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Model of Market Participation:
Evidence from Kenya and Ethiopia

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Abstract

Do rural households in developing countries make market participation and volume decisions simultaneously or sequentially? This article develops a two-stage econometric model that allows testing between these two competing hypotheses regarding household-level market behavior. The first stage models the household's choice of whether to be a net buyer, autarkic, or a net seller in the market. The second stage models the quantity bought (sold) for net buyers (sellers) based on observable household characteristics. Using household data from Kenya and Ethiopia on livestock markets, we find evidence in favor of sequential decision-making, the welfare implications of which we discuss.

JEL Classification Codes: C34, D19, O12, Q12.

1   Introduction

Do rural households in developing countries make market participation and volume decisions simultaneously or sequentially? That is, does the household head decide whether to be a net buyer, autarkic, or a net seller, and then decides how much to buy or sell only once it gets to market, conditional on having chosen not to be autarkic, or does the household head make either decision before leaving for market? This seemingly esoteric question addresses a critical issue of market power that has bedeviled development economics for decades. If poor households make participation and volume decisions simultaneously, they effectively precommit to a volume before learning information available to them only once they arrive at market. This *ex ante* decision-making effectively gives the traders with whom the household interacts market power by rendering the household's demand (supply) inelastic with respect to new market (e.g., price) information they discover, leaving poor, pre-committed households vulnerable to exploitation by astute traders. If, however, households make marketing decisions sequentially, then they retain greater flexibility once they arrive in a market, making their purchases or sales volume decisions *ex post* based on new information they only discover at market, thereby reducing the likelihood of exploitative transactions that empower traders to extract much of the gains from trade. Given longstanding popular assumptions that traders exert market power over poor sellers and buyers in rural dyadic markets, it seems appropriate to test this hypothesis directly while estimating household market participation behavior. This article is, to the best of our knowledge, the first to accomplish that objective.

The research on market participation has been scant, especially in developing country

settings where significant frictions make this question most salient. Goetz (1992) studied the participation of Senegalese agricultural households in grain markets, using a probit model of households' discrete decision to participate in the market (either as buyers or sellers, without distinction) followed by a second-stage switching regression model of the continuous extent of market participation decision (i.e., transaction volume). Key et al. (2000) developed a structural model to estimate structural supply functions and production thresholds for Mexican farmers' participation in the maize market, based on a censoring model with an unobserved censoring threshold. Their model differentiates between the effects of fixed and proportional (i.e., variable) transactions costs. Holloway et al. (2005) used a Bayesian double-hurdle model to study participation of Ethiopian dairy farmers in the milk market when non-negligible fixed costs lead to non-zero censoring, as in Key et al., but distinguishing between the discrete participation decision and the continuous volume marketed decision, as in Goetz.

These extant articles on household marketing behavior in developing countries thus begin from fundamentally different assumptions on the nature of households' market participation choices. Goetz and Holloway et al. explicitly assume sequential choice: households initially decide whether or not to participate in the market, then decide on the volume purchased or sold conditional on having chosen market participation. Key et al., by contrast, implicitly model the household as making the discrete market participation choice simultaneously with the continuous decision as to volumes purchased or sold. None of the previous articles allows for the possibility that households could make marketing decisions either sequentially or simultaneously.

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Our contribution is thus threefold. First, and of most general value, we introduce a method that nests within it both the simultaneous and sequential formulations of household marketing behavior, allowing for direct testing of which assumption the data most support. The estimation method we introduce can be applied to a relatively broad range of problems, as we briefly discuss in the concluding section. Second, we add new empirical results to the thin literature on market participation, in our case looking at pastoralists' participation in livestock markets in southern Ethiopia and northern Kenya. This new application adds insights from markets for durable assets — livestock — to the extant literature on grain and milk marketing. Finally, our data also permit us to offer some interesting albeit tentative empirical insights related to possible behavioral anomalies in household marketing behavior.

The rest of the article is structured as follows. In section 2, we lay out a simple theoretical model of household marketing behavior, highlighting the implications of different assumptions about households' (discrete) participation and (continuous) volume decisions. Then, in section 3, we present the ordered tobit estimator, a two-stage econometric model that treats both sales and purchases as censored dependent variables, but models the actual participation decision as an ordered decision by partitioning the real line into three mutually exclusive and collectively exhaustive positions *vis-à-vis* the market: net buyer, autarkic and net seller. After briefly describing the data in section 4, section 5 then reports the estimation results from applying this novel method to study livestock marketing behavior among a population of poor herders in east Africa. The concluding section focuses on both the policy and welfare implications of our empirical findings and prospective other uses of the ordered tobit estimator.

