Nuclear Structure I: Basic Facts and Models

J. Engel

University of North Carolina

June 13, 2005

Outline

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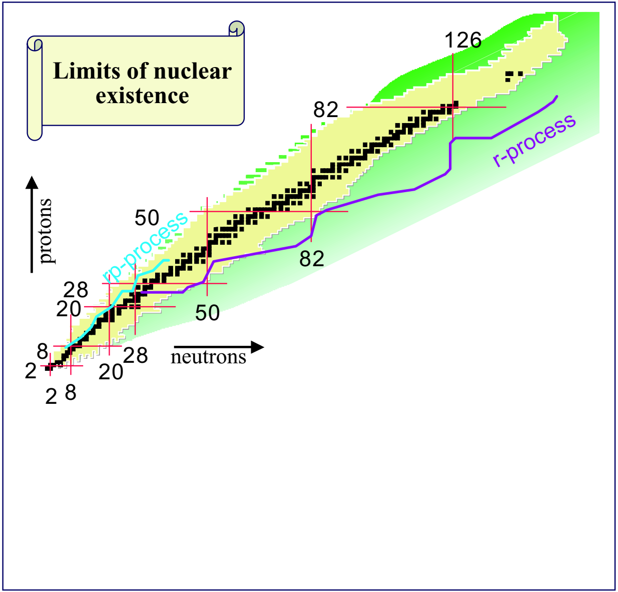
The Nuclear Landscape



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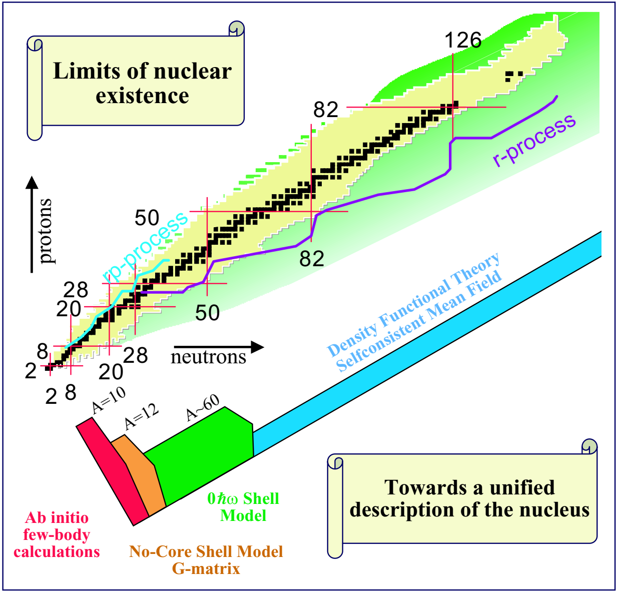
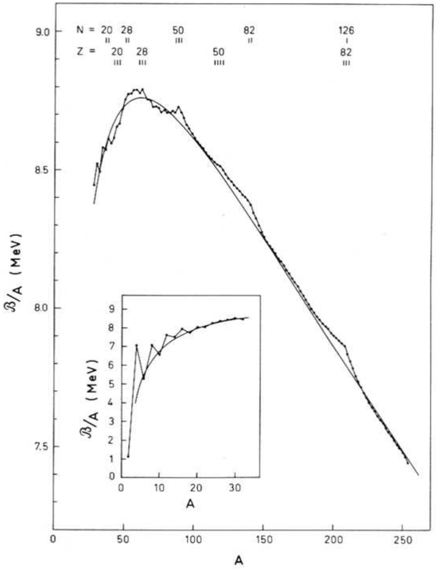


Binding Energies



From Bohr and Mottelson, *Nuclear Structure*, v.1, p. 168.

