A Smoothed Two- and Three-Dimensional Interface Reconstruction Method

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Algorithm Outline

- **Extension of the Youngs' 3D algorithm:**

  DL Youngs, *"An Interface Tracking Method for a Three-Dimensional Hydrodynamics Code",* Technical Report 44/92/35, (AWRE 1984)

  - Approximate interface normal by –Grad(Vf)

  - Position planar interface (polygon) in element to conserve volume

  - not spatially second-order accurate

- **Reconstruction uses:**

  - accurately move materials through the computational mesh

  - visualization

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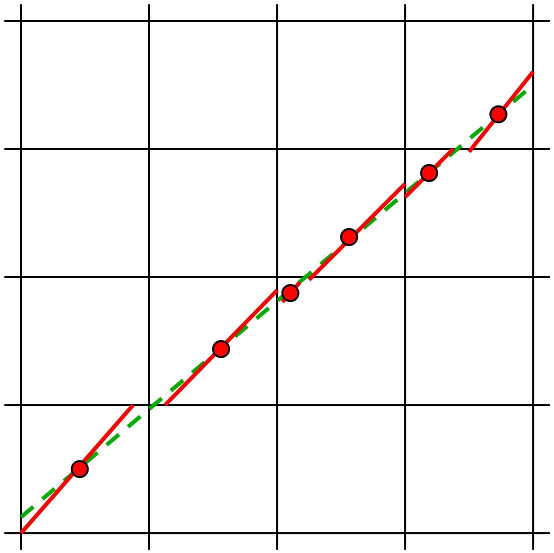
3D Smoothing Algorithm

- **2D Smoothing using Swartz Stability Points**

  SJ Mosso, BK Swartz, DB Kothe, RC Ferrell, *"A Parallel Volume Tracking Algorithm for Unstructured Meshes",* Parallel CFD Conference Proceedings, Capri, Italy, 1996.
- **The centroid of each interface is a 'stable' position**
- **Fit a surface to the neighboring centroids and compute the normal to the fit.**
- **Iterate to convergence.**
- **Volume conserving**



3D Interface Fitting Algorithm

Least-Squares fit of a plane to the immediate 3D neighborhood

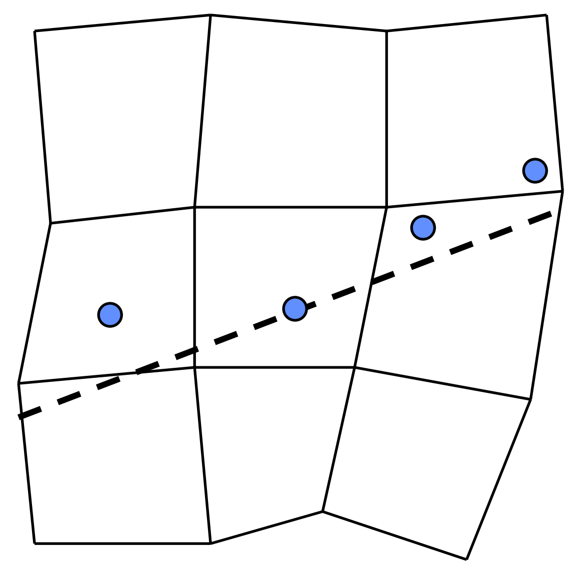
- Equation of a plane: $\hat{n} \circ \vec{X} - p = 0$
- Normal distance of a point S from the plane: $dist = \hat{n} \circ \vec{S} - p$
- Objective Function:

$$Obj = \sum_{neighbors} (W_i * dist_i)^2 = \sum_{neighbors} \left[W_i \left(\hat{n} \circ \vec{S}_i - p \right) \right]^2$$

Fitted interface passes through the home stability point (reduces # variables to 2)

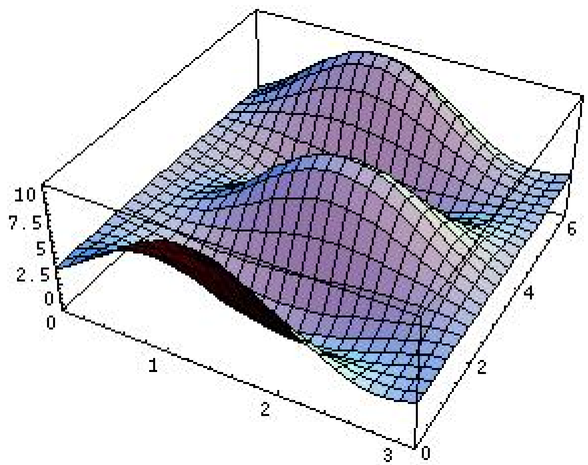
$$Obj = \sum_{neighbors} \left[W_i \left(\hat{n} \circ \vec{S}_i - \overset{0}{\not{p}} \right) \right]^2$$

