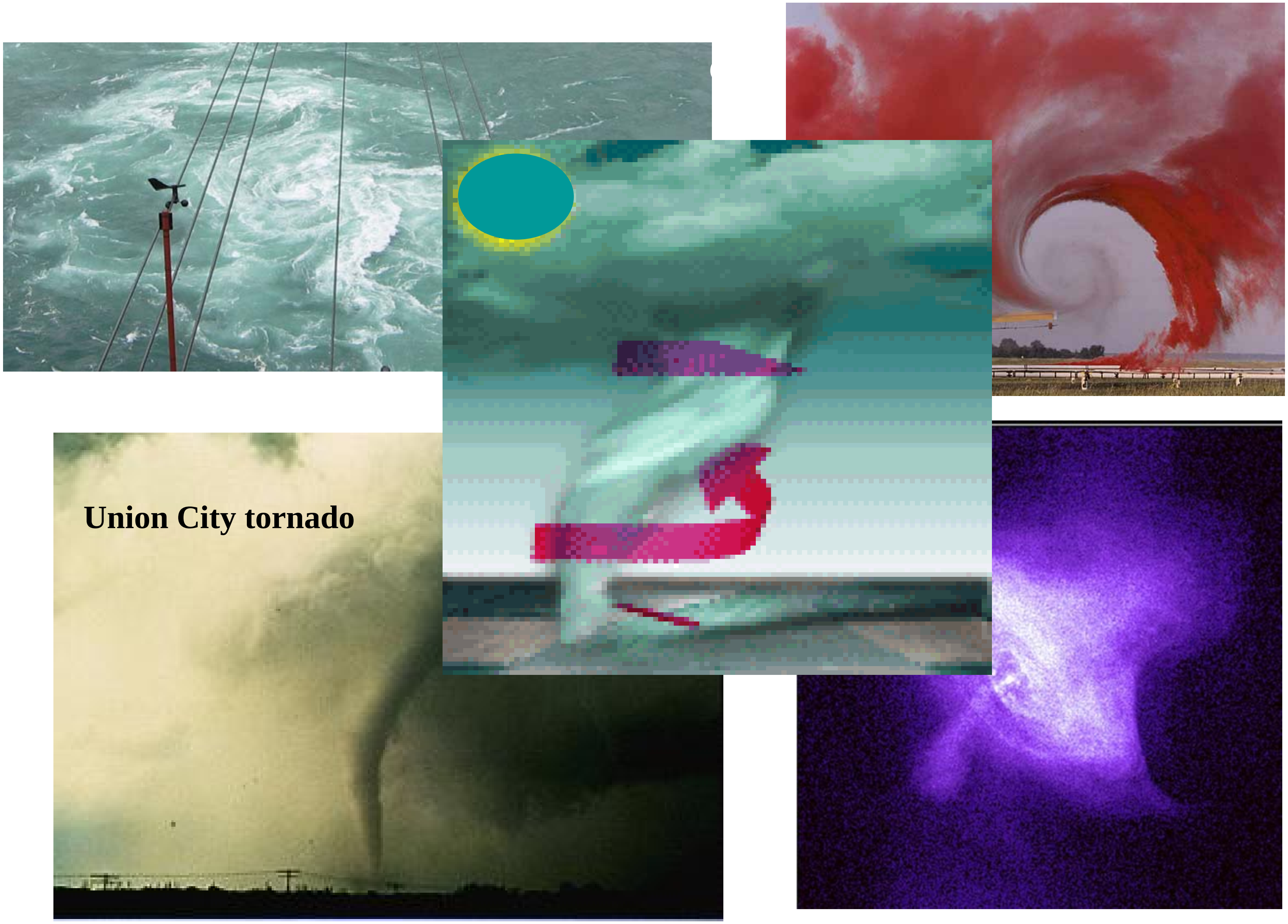


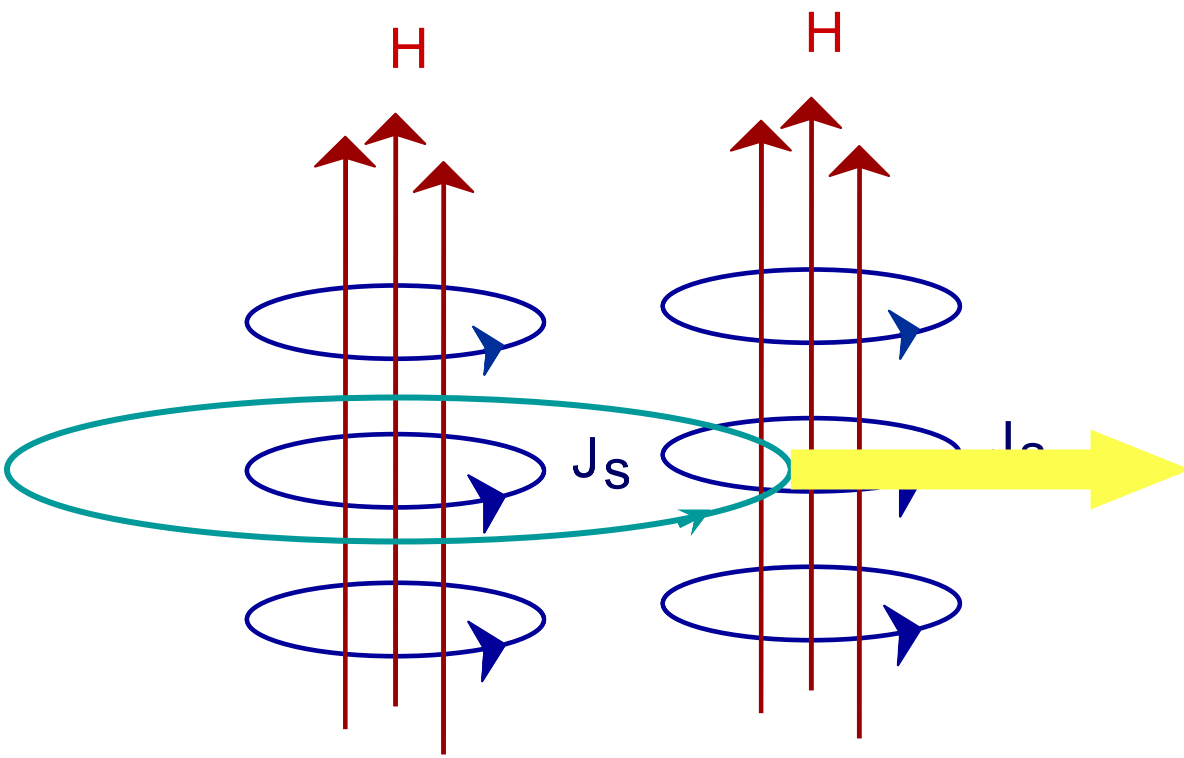
Abrikosov vortices



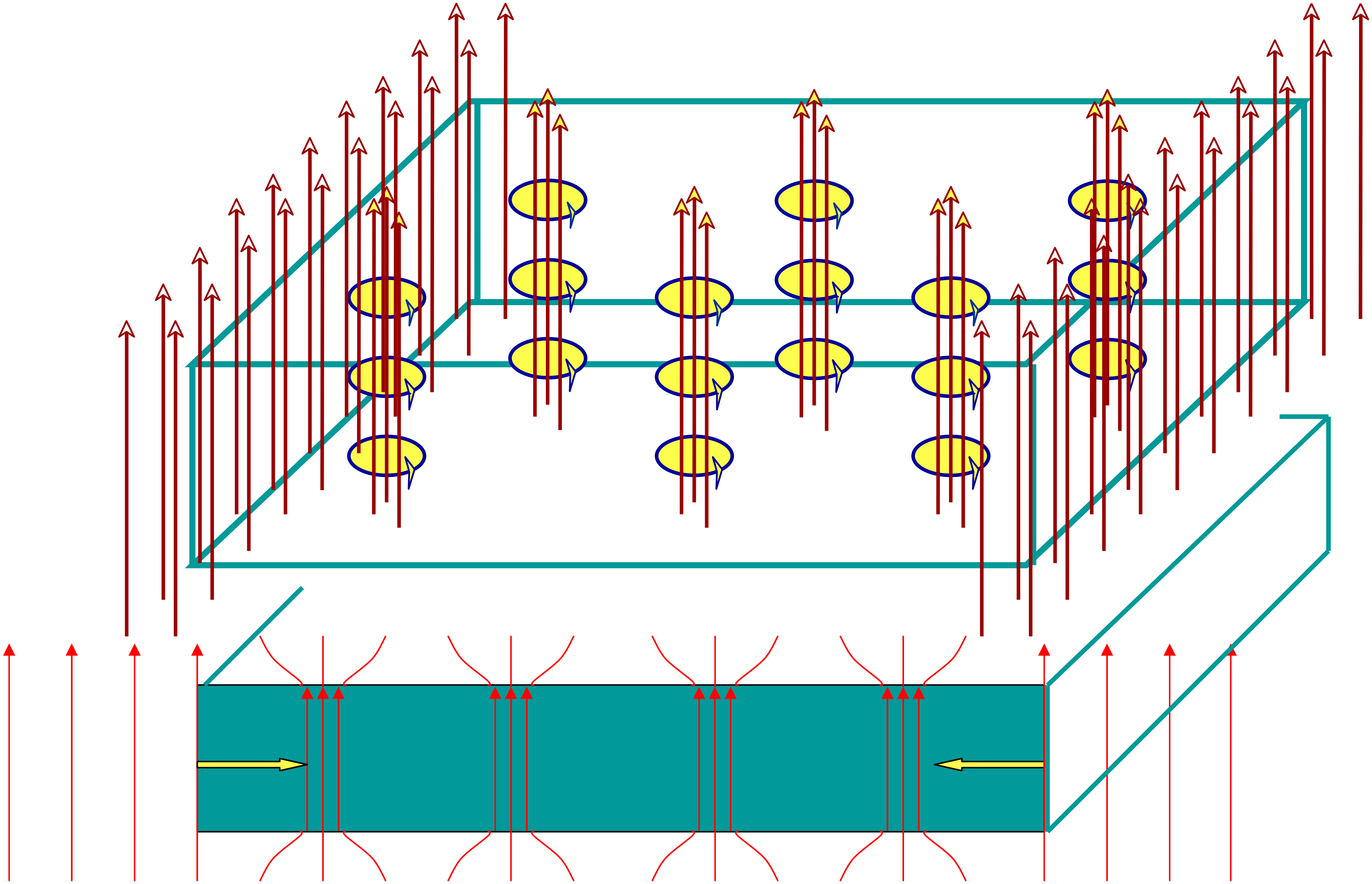
$$\lambda \gg \xi$$

… vortices…



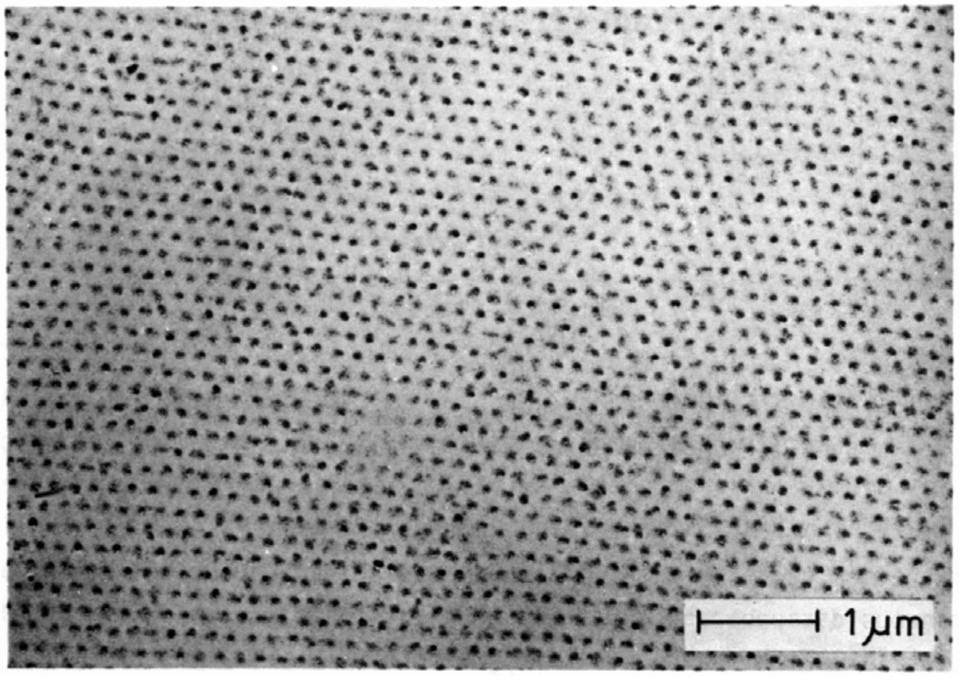


H
H
J_S

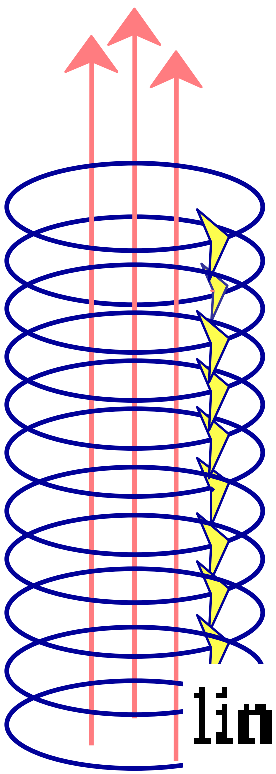


Vortex lattice

decoration by magnetic smoke
image in electron microscope



Nb at T= 1.2 K H = 985 G
U. Essmann, H. Träuble, Phys. Lett. **24A**, 526 (1967)



