Options for Robust Airfoil Optimization Under Uncertainty

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Needed: Uncertainty-based Methods

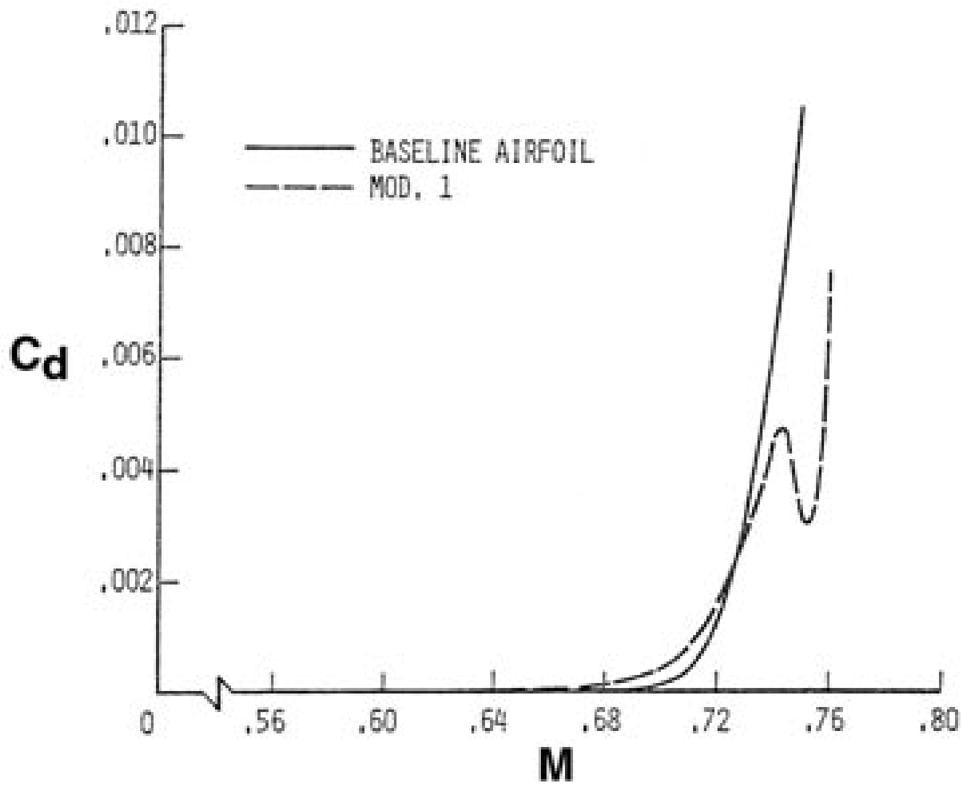
- Aerospace design examples
  - During early design stages, parameters such as cruise Mach number are not precisely specified.
  - During later design stages, parameters such as payload weight are specified by upper and lower bounds.
- Airfoil shape optimization example
  - Possible uncertain parameters are required lift, Mach number, or Reynolds number
  - Lessons learned with this example will guide future work in uncertainty-based methods.

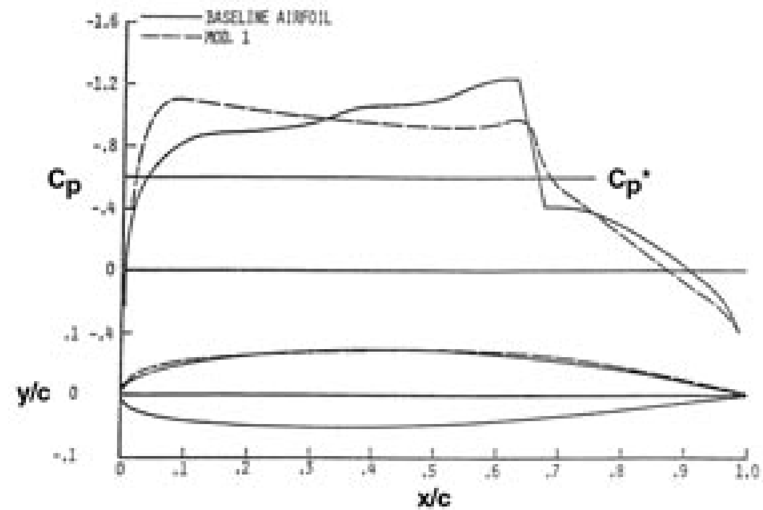
Outline

- Motivation
  - Airfoil Shape Optimization
  - Sample Results of 2-D Demo Problems
- Robust Airfoil Optimization Method
  - Algorithm Details and Options
  - Illustrative Examples

Observation

Drag minimization at one M has unintended effects at off-design points

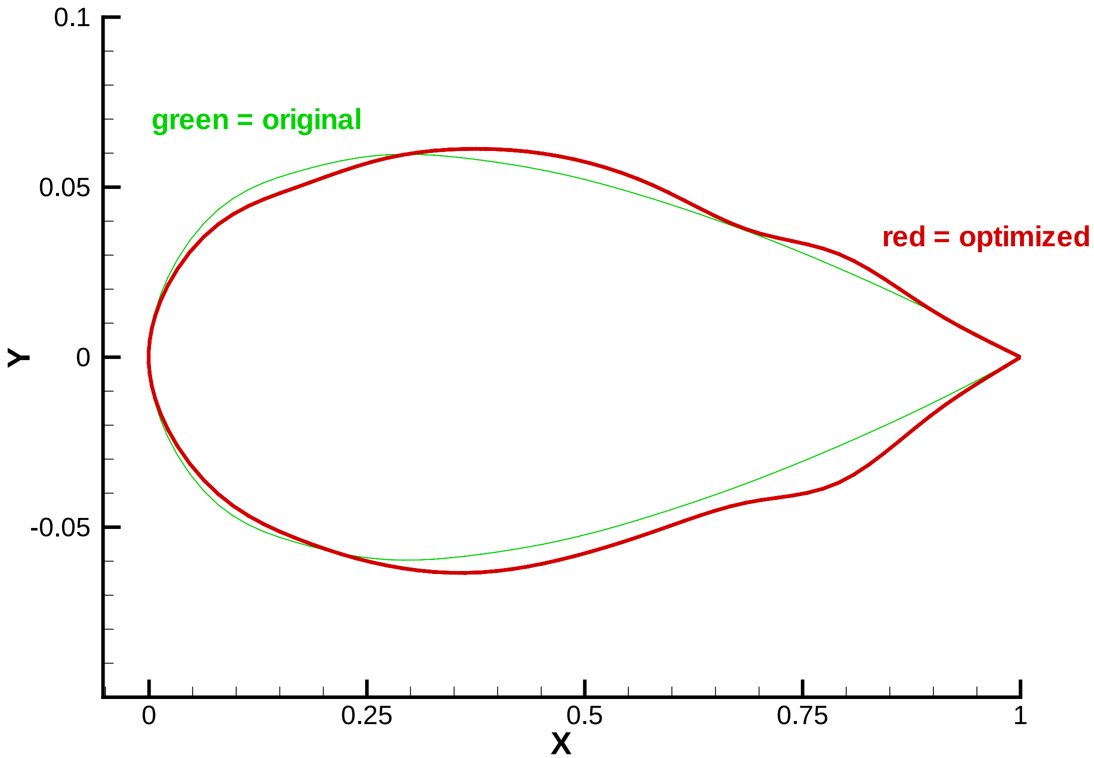




Hicks and Vanderplaats (1977)
"Application of Numerical Optimization to the Design of Supercritical Airfoils without Drag Creep" SAE Paper 770440.

Observation

Airfoil smoothing is often necessary

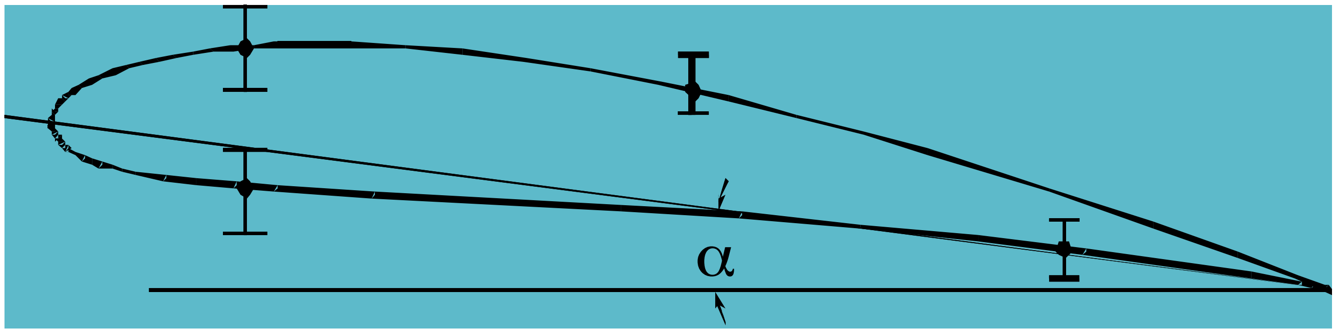


Airfoil Shape Optimization

- Required Characteristics
  - Reduce drag over range of Mach numbers
  - Produce smooth airfoils without post-processing
  - Succeeds with moderate number of function evaluations
- Previous Airfoil Optimization Studies
  - Multipoint = Minimize weighted sum of objectives
    - Hicks & Vanderplaats (1977) - Suggest off-design pt constraints
    - Mark Drela (1998) - Multipoint pros & cons discussed
    - Reuther *et.al*. (1999) - Discuss need for airfoil smoothing
  - Robust = Minimize expected value
    - Huyse *et.al.* (AIAA J Sept 2002) - Airfoil optimization ideas borrowed from civil engineering uncertainty-based design
    - Li *et.al.* (J Structural & Multi Opt Aug 2002) - Robust airfoil opt.

Demonstration Case

2-D Airfoil Shape Optimization Using Inviscid Euler Code



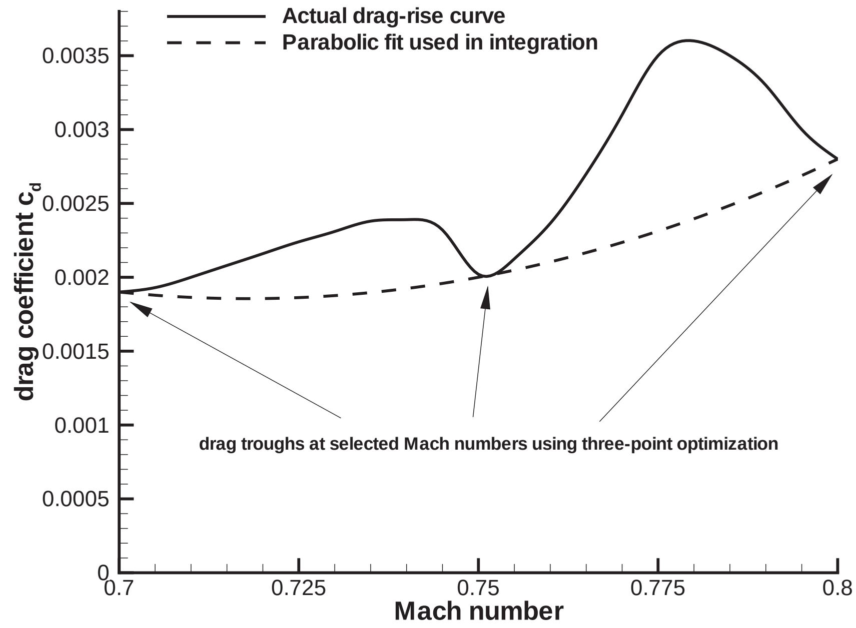
$$\min_{d \in D} E_M\left[C_d(d,M)\right] = \min_{d \in D} \int_M C_d(d,M)\, f_M(M)\, dM$$

$$\textit{subject to} \quad C_l \geq C_l^{required} \quad \text{for all } M$$

Minimize drag over a range of Mach numbers [0.7, 0.8] using 20 bounded spline coefficients and angle-of-attack

Multi-point vs Robust Optimization

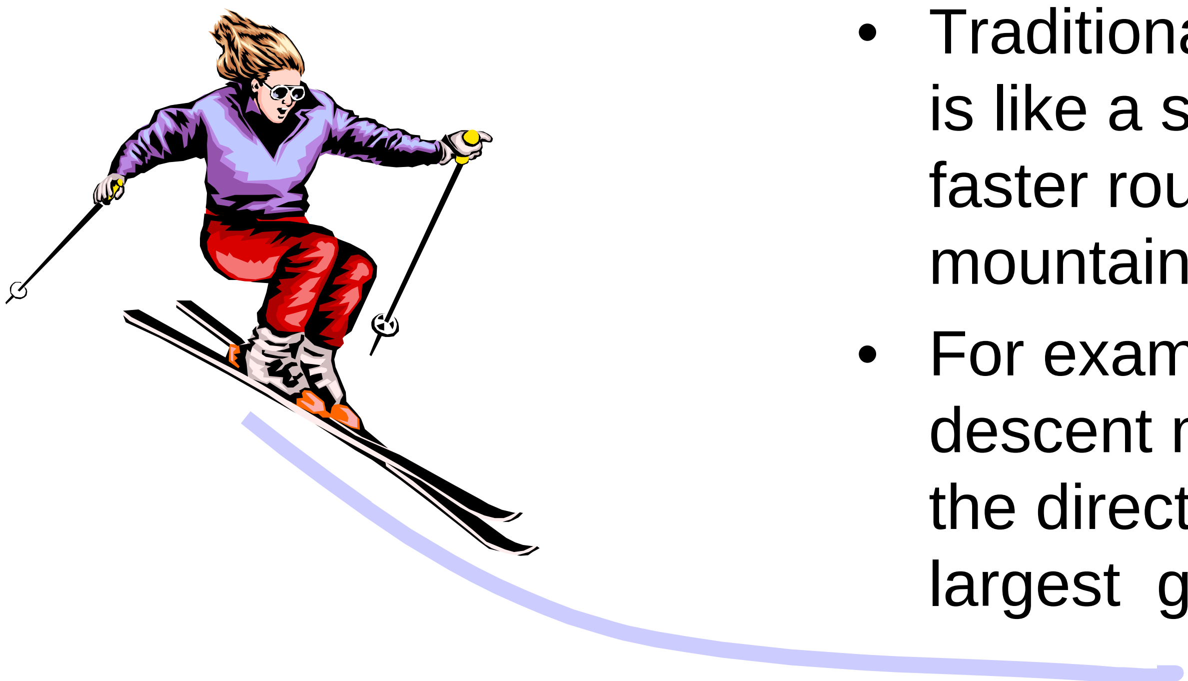
- Multi-point reduces drag at specific Mach numbers
- Robust minimizes drag over a range of Mach numbers
- Results in this presentation use uniform PDF



Huyse: "Minimize the area under the curve"

Li: " Choose descent direction wisely"

Choice of Descent Direction



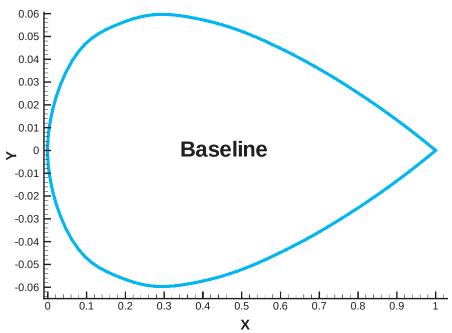
- Traditional optimization is like a skier finding the faster route down the mountain
- For example, steepest descent method picks the direction with the largest gradient

Robust Optimization

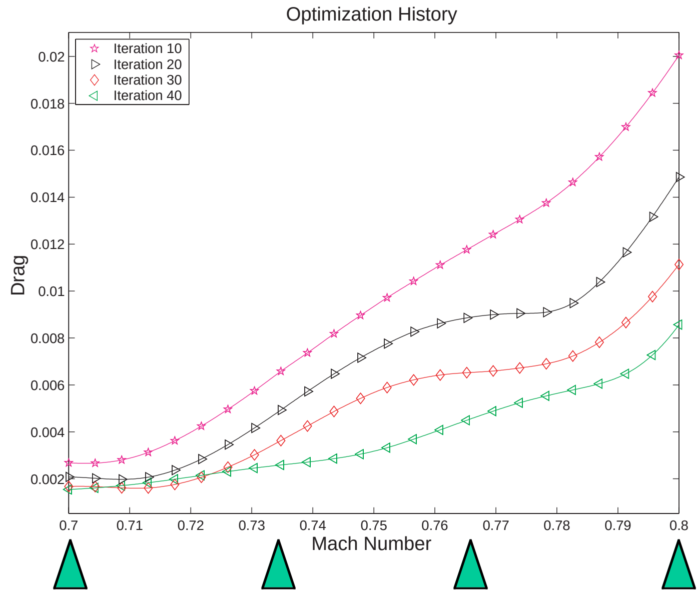
- Robust optimization is like many skiers in a formation
- They pick a descent direction so that all individuals descend at the same rate

Results: Demo Problem

20 geometric design variables

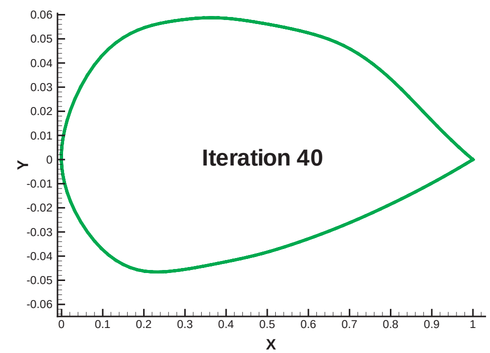


4 Euler CFD solutions per iteration



4 design pts

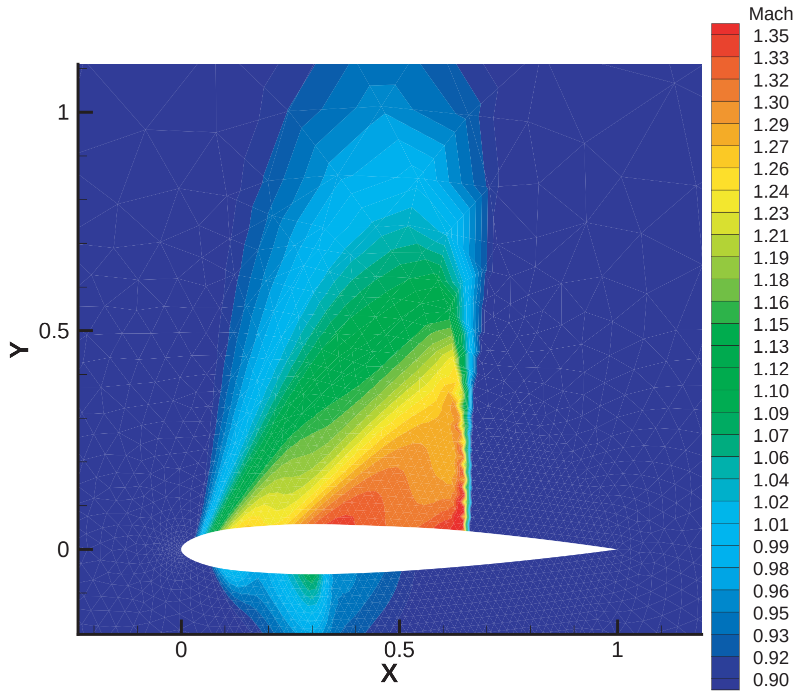
Optimized airfoil is more cambered



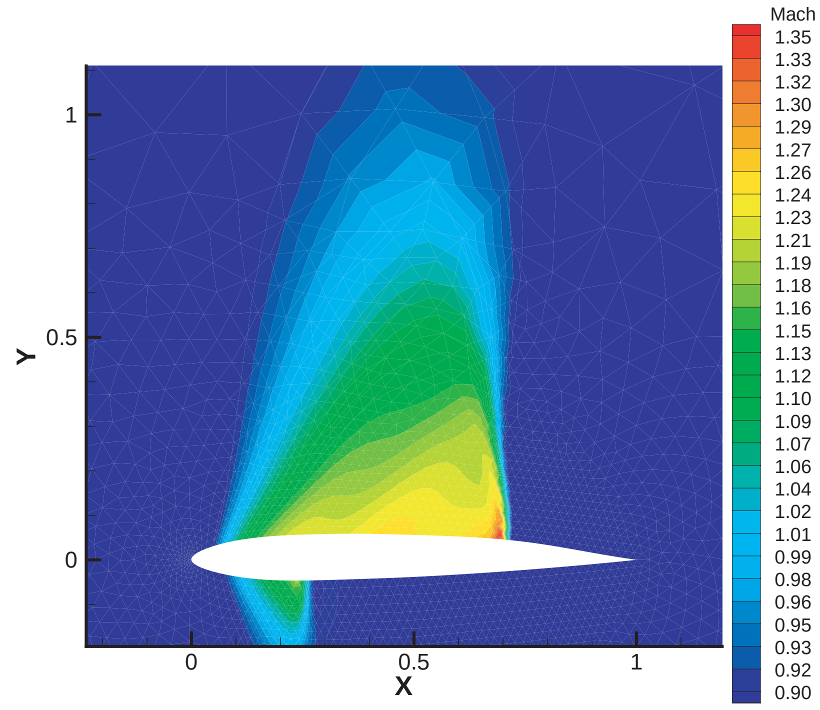
- What Has Been Accomplished?
  - Robust optimization directly minimizes wave drag for 0.7< Mach # <0.8
  - User can adjust optimization for aggressive improvement or conservative modification to a baseline design
  - No smoothing of optimized airfoil shape is required

Comparison of Mach Contours

Design Point 4 M=0.8



Iteration 10



Iteration 40

Notes

- Good results are possible because of FUN2D. This dependable CFD code provides derivatives that are consistent with lift and drag function evaluation.
- Dependable automatic grid movement for each modified airfoil is important.
- Published demonstration problem uses coarse grid and inviscid Euler code.
- Need to test method with better grid, more realistic geometry and viscous CFD.

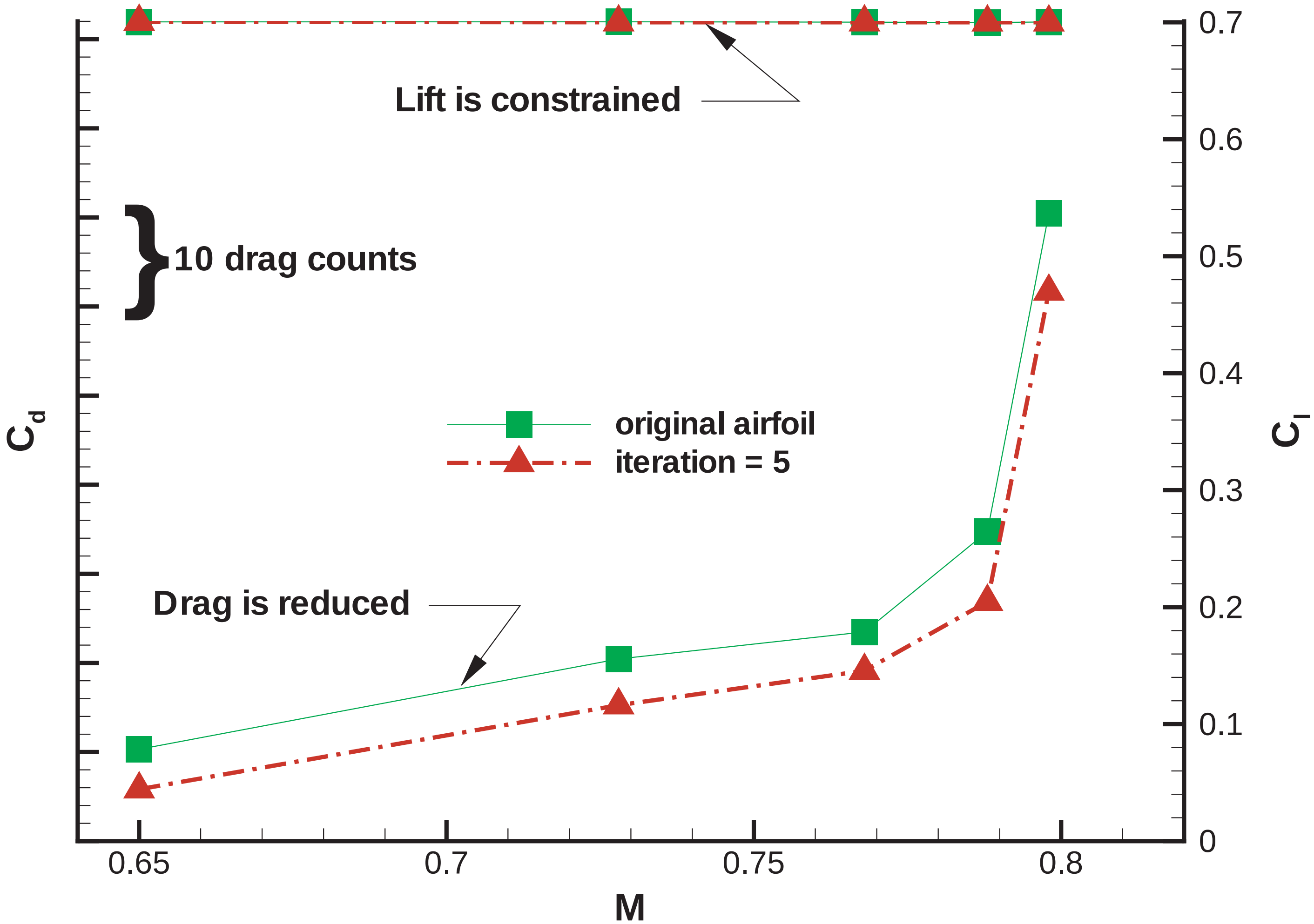
Challenging Test Problem

2-D Airfoil Optimization Using Viscous NS Code

- Advanced airfoil and design specifications provided by Aerodynamics experts
  - Experts specify 5 design points
  - Design variables are 82 spline coordinates
  - Experts provide FUN2D grid for viscous flow calculations
- Minimize expected value of drag with lift constraints
- Thickness constraints are added to our procedure

Successful Demo for Advanced 2-D Airfoil

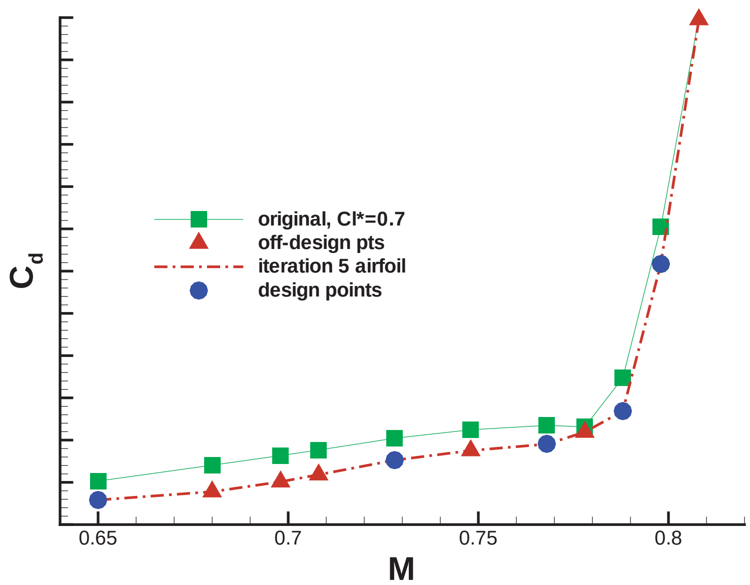
Reduction of 5-9 Drag Counts at Five Design Points



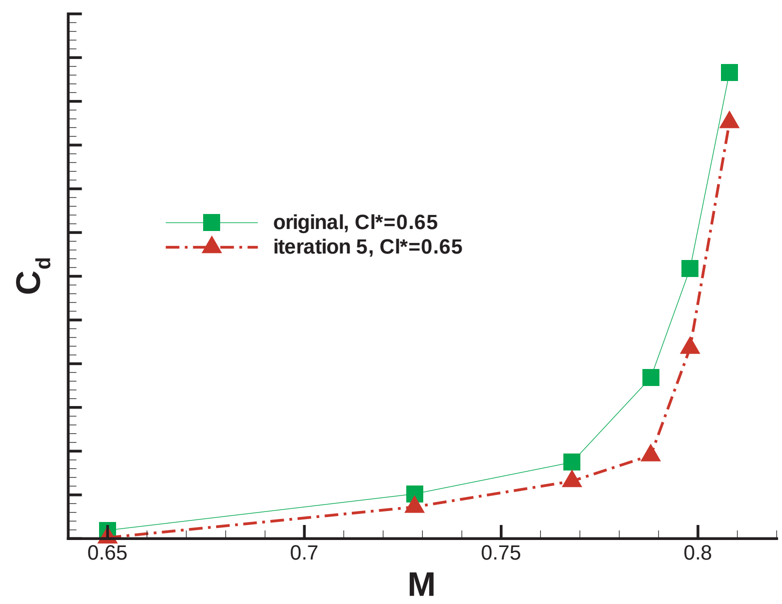
Successful Demo for Advanced 2-D Airfoil

Drag Reduction at Off-design Points

Lift constraint = 0.7



Lift constraint = 0.65



Note: Angle-of-attack is adjusted to satisfy lift constraint.

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Details of Robust Optimization Algorithm

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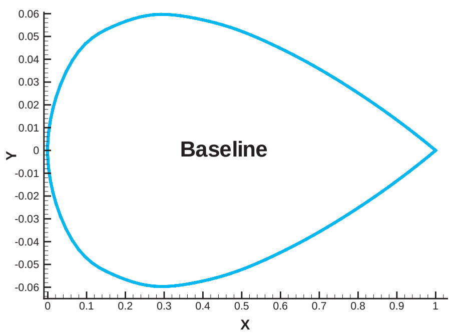
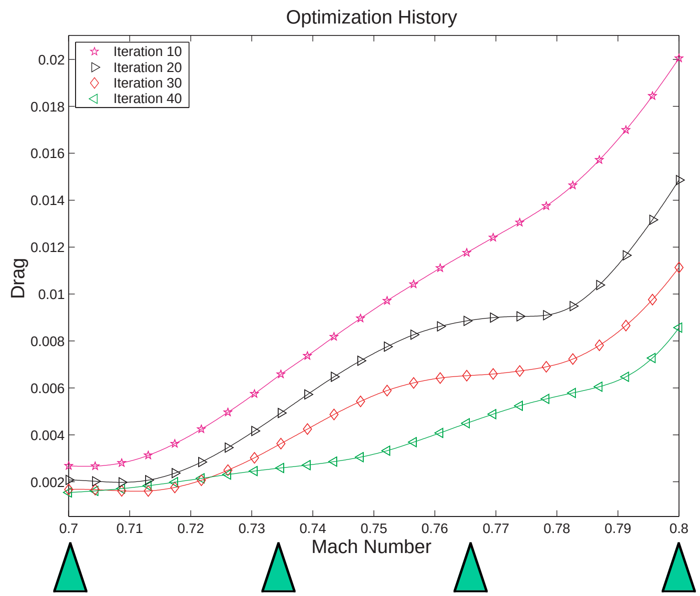
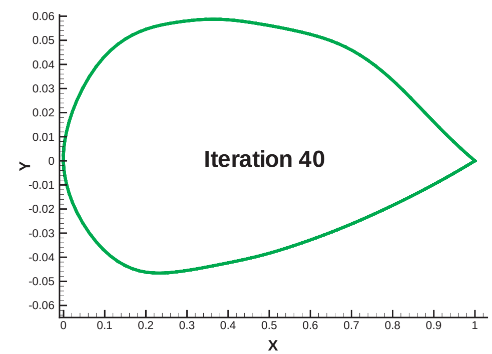




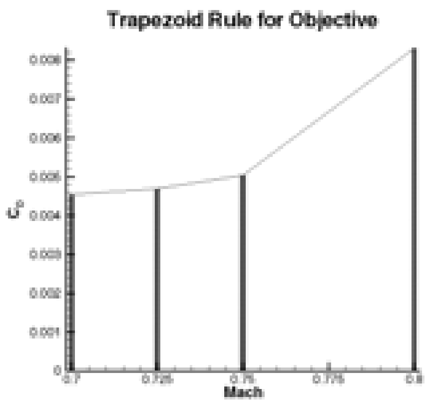
4 design pts

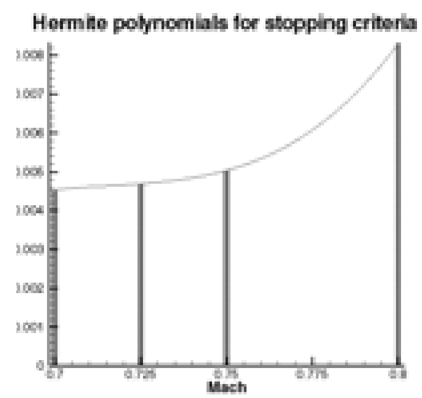
Optimized airfoil is more cambered

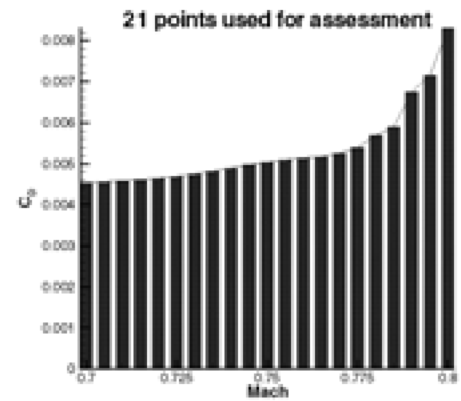


Assessing Expected Value Improvement

- Select Mach numbers - fixed (Li *et.al*.) or random (Huyse *et.al*.)
- Objective - Area under the curve estimated by trapezoid rule
- Estimate of actual improvement using Hermite polynomials
- Final solution assessment uses additional Mach numbers
- Multi-point with 21 Mach numbers should agree with robust

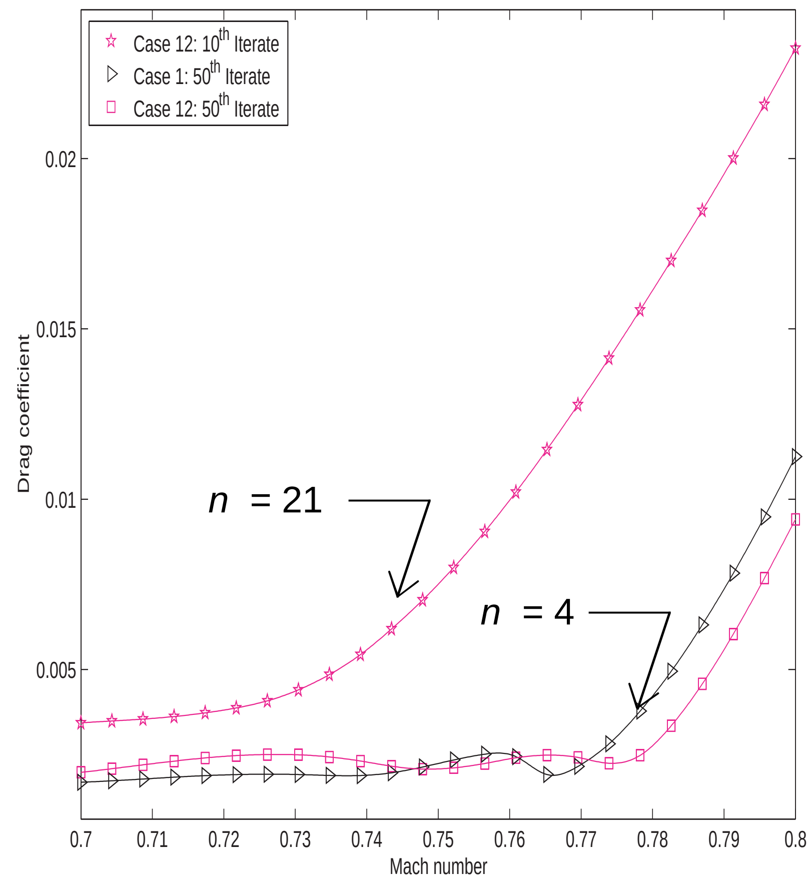






Number of M_i Design Points Needed

- For *m* design variables, *n*=*m*+1 Mach numbers suggested by Drela
- Yet, we use *n*=4 Mach points when *m*=20 and 5 Mach points when m=82 !
- Compare robust solutions for *n*=4 with multi-point *n*=21
- Note that 10 iterations with *n*=21 equals computational effort of 50 iterations with *n*=4