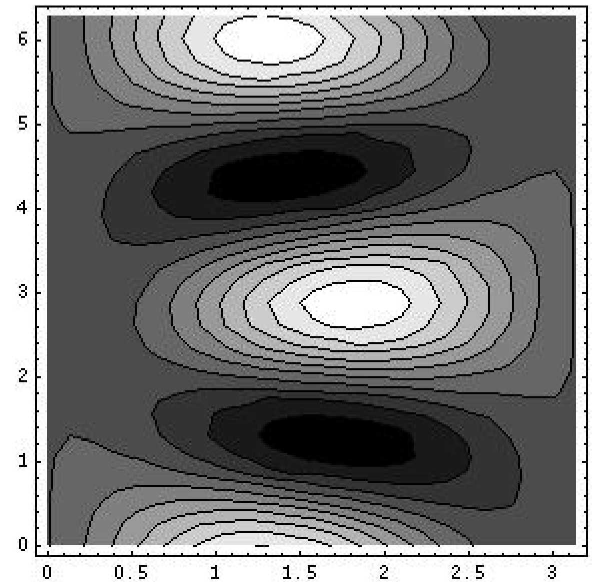
$$\hat{n} = \cos\phi \sin\theta \,\hat{i} + \sin\varphi \sin\theta \,\hat{j} + \cos\theta \,\hat{k}$$



Iterate until $Global\ Min(\hat{n}_{previous} \circ \hat{n}_{current}) > (1.0 - \varepsilon)$

Plots of the Objective Function



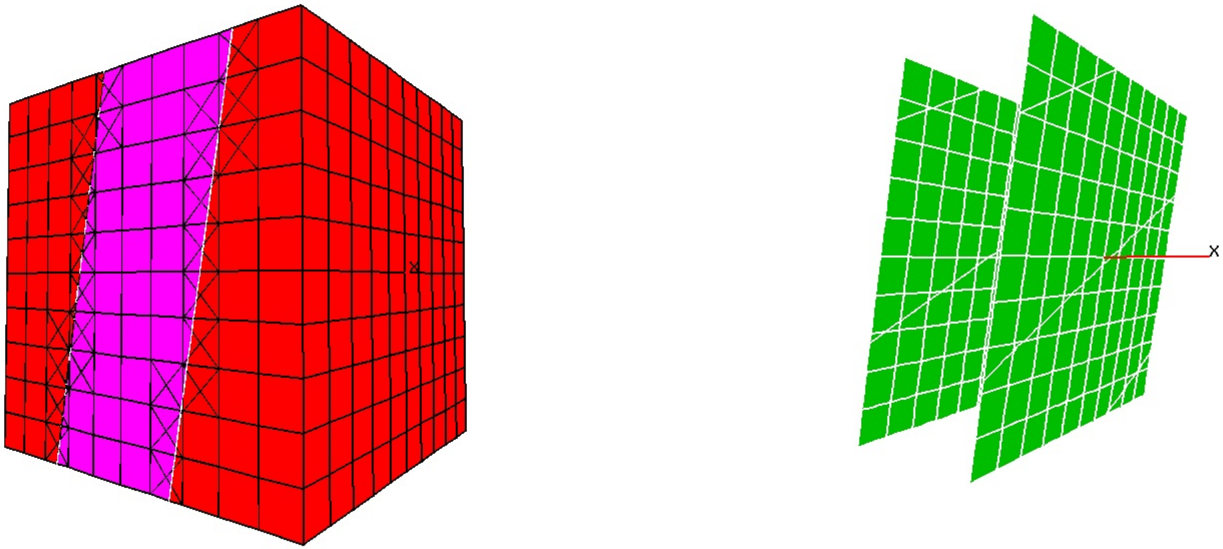


Objective Function Sample for Three Stability Points

Minimization Procedure

- Start with values of phi and theta from Youngs' reconstruction.
- Use Steepest Descent to descend toward the minima.
- To speed convergence, when Steepest Descent progress slows, switch to a Newton's Method finish the descent.