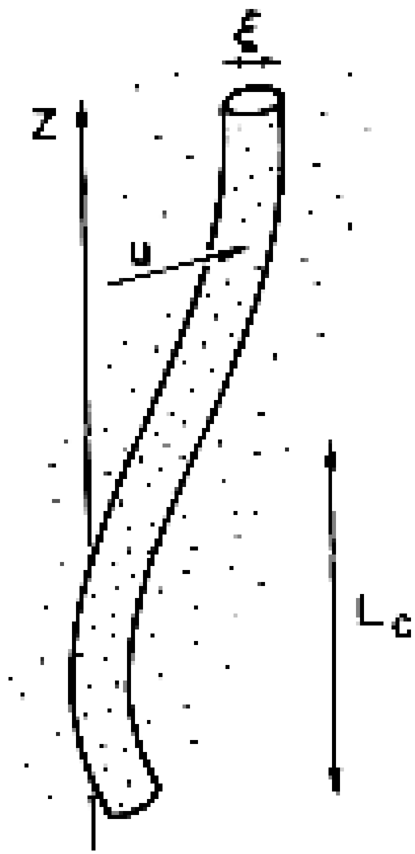
$$e_l = \varepsilon_0 \ln\frac{\lambda}{\xi} = \frac{\Phi_0}{4\pi} H_{c1}$$

$$\varepsilon_0 = \left(\frac{\Phi_0}{4\pi\lambda}\right)^2$$

line tension ε_l is equal to the line energy e_l

$$\mathcal{F} = \sum_i \int dz \frac{\varepsilon_l}{2}\left(\frac{\partial \mathbf{u}_i}{\partial z}\right)^2 + \sum_{i,j}\int dz\, dz'\, v[\mathbf{u}_i(z)-\mathbf{u}_j(z')]$$

Disorder



Single vortex line pinned by the collective action of many weak pointlike pinning centers. Only fluctuations in the pin density are able to pin the vortex. In order to accommodate optimally to the pinning potential, the vortex line deforms by ξ (the minimal transverse length scale the vortex core is able to resolve equals the scale of the pinning potential) on a longitudinal length scale L_c, the collective pinning length.

Exemplary system: elastic medium in random environment

Models a wealth of physical systems and phenomena:

$$\mathcal{H} = \int d^D x \left[\frac{C}{2} \left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}} \right)^2 + V(\mathbf{x}, \mathbf{u}) - \mathbf{F} \cdot \mathbf{u} \right]$$

Dislocations in crystals

CDW and SDW

Domain walls

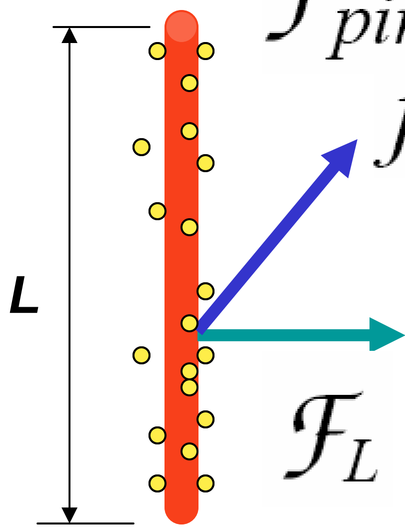
Interacting electrons

Wigner crystals on disordered substrates

Spin- and other glasses…

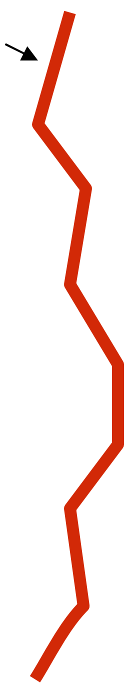
Collective pinning

$$\mathcal{F}_{pin} \simeq (f_{pin}^2 n_i \xi^2 L)^{1/2}$$



$$\mathcal{F}_L \simeq j\Phi_0 L/c$$

L_c

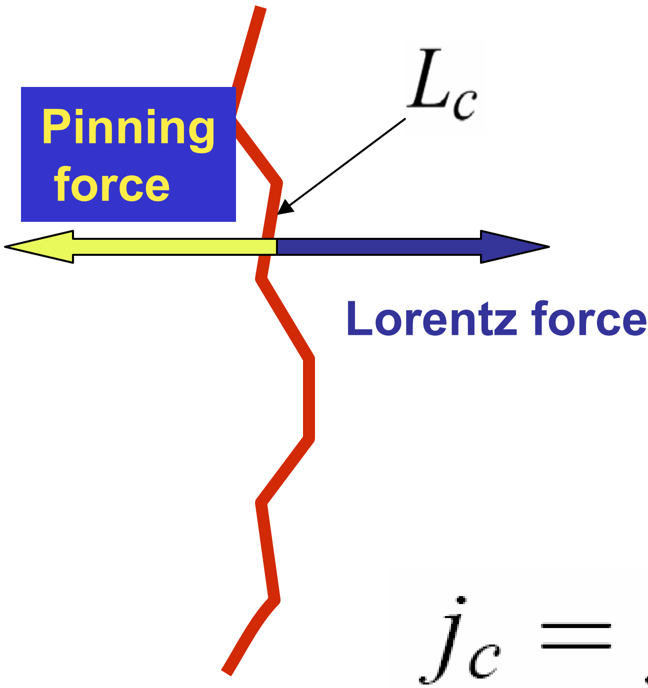


$$\mathcal{E}_{el}(L_c) \simeq \varepsilon_\circ \xi^2/L_c \sim \mathcal{E}_{pin}(L_c)$$

$$\mathcal{F}_{pin} \sim L/L_c$$

recovers pinning

Critical current is defined by the relation:



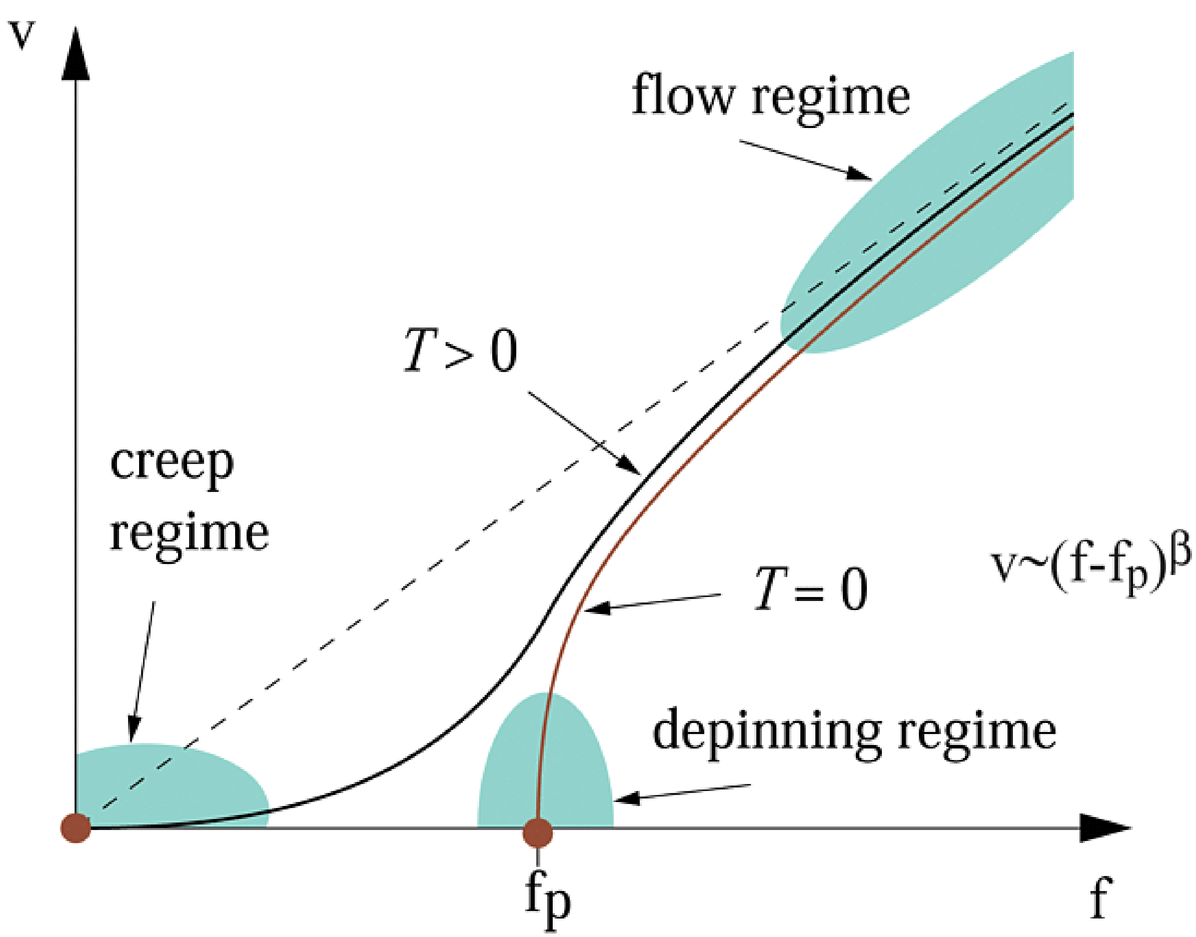
$$\mathcal{F}_{pin}(L_c) = \mathcal{F}_L(L_c)$$

and reads:

$$j_c = j_\circ(\xi\gamma/\varepsilon_\circ^2)^{2/3},\ \gamma \simeq n_i \xi^2 f_{pin}^2$$

vortex dynamics

Three regimes of vortex motion:



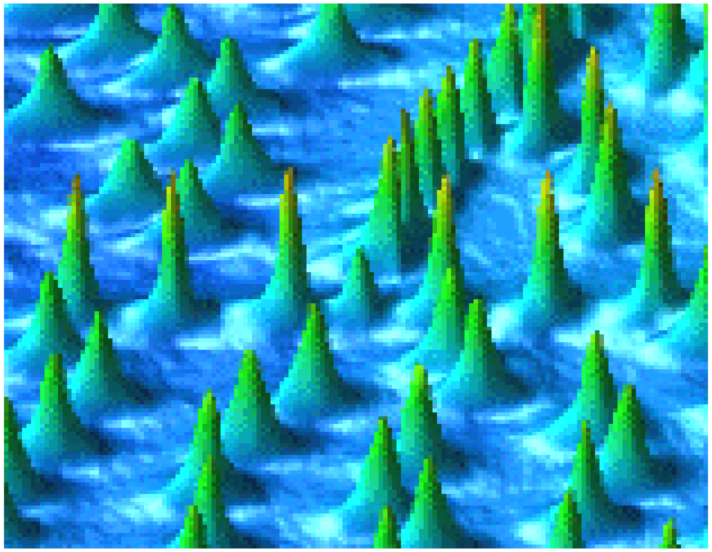
T. Nattermann and S. Scheidl, Vortex glass phases in type II superconductors, Advances in Physics, 2000, vol. 49, No. 5, 607, 704

We consider creep regime

- On n'est pas sérieux, quand on a dix-sept ans
  Et qu'on a des tilleuls verts sur la promenade.

(One isn't serious when one is seventeen,
And when there are green lime-trees on one's
promenade.)

Arthur Rimbaud



Linear response

Experiments show that if a *force* $\vec{F}$ is imposed to a system, its response is a *current* $\vec{j}$ vanishing as the force vanishes. Thus for $\vec{F}$ small

$$\vec{j} = L \cdot \vec{F} + O(\vec{F}^2),$$

Here L is a matrix of *transport cœfficients*.

$$X_a = -\frac{\partial S}{\partial x_a}$$

$$\frac{\partial x_a}{\partial t} = -\sum_b \gamma_{ab} X_b$$

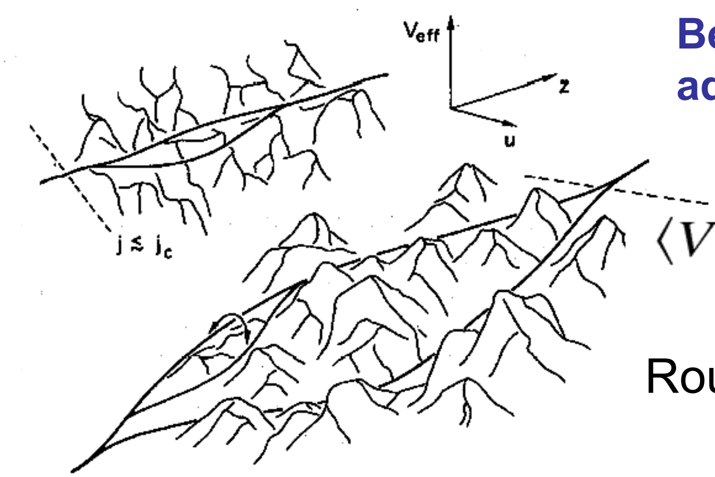
Examples:

1. FOURIER's law: a temperature gradient produces a heat current $\vec{j}_{heat} = -\lambda \vec{\nabla} T$.
2. OHM's law: a potential gradient (electric field) produces an electric current $\vec{j}_{el} = -\sigma \vec{\nabla} V$.
3. FICK's law: a density gradient produce a flow of matter $\vec{j}_{matter} = -\kappa \vec{\nabla} \rho$.

