Robust Airfoil Optimization

to achieve a consistent drag reduction over a range of Mach numbers

Wu Li, Sharon Padula, and Luc Huyse

Old Dominion University
ICASE
NASA Langley Research Center

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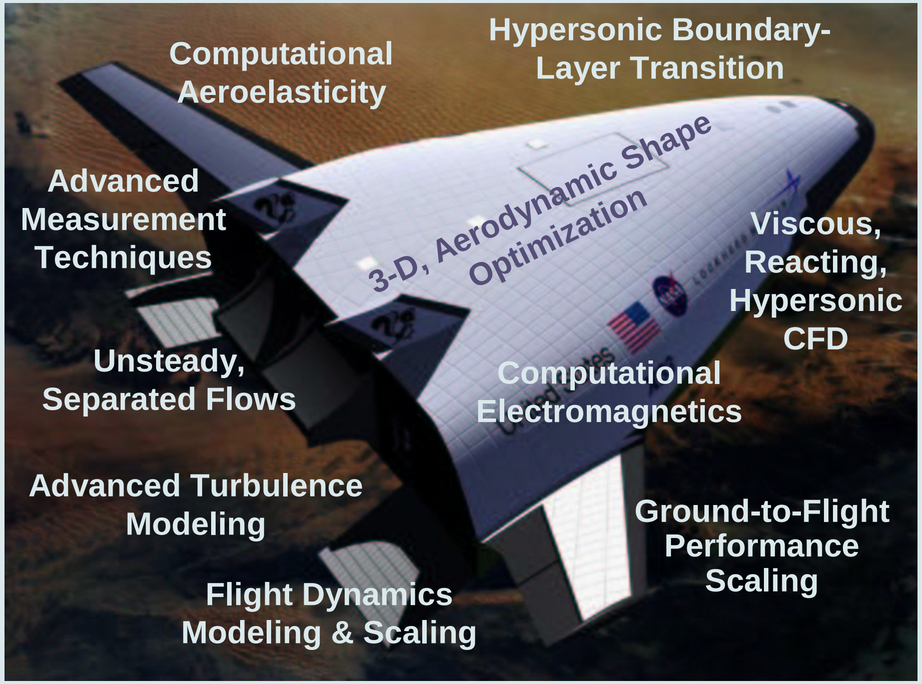
1. Introduction

ASCoT Project: Aerospace Vehicle Systems Technology

Project Vision

Provide next-generation modeling, simulation, and design tools to increase confidence and reduce development time in aerospace vehicle designs



NASA Langley Risk-based Design Group

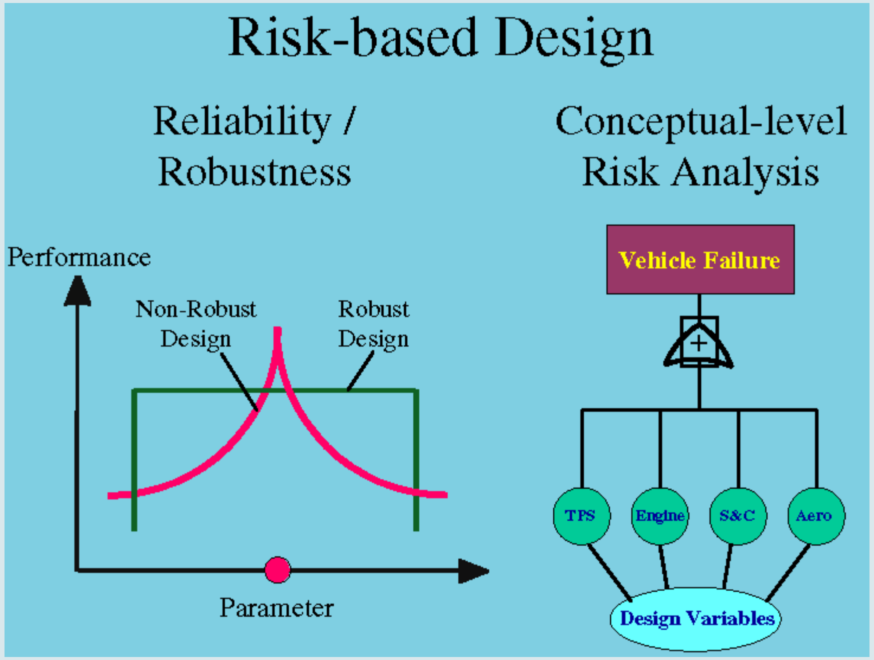
Vision: Develop and validate design methods and tools for aerospace vehicle systems that incorporate reliability, robustness and risk (the 3 R's) concepts in all phases of design

Technical Challenges

- Include risk & reliability at early design phases
- Reduce enormous testing costs and design cycle times via reliability assessment with bounded estimates of uncertainty
- Reduce intractable computational cost of probabilistic methods
- **Create optimization and robust control methods which include uncertainties**

Approach

- Develop multidisciplinary probabilistic-oriented analysis tools and design methods
- Develop technology to construct failure scenarios at conceptual design stage
- Develop and validate structural reliability and robust control synthesis techniques
- **Develop effective algorithms for optimization under uncertainties**



Partners

- DoD, FAA, Boeing, Sandia, NASA (ISE , Propulsion, 3rd Gen RLV, Design for Safety, HPCC, OSMA) — Leverage related programs
- SwRI, ARA, SGI — collaboration on commercial software development
- **Universities — basic research**

2. Airfoil Optimization and Related Topics
(samples of past research works)

- Airfoil shape optimization: high fidelity CFD codes, reliable grid generation, and numerically efficient sensitivity calculations

  Anderson and Venkatakrishnan (1997), Drela (1998),
  Nielsen and Anderson (1998), Anderson and Bonhaus (1999)

- Optimization of 3-D wings

  Elliott and Peraire (1997,1998), Nielsen and Anderson (2001)

- Coupled structural-aerodynamic optimization

  Gumbert, Hou, and Newman (2001)

Except Drela's paper, uncertainty was not considered in these works.

3. Two Robust Optimization Formulations

- What is Robust Optimization?
  - Identify designs that minimize the variability of the performance under uncertain operating conditions. (Taguchi methods)
  - Mitigate the detrimental effects of the worst-case performance. (Ben-Tal and Nemirovski)
  - Provide the best overall performance of a system by maximizing the expected value of its utility. (Huyse and Lewis)
  - Achieve consistent improvements of the performance over a given range of uncertain parameters.

Lift Constrained Drag Minimization

- Expected Value Optimization

$$\min_{D,\alpha(\cdot)} \int_{M_{\min}}^{M_{\max}} c_d(D, M, \alpha(M))\, p(M)\ dM$$

subject to

$$c_l(D, M, \alpha(M)) \geq c_l^* \quad \text{for } M_{\min} \leq M \leq M_{\max}.$$

- Continuous Minimax Optimization

$$\min_{D,\alpha(\cdot)} \max_{M_{\min} \leq M \leq M_{\max}} \rho(M)\, c_d(D, M, \alpha(M))$$

subject to

$$c_l(D, M, \alpha(M)) \geq c_l^* \quad \text{for } M_{\min} \leq M \leq M_{\max}.$$

4. Approximations of Robust Optimization Formulations

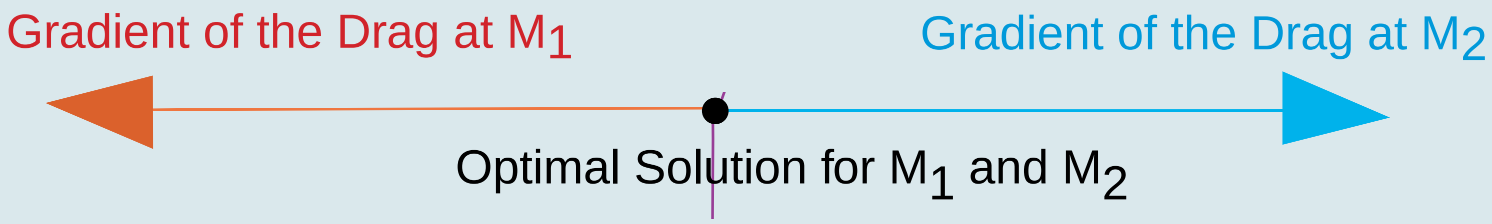
- Multipoint Optimization With Lift Constraints

$$\min_{D,\alpha_1,\ldots,\alpha_r} \sum_{i=1}^{r} w_i \, c_d(D, M_i, \alpha_i) \;\text{ st }\; c_l(D, M_i, \alpha_i) \geq c_l^* \;\text{ for }\; 1 \leq i \leq r.$$

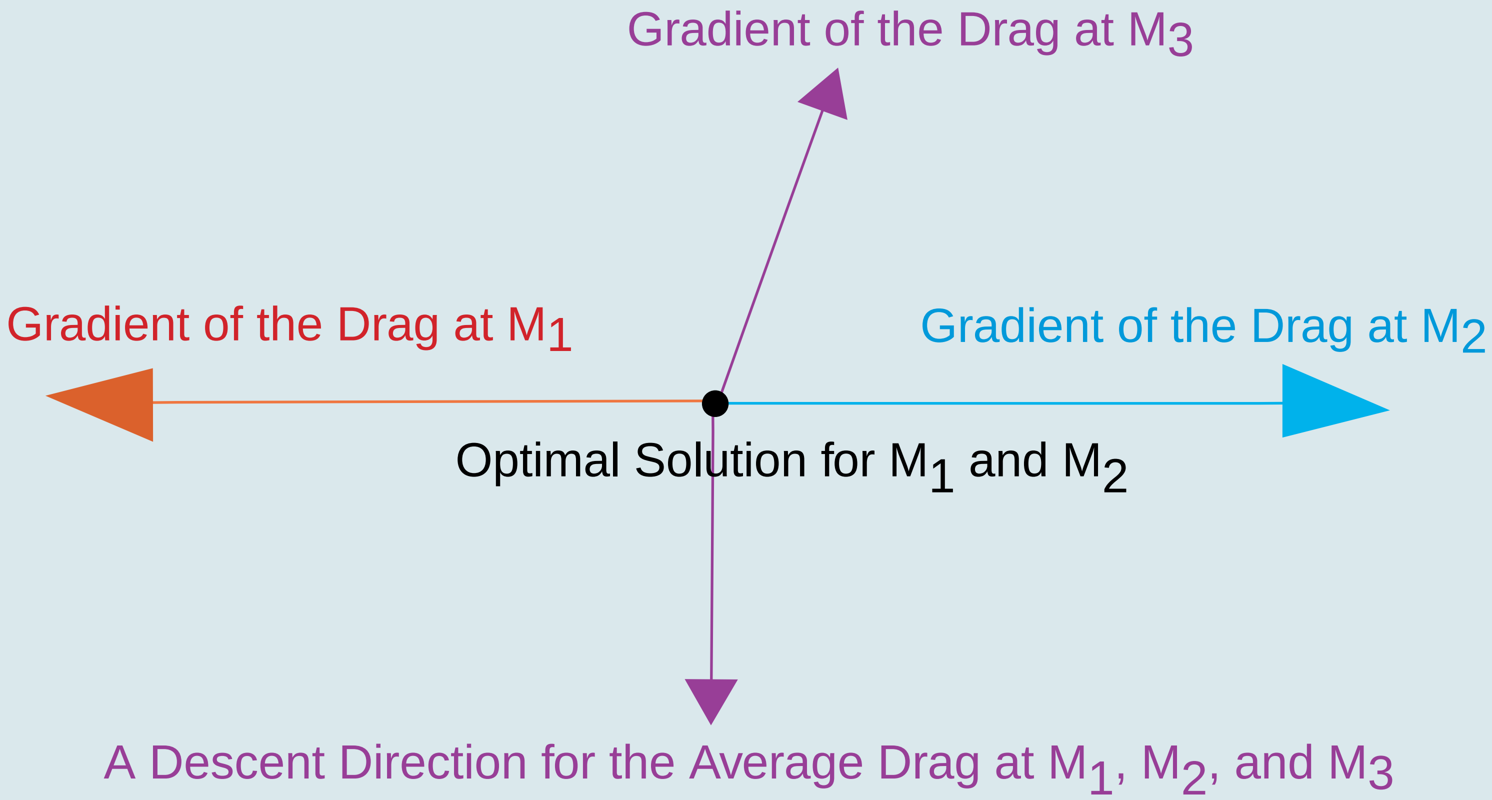
- Discrete Minimax Optimization With Lift Constraints

$$\min_{D,\alpha_1,\ldots,\alpha_r} \max_{1\leq i\leq r} \rho_i \, c_d(D, M_i, \alpha_i) \;\text{ st }\; c_l(D, M_i, \alpha_i) \geq c_l^* \;\text{ for }\; 1 \leq i \leq r.$$

5. Critical Number of Design Points for the Multipoint Optimization

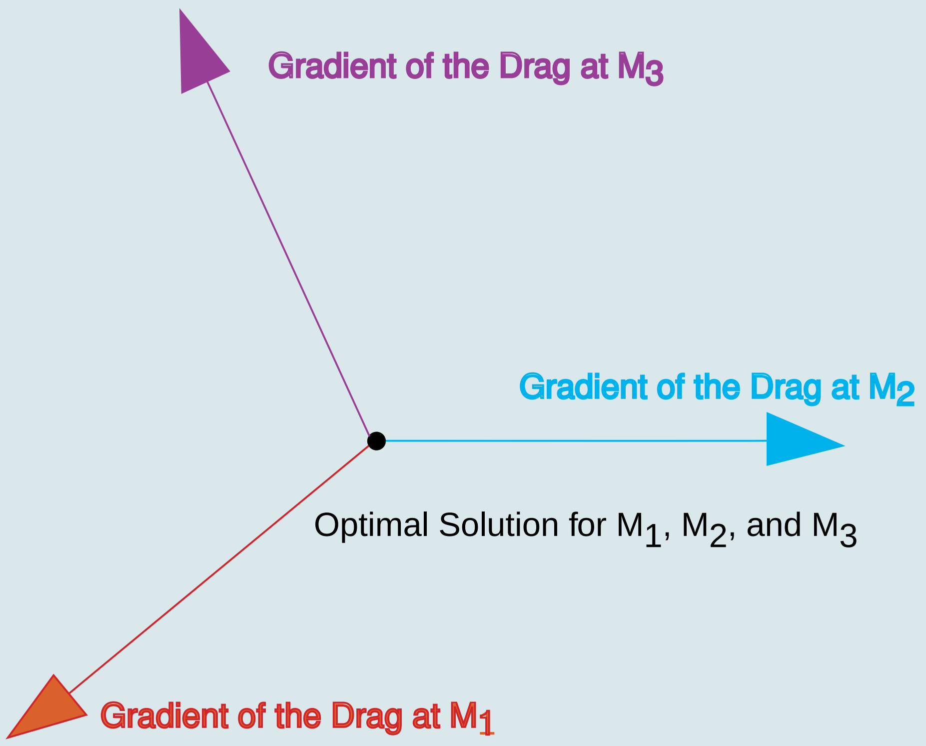


5. Critical Number of Design Points for the Multipoint Optimization



Increases of the drag at M_1 and M_2 in order of t^2 versus a reduction of the drag at M_3 in order of t

For the multipoint optimization method, it is necessary to use (m+1) design points, where m is the number of free-design variables.



A first-order decrease of the drag for any Mach number leads to a first-order increase of the drag for another Mach number.

6. Equivalence of Two Approximation Formulations

$$\min_{D,\alpha_1,\dots,\alpha_r} \sum_{i=1}^{r} w_i\, c_d(D, M_i, \alpha_i) \;\text{ st }\; c_l(D, M_i, \alpha_i) \geq c_l^* \;\text{ for }\; 1 \leq i \leq r.$$

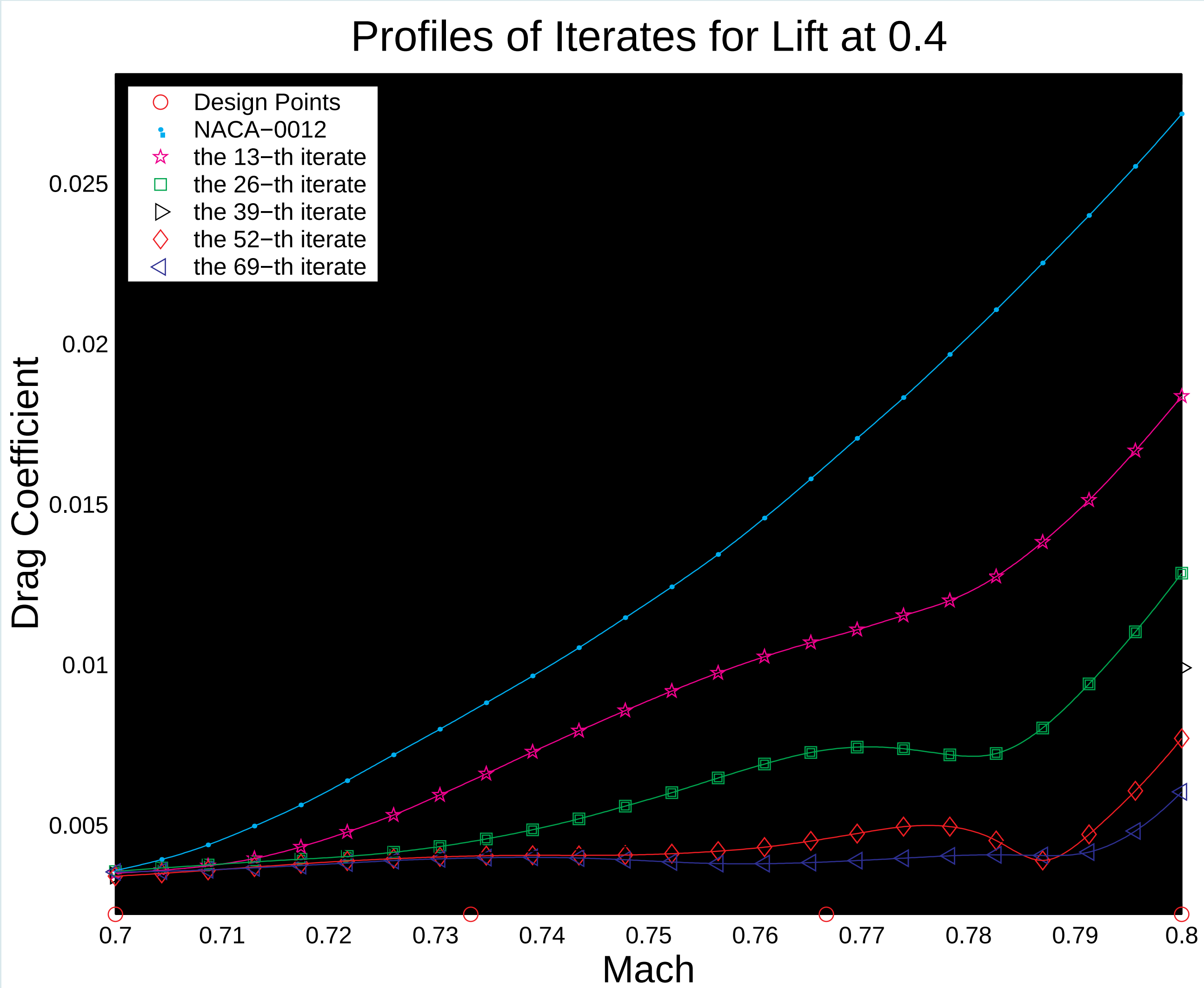
$$\min_{D,\alpha_1,\dots,\alpha_r} \max_{1\leq i\leq r} \rho_i\, c_d(D, M_i, \alpha_i) \;\text{ st }\; c_l(D, M_i, \alpha_i) \geq c_l^* \;\text{ for }\; 1 \leq i \leq r.$$

Under the strict complementarity condition, the equivalence of the above two problems can be established by using the following conversion formulas:

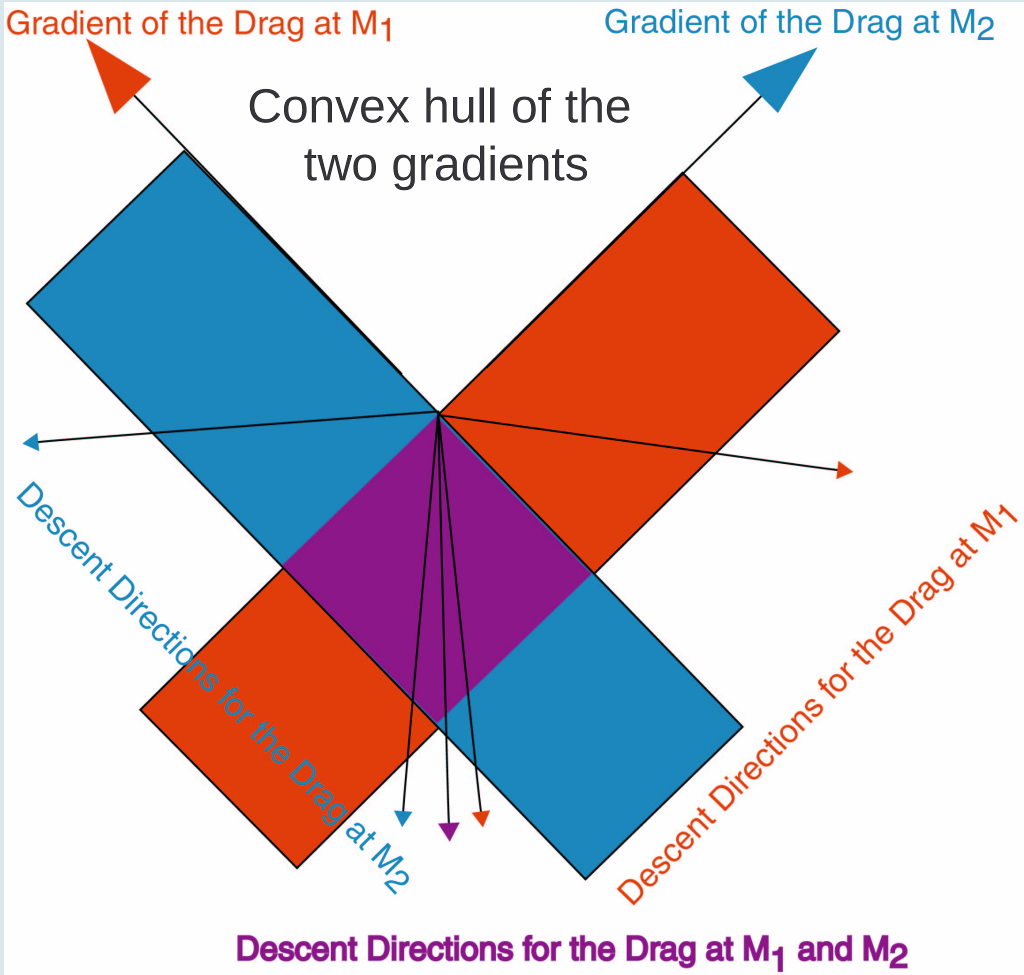
$$\hat{\gamma} := \sum_{i=1}^{r} w_i c_d(\hat{D}, M_i, \hat{\alpha}_i) \;=\; \max_{1\leq i\leq r} \rho_i c_d(\hat{D}, M_i, \hat{\alpha}_i),$$

$$\rho_i := \frac{\hat{\gamma}}{c_d(\hat{D}, M_i, \hat{\alpha}_i)} > 0, \quad \theta_i := \frac{w_i}{\rho_i} \quad \text{for } 1 \leq i \leq r.$$

7. Profile Optimization for a Consistent Drag Reduction over a Mach Range



A Descent Direction for a Consistent Drag Reduction over a Mach Range



The Profile Optimization Method

1) Initialize the angles of attack at r Mach numbers M_1, $M_2,\dots,M_r$ when started.
2) Adjust weights: $\rho_i = 1/c_d(D^k,\alpha_{i,k},M_i)$ for $1\le i\le r.$
3) Find the size of a trust region for an LP subproblem of the discrete minimax optimization problem so that a predicted percentage reduction of the drag for all sampled Mach numbers is achieved.
4) Compute the least norm solution of the LP subproblem: $(\Delta D^k, \Delta\alpha_{1,k},\dots, \Delta\alpha_{r,k})$.
5) Generate the new iterate:

   $D^{k+1} := D^k + \Delta D^k$, and $\alpha_{i,k+1} := \alpha_{i,k} + \Delta\alpha_{i,k}$ for $1\le i\le r.$
6) Repeat the process until a termination criterion is satisfied.

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Choose a Trust Region and Compute the Least Norm Solution

$$\min_{\Delta D, \Delta\alpha_i, \gamma} \ \gamma \quad \text{such that} \quad -\sigma_j\delta \leq \Delta D_j \leq \sigma_j\delta \quad \text{for } 1 \leq j \leq m,$$

$$c_l(D^k, \alpha_{i,k}, M_i) + \left\langle \frac{\partial c_l}{\partial D}(D^k, \alpha_{i,k}, M_i), \Delta D \right\rangle + \frac{\partial c_l}{\partial \alpha}(D^k, \alpha_{i,k}, M_i)\Delta\alpha_i \geq c_l^*,$$

$$c_d(D^k, \alpha_{i,k}, M_i) + \left\langle \frac{\partial c_d}{\partial D}(D^k, \alpha_{i,k}, M_i), \Delta D \right\rangle + \frac{\partial c_d}{\partial \alpha}(D^k, \alpha_{i,k}, M_i)\Delta\alpha_i \leq \frac{\gamma}{\rho_i},$$

$$-\alpha_{i,k} \leq \Delta\alpha_i \leq \alpha_{\max} - \alpha_{i,k}, \qquad \text{for } 1 \leq i \leq r,$$

- Choose δ such that the optimal objective function value of the above LP subproblem is $(1 \quad \eta)$, where η is the predicted percentage reduction of the drag.
- Solve a quadratic perturbation of the above LP subproblem to get the least norm solution. (Mangasarian 1984)

Heuristic Termination Criterion

Let ϵ_k be the accumulative reduction of the drag at the design points defined by

$$\epsilon_k := \sum_{i=1}^{r} \left(c_d(D^k, \alpha_{i,k}, M_i) - c_d(D^{k+1}, \alpha_{i,k+1}, M_i)\right).$$

If

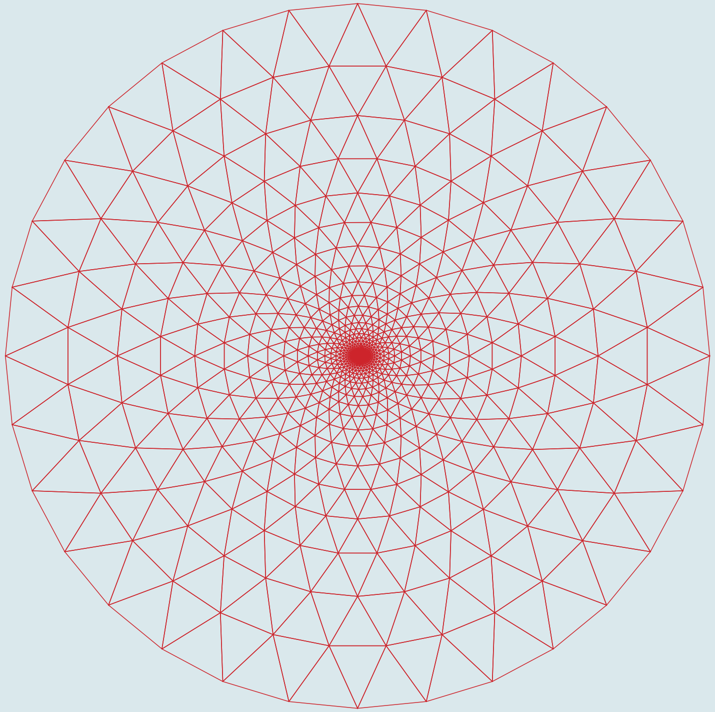
$$\max_{1 \le i \le r} \; \rho_i \, c_d(D^{k+1}, \alpha_{i,k+1}, M_i) > 1 \quad \text{and}$$