Slanted Slab Test



Slab is 0.4 thick and rotated 10^{o} about y-axis and 5^{o} about z-axis

Reconstruction Accuracy for Slanted Slab

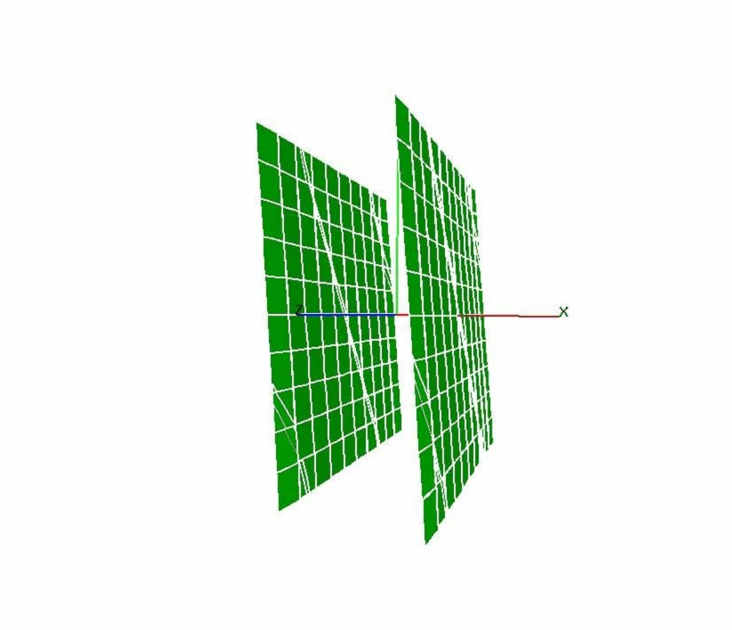
Method	Coarse Grid Error	Fine Grid Error	
Unsmoothed	$1.71280*10^{-4}$	$7.17863*10^{-5}$	
Smoothed	$4.38547*10^{-10}$	$5.76368*10^{-10}$	(3 iterations)

Number of mixed elements: Coarse Mesh = 254
Fine Mesh = 1012

Coarse grid size = 1/10, fine grid size = 1/20
Courant number 1/10 for both

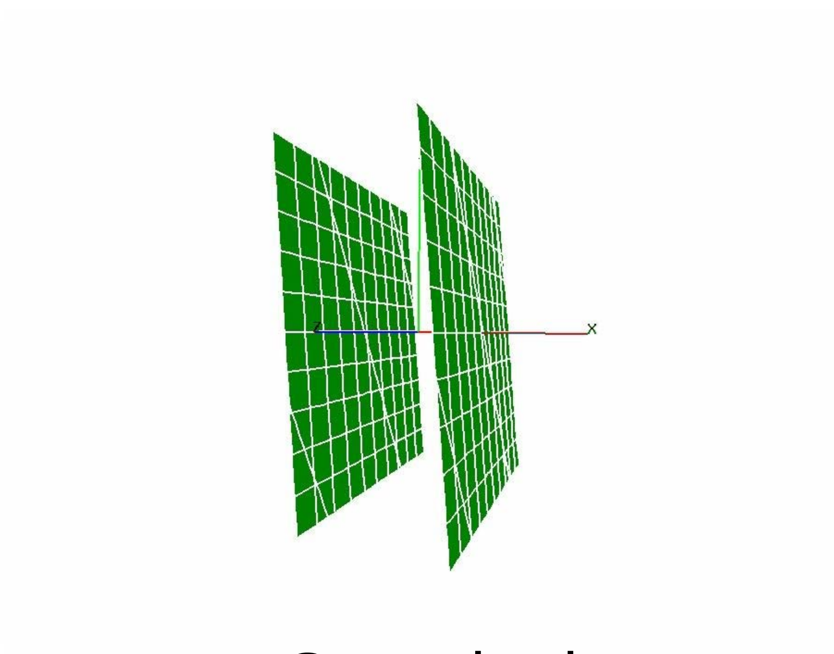
Method	L_{inf} Error	L_{inf} Error
Unsmoothed	$6.91118*10^{-6}$	$1.19250*10^{-6}$
Smoothed	$4.35268*10^{-10}$	$1.04547*10^{-10}$