Bethe-Weizsacker Formula

$$\mathcal{B}(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(Z-N)^2}{A}$$

with

$$a_V = 15.65 \text{ MeV}$$
$$a_s = 17.23 \text{ MeV}$$
$$a_C = .698 \text{ MeV}$$
$$a_{\text{sym}} = 28.1 \text{ MeV}$$

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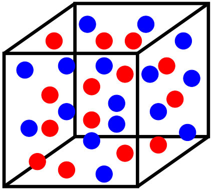
Simplest Possible Model: Nucleons in a "Box"



Periodic boundary conditions:

$$k_x = \frac{2\pi n_x}{L}, \qquad \Delta k_x = \frac{2\pi}{L}, \qquad \Delta^3 k = \frac{(2\pi)^3}{L^3} = \frac{(2\pi)^3}{V}$$

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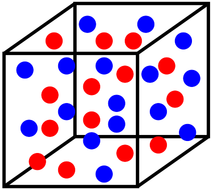

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$$\rho_0 \quad = \quad (1/V)\sum_{\vec{k},s}\theta(k_F - k) \quad = \quad (1/V)\sum_{\vec{k},s}\frac{V}{(2\pi)^3}\,\Delta^3 k\;\theta(k_F - k)$$

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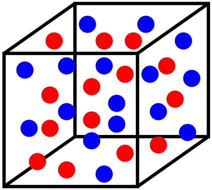

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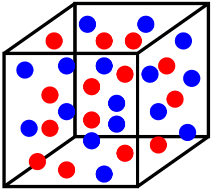

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From $\rho \approx 1.72\ \mathrm{fm}^{-3}$ get $k_F = 1.36\ \mathrm{fm}^{-1}$, $\epsilon_F = 38\mathrm{MeV}$.

Energy density:

$$\mathcal{E}_0 = \frac{2}{(2\pi)^3} 4\pi \int_0^{k_F} \frac{k^4}{2m}\, dk = \frac{k_F^5}{10m\pi^2} = \frac{3}{5}\frac{k_F^2}{2m}\rho_0 = 3/5\epsilon_F\rho_0$$

and $\bar{\epsilon} \equiv \mathcal{E}_0/\rho_0 = (3/5)\epsilon_F \approx 23$ MeV.

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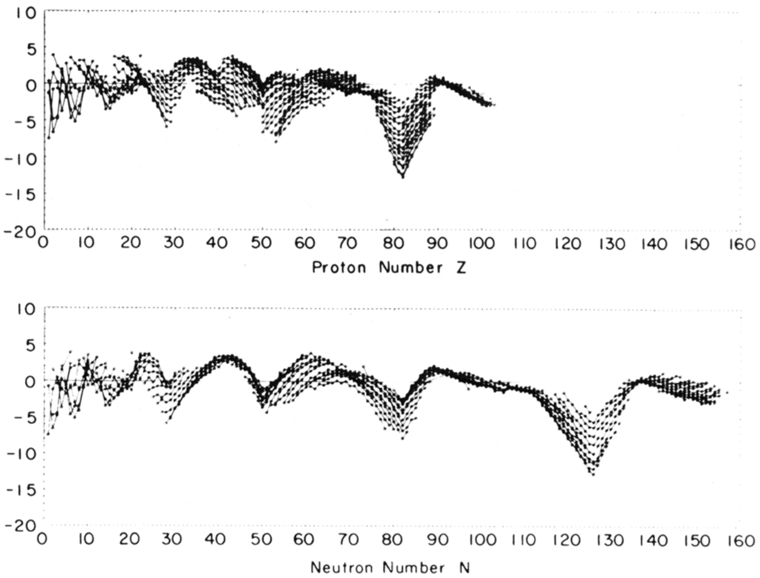
Homework problem: Derive the value of a_{sym} above.

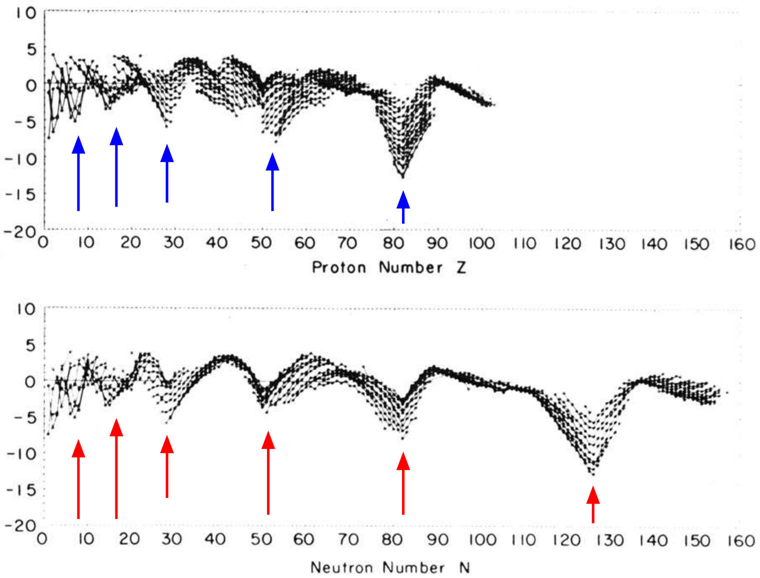
Outline

Binding-Energy Deviations



From W.D. Meyers and W.J. Swiatecki, Nucl. Phys. **81**, 1 (1966).

Binding-Energy Deviations



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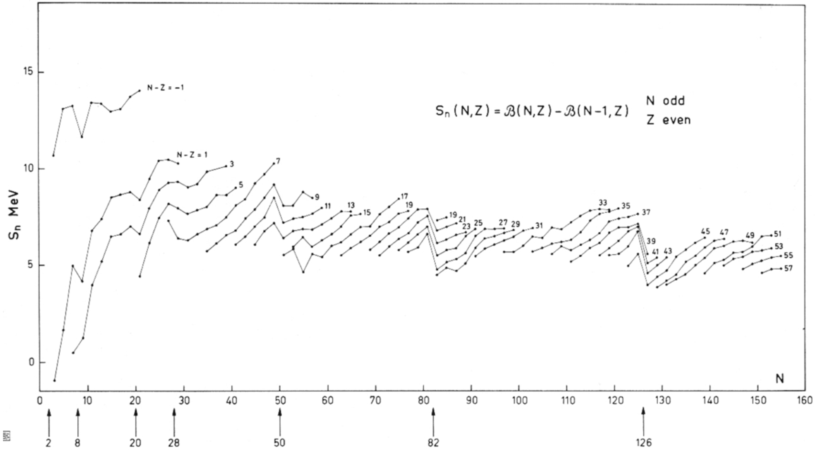
Neutron Separation Energies



From Bohr and Mottelson, *Nuclear Structure*, v.1, p. 193.

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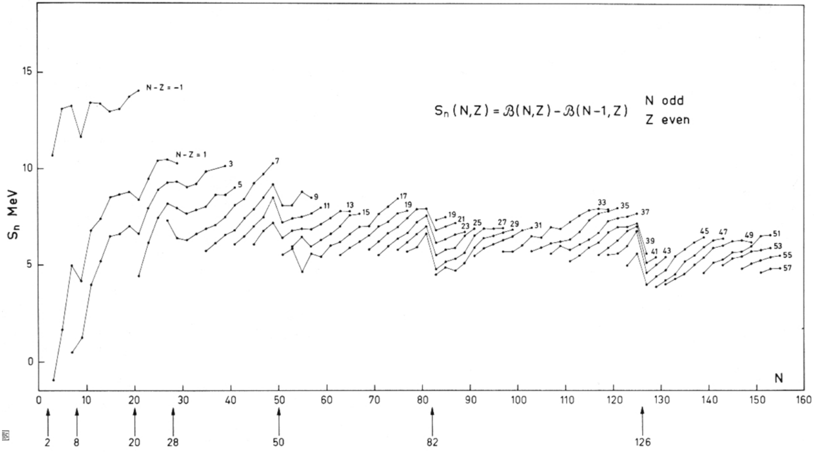
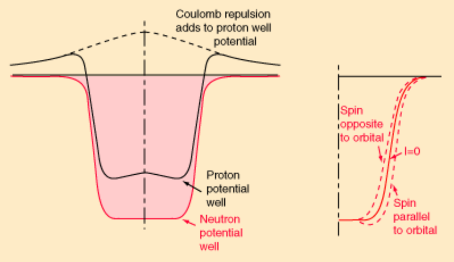


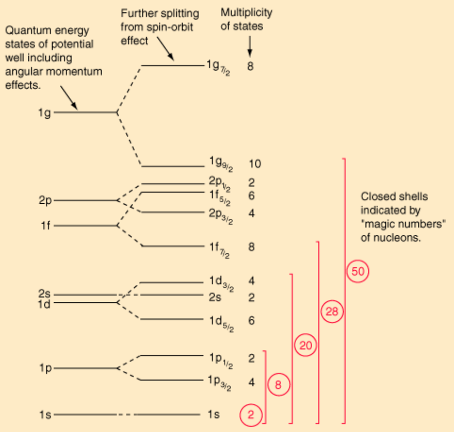
From Bohr and Mottelson, *Nuclear Structure*, v.1, p. 193.

Levels not evenly spaced: shell gaps at "magic numbers".

Shell Model of Nucleus

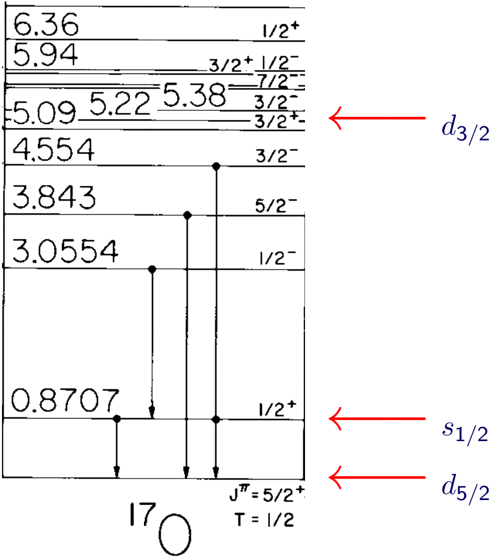


http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/shell.html

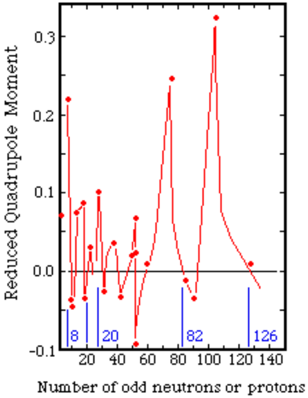


Replace spherical box with more sophisticated potential. Get magic numbers: 2, 8, 20, 28, 50, 126

An Example


6.36 1/2⁺
5.94 3/2⁺ 1/2⁻
7/2⁻
5.09 5.22 5.38 3/2⁻
3/2⁺
4.554 3/2⁻
3.843 5/2⁻
3.0554 1/2⁻
0.8707 1/2⁺
J^π = 5/2⁺
T = 1/2
17O
$d_{3/2}$
$s_{1/2}$
$d_{5/2}$

Shell Model Doesn't Explain Collective Rotation. . .



From Booth and Combey, http://www.shef.ac.uk/physics/teaching/phy303/phy303-3.html and E. Ormand, http://www.phy.ornl.gov/npss03/ormand1.ppt

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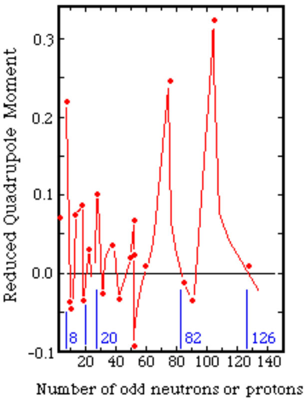

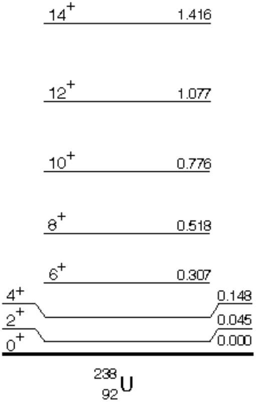

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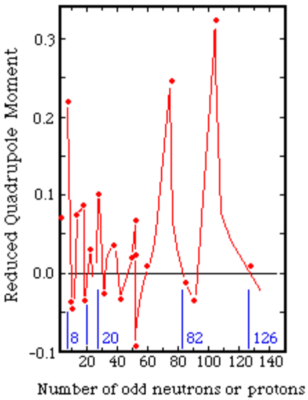

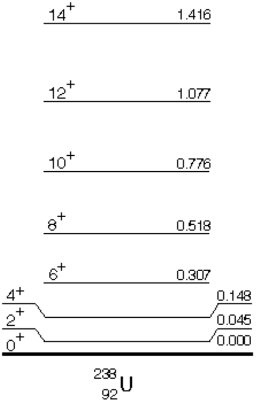

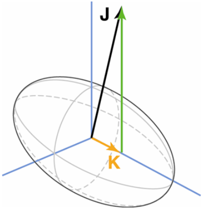

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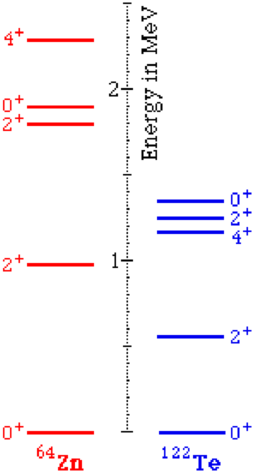
Some sort of collective rotation going on between magic numbers!

And Doesn't Explain Collective Vibrations



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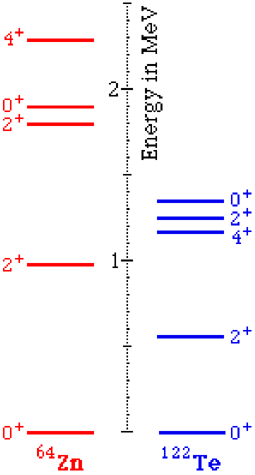
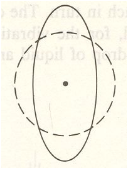


Two vibrational "phonons" with angular momentum 3 give states with angular momentum 0, 2, or 4.



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Alternative: "Liquid Drop" Model

Low-Lying Quadrupole Collective States

Protons and neutrons move together; volume is conserved, surface changes shape.

Ansatz for surface:

$$R(\theta,\phi) = R_0\left(1 + \sum_m \alpha_m Y_{2,m}(\theta,\phi)\right)$$

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$$H \approx 1/2B\sum_m |\dot{\alpha}_m|^2 + 1/2C\sum_m |\alpha_m|^2$$

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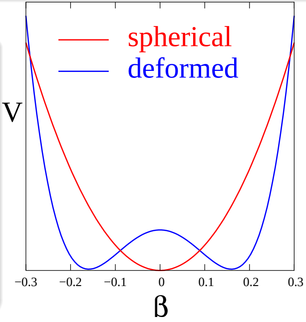
This reproduces the average trend of $E = \omega$ with A, but real life is more complicated, with frequencies depending significantly on how nearly magic the nucleus is.

Deformation in Liquid Drop Model

Restoring parameter C becomes negative if Coulomb effects overcome surface tension. Nucleus can then deform. Go to "intrinsic frame," replacing 5 intrinsic-frame α's with

$\beta \equiv \sqrt{\alpha_0^2 + 2\alpha_2^2}$ and $\gamma \equiv \tan^{-1}[\sqrt{2}\alpha_2/\alpha_0]$

Wave funtion: $|\Psi\rangle \approx D^{J*}_{MK}(\theta,\phi,\psi)\Phi_{\text{int.}}(\beta,\gamma)$.



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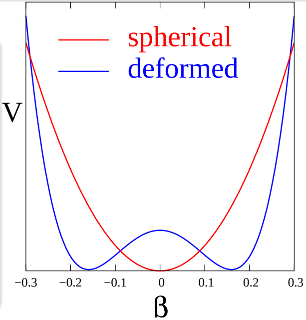
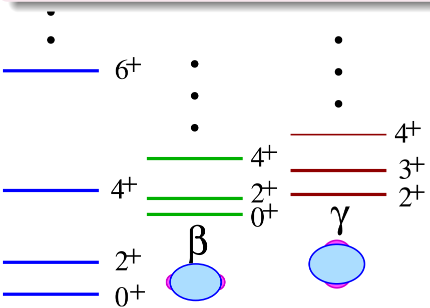


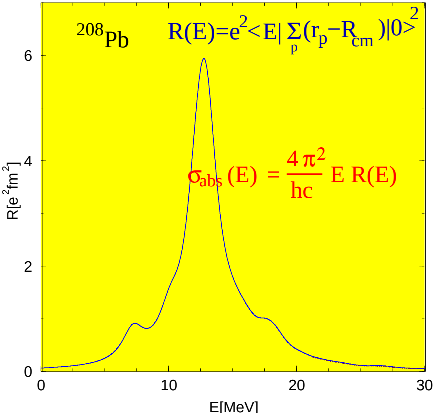


Low-lying states in deformed nuclei

1. Rotations of deformed nucleus with moment of inertia between that of rigid body and "irrotational flow"
2. Surface vibrations along or against symmetry axis

Density Oscillations

Photoabsorption cross section proportional to “isovector” dipole strength, which lies mainly at higher energy than surface modes.



From N. Paar, http://www1.physik.tu-muenchen.de/~npaar/pygmy/

Giant dipole resonance

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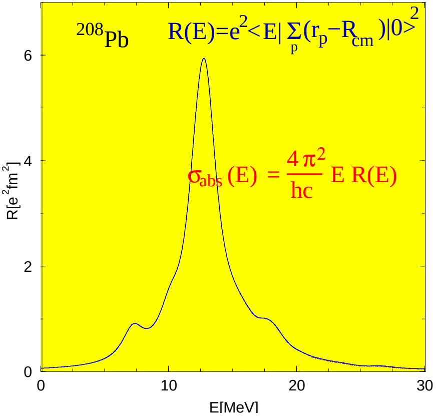

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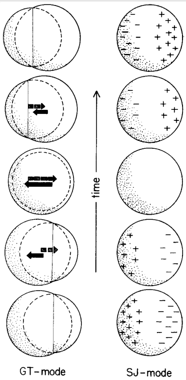


From W D. Myers et al., Phys. Rev. C 15, 2032 (1977).

Hydro for SJ mode gives

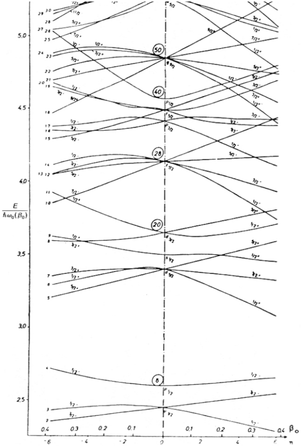
$$E \approx 2\hbar\sqrt{\frac{8a_{\rm sym}NZ}{mA^3R_0^2}} \approx 12 \text{ MeV in } ^{208}\text{Pb}.$$

Outline

Early Attempts at "Unification": Nilsson Model

$$|\Psi\rangle \approx D^{j*}_{m\Omega}(\theta,\phi,\psi)|N,l,m_l,\Omega\rangle_{\text{int.}}$$

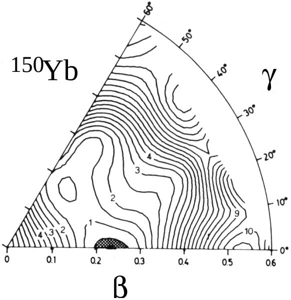
Ω is like K. Valence intrinsic level is ground-state rotational bandhead.



From Eisenberg and Greiner, *Nuclear Models* p. 542

Shell model in deformed potential

- Part of deformation energy from "deformed Fermi gas".
- Can map out "energy surface" to determine ground-state deformation and energies of vibrations.



From C.G. Andersson et al, Nucl. Phys. **A268**, 205 (1976)

Microscopic Goldhaber-Teller Picture

Assume potential is harmonic oscillator

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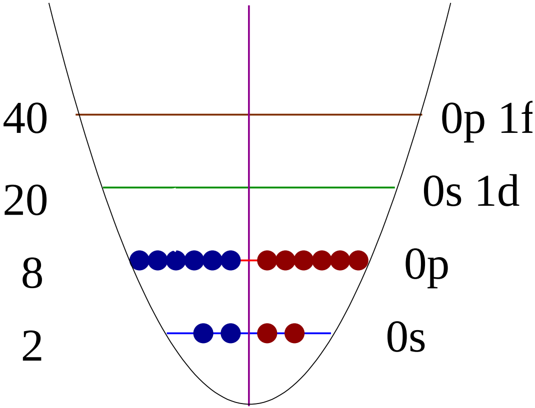

The operator $\sum_p (\vec{r}_p - \vec{R}_{\text{cm}})$ acts on the ^{16}O ground state

...

Microscopic Goldhaber-Teller Picture

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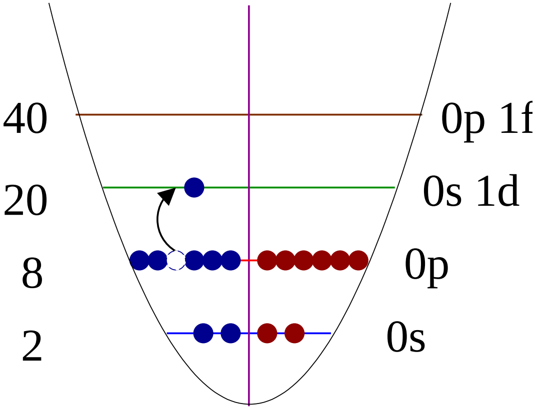
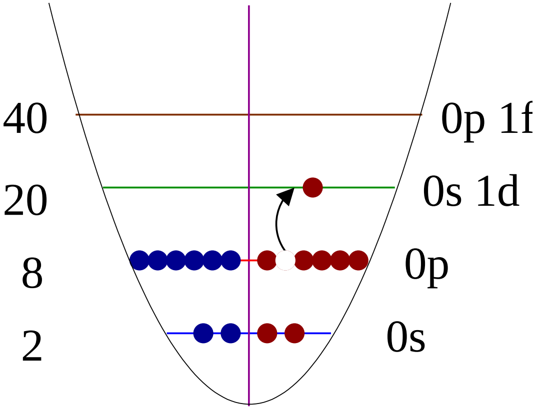


promoting a proton up one oscillator shell . . .

Microscopic Goldhaber-Teller Picture

Assume potential is harmonic oscillator



and a neutron . . .

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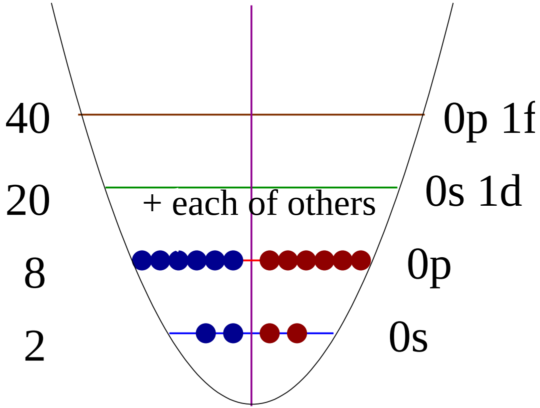
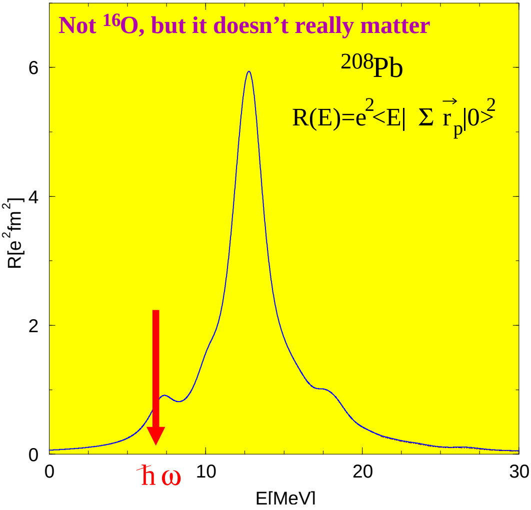
Assume potential is harmonic oscillator



and doing this "coherently" (with a common amplitude for protons and another for neutrons) to all the p-shell particles. . .

Microscopic Goldhaber-Teller Picture



creating the resonant state with energy about $1\ \hbar\omega$.

⋮

Well, though I haven't shown it to you, at least the size of the peak turns out about right. Something's still missing from the energy, though.

Next Time . . .

These early attempts to merge the shell-model and liquid-drop pictures — to understand the "microscopy" of collective nuclear motion — provide insight but are crude. Next time we'll start to examine new methods that let us do the job better, starting from the nucleon-nucleon interaction. Then it's on to the the drip lines, and after that to the role nuclear structure in the search for new fundamental physics.