For vortices: $v = \mu F$

find μ ?

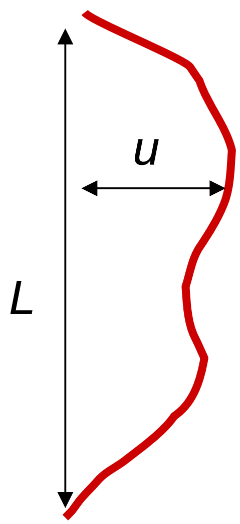
Being placed in disordered medium elastic object adjusts itself to rugged potential relief



$$\langle V(\mathbf{x},\mathbf{u})V(\mathbf{x}',\mathbf{u}')\rangle = \Delta^2\delta^D(\mathbf{x}-\mathbf{x}')f(|\mathbf{u}-\mathbf{u}'|/\xi)$$

Roughness: $w(L) = \langle[\mathbf{u}(\mathbf{x}+\mathbf{L})-\mathbf{u}(\mathbf{x})]^2\rangle^{1/2}$

On the intermediate scales where $u < a_0$



$$w \sim \xi\left(\frac{L}{L_c}\right)^{\varsigma}$$

$\zeta < 1$ is the roughness exponent

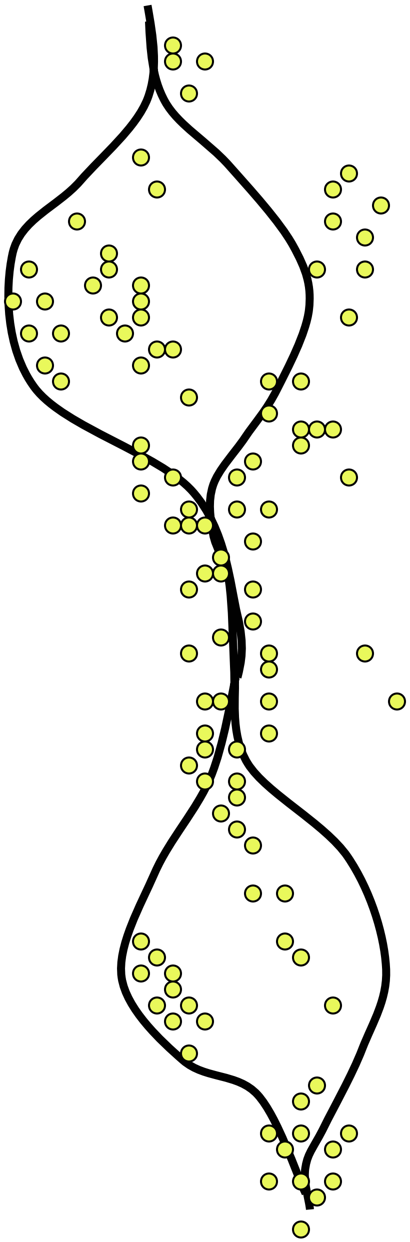
$$E_{barrier} = E_p\left(\frac{L}{L_c}\right)^{\chi} f\left(\frac{x-x'}{w(L)}\right)$$

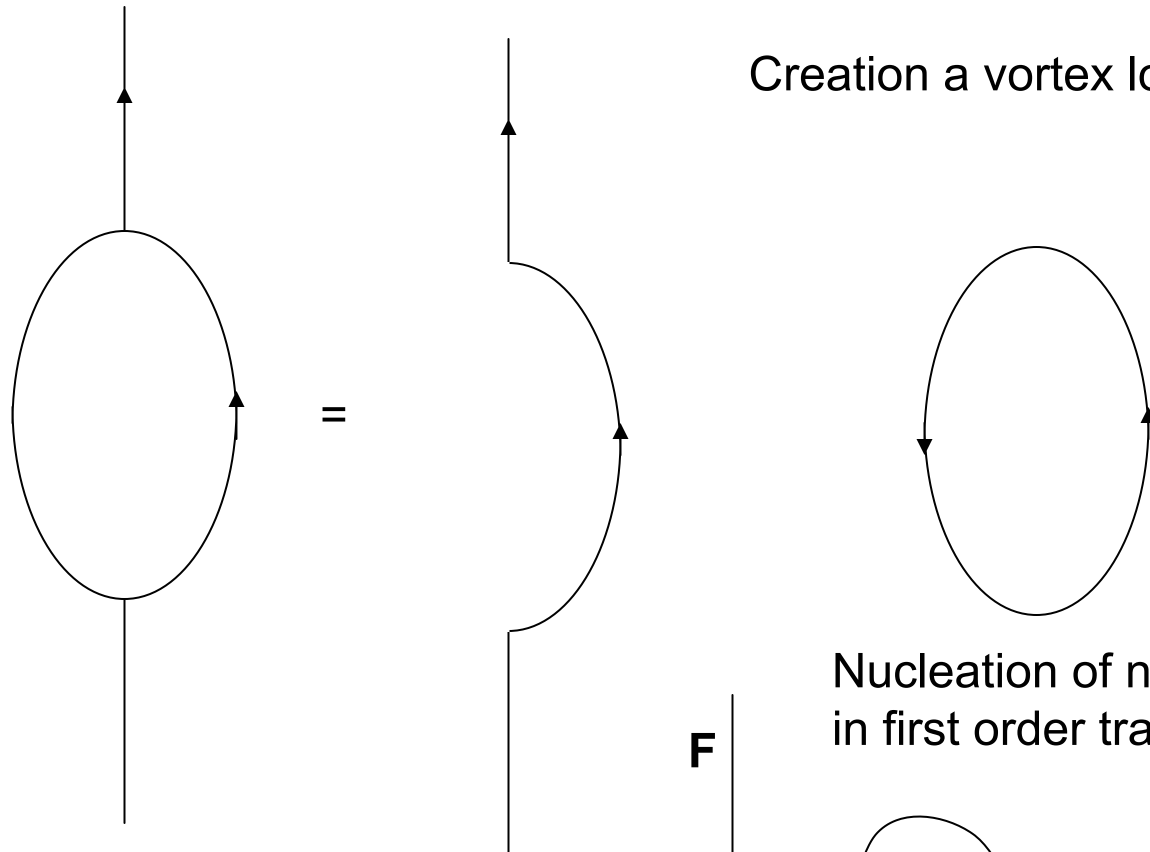
$$L_c = (C\xi^2/\Delta)^{2/(4-D)}$$

$$E_p = a_0\Delta L_c^{D-2}$$

$$\chi = D-2+2\zeta$$

Thermally activated vortex dynamics in random media

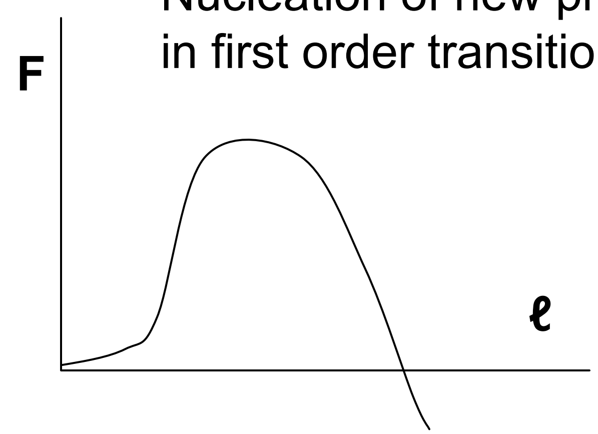




Activated vortex motion:

Creation a vortex loop

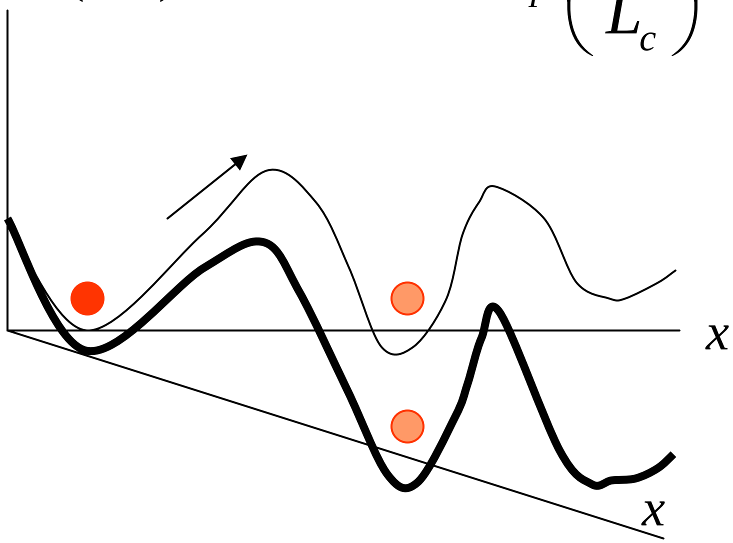
Nucleation of new phase
in first order transitions



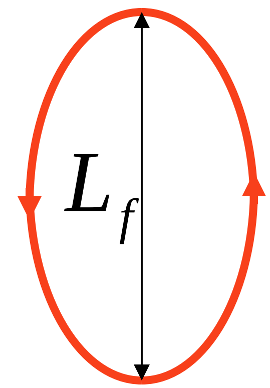
Driving force *f* :

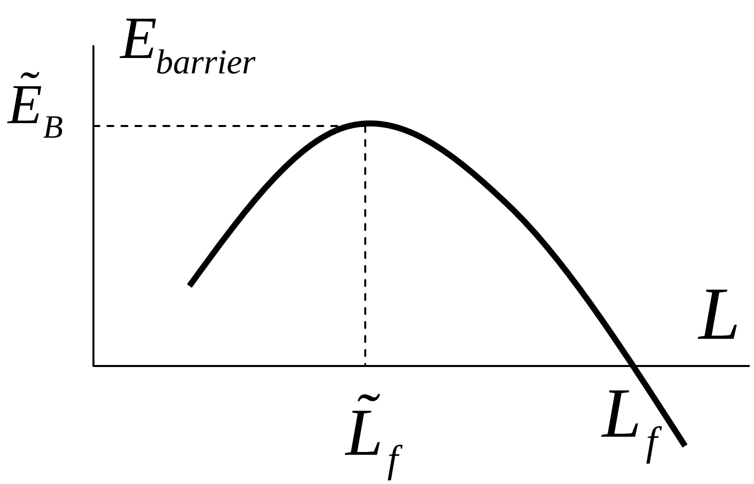
$$E^{(0)}_{barrier} = E_p \left(\frac{L}{L_c}\right)^{\chi}$$

$$E_{barrier} \simeq E_p \left(\frac{L}{L_c}\right)^{\chi} - w(L)\, f L^D = E_p \left(\frac{L}{L_c}\right)^{\chi} \left[1 - \left(\frac{L}{L_f}\right)^{2-\zeta}\right]$$



$$L_f = L_c \left(\frac{f_p}{f}\right)^{1/(2-\varsigma)}$$





Meaning of L_f :

At distances $L > L_f$

pinning is not effective

$$E_{barrier}(L_f) \equiv \tilde{E}_B = E_p \left(\frac{f_p}{f}\right)^{\mu}$$

Thermally activated dynamics of random systems:

Governed by the wide distribution of barriers

time to overcome barrier E:

$$\tau \simeq \tau_0 \exp\left(\frac{E}{T}\right)$$

Basic law of relaxation in random (glassy) systems:

$$E \simeq T \ln(\tau / \tau_0)$$

Consequences of general law of relaxation: CREEP

$$\tilde{E}_B = E_p \left(\frac{f_p}{f} \right)^{\mu} \simeq T \cdot \ln(\tau/\tau_0)$$

Motion under Constant force:

$$v \sim \tau^{-1}$$

$$v \sim \exp\left[-\frac{E_p}{T} \left(\frac{f_p(T)}{f} \right)^{\mu} \right]$$

$$\mu = \frac{\chi}{2-\varsigma}$$

No linear response!

Current (magnetization relaxation:

$$f = \frac{n_v \Phi_0}{c} J$$

$$\Rightarrow$$

$$J(t) \sim \frac{J_0}{\left[\ln(t/\tau_0)\right]^{1/\mu}}$$

O Snail,
Climb Fuji slope, slowly, slowly
Up to the top…
Matsuo Bashō

Experiment Unambiguous proof: divergent barriers: $E \propto J^{-\mu}$

PHYSICAL REVIEW B VOLUME 42, NUMBER 4 1 AUGUST 1990

Dependence of flux-creep activation energy upon current density in grain-aligned $YBa_2Cu_3O_{7-x}$

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(Received 2 April 1990; revised manuscript received 21 May 1990)

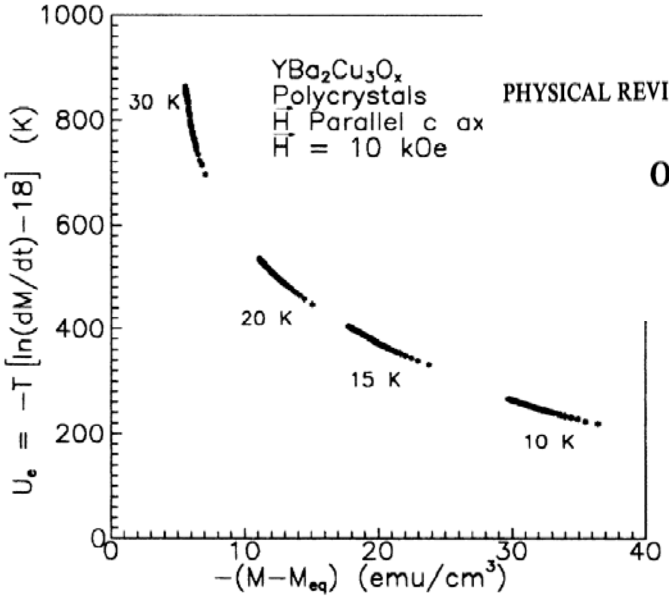

FIG. 4. (a) Plots of nonthermally cycled data: $T\ln(dM/dt)$ vs $M-M_{eq}$ for temperatures 10, 15, 20, and 30 K. (b) Plots of the same data shown in (a) with 18 T added to each data set where T is the temperature. As discussed in the text, this represents U_e vs $M-M_{eq}$.

RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 47, NUMBER 9 1 MARCH 1993-I

Observation of flux creep through columnar defects in $YBa_2Cu_3O_7$ crystals

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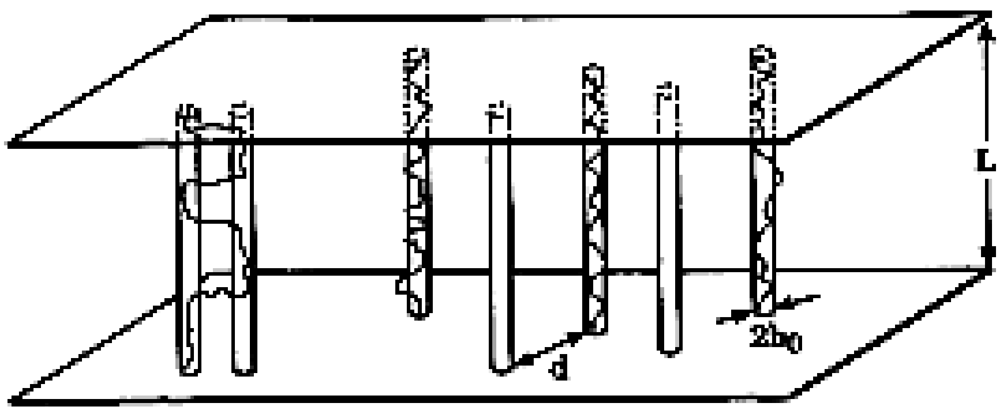
et al

BOSE GLASS:
VORTICES + COLUMNAR DEFECTS

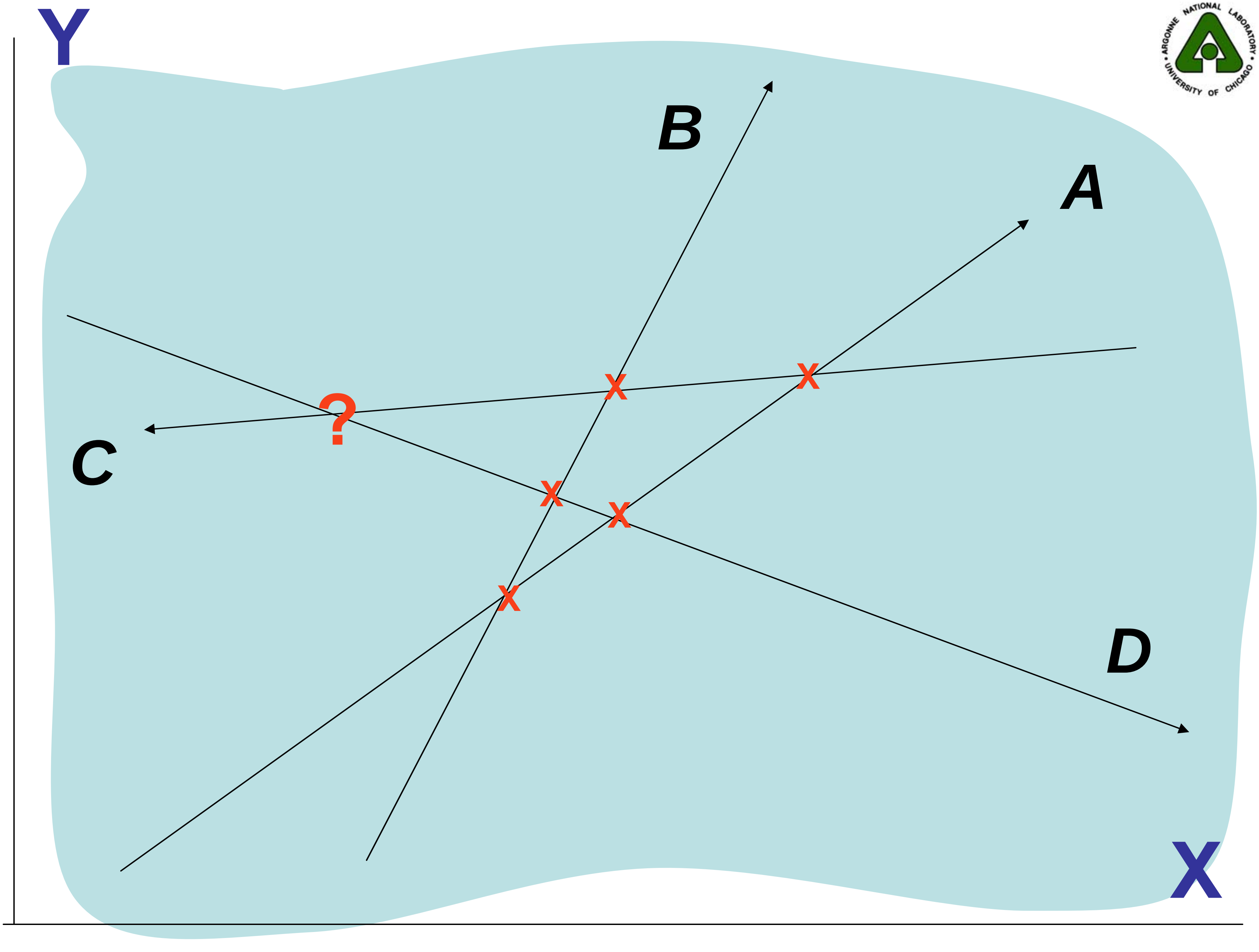


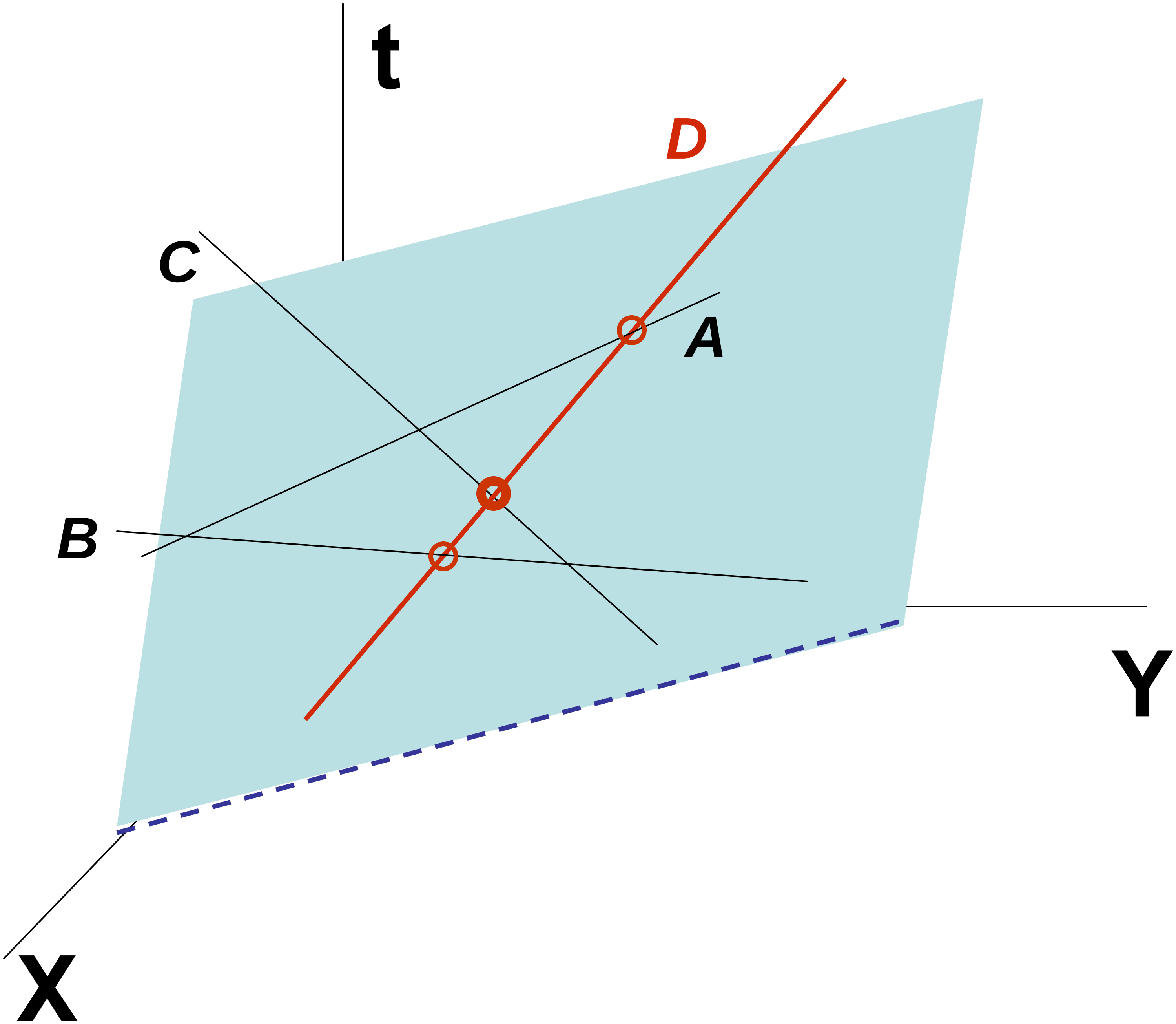
A PROBLEM:

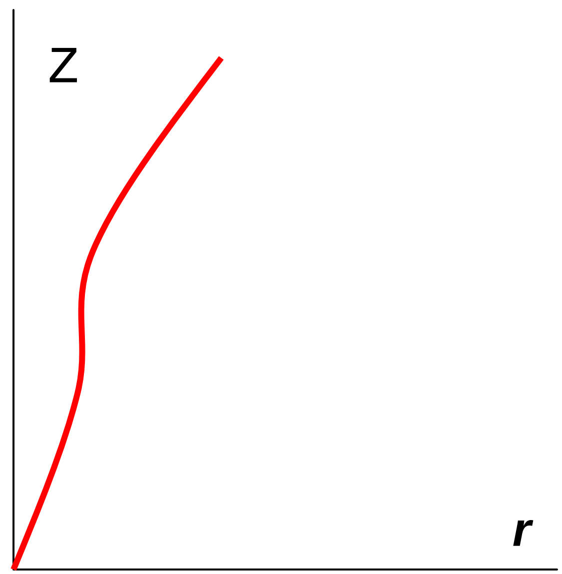
Four ships,

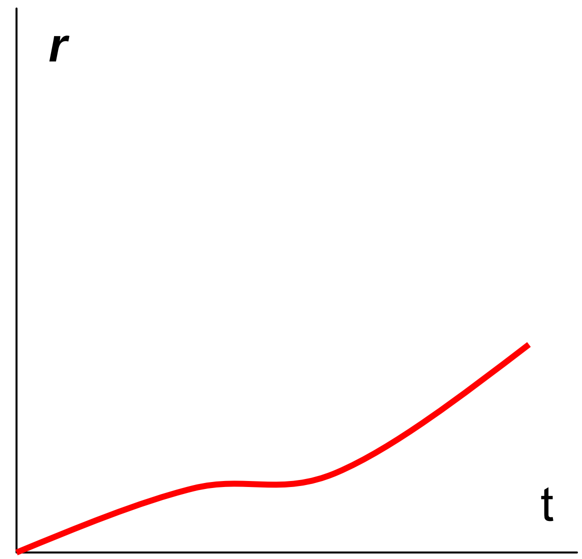
sail over the sea. The paths are straight lines and all the velocities are different. Ship *A* 'collided' with ships *B*, *C*, and *D; B* collided with *C* and *D* (triple collisons excluded).

