THE ATTENUATION OF ELECTROMAGNETIC WAVES BY MULTIPLE KNIFE-EDGE DIFFRACTION

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Starting from work by Furutsu, a multiple knife-edge attenuation function is derived. A series representation of the function is developed which is amenable to computer implementation. Comparisons of computer-generated numerical values with known results are presented and discussed.

Key words: attenuation calculations; microwave propagation, multiple knife-edge diffraction

1. INTRODUCTION

For the propagation of radio signals over irregular terrain at microwave frequencies, it appears reasonable to assume that the terrain obstacles along the path are approximately equivalent to knife-edge obstacles because of the short wavelengths involved. In fact this has been suggested as a possible propagation mechanism by many authors. Unless the path contains large portions of calm water, the terrain features of an actual path are very seldom smooth rounded obstacles at microwave frequencies.

Single knife-edge diffraction theory has been found to give good agreement with observed measurements of propagation over paths consisting of essentially one isolated hill (Kirby et al., 1955). Similarly, a double knife-edge theory has been developed and shows excellent agreement with recent test measurements (Ott, 1979). Multiple knife-edge theory for more than two knife-edges has not been available up to now, although recently suggested approximations have been compared with observed data (Meeks and Reed, 1981).

It is the purpose of this paper to derive an expression for the multiple knife-edge attenuation function. This equation, in the form of a multiple integral, is then developed into a series which is amenable to computer implementation. Computer generated numerical values are compared with known results as a means of computational verification.

The derivation starts from some basic results pertaining to propagation over irregular terrain obtained by Furutsu (1963). The expression from which the work

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in the present paper proceeds is a generalized residue series formulation for the propagation of radio signals over smooth, rounded obstacles. No attempt is made here to describe the work leading up to this expression because the details are given by Furutsu (1956, 1963).

2. THE MULTIPLE KNIFE-EDGE ATTENUATION FUNCTION

In the derivation of the attenuation function for propagation over irregular terrain, Furutsu (1963; p. 55) assumes a path profile consisting of a series of rounded obstacles as shown in Figure 1. The obstacles are characterized by radii of curvature, a_m, diffraction angles, θ_m, electromagnetic parameters, q_m, and separation distances, r_m. The quantity, q_m, is a function of the radius and electrical ground constants of the mth obstacle, and the wavelength λ and polarization of the wave.

For a path having N obstacles and for both transmitting and receiving antennas well away from any diffracting surface, the attenuation of the field strength relative to free-space, A, over a total path distance, r_T, is given by equation (3.1) of the Furutsu paper:

$$A = C_N' \sum_{t_1} \cdot \cdot \cdot \sum_{t_N} \left\{ \prod_{m=1}^{N} T_1(\xi_m) \right\} \left\{ \prod_{m=1}^{N+1} T_2(r_m) \right\}, \quad N \geq 1 \quad , \tag{1}$$

where

$$C_N' = \left[r_T/(k^N r_1 \cdot r_2 \cdot \cdot \cdot r_{N+1}) \right]^{1/2} \quad , \tag{2}$$

$$T_1(\xi_m) = (2\sqrt{2\pi}) e^{-i3\pi/4} (ka_m/2)^{1/3} f(t_m) e^{-i\xi_m t_m}$$

$$\sim (\pi/2)^{1/2} e^{-i\pi/4} (ka_m/2)^{1/3} t_m^{-1/2} e^{-i\xi_m t_m} \quad , \tag{3}$$

$$T_2(r_m) = \exp\left[-i(2kr_m)^{-1} \left\{ (ka_{m-1}/2)^{1/3} t_{m-1} - (ka_m/2)^{1/3} t_m \right\}^2 \right], \tag{4}$$

and $$\xi_m = (ka_m/2)^{1/3} \theta_m \quad , \tag{5}$$

with $k = 2\pi/\lambda$ denoting the wave number.

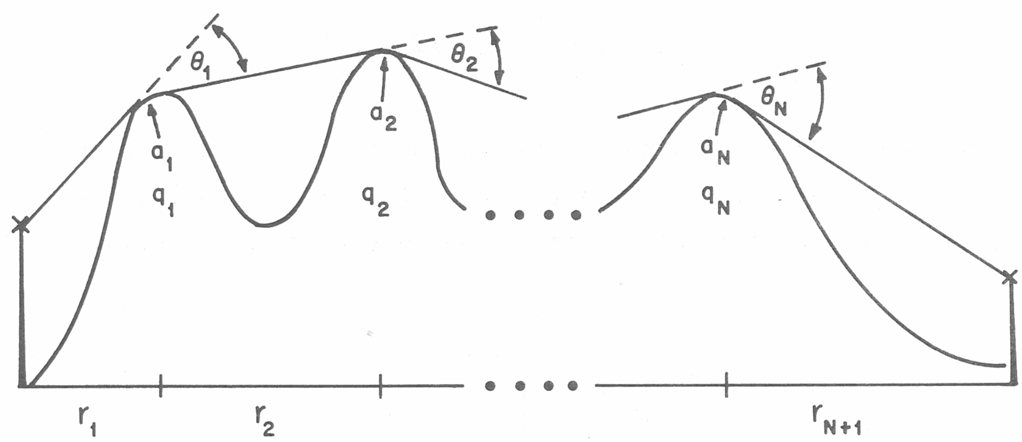

Figure 1. Representative path profile and geometry for equation (1).

The symbol t_m is here used as shorthand notation for the set of roots of the equation

$$W'(t) - q_m W(t) = 0 \quad , \tag{6}$$

where $W(t)$ is the Airy function as defined in equation (1.2) of Furutsu (1963). Thus, a summation over t_m should be interpreted as a summation over all the roots of (6). Also, t_0 and t_{N+1} as they enter in (4) are defined to be identically zero.

The function, $f(t_m)$, in (3) is

$$f(t_m) \equiv \left[(t_m - q_m^2)\, W^2(t_m) \right]^{-1} \sim e^{i\pi/2}/(4t_m^{\;1/2}) \quad , \tag{7}$$

where the approximation is obtained by taking the first term of the asymptotic expansion of $W(t_m)$, valid for $0 < \arg t_m < 4\pi/3$ (Furry and Arnold, 1945).

Equation (1) can be put into a more convenient form if we define the parameters

$$\eta_m = (ka_m/2)^{1/3} \left[\frac{2(r_m + r_{m+1})}{kr_m r_{m+1}} \right]^{1/2} \quad , \tag{8}$$

$$\gamma_m = (ka_m/2)^{1/3}\, (ka_{m+1}/2)^{1/3}/(kr_{m+1}) \quad . \tag{9}$$

Then

$$\prod_{m=1}^{N} T_1(\xi_m) = (\pi/2)^{N/2}\, e^{-iN\pi/4} \left\{ \prod_{m=1}^{N} (ka_m/2)^{1/3} t_m^{\;-1/2} \right\} \exp\left[-i \sum_{m=1}^{N} \xi_m t_m \right], \tag{10}$$

$$\prod_{m=1}^{N+1} T_2(r_m) = \exp\left[-i \sum_{m=1}^{N} \left\{ (\eta_m/2)^2 t_m^{\;2} - \gamma_m t_m t_{m+1} \right\} \right], \tag{11}$$

and

$$A = (\pi/2)^{N/2}\, e^{-iN\pi/4}\, C_N' \sum_{t_1} \cdots \sum_{t_N} \left\{ \prod_{m=1}^{N} (ka_m/2)^{1/3}\, t_m^{\;-1/2} \right\} e^{-F_N} \quad , \tag{12}$$

$$F_N = i \sum_{m=1}^{N} \left\{ (\eta_m/2)^2 \; t_m{}^2 + \xi_m t_m - \gamma_m t_m t_{m+1} \right\} \quad . \tag{13}$$

As long as the obstacle radii, $a_m(m = 1, \ldots, N)$, are not too small, only the first few terms in the summations of (12) are required in order to compute the attenuation. However, if the obstacles are to represent knife-edges, which is our present concern, the a_m must decrease to zero. In this case the series converges very slowly and many terms must be calculated.

In the usual approach the summations are transformed into integrals which, it is hoped, are more amenable to computation. And in fact for the case of one knife-edge, the transformation results in the well-known Fresnel knife-edge diffraction function. A rigorous derivation of the transformation has been discussed by many authors, e.g., Bremmer (1949), Wait (1961), Furutsu (1963). In the present paper a less rigorous but quicker method will be used which leads to the same result.

The parameter, q_m, appearing in (6) is proportional to $a_m{}^{1/3}$ and, consequently, tends to zero as the radius becomes very small. It is known that a good approximation to the roots for the case of $q = 0$ is given by (Bremmer, 1949):

$$t_s = \left\{ (3\pi/2)(s + 1/4) \right\}^{2/3} \; e^{-i\pi/3}, \; s = 0, 1, 2, \ldots \quad . \tag{14}$$

Thus, for a given function ϕ, we have

$$\sum_{t_s} \phi(t_s) \equiv \sum_{s=0}^{\infty} \phi(t_s)$$

$$\sim \int_0^{\infty} \phi(t) ds \sim \int_0^{\infty} (ds/dt) \; \phi(t) dt \quad , \tag{15}$$

where, in the integral expressions, t and s are now considered to be continuous variables related by

$$t = \left\{ (3\pi/2)(s + 1/4) \right\}^{2/3} \; e^{-i\pi/3} \quad , \tag{16a}$$

$$ds = (1/\pi) \; e^{i\pi/2} \; t^{1/2} \; dt \quad . \tag{16b}$$

With the definitions

$$\Phi = (\pi/2)^{N/2}\, e^{-iN\pi/4}\, C_N' \left\{ \prod_{m=1}^{N} (ka_m/2)^{1/3}\, t_m^{-1/2} \right\} e^{-F_N} \quad , \tag{17}$$

$$t_m = \left\{ (3\pi/2)(s_m + 1/4) \right\}^{2/3} e^{-i\pi/3} \tag{18a}$$

$$(ds_m/dt_m) = (1/\pi)\, e^{i\pi/2}\, t_m^{1/2} \tag{18b}$$

the attenuation as given by (12) becomes

$$A = \sum_{t_1} \cdot \cdot \cdot \sum_{t_N} \Phi \sim \int_o^\infty \cdot \cdot \cdot \int_o^\infty \left\{ \prod_{m=1}^{N} (ds_m/dt_m) \right\} \Phi\, dt_1 \ldots dt_N$$

$$= (2\pi)^{-N/2}\, e^{iN\pi/4}\, C_N' \left\{ \prod_{m=1}^{N} (ka_m/2)^{1/3} \right\} \int_o^\infty \cdot \cdot \cdot \int_o^\infty e^{-F_N}\, dt_1 \ldots dt_N \quad , \tag{19}$$

where F_N and C_N' are defined in (13) and (2), respectively.

We now introduce the change of variable

$$(\eta_m/2) e^{i\pi/4}\, t_m = \tau_m, \text{ if } m \leq N, \text{ and } \tau_{N+1} \equiv 0 \quad , \tag{20a}$$

$$dt_m = (2/\eta_m) e^{-i\pi/4}\, d\tau_m = (ka_m/2)^{-1/3} \left[\frac{2kr_m r_{m+1}}{r_m + r_{m+1}} \right]^{1/2} e^{-i\pi/4}\, d\tau_m \quad , \tag{20b}$$

and define

$$i\, \xi_m t_m = 2(\xi_m/\eta_m) e^{i\pi/4}\, \tau_m \equiv 2\, \beta_m\, \tau_m \quad , \tag{21}$$

$$i\, \gamma_m t_m t_{m+1} = 4(\gamma_m/\eta_m \eta_{m+1})\, \tau_m\, \tau_{m+1} \equiv 2\, \alpha_m \tau_m \tau_{m+1} \quad , \tag{22}$$

where $$\beta_m = \theta_m \left[\frac{ikr_m r_{m+1}}{2(r_m + r_{m+1})} \right]^{1/2} , \; m = 1, \ldots, N, \tag{23}$$

$$\alpha_m = \left[\frac{r_m r_{m+2}}{(r_m + r_{m+1})(r_{m+1} + r_{m+2})} \right]^{1/2} , \quad m = 1, \ldots, N - 1 \quad . \tag{24}$$

The attenuation, A, in (19) now takes the form

$$A = (1/\pi)^{N/2} \, C_N \int_0^\infty \cdot \cdot \cdot \int_0^\infty e^{-F_N} \, d\tau_1 \ldots d\tau_N \quad , \tag{25}$$

where $$F_N = \sum_{m=1}^{N} \left\{ \tau_m^{\,2} + 2 \, \beta_m \, \tau_m - 2 \, \alpha_m \, \tau_m \, \tau_{m+1} \right\} \quad , \quad \tau_{N+1} \equiv 0 \quad , \tag{26}$$

$$C_N = \begin{cases} 1 & , \; N = 1 \qquad (27a) \\ \left[\dfrac{r_2 r_3 \ldots r_N r_T}{(r_1 + r_2)(r_2 + r_3) \ldots (r_N + r_{N+1})} \right]^{1/2} & , \; N \geq 2 \qquad (27b) \end{cases}$$

$$r_T = r_1 + r_2 + \ldots + r_{N+1} \quad . \tag{28}$$

Finally, with $x_m = \tau_m + \beta_m$ and $d\tau_m = dx_m$, the attenuation function for a path consisting of N knife-edges may be expressed as

$$\begin{aligned} A &= (1/\pi)^{N/2} \, C_N \, e^{\sigma_N} \int_{\beta_1}^\infty \cdot \cdot \cdot \int_{\beta_N}^\infty e^{-F_N} \, dx_1 \ldots dx_N \\ &= (1/2^N) \, C_N \, e^{\sigma_N} \, (2/\sqrt{\pi})^N \int_{\beta_1}^\infty \cdot \cdot \cdot \int_{\beta_N}^\infty e^{2f} \, e^{-(x_1^{\,2} + \ldots + x_N^{\,2})} \, dx_1 \ldots dx_N , \end{aligned} \tag{29}$$

where $$f = \begin{cases} 0 & , \; N = 1 \qquad (30a) \\ \displaystyle\sum_{m=1}^{N-1} \alpha_m (x_m - \beta_m)(x_{m+1} - \beta_{m+1}) & , \; N \geq 2 \qquad (30b) \end{cases}$$

$$\sigma_N = \beta_1^{\,2} + \ldots + \beta_N^{\,2} \quad . \tag{31}$$

The quantities C_N, α_m, and β_m are defined in (27), (24), and (23), respectively.

Notice that for N = 1, (29) becomes the well-known single knife-edge diffraction function (Baker and Copson, 1950; Wait and Conda, 1959)

$$A(N = 1) = (1/2)\, e^{\beta_1^2}\, (2/\sqrt{\pi}) \int_{\beta_1}^{\infty} e^{-x^2}\, dx \quad , \tag{32}$$

$$\beta_1 = \theta_1 \left[\frac{ikr_1 r_2}{2(r_1 + r_2)} \right]^{1/2} \quad . \tag{33}$$

For N = 2 the equivalent of (29) is given by Furutsu (1956). In that paper the equation is transformed into yet another form from which series expansions are developed to compute double knife-edge attenuation (see Furutsu, 1963). The development is quite different from the one used in the present paper and serves as an excellent check. A similar formulation for the double knife-edge case based on repeated Kirchhoff integrals at each knife-edge aperture has also been derived by Millington et al., (1962).

As far as the author is aware, no explicit formulation of the knife-edge attenuation function for $N \geq 3$ has been published previously. Very general discussions indicating possible approaches to the problem have appeared, but detailed analyses are lacking. Approximate solutions based on linear combinations of the single knife-edge function have been developed by Deygout (1966) and by Meeks and Reed (1981). Furthermore, an unpublished computer program to compute triple knife-edge attenuation based on an extension of Furutsu's approach to the double knife-edge case is available. This latter program has been used to check the validity of the results of the present paper.

In order to evaluate the attenuation function as given by (29), a number of approaches were tried including a straightforward numerical integration of the expression as it stands. However, once past the double knife-edge case, the complexity of the solutions increases greatly. Finally, an approach was adopted which made use of repeated integrals of the error function. The latter have been thoroughly studied, and a number of computational algorithms are available. The following section discusses the derivation of the equations used for numerical evaluation of the multiple knife-edge attenuation function, (29).