Motion of Unsmoothed and Smoothed Reconstructions



Unsmoothed

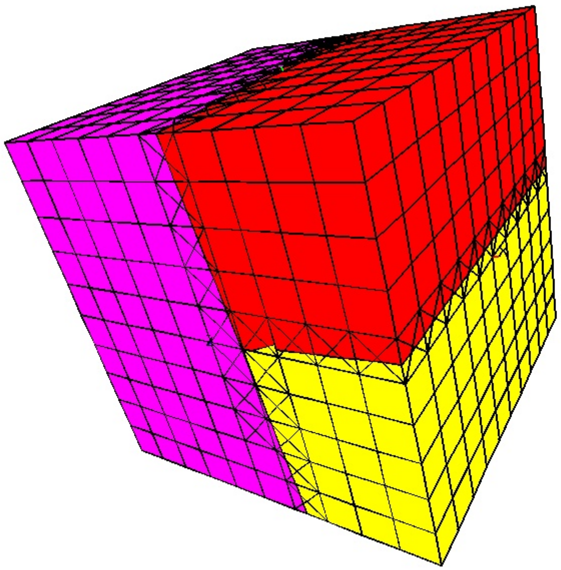
1:24

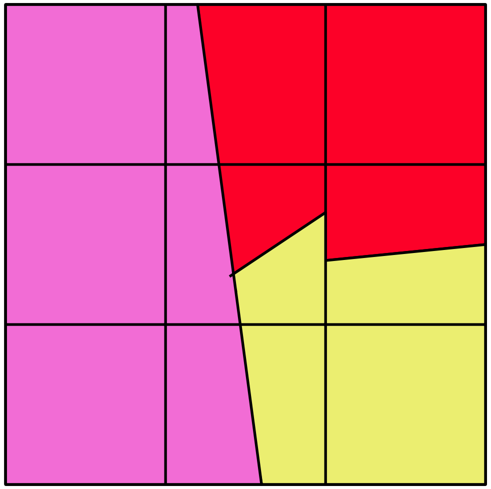


Smoothed

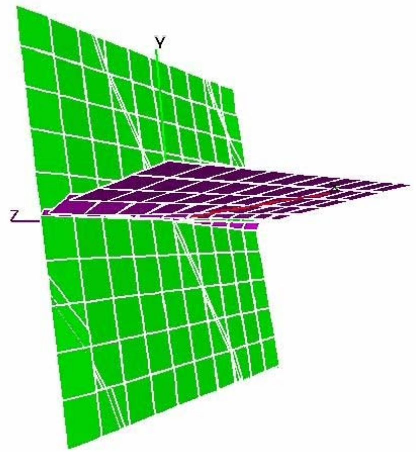
3:05

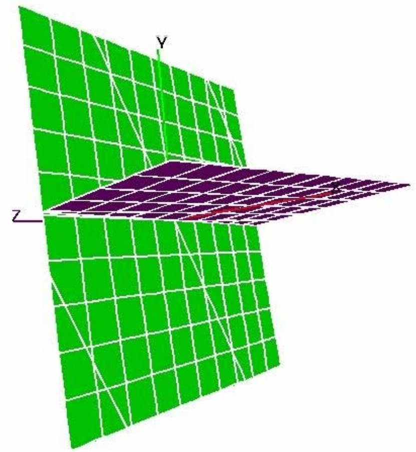
Intersecting Material Interfaces





Intersecting Material Interfaces



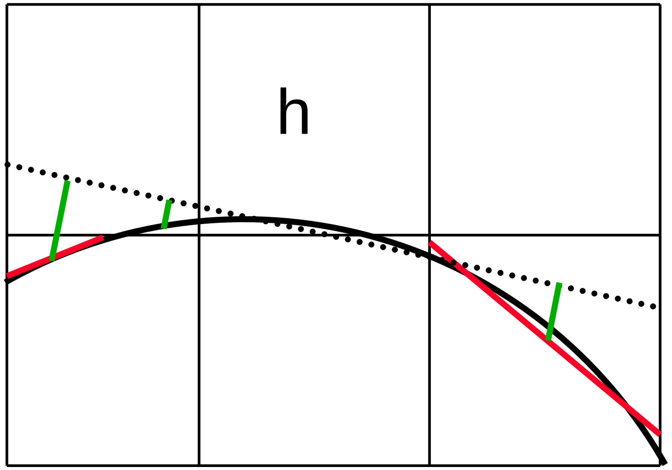


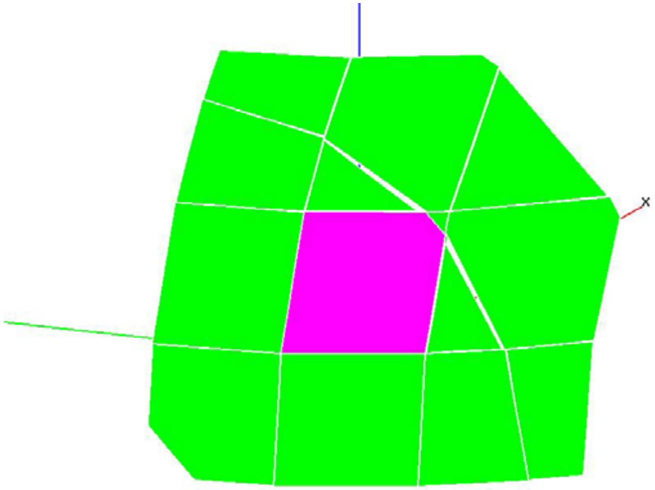
Unsmoothed

Smoothed

Planar Smoothing Accuracy for Sphere

- *Planar* smoothing inaccuracy:
  - Least squares fit of spherical arc produces poor interface normal.
  - Inaccuracies due to linearization are small