Prove that *C* collided (or will collide) with *D*


Y
B
A
C
?
X
X
X
X
X
D
X


t
D
C
A
B
Y
X





$$\exp\left[-\frac{1}{T}\int dz\,\frac{\varepsilon_\ell}{2}\left(\frac{dr}{dz}\right)^2\right]$$

$$\exp\left[\frac{i}{\hbar}\int dt\,\frac{m}{2}\left(\frac{dr}{dt}\right)^2\right]$$

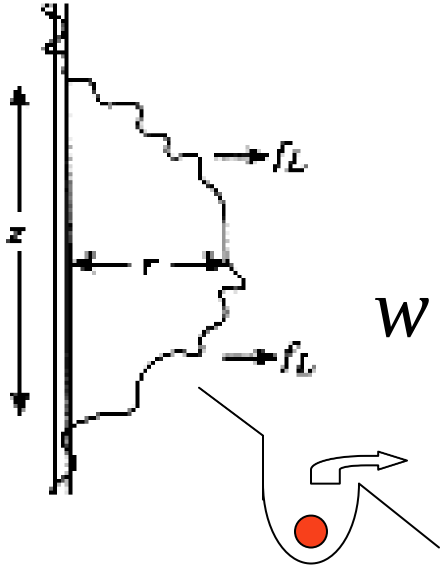
$$F_N = \int dz\left[\frac{\varepsilon_1}{2}\sum_i\left(\frac{dr_i}{dz}\right)^2 + U(r_i) + \sum_{i<j} V(r_{ij})\right]$$

Quantum mechanical mapping

Vortex trajectories can be mapped onto world lines of the 2D Bosons

TABLE I. Boson analogy applied to vortex transport.

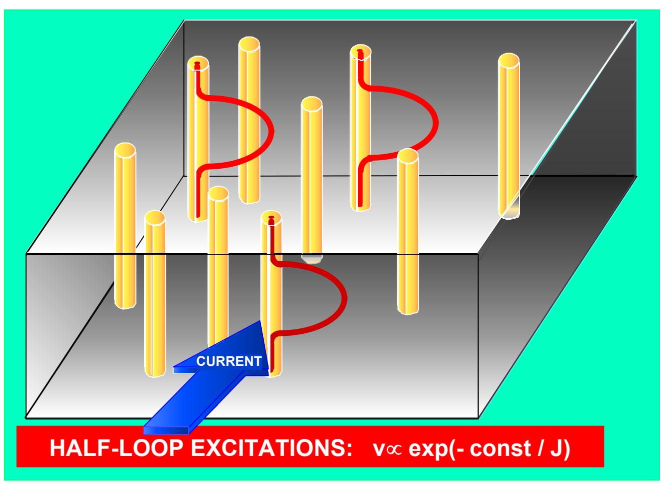
Charged bosons	Mass	$\hbar$	$\beta\hbar$	Pair potential	Charge	Electric field	Current
Superconducting Vortices	$\tilde{\varepsilon}_1$	T	L	$2\varepsilon_0 K_0(r/\lambda_{ab})$	ϕ_0	$\frac{\hat{\mathbf{z}}\times\mathbf{J}}{c}$	$\mathscr{E}$



$$w \sim \exp\left(-\frac{E_k}{T}\frac{J_1}{J}\right) \equiv \exp\left(-\frac{4\sqrt{2}}{3}\frac{c\varepsilon_l^{1/2}U_0^{3/2}}{\Phi_0 J}\right)$$

TABLE I. Boson analogy applied to vortex transport.

Charged bosons	Mass	$\hbar$	$\beta\hbar$	Pair potential	Charge	Electric field	Current
Superconducting Vortices	ε_l	T	L	$2\varepsilon_0 K_0(r/\lambda_{ab})$	ϕ_0	$\frac{\hat{z}\times \mathbf{J}}{c}$	$\mathcal{E}$



$$w \simeq \exp\left(-\frac{4\sqrt{2}}{3}\frac{m^{1/2}|U|^{3/2}}{\hbar|e|E}\right)$$

Variable range vortex hopping

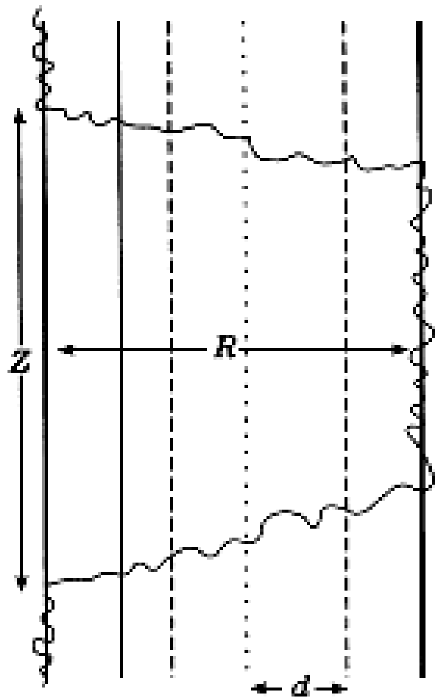


FIG. 15. Double-superkink configuration required for variable-range hopping. The "tongue" of vortex line seeks out a compatible low-energy pin so that the line can spread.

$$\rho \sim \exp\left[-\frac{E_k}{T}\left(\frac{J_0}{J}\right)^{1/3}\right]$$

Variable range hopping
(Shklovskii formula for semiconductors)

BOSE GLASS DYNAMICS

PHYSICAL REVIEW B VOLUME 51, NUMBER 6 1 FEBRUARY 1995-II

Experimental evidence for Bose-glass behavior in $Bi_2Sr_2CaCu_2O_8$ crystals with columnar defects

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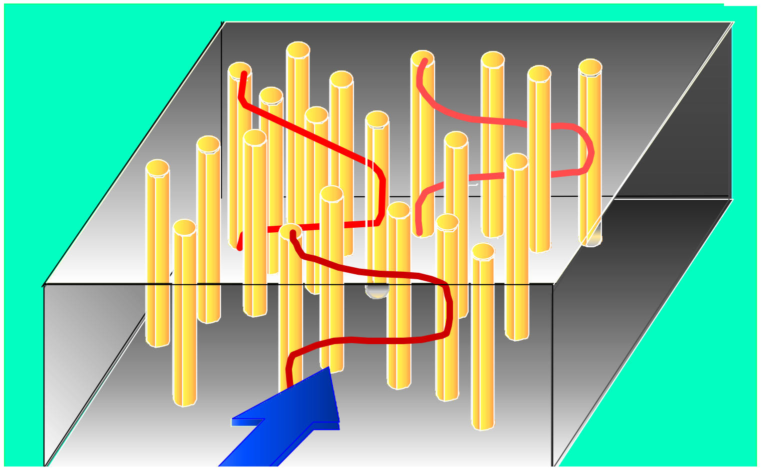
et al



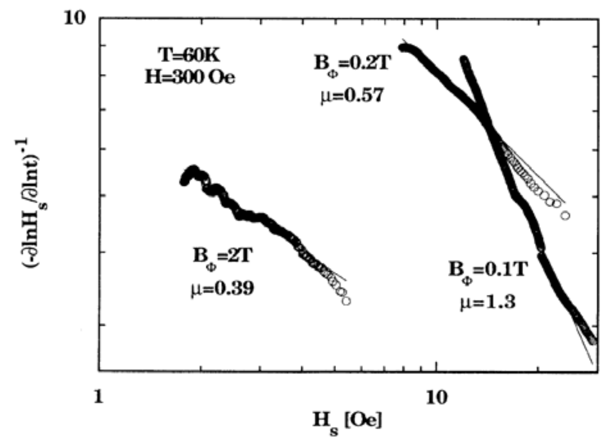


FIG. 4. Energy barrier for flux creep ($1/S$) vs stray field H_s recorded at 300 Oe and 60 K on BSCCO:2212 samples irradiated to various doses.

VOLUME 78, NUMBER 16 PHYSICAL REVIEW LETTERS 21 APRIL 1

Superfast Vortex Creep in $YBa_2Cu_3O_{7-\delta}$ Crystals with Columnar Defects: Evidence for Variable-Range Vortex Hopping

J. R. Thompson,[1] L. Krusin-Elbaum,[2] L. Civale,[3] G. Blatter,[4] and C. Feild[2]

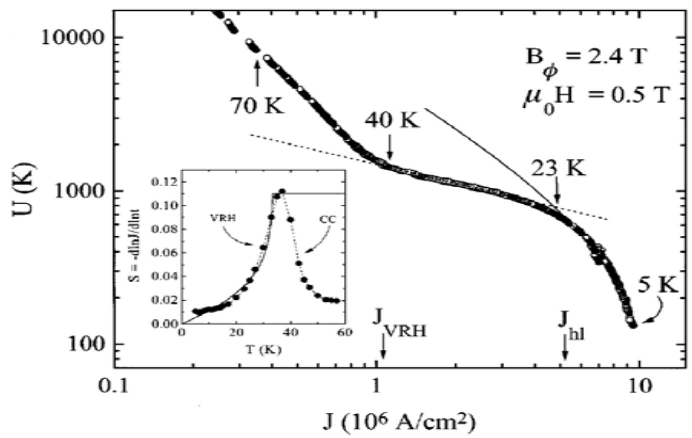
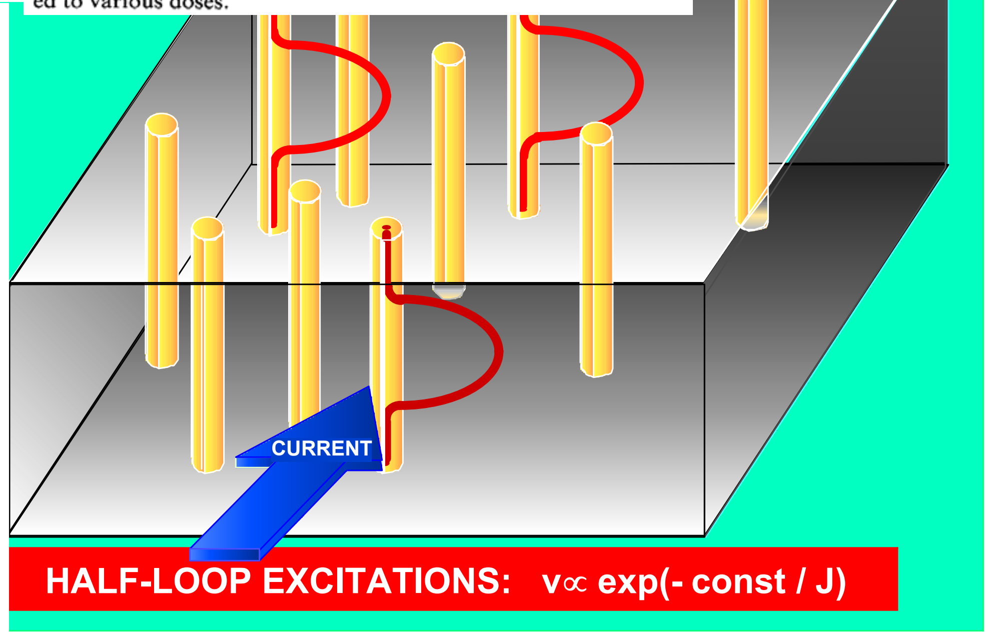


FIG. 4. $U(J)$ for the $B_\Phi = 2.4$ T crystal in a 0.5 T magnetic field. The solid line is the fit to the full glassy expression for $U(J)$ (see text) with $\mu \simeq 1$. The slope $\mu \sim 1/3$ (dashed line) fits the data well between 23 and 40 K. Crossover currents are indicated by the arrows. Inset: Fit to variable-range hopping [Eq. (3)] with $\mu = 1/3$ (see text) is shown as the solid line. The decreased rate on the high-T side of the peak is due to slower creep in the collective regime.



Creep is a general phenomenon

VOLUME 80, NUMBER 4 PHYSICAL REVIEW LETTERS 26 JANUARY 1998

Domain Wall Creep in an Ising Ultrathin Magnetic Film

S. Lemerle,[1] J. Ferré,[1] C. Chappert,[2] V. Mathet,[2] T. Giamarchi,[1] and P. Le Doussal[3]

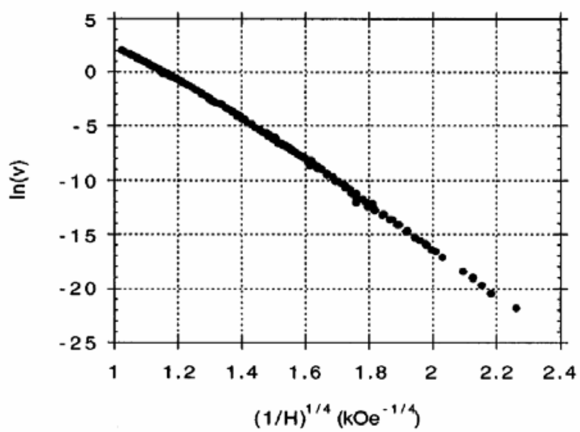


FIG. 3. Natural logarithm of MDW velocity as a function of $(1/H)^{1/4}$ (room temperature, $H \leq 955$ Oe).

VOLUME 89, NUMBER 9 PHYSICAL REVIEW LETTERS 26 AUGUST 2002

Domain Wall Creep in Epitaxial Ferroelectric $Pb(Zr_{0.2}Ti_{0.8})O_3$ Thin Films

T. Tybell,[1,2] P. Paruch,[1] T. Giamarchi,[3] and J.-M. Triscone[1]

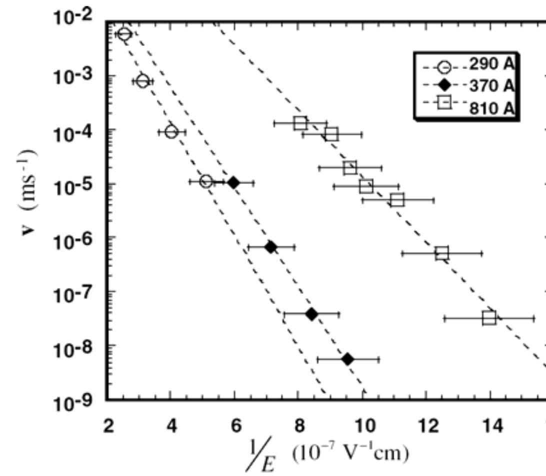
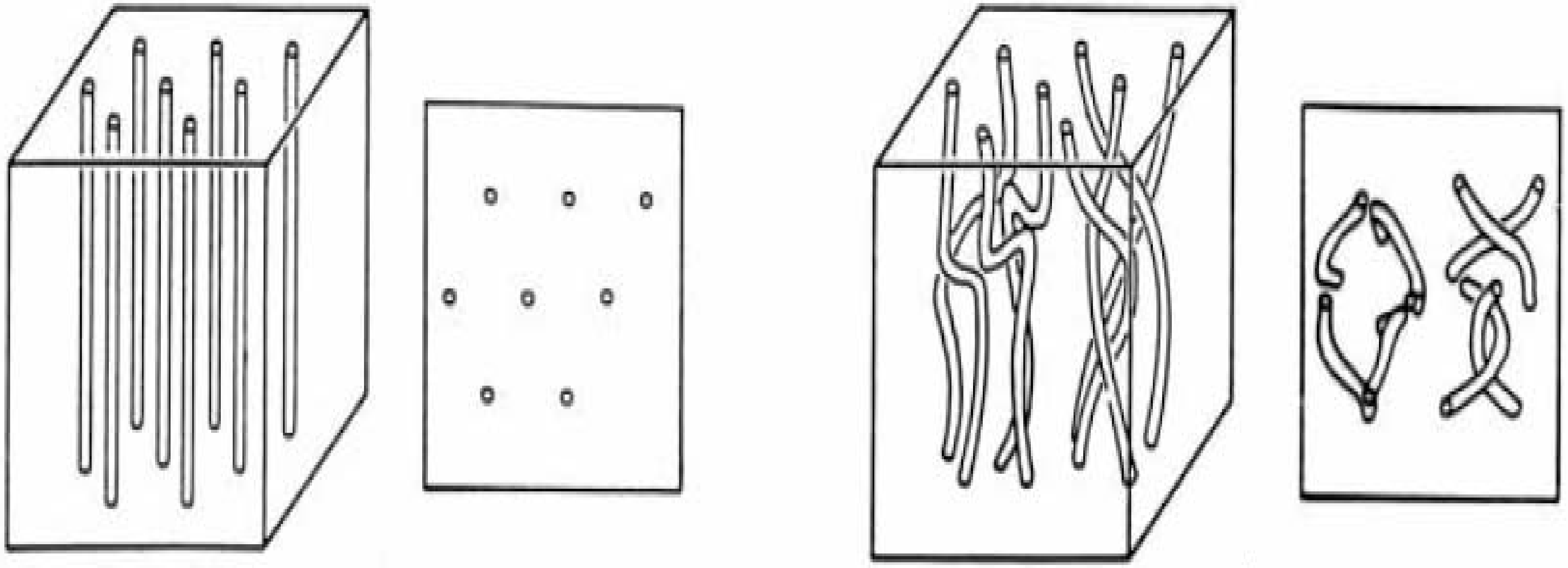


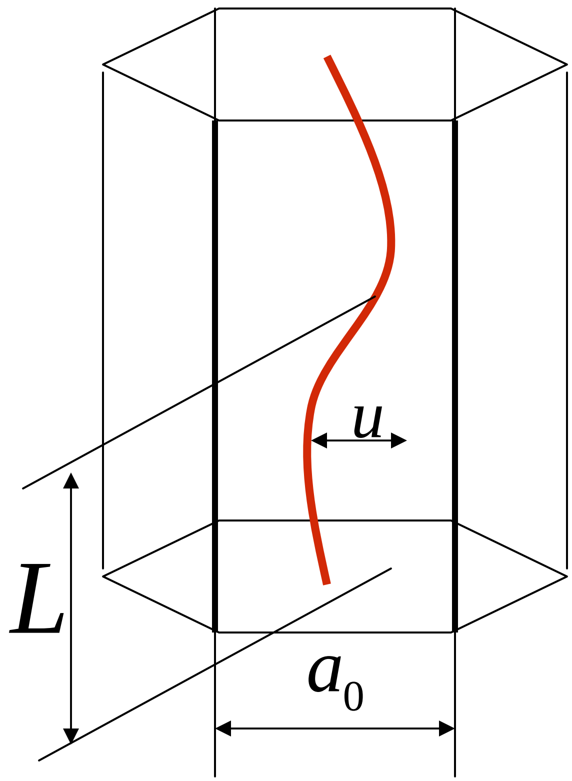
FIG. 3. Domain wall speed as a function of the inverse applied electric field for 290, 370, and 810 Å thick samples. The data fit well to $v \sim \exp[-\frac{R}{k_B T}(\frac{E_0}{E})^{\mu}]$ with $\mu = 1$, characteristic of a creep process.

Other random systems?
All of them?

Vortex lattice melting



Vortex lattice (1957)　　Vortex liquid (1988)



Mean-field (cage) approach

$C_{66}u^2L$ - elastic energy to deform vortex in a cage

T - energy of thermal fluctuations

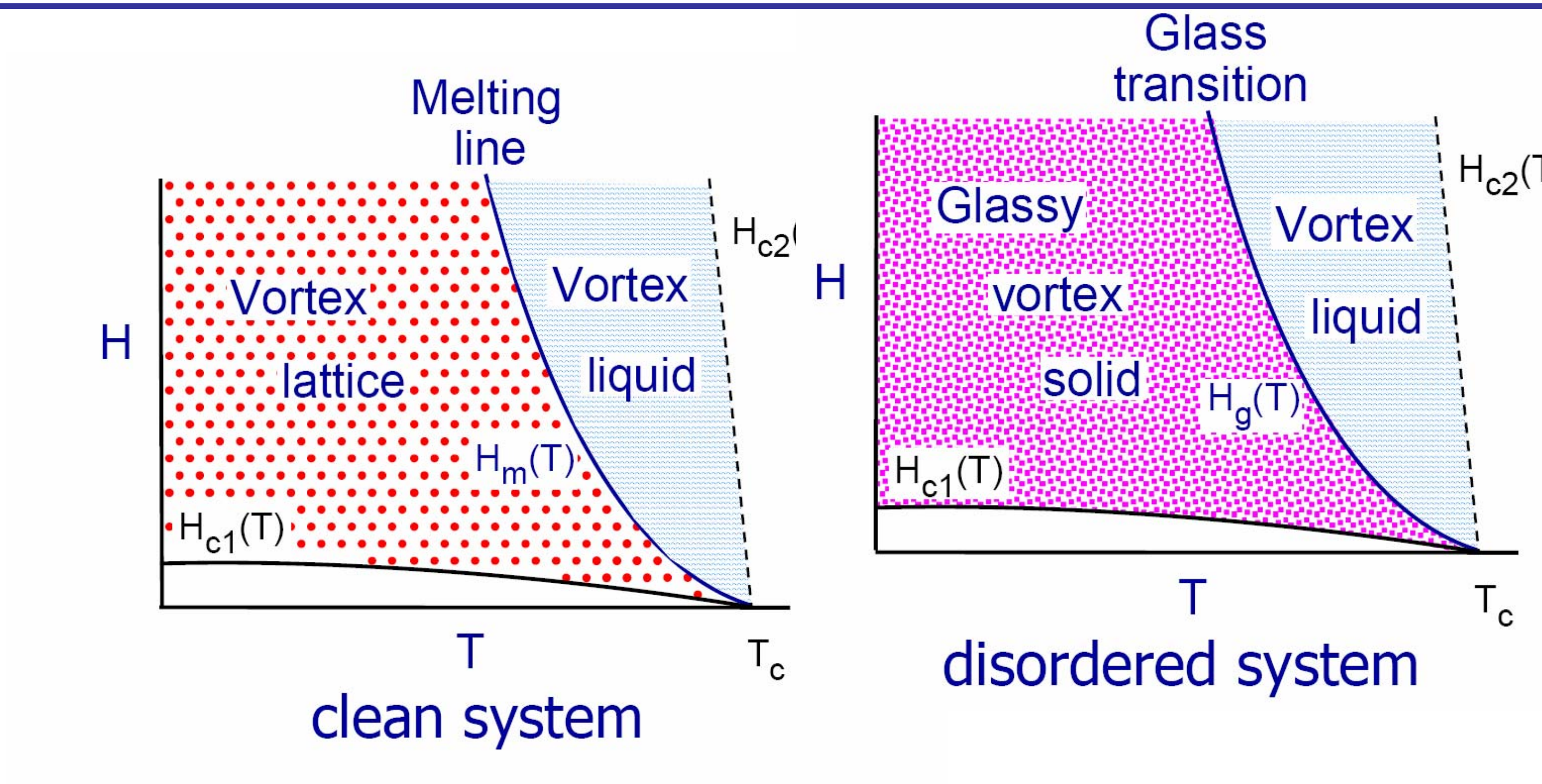
$$C_{66}u^2L \simeq \varepsilon_\ell \frac{u^2}{L}, \quad C_{66} \approx \varepsilon_\ell / a_0^2 \Longrightarrow L \approx a_0$$

Energy balance: $C_{66}u^2a_0 \simeq T$

Melting condition: $C_{66}a_0^3 \simeq T \Longrightarrow u \simeq c_L\, a_0$

$$c_L \approx 0.1 \div 0.2$$

melting and disorder

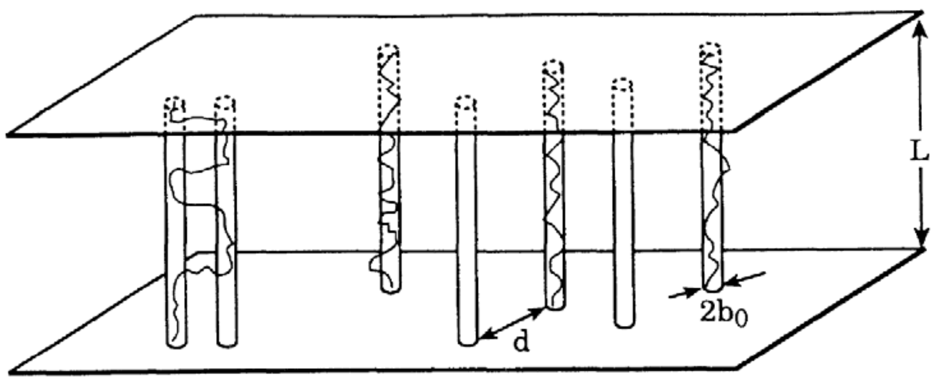


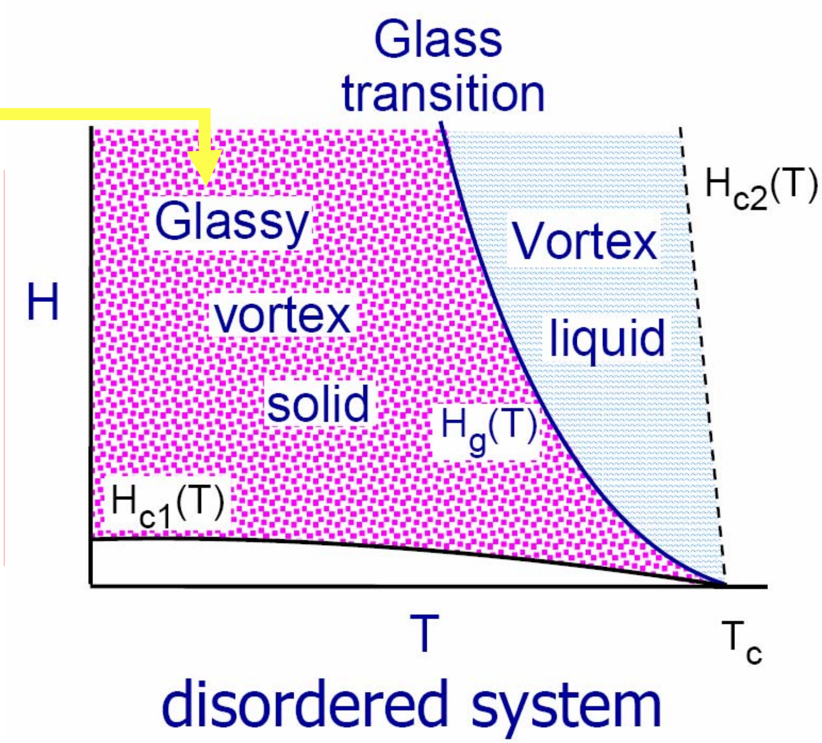
Weak disorder does not influence melting much since disorder becomes relevant on the scales of the order L_c, and melting characteristic distances are a_0 , and

$$L_c \gg a_0$$

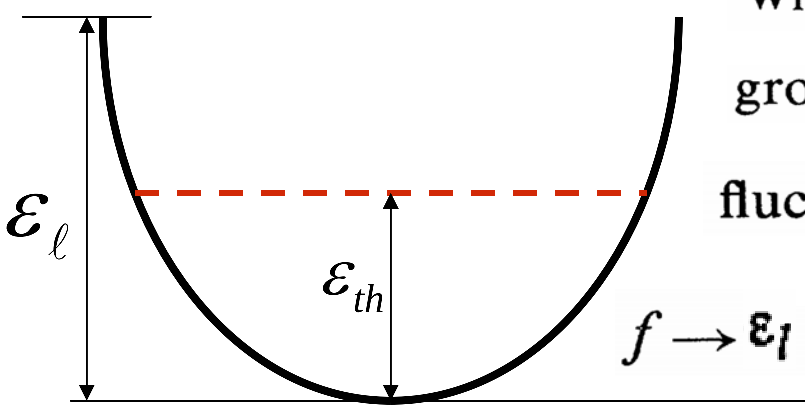
?