Options for Robust Optimization

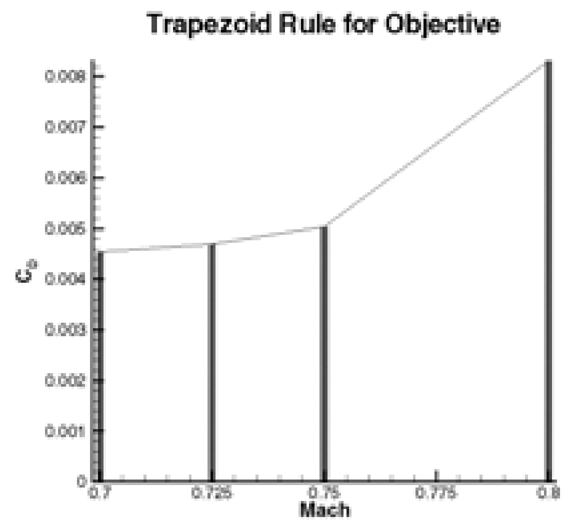
- Choose a set of Mach numbers, M_i
- Find angle-of-attack, α, to satisfy lift constraints
- Calculate objective, constraints and gradients
- Find a solution of the linear subproblem with the *smallest change* in design variables
- Adjust *trust region size* to achieve specified predicted decrease in drag
- Update design variables based on linear subproblem
- Iterate or terminate

Selecting Trust Region Size

- Linear subproblem is solved to find next optimization step
- Allowable change in any C_D based on γ_{min}
- Required predicted decrease in objective based on γ_{obj}
- Trust region size is adjusted based on γ_{obj}

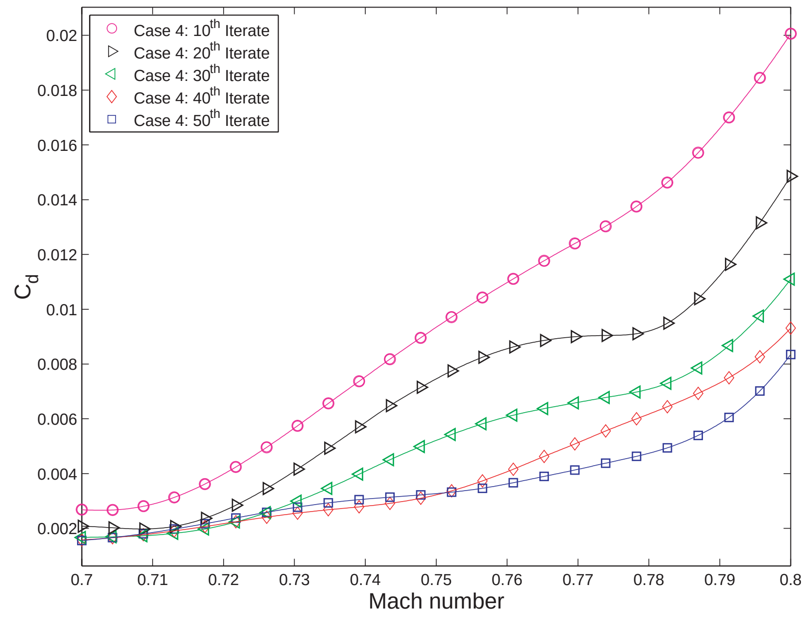
$$C_D^{new} \leq C_D^{old}(1-\gamma_{\min})$$

$$Obj^{new} \leq Obj^{old}(1-\gamma_{obj})$$



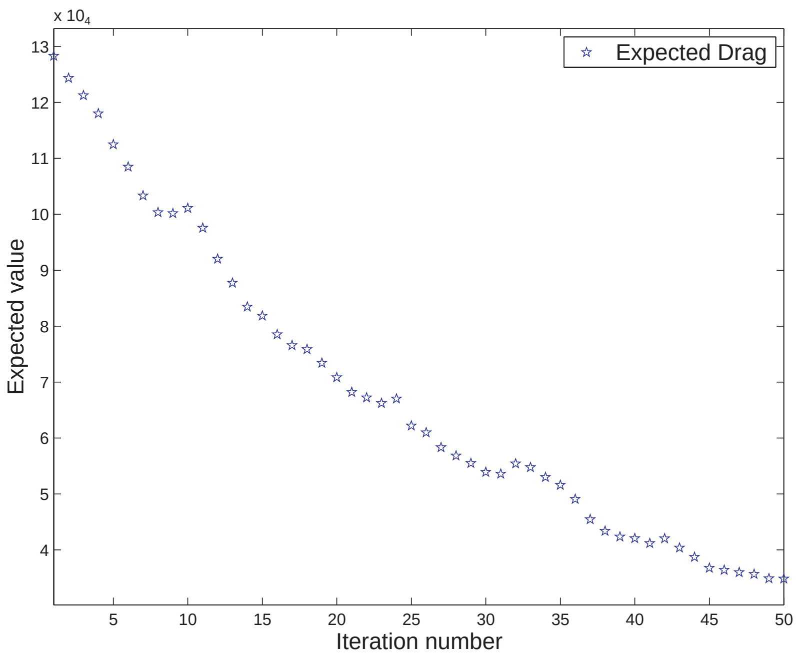
Successful Approach - Conservative

- Fixed M_i
- Some decrease in each C_d is required
- Adaptive trust region size, $\gamma_{obj} = \gamma_{min} = 3\%$
- Good, consistent convergence
- Solution may be overly conservative due to requirement for simultaneous reduction



Successful Approach - Exploratory

- Random M_i
- Decrease in each iteration depends on which M_i are selected
- May discover excellent new designs because of new convergence route



Conclusions

- Heuristic airfoil shape optimization method is quite successful for problem suggested by aero experts.
- Random and fixed design points plus several γ options are tested successfully.
- Fixed approach similar to Li *et.al.* tends to produce improved designs with smallest change to original airfoil.
- Random approach similar to Huyse *et.al.* converges less smoothly but can find unexpected designs
- Choice of options depends on needs of design team