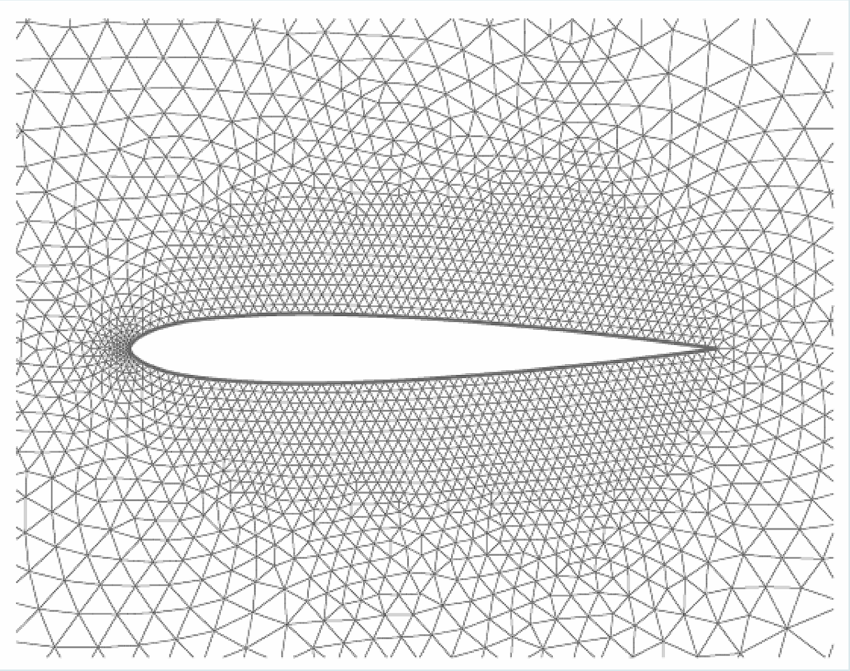
$$\epsilon_k < \epsilon \sum_{i=1}^{r} c_d(D^k, \alpha_{i,k}, M_i),$$

then output D^k as an optimal solution and terminate the algorithm.

8. Numerical Results

The grid for solving the Euler equation



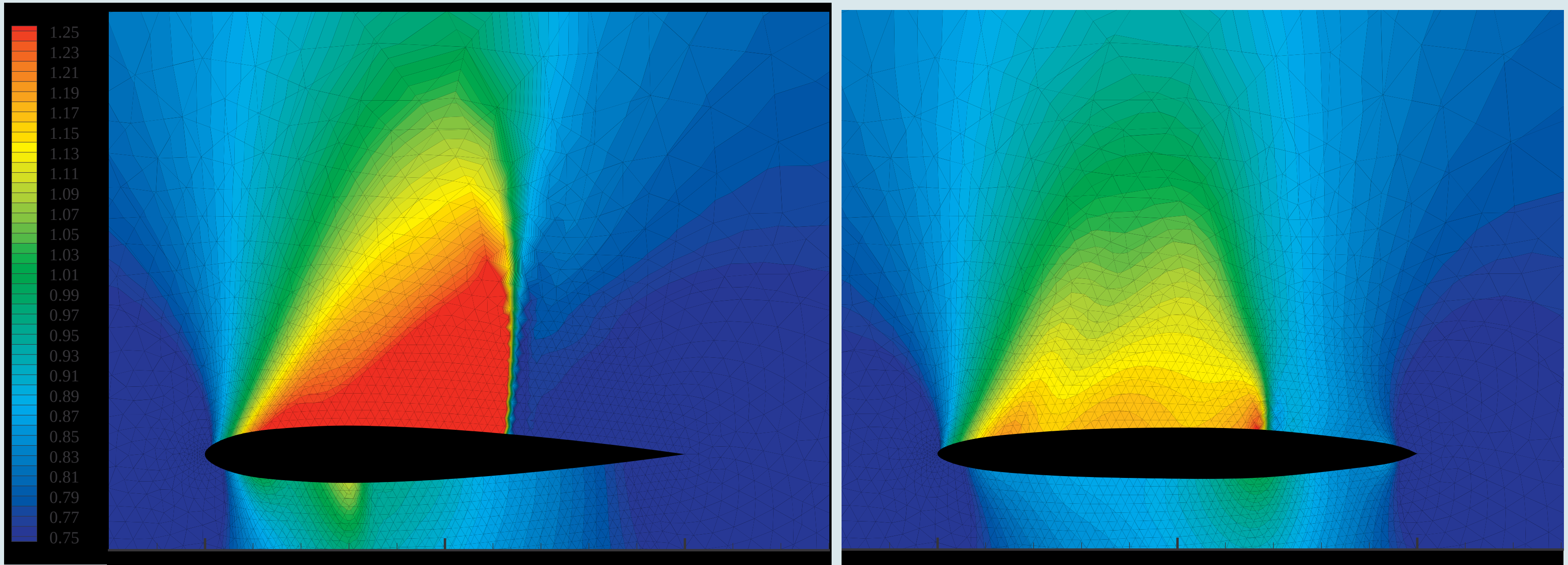


Airfoil shape is represented by spline functions with 20 free-design parameters.

Typical Mach Contours, for $M_\infty = 0.796$

A strong shock wave for NACA-0012 is reduced to several weaker shock waves for an optimal airfoil generated by the profile optimization.

$c_\ell = 0.4$, $c_d = 0.0255$ $c_\ell = 0.4$, $c_d = 0.0048$



(a) NACA-0012 (b) Profile Optimization

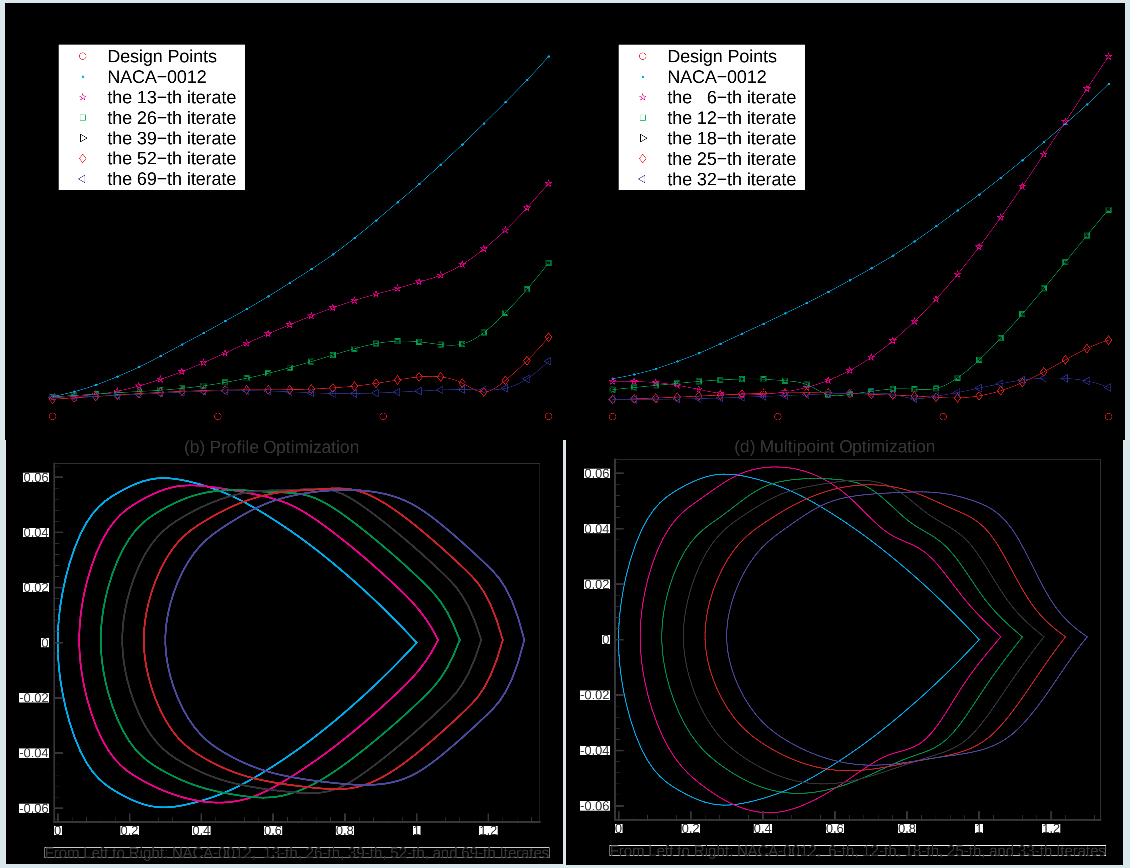
Comparison of Optimal Solutions

Table 1. Relative drag reduction rates (in percentage)

	Max	Min	Average
Profile Optimization (3 Points)	82.9	5.1	70.2
Profile Optimization (4 Points)	82.7	1.4	69.7
Profile Optimization (8 Points)	82.6	4.1	67.9
Multipoint Optimization (4 Points)	89.4	46.2	81.1
Expected Value Optimization (4 Points)	89.4	49.6	82.6

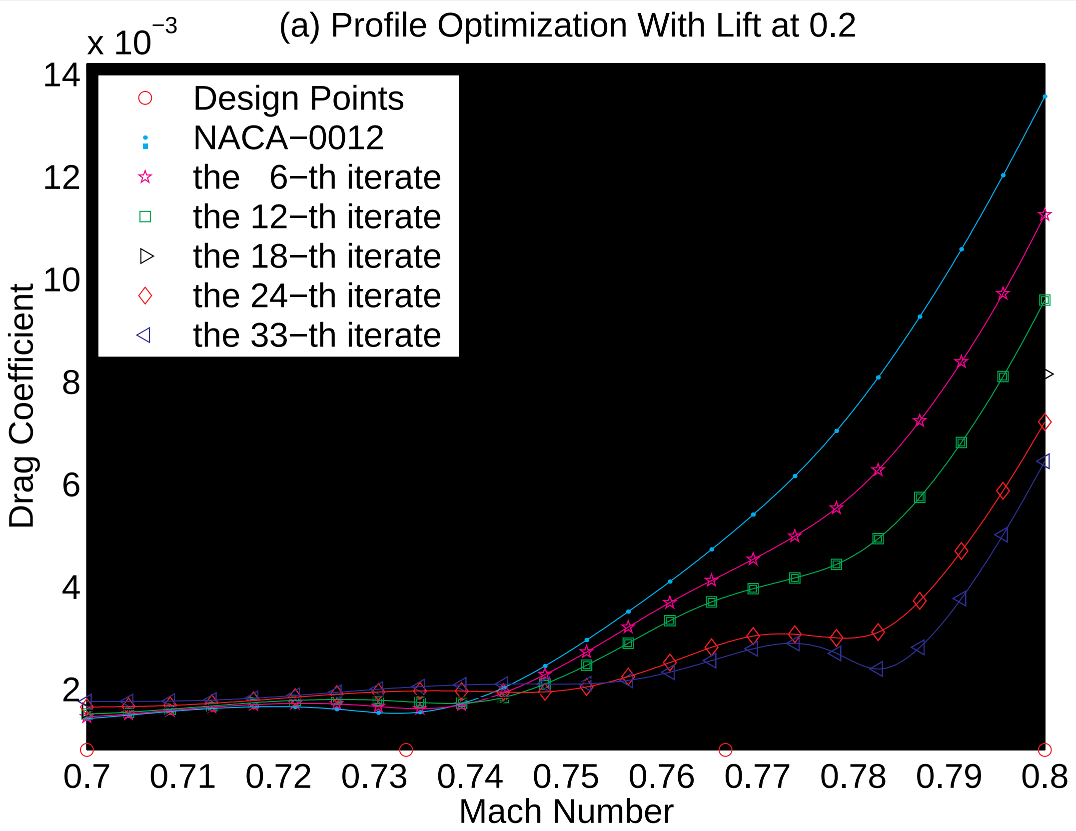
- **The profile optimization method uses a very conservative optimization strategy to achieve the robustness of the optimal solution.**
- **The number of design points has no impact on the profile optimization method.**

Comparison of Optimal Airfoils With Lift at 0.4



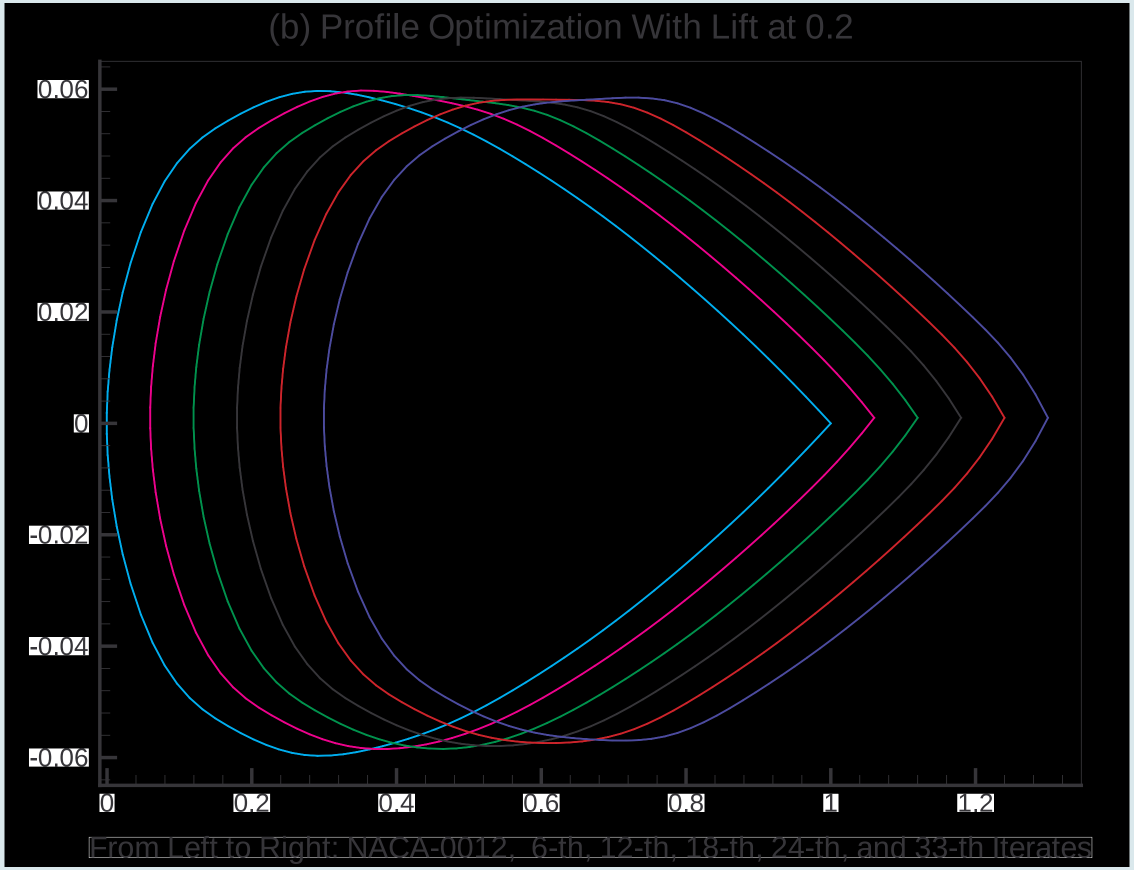
For the profile optimization method, far fewer design points (4) than free-design variables (20) are needed to generate robust optimal solutions and there is no random airfoil shape distortion.

Case Study II: Drag Reduction for Lift at 0.2



The only compromise made by the optimal airfoil to the NACA-0012 is a small increase of the drag around the original drag bucket for the NACA-0012.

Case Study II: Changes of the Airfoil Shape



There is no random distortion of airfoil shapes during the optimization process.

9. Conclusion

1) For the multipoint optimization method, in order to avoid any degradation in the off-design performance, it is necessary to use more design points than the number of free-design variables.
2) With adaptive adjustment of weights, the profile optimization method generates robust optimal solutions for airfoil shape optimization under uncertain flight conditions with far fewer design points (4) than free-design variables (20).
3) The profile optimization method finds a drag reduction direction for all design conditions, which leads to a consistent drag reduction over a given Mach range from iteration to iteration.
4) There is no random airfoil shape distortion for any iterate generated by the profile optimization method.
5) The profile optimization method allows a designer to make a trade-off between a truly optimized airfoil and the amount of computing time consumed.
6) **The profile optimization method has the potential of becoming a practical design tool for optimization under uncertainty.**

Acknowledgement

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Download the paper from www.math.odu.edu/~wuli/