Bose glass transition and vortex localization





Energy Lindemann criterion



Within a harmonic approximation $[V(R)=fR^2/2]$

ground-state energy: $\hbar\omega/2$ $\quad \omega=\sqrt{f/m}$

fluctuation amplitude $\langle R^2\rangle \simeq \hbar\omega/f$

$f \to \varepsilon_l/a_0^2, \quad m \to \varepsilon_l, \quad \hbar \to T, \quad \omega \to 1/a_0$

$$\varepsilon_{th} \simeq \frac{T}{a_0} \qquad \left\langle u_{th}^2 \right\rangle \simeq \frac{Ta_0}{\varepsilon_\ell}$$

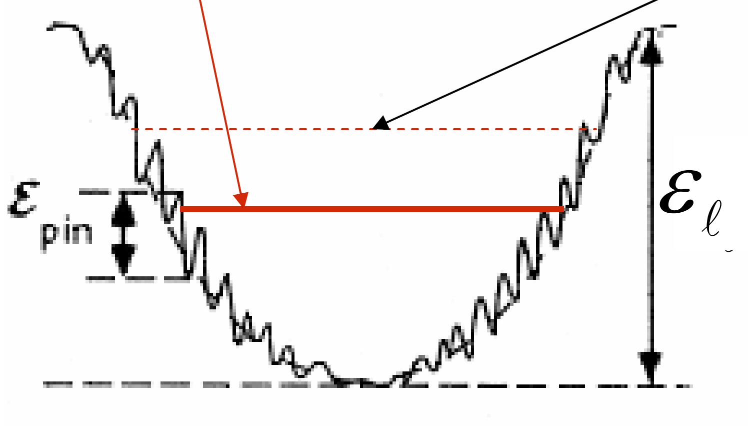
Lindemann criterion: $\left\langle u_{th}^2 \right\rangle = c_L^2 a_0^2 \Longrightarrow T_m = c_L^2 \varepsilon_\ell a_0$

Energy criterion: $\varepsilon_{th}(T_m) = c_L^2 \varepsilon_\ell \Longrightarrow T_m = c_L^2 \varepsilon_\ell a_0$

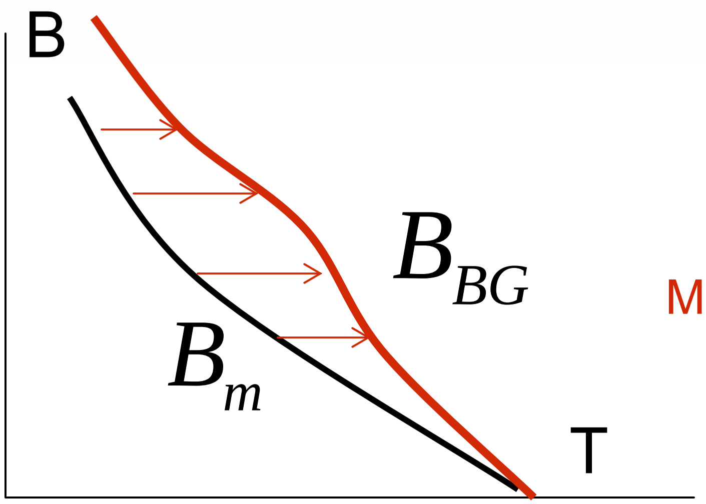
Shift of the melting line due to columnar defects

Bound state in the rippled parabolic well

Bound state in the parabolic well



In the first order with respect to disorder the correction to the energy is zero (after averaging with respect to disorder). Thus the first correction appears in the second order of perturbation theory. But the second order correction is always negative. Therefore, the bound state in the rippled well is more deep.



Melting line shifts upwards

Is that all?

Bose glass melting vs. delocalization process

Formation of the Bose glass phase is equivalent to localization of 2D quantum particles in the random field of point defects. Melting into a liquid phase corresponds to the delocalization effect

Then little Gerda shed burning tears; and they … thawed the lumps of ice, and consumed the splinters of the Bose-glass…

Hans Christian Andersen, The Snow Queen

c_{44} (tilt modulus) is related to the superfluid density of bosons

$$c_{44} = \left(B^2 / 4\pi\right)\left[1 + (4\pi n_s \lambda\ )^{-1}\right]$$

Bose glass transition takes place at *$n_s = 0$*

Consider vortex liquid state → equivalent to superfluid state of 2D bosons

Disorder-induced depletion of superfluid density ⟷ Stiffening of the tilt modulus

$$n_s = n - 4n_2/3$$

$$n_2 = \frac{\kappa}{4\pi}\frac{a^2 m^{3/2}}{\sqrt{\mu}}$$

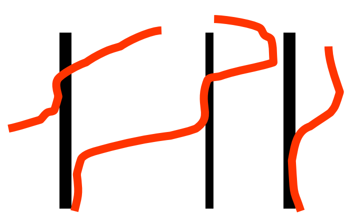
$$c_{44} \simeq (B^2/4\pi)[1 + (4\pi\lambda^2 n_s)^{-1}]$$

$$\frac{1}{c_{44}^{vR}} = \frac{1}{c_{44}^{v}} - \frac{T^4 n \Delta_1}{(c_{44}^{v})^2 \epsilon_1}\int \frac{d^2q\, q^4}{\varepsilon^4(q)}$$

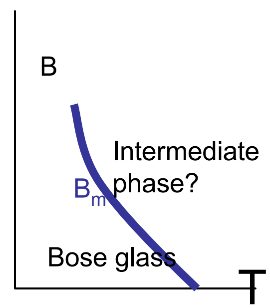
⟷

Any disorder reduces n_s.

May be interpreted as the fact that some fraction of the vortices is always localized (partially pinned)



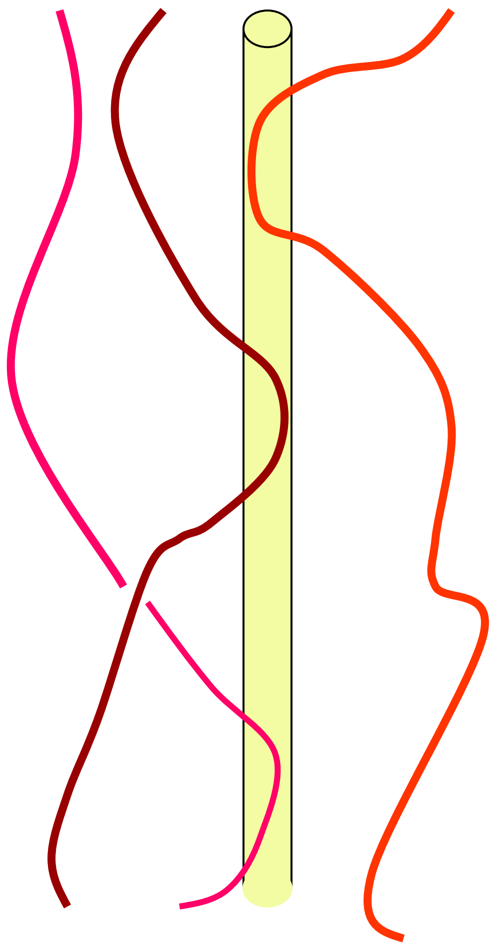
Intermediate vortex state liquid+pinned?



What happens to pinned vortices as we raise temperature?

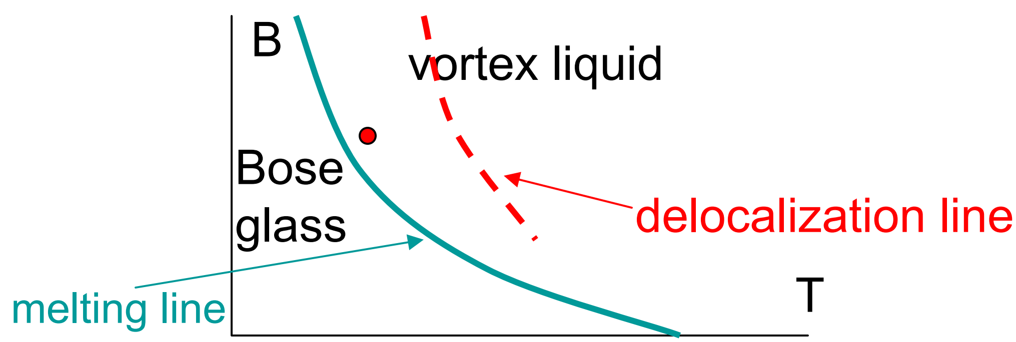
There are always bound states in 2D, therefore one vortex is always pinned by one columnar defect.

What happens if there are many interacting vortices (take high enough temperatures where binding is exponentially weak)?



Naïve picture:
Vortices wander freely and screen each other out from columnar defect. Thus, if the defect potential is not sufficiently strong, vortices may get depinned.

Increasing temperature effectively suppresses pinning potential. Therefore, by increasing temperature, one can depin vortices from columnar defects



Calculations: finding the occupation number *n* for the columnar defects

Hamiltonian of bosons:

$$\hat{H} = \int d^2 r\, \psi^+ \left[p^2/2m - \mu + U(r) \right] \psi + \int d^2 r_1\, d^2 r_2\, \psi^+(r_1)\psi(r_1)\, V(r_{12})\, \psi^+(r_2)\psi(r_2)$$

$$U(r) = \sum_i u(r - r_i)$$

Low defect concentration:
defects can be considered separately

$$u(r) \cong \varepsilon_0 \frac{r_0^2}{r^2 + 2\xi^2} \qquad \varepsilon_0 = \frac{\phi_0^2}{(4\pi\lambda)^2}$$

Single defect pinning energy: $E_1 \sim \dfrac{T^2}{\varepsilon_1 \xi^2} e^{-1/\sqrt{\beta}}$ $\quad \beta = \varepsilon_0 \varepsilon_1 r_0^2 / 4T^2$

Localization length: $\ell_\perp \sim \dfrac{T}{|E_1|\varepsilon_1}$

Now we use the basis of the exact eigenstates of the *noninteracting* problem:

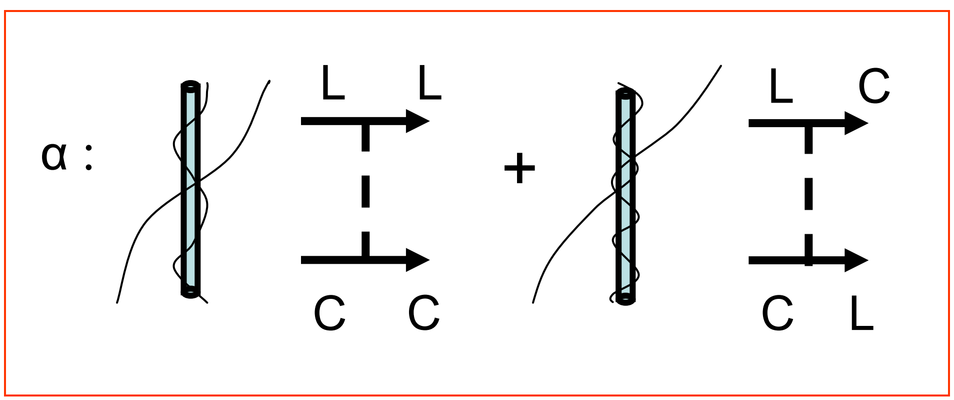
$$\psi = \varphi_0 \hat{b}_0 + \varphi_1 \hat{b}_1 + \sum_k \varphi_k \hat{b}_k$$

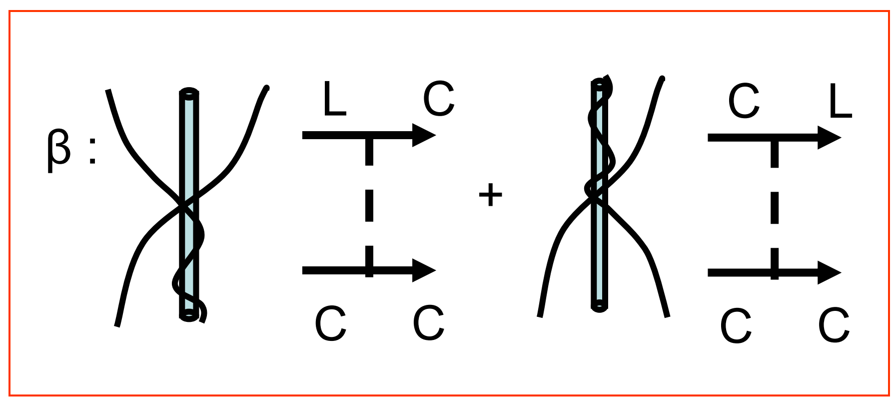
$\hat{b}_0$ – condensate, $\hat{b}_1$ – localized state, $\hat{b}_k$ – excitations

In case of strong vortex interaction double occupation is prohibited $\hat{b}^{+}\hat{b}^{+} = 0$

Effective model that describes occupation of the localized sates:

$$\hat{H}_{eff} = \hat{b}_1^{+}(E_1 + \alpha - \mu)\,\hat{b}_1 + \beta(\hat{b}_1^{+} + \hat{b}_1)$$





$$\psi = a_0|0\rangle + a_1|1\rangle \Longrightarrow E_\pm = \frac{1}{2}\left(E \pm \sqrt{E^2 + 4\alpha^2}\right)$$

$$E = E_1 + \alpha - \mu$$

Occupation of the lowest state: $n = a_1^2 = \dfrac{2\alpha^2}{4\alpha^2 + E^2 + E\sqrt{E^2 + 4\alpha^2}}$

$n \to 1$ when $E/\alpha \ll -1$ $\quad$ $n \to 0$ when $E/\alpha \gg 1$

At $E = 0$ localization-delocalization crossover occurs

$$E = E_1 + \frac{\mu}{v_0}\int d^2 r_1 d^2 r_2 \phi_1(r_1) v(r_1 - r_2)\phi_2(r_2)$$

$$v_0 = \int d^2 r v(r)$$

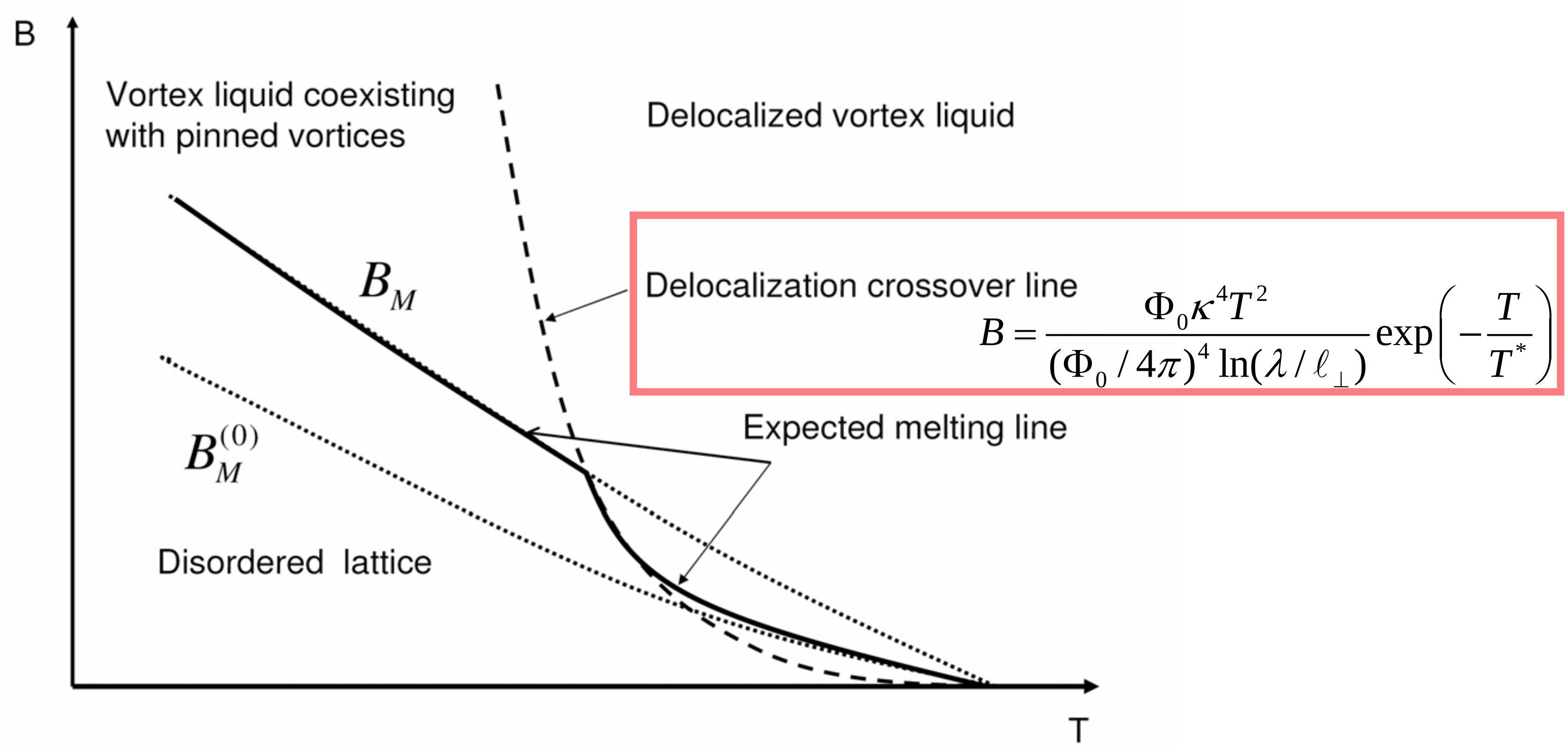
$$n_V \ell_\perp^2 \varepsilon_0 \ln(\lambda / \ell_\perp)$$

Localization-delocalization crossover:

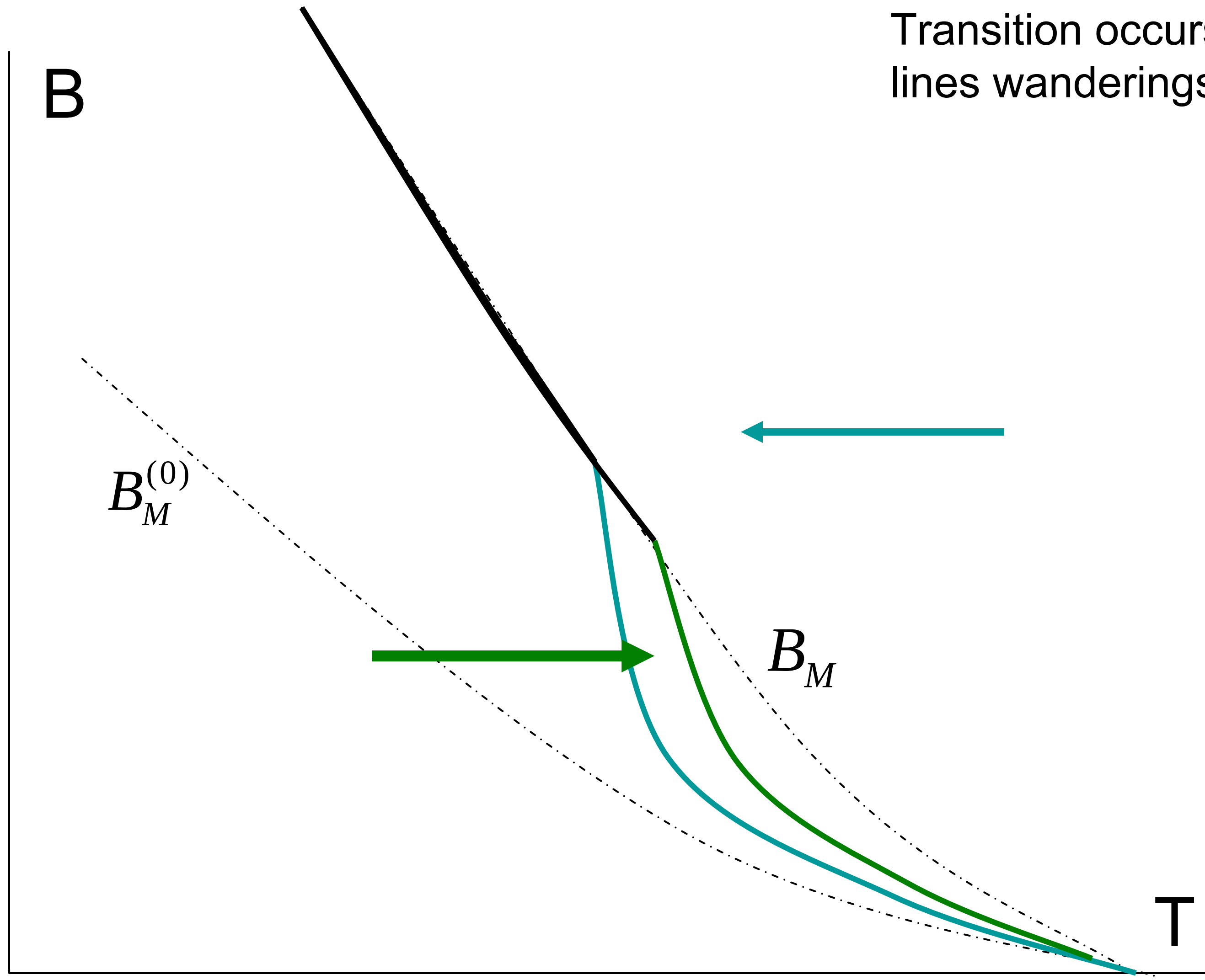
$$n_V \equiv \frac{B}{\Phi_0} \simeq \frac{T^2}{\varepsilon_0 \varepsilon_1 \ell_\perp^4 \ln(\lambda/\ell_\perp)} \simeq \frac{\kappa^4 T^2}{(\Phi_0/4\pi)^4 \ln(\lambda/\ell_\perp)} \exp\left(-\frac{T}{T^*}\right)$$

$$T^* = (1/8) r_0 \sqrt{\varepsilon_1 \varepsilon_0}$$

VORTEX PHASE DIAGRAM

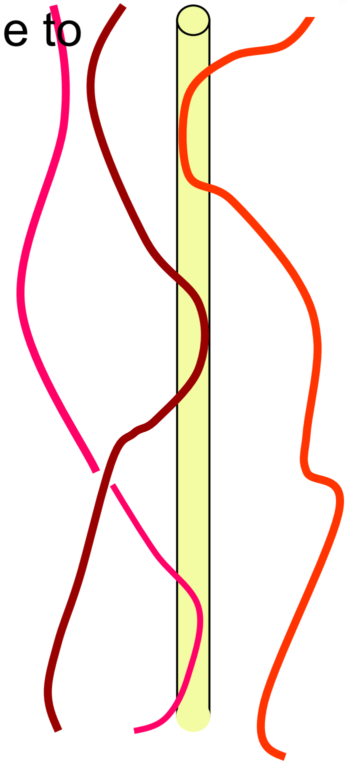


Schematic phase diagram for the vortex system with columnar defects. The melting line under assumption of pinning is denoted by B_M, while the melting line of the pristine lattice is denoted by $B_M^{(0)}$.



FIRST ORDER TRANSITION

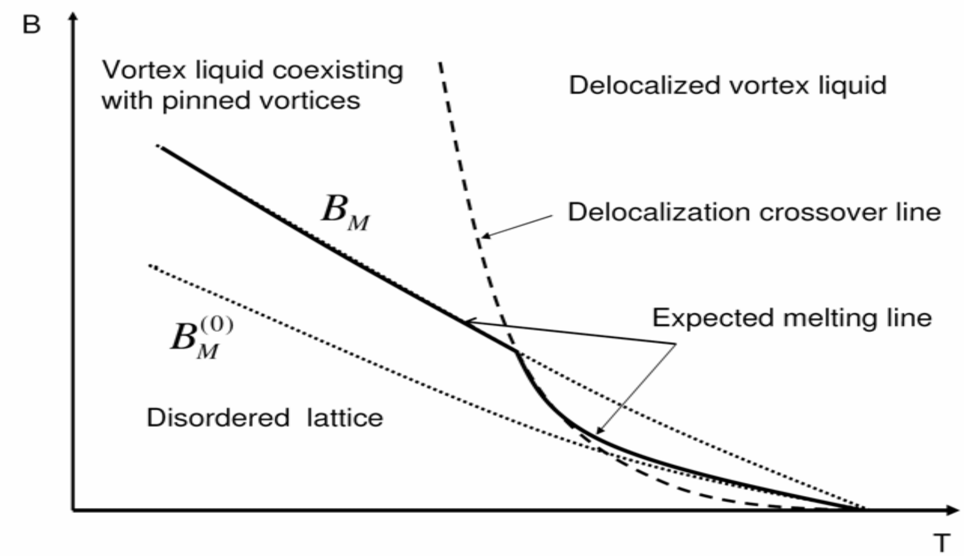
Transition occurs due to lines wanderings



In a liquid vortices fluctuate much easier, thus we can expect the transition to shift to higher temperatures, if we go from the solid side!

$$v \sim \exp\left[-\frac{E_p}{T}\left(\frac{f_p(T)}{f}\right)^{\mu}\right] \qquad \mu = \frac{\chi}{2-\varsigma}$$

Competing localization



Schematic phase diagram for the vortex system with columnar defects. The melting line under assumption of pinning is denoted by B_M, while the melting line of the pristine lattice is denoted by $B_M^{(0)}$.