Spherical Smoothing Accuracy for Sphere

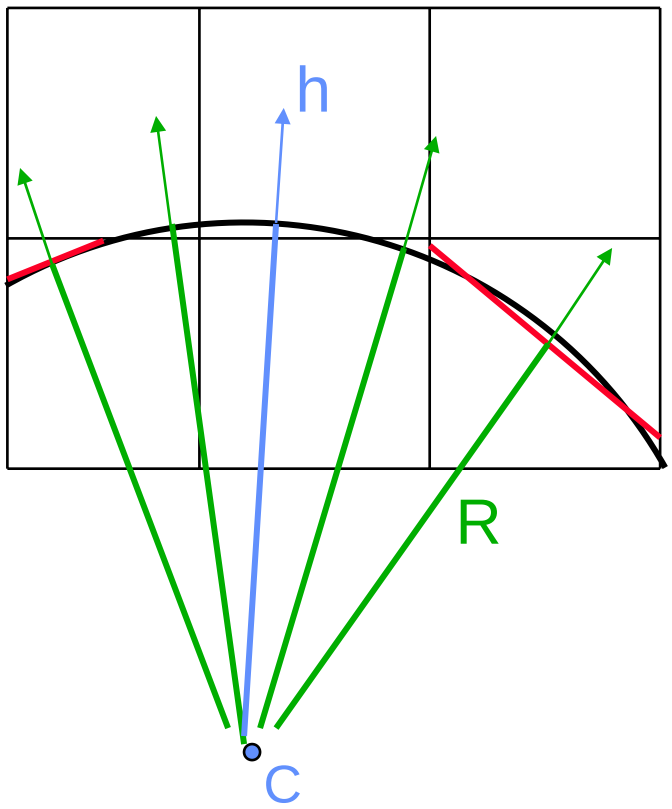
- **Compute average radius of neighborhood:**

$$\overline{R} = \frac{1}{\# ngbrs} \sum_{ngbrs} \frac{L}{\sqrt{2(1-\hat{h} \circ \hat{n})}}$$

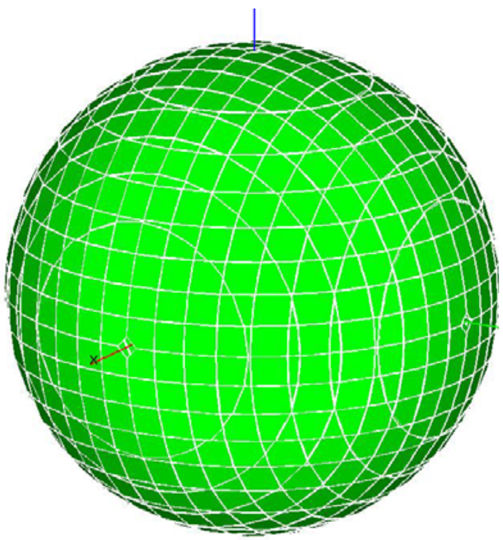
- **Project radial line back from each stability points along normal to compute center, C**
- **New normal of home element is**

$$\hat{n} = \frac{\vec{H} - \vec{C}}{\left|\vec{H} - \vec{C}\right|}$$

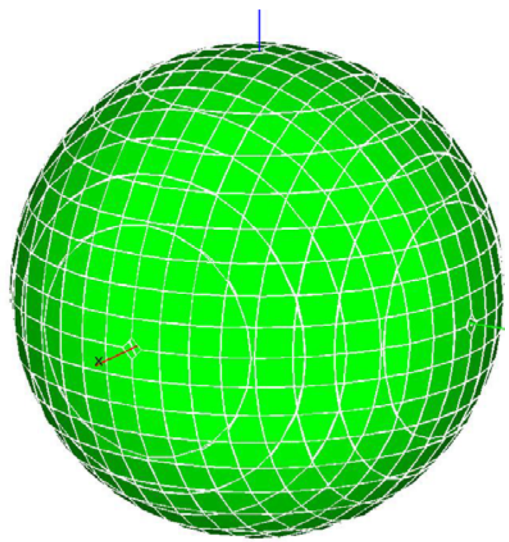
where H is the home element's stability point.



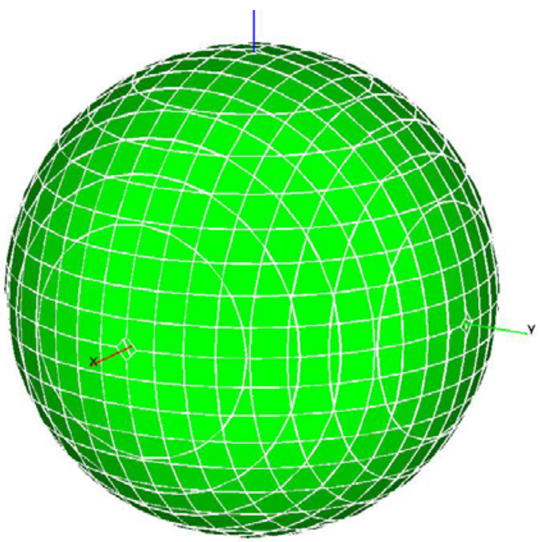
Smoothed Reconstruction of Sphere



unsmoothed

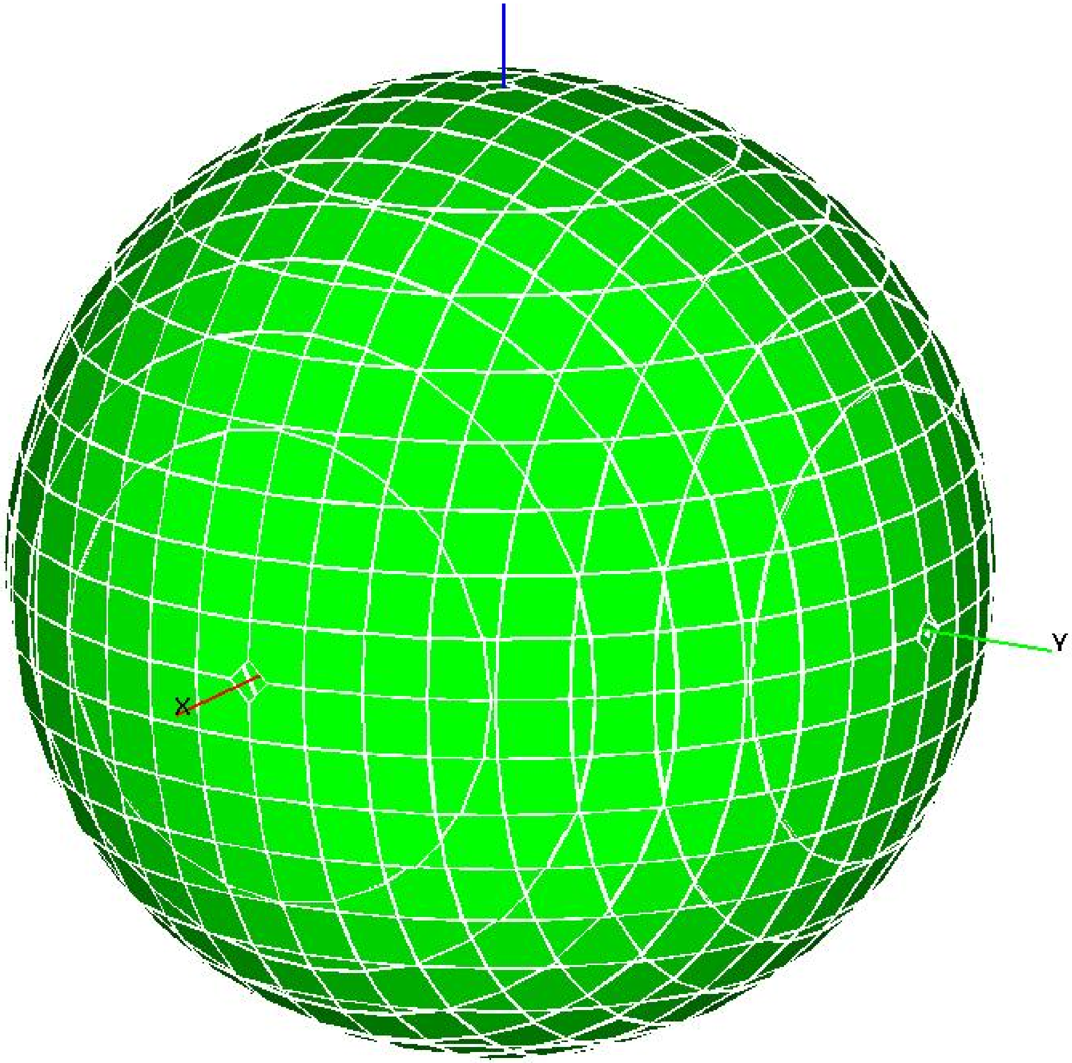


smoothed

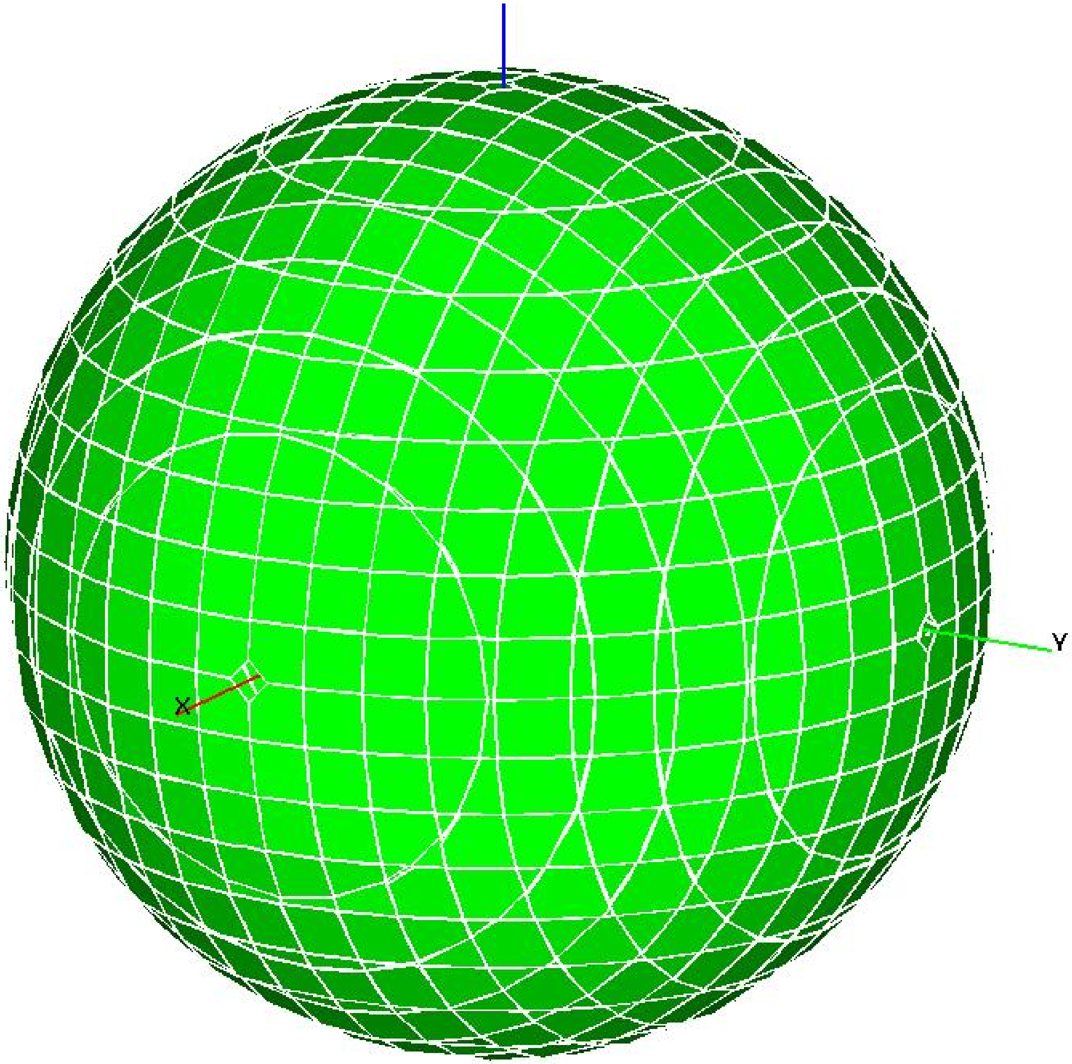


perfect

Unsmoothed Sphere Reconstruction



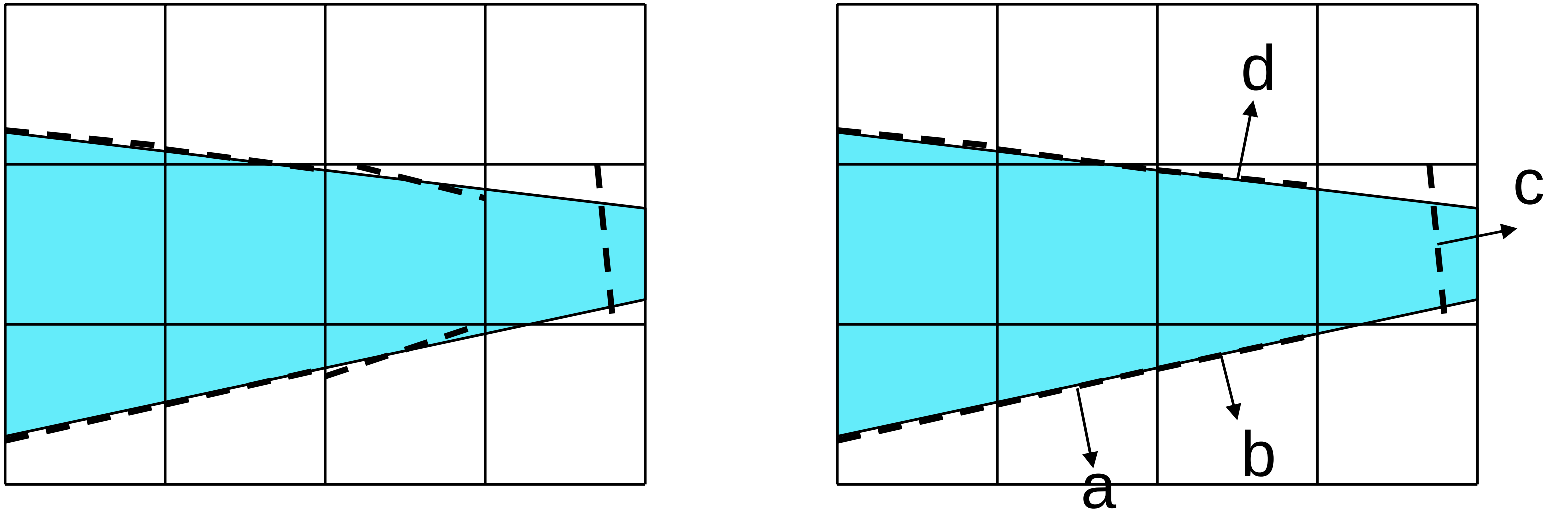
Smoothed Sphere Reconstruction



Cautions for Smoothed Reconstruction:

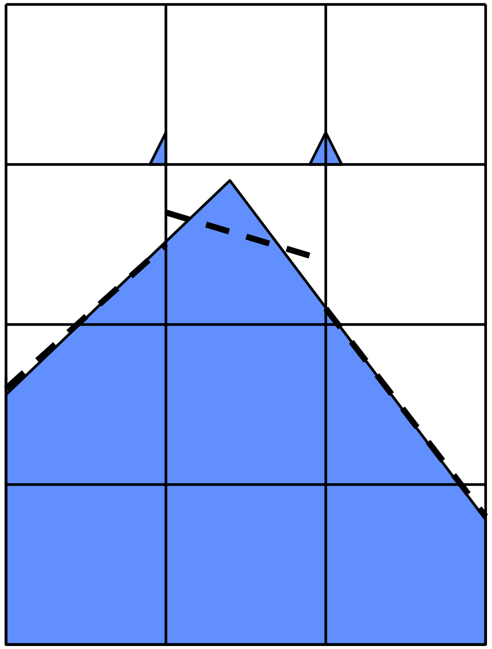
- **Thin layers (with same material on both sides) require caution**
- **Minimized through use of 'dot product' filter. For element 'b',**

$\hat{a} \circ \hat{b} > \lim, \quad \hat{c} \circ \hat{b} < \lim, \quad \hat{d} \circ \hat{b} < \lim$



Cautions for Smoothed Reconstruction:

- **Smoothing is 'sensitive' to interface fragmentation**
  - **Fragmentation is caused by discontinuity of interface representation for high curvature interface problems (smoothed reconstruction is still better than unsmoothed reconstruction).**
  - **Fragment neighbors can be excluded from use in iteration by a fragment recognition algorithm**
  - **Increased Mesh refinement**



Fragmented corner example

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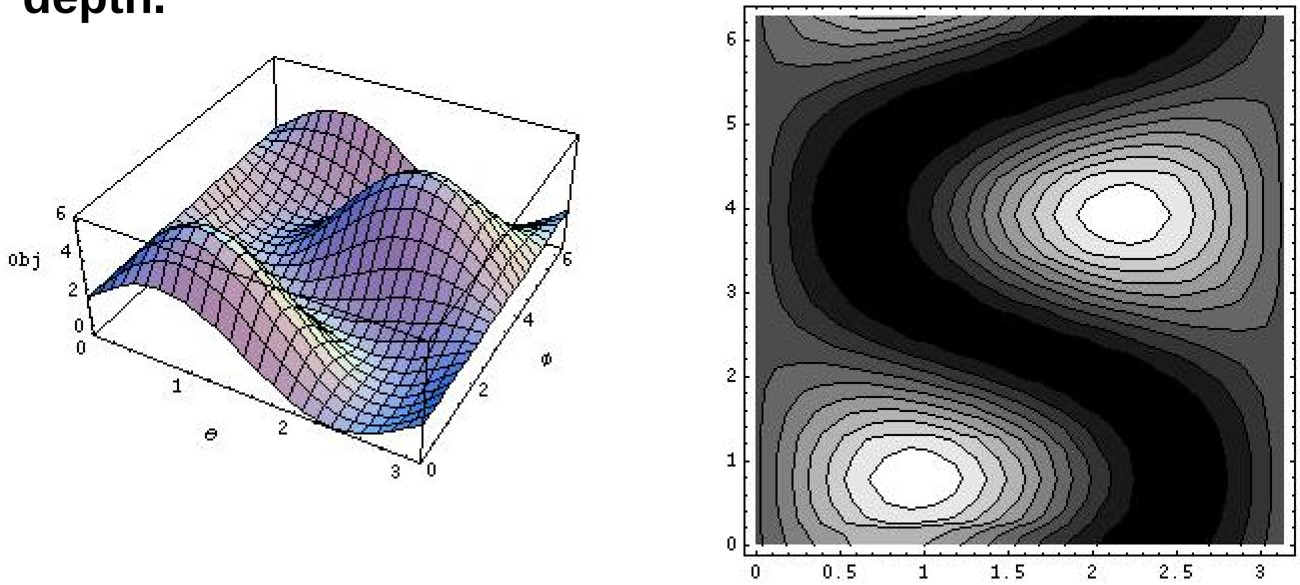
Cautions for Planar Smoothed Reconstruction:

- **Degenerate (collinear) neighbors**
  - **Usually caused by 3D interfaces on problem boundary with no depth.**



Objective Function for Collinear Stability Points

Summary

- **A 2D and 3D smoothed interface reconstruction algorithm has been developed whose error is several orders of magnitude lower than gradient normal methods.**
- **Method will be presented in a future publication and Sandia National Lab webpage.**
- **Future work will use interface shapes other than planar polygons in 3D (spherical and corner shapes)**
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