2 A Theoretical Model of Market Participation

Pastoralist households in the drylands of East Africa routinely make decisions as to whether to buy or sell livestock, the principal form of wealth in the region. Under the maintained hypothesis that market behavior is driven by a household's objective of maximizing the discounted stream of consumption it enjoys, one can usefully focus attention on the choice problem that relates optimal (non-negative) quantities bought and sold, Q^{b*} and Q^{s*}, respectively, to household attributes and the environmental factors that condition consumption and market behaviors. For a representative household, let C_t represent discretionary consumption over period t. The household possesses a vector of assets at the beginning of period t. Let W_t be liquid but non-productive household wealth, H_t reflect the size of a household's herd, and A_t equal the amount of cultivable land it operates. The productive assets — herd and land size — generate income over period t according to the mapping $Y_t = y(H_t, A_t)$, where income is measured in units of the numéraire consumption good.[1] The household may also incur obligatory, norms-driven ceremonial expenses, X_t (likewise measured in consumption good units), associated with births and deaths, which we treat as exogenously determined.

Under the assumption that the household makes its market participation and marketed amount[2] decisions simultaneously, household livestock marketing behavior can thus be

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[1] We use uppercase letters to reflect household attributes and lowercase letters to represent community-level conditions or functional relationships.

[2] We use the terms "amount", "extent" and "volume" interchangeably to represent the nonnegative continuous variable reflecting net sales or net purchases. We also abstract from the possibility that households could be both buyers and sellers in the same period. In the data set we use, there were no such observations. Further, in places where transactions costs drive a significant wedge between buyer

described by

$$\max_{C_t, Q_t^j} E_t \sum_{t=0}^{\infty} \delta^t U(C_t) \ \forall j \in \{b, s\} \tag{1}$$

$$\text{s.t.} \ \ C_t \le y(H_t, A_t) + W_t - X_t \tag{2}$$

$$H_{t+1} = H_t + g(H_t, e_t) + Q_t^b - Q_t^s \ge 0 \tag{3}$$

$$W_{t+1} = W_t - X_t - C_t + y(H_t, A_t) + p_t^{*s} Q_t^s - p_t^{*b} Q_t^b \ge 0 \tag{4}$$

where E is the expectation operator, δ is the household's discount rate and $g(H_t, e_t)$ represents the biological recruitment (growth) rate of the herd as a function of beginning period herd size and current local environmental conditions, e_t. This model is essentially the dynamic generalization of the structural model presented in Key et al. (2000). The p^{*j} are the shadow prices for purchases ($j = b$) and sales ($j = s$). The shadow prices reflect the boundaries of the "price band" that defines household endogenous valuation of a resource that may or may not be traded (de Janvry et al., 1991). At the upper boundary of the price band, households buy, paying a shadow price that adds the fixed and variable transactions costs of market participation to the underlying market price, p_t^m. At the lower boundary, households sell, receiving net unit value equal to p_t^m less the fixed and variable transactions costs of market participation. Thus,

$$p_t^{*b} = (1 + vc_t)p_t^m + fc_t \tag{5}$$

$$p_t^{*s} = (1 - vc_t)p_t^m - fc_t \tag{6}$$

where vc represents the (proportional) variable costs, $vc_t \in [0, \infty)$, such as market taxes and transport fees per unit sold, and fc summarizes non-negative fixed costs, including

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and seller shadow prices, there should not be observations of both purchases and sales within the same (sufficiently disaggregated) period.

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the cost of the person's transport to and from market, search, screening and negotiation costs, etc. Controlling for random variation in prices described by the stochastic term z^k for $k \in \{p, fc, vc\}$ — future market prices, fixed costs and variable costs follow a random walk:

$$p_{t+1}^m = p_t^m + z_t^p \tag{7}$$

$$fc_{t+1} = fc_t + z^{fc} \tag{8}$$

$$vc_{t+1} = vc_t + z^{vc} \tag{9}$$

Rewriting this dynamic optimization problem as a Bellman equation (not shown) one can derive the reduced form of the household's optimum marketing decisions as

$$Q_t^{b*} = q^b(A_t, H_t, W_t, X_t, e_t, fc_t, p_t^m, vc_t, \delta, z_t^k), \text{ and} \tag{10}$$

$$Q_t^{s*} = q^s(A_t, H_t, W_t, X_t, e_t, fc_t, p_t^m, vc_t, \delta, z_t^k). \tag{11}$$

The theoretical predictions of this model are several, as applied to the east African range-lands context we study. First, one would expect that Q_t^{b*} (Q_t^{s*}) is decreasing (increasing) in A_t because if a household cultivates, its mobility is restricted, thereby limiting the size of the herd it can manage sustainably, given local forage and water resources. Second, because income is increasing in herd size (i.e., income is not a stationary process), the usual Friedmanite consumption smoothing behavior breaks down. So long as $\text{E}[\partial g(H_t, e_t)/\partial H_t] > 0$ (i.e., expected capital gains in livestock exceed those for other assets), households have an incentive for herd accumulation that will limit their use of livestock to smooth consumption (McPeak 2004). This can lead to both a positive (negative) relation between ex ante herd size and livestock sales (purchases) and a potentially negative (positive) relation between livestock sales (purchases) and income from sources

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other than livestock sales. Third, given the liquidity constraints these households face, they can only satisfy current consumption from asset sales. Thus livestock sales (purchases) should be increasing (decreasing) in household demographic shocks that necessitate ceremonial expenditures, X, and sales could be negatively related to price (i.e., the supply curve could bend backwards) as households liquidate only as many animals as are necessary, given prevailing prices, to meet immediate expenditure needs. Fourth, both sales and purchases should be decreasing in fixed and variable costs. Fifth, there should be a positive relationship between wealth and purchases since the budget constraint limits poorer households' capacity to buy livestock. This system of reduced form equations described by equations (10) and (11) is estimable as a bivariate tobit model.

However, the preceding specification relies on the potentially strong assumption that the discrete household choice to participate in the market is made simultaneously with the continuous choice as to the number of animals to buy or sell conditional on having chosen to go to market. If, however, participation and volume choices are made sequentially, as other articles in this literature assume (Goetz, 1992; Holloway et al., 2005), then the preceding model will be misspecified. If households make decisions sequentially, we need to break each period down into sub-periods.

In the interests of parsimony, we break each period t into only two sub-periods: $r = 0$ when the household makes the discrete participation decision, not yet knowing information available only at the market, and $r = 1$ when those households that have chosen to participate in the market as either net buyers or net sellers have arrived at market, received additional information, and make their continuous decision as to net sales or

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purchase volume. This changes the household's optimization problem to

$$\max_{C_{rt}, I_{rt}^j, Q_{rt}^j} E_{rt} \sum_{r=0}^{1} \sum_{t=0}^{\infty} \delta^t U(C_{rt}) \; \forall j \in \{b, s\} \tag{12}$$

$$\text{s.t.} \;\; C_{rt} \leq y(H_{rt}, A_{rt}) + W_{rt} - X_{rt} \tag{13}$$

$$H_{1t} = H_{0t} + g(H_{0t}, e_{1t}) \geq 0 \tag{14}$$

$$H_{0t+1} = H_{1t} + g(H_{1t}, e_{0t}) + I_{1t}^b Q_{1t}^b - I_{1t}^s Q_{1t}^s \geq 0 \tag{15}$$

$$W_{1t} = W_{0t} + X_{0t} - C_{0t} + y(H_{0t}, A_{0t}) + \widetilde{p}_t^s I_{1t}^s Q_t^s - \widetilde{p}_t^{*b} I_{1t}^b Q_t^b \geq 0 \tag{16}$$

$$W_{0t+1} = W_{1t} - X_{1t} - C_{1t} + y(H_{1t}, A_{1t}) - (I_{1tt}^{s\,s} + I_{1t}^b) fc_t \geq 0 \tag{17}$$

where the indicator variable $I_{rt}^b = 1$ if the household chooses to be a net buyer ($I_{rt}^b = 0$ otherwise) and $I_{rt}^s = 1$ if it chooses to be a net seller ($I_{rt}^s = 0$ otherwise), with a complementary slackness condition that $I_{rt}^b \cdot I_{rt}^s = 0$. This implies that $I_{0t}^b = I_{1t}^b$ and $I_{0t}^s = I_{1t}^s$, with $Q_{0t}^b = Q_{0t}^s = 0$. In this formulation, the information set differs between the discrete and continuous decisions. Furthermore, the boundary shadow prices no longer include the fixed costs of market participation, since those are paid in subperiod 0 when the household makes the discrete market participation choice. So the relevant marginal cost or revenue per animal bought or sold, respectively, is

$$\widetilde{p}_t^b = (1 + vc_t) p_t^m \tag{18}$$

$$\widetilde{p}_t^s = (1 - vc_t) p_t^m \tag{19}$$

The household's optimum continuous marketing decisions under the assumption of sequential decision making therefore does not include the fixed costs already incurred:

$$I_{rt}^{b*} = i^b(A_t, H_t, W_t, X_t, e_t, p_t^m, fc_t, vc_t, z_t^k) \tag{20}$$

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$$I_{rt}^{s*} = i^s(A_t, H_t, W_t, X_t, e_t, p_t^m, fc_t, vc_t, z_t^k) \tag{21}$$

$$Q_{rt}^{b*} = q^b(A_t, H_t, I_{rt}^b, W_t, X_t, e_t, p_t^m, vc_t, z_t^k) \tag{22}$$

$$Q_{rt}^{s*} = q^s(A_t, H_t, I_{rt}^s, W_t, X_t, e_t, p_t^m, vc_t, z_t^k) \tag{23}$$

The relationship between the purchase or sales quantities and the discrete market participation choice is a form of selectivity correction akin to that on which Goