HYP links for dynamical fermion simulations

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Domain Wall Fermions at Ten Years

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Outline

- Why fat links?
- HYP links suitable for dynamical fermion simulations
- difference to standard HYP: Projection of U(3) instead of SU(3)
- tests for dynamical clover Wilson and overlap

Why fat links?

short range fluctuations of the gauge field make fermions expensive

- Wilson: exceptional configurations, phase structure
- staggered: taste breaking
- overlap: cost of construction (low modes of the kernel operator)
- DWF: explicit chiral symmetry breaking

Working hypothesis:
large cut-off effects in fermion sector due to dislocations

Improved gauge actions

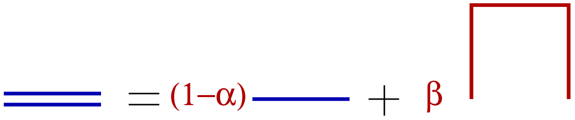
Possible cure: Improved gauge actions (DBW2, Iwasaki)

- suppress short range fluctuations
- possibly large cut-off effects from gauge sector
- large auto-correlation time in MD simulations
  need dislocations to change topology

part of the solution

Fat links

construct Dirac operator from sum over extended paths



- less vulnerable to short range fluctuations
- examples: APE, Asqtad, stout
- iterate to make it more efficient
- large spatial extend can cause large cut-off effects
- improves scaling (if not over-done)

Main building blocks

APE Albanese et al'87

$$\widetilde{U}_\mu(x) = \mathrm{Proj}_{SU(3)}\left[(1-\alpha)U_\mu(x) + \frac{\alpha}{6}V_\mu(x)\right]$$

- Projection not easily differentiable $\rightarrow$ not usable in MD

STOUT Morningstar, Peardon'04

$$\widetilde{U}_\mu(x) = \exp(iQ_\mu)U_\mu(x)\ ;\ Q_\mu(x) = \left[\frac{\rho}{i}V_\mu(x)U_\mu^+(x)\right]_{\mathrm{TH}}$$

- differentiable everywhere
- $6\rho \longleftrightarrow \alpha$

Main building blocks

n-APE

$$\widetilde{U}_\mu(x) = \mathrm{Proj}_{U(3)} \left[(1-\alpha) U_\mu(x) + \frac{\alpha}{6} V_\mu(x) \right]$$

$$\mathrm{Proj}_{U(3)} A = A (A^\dagger A)^{-1/2}$$

- differentiable everywhere if A non-singular
  $\Longrightarrow$no problem in practice
- projection has been used in the past: Kentucky'93, FLIC, Narayanan&Neuberger'06
- force term can be computed exactly (à la stout)
- the projection costs about the same as stout smearing

HYP links

Hasenfratz, Knechtli'01

$$
\begin{aligned}
V_{n,\mu} &= \mathrm{Proj}_{SU(3)}\Big[(1-\alpha_1)U_{n,\mu} + \frac{\alpha_1}{6}\sum_{\pm\nu\neq\mu} \widetilde{V}_{n,\nu;\mu}\,\widetilde{V}_{n+\hat{\nu},\mu;\nu}\,\widetilde{V}^{\dagger}_{n+\hat{\mu},\nu;\mu}\Big] \\
\widetilde{V}_{n,\mu;\nu} &= \mathrm{Proj}_{SU(3)}\Big[(1-\alpha_2)U_{n,\mu} + \frac{\alpha_2}{4}\sum_{\pm\rho\neq\nu,\mu} \overline{V}_{n,\rho;\nu\,\mu}\,\overline{V}_{n+\hat{\rho},\mu;\rho\,\nu}\,\overline{V}^{\dagger}_{n+\hat{\mu},\rho;\nu\,\mu}\Big] \\
\overline{V}_{n,\mu;\nu\,\rho} &= \mathrm{Proj}_{SU(3)}\Big[(1-\alpha_3)U_{n,\mu} + \frac{\alpha_3}{2}\sum_{\pm\eta\neq\rho,\nu,\mu} U_{n,\eta}\,U_{n+\hat{\eta},\mu}\,U^{\dagger}_{n+\hat{\mu},\eta}\Big]
\end{aligned}
$$

- Standard HYP: iterate projected APE smearing three times
- restrict contributions to fat link to the hypercube
- local and efficient, widely used
- improves scaling

How to use it in dynamical simulations?

HYP links for dynamical fermions

$$
\begin{aligned}
V_{n,\mu} &= \mathrm{Proj}_{U(3)}[(1-\alpha_1)U_{n,\mu} + \frac{\alpha_1}{6}\sum_{\pm\nu\neq\mu} \widetilde{V}_{n,\nu;\mu}\widetilde{V}_{n+\hat{\nu},\mu;\nu}\widetilde{V}^{\dagger}_{n+\hat{\mu},\nu;\mu}] \\
\widetilde{V}_{n,\mu;\nu} &= \mathrm{Proj}_{U(3)}[(1-\alpha_2)U_{n,\mu} + \frac{\alpha_2}{4}\sum_{\pm\rho\neq\nu,\mu} \overline{V}_{n,\rho;\nu\,\mu}\overline{V}_{n+\hat{\rho},\mu;\rho\,\nu}\overline{V}^{\dagger}_{n+\hat{\mu},\rho;\nu\,\mu}] \\
\overline{V}_{n,\mu;\nu\,\rho} &= \mathrm{Proj}_{U(3)}[(1-\alpha_3)U_{n,\mu} + \frac{\alpha_3}{2}\sum_{\pm\eta\neq\rho,\nu,\mu} U_{n,\eta}U_{n+\hat{\eta},\mu}U^{\dagger}_{n+\hat{\mu},\eta}]
\end{aligned}
$$

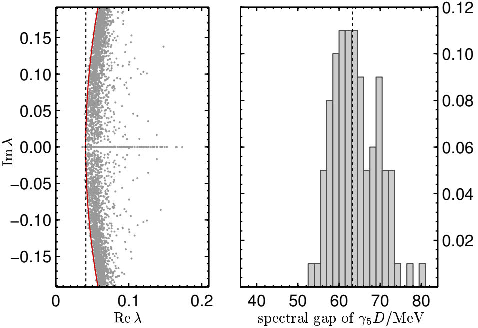
- n-HYP: same as HYP with projection to U(3)
- virtually indistinguishable from standard HYP
- more efficient than if built from stout smearing

How does the projection perform in MD simulations?

Tests: Dynamical clover Wilson

- clover Wilson with $c_{SW} = 1$
- standard HYP parameters: $\alpha_1 = 0.75$, $\alpha_2 = 0.6$, $\alpha_3 = 0.3$
  no tuning necessary
- Lüscher-Weisz gauge action
- $12^3 \times 24$
- $a \approx 0.13\text{fm}$
- $m_{PS}/m_V \approx 0.6$

Dynamical clover Wilson



- large spectral gap ⇒ smaller quark masses possible
- fat link cost: 11% of total budget
- gain on inversions

Overlap: Locality

$$D_{ov} = R\big[1 + \gamma_5 \, \mathrm{sign}(H_W(-R))\big]$$

- use dynamical clover configurations
- valence overlap with Wilson kernel
  $-R$ tuned for optimal locality
- compare to iterated stout at $6\rho = 0.9$

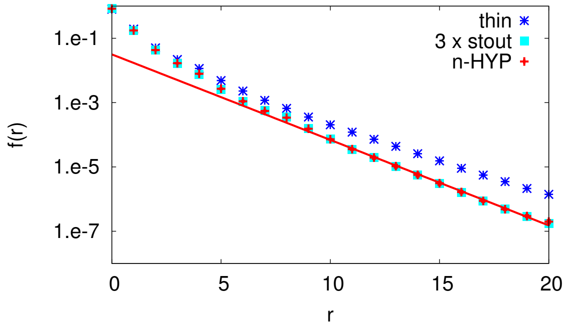
Hernandez, Jansen, Lüscher '99

$$\psi = D_{\mathrm{ov}}\eta \quad \text{with} \quad \eta(x) = \delta_{x,x_0}$$

$$f(r) = \max\{||\psi(x)|| : \mathrm{dist}(x, x_0) = r\}$$

Overlap: Locality

If local: $f(r) \propto e^{-\nu r}$ for large r



smearing	ν
thin	0.50(1)
1stout	0.58(1)
2stout	0.60(1)
3stout	0.61(1)
n-HYP	0.62(1)

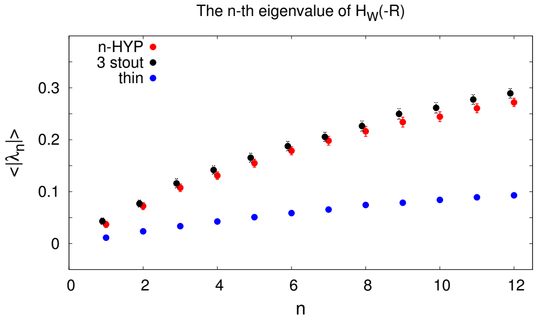
Any smearing improves locality equally well.

Overlap: Cost

$$D_{ov} = R\left[1 + \gamma_5\, \mathrm{sign}(H_W(-R))\right]$$

$$\mathrm{sign} H_W \approx \sum_\lambda \mathrm{sign}\lambda\, P_\lambda \;+\; (1 - \sum_\lambda P_\lambda)\, \mathrm{sign}_{\mathrm{app}} H_W$$

- lower eigenmode density of $H_W(-R)$
  $\Rightarrow$ easier to approximate sign function; lower cost
- n-HYP as good as $3\times$ stout at $6\rho = 0.9$



Overlap: Cost II

	$\langle|\lambda_1|\rangle$	$\langle|\lambda_{12}|\rangle$	rel. cost of D_{ov}
thin	0.011(2)	0.093(3)	1
1stout	0.019(3)	0.156(5)	0.59
2stout	0.031(5)	0.217(6)	0.43
3stout	0.043(6)	0.289(9)	0.29
n-HYP	0.037(6)	0.272(8)	0.32

Conclusions I

- smeared links can greatly reduce the cost of chiral fermion simulations
- Danger: too much smearing can introduce large cut-off effects
- stay as local as possible
- HYP smearing improves scaling in quenched
  $\Rightarrow$ use in dynamical too
- no negative effect on auto-correlation times expected

Conclusions II

- no parameter tuning required
- as efficient as $3\times$stout at $6\rho = 0.9$
- computational overhead small, even for clover Wilson
- clover Wilson stable at $a = 0.13$fm, m_{PS}/m_V=0.6
- next: smaller quark mass, larger volume, dynamical overlap
- More details in
  A. Hasenfratz, R. Hoffmann, St.S. hep-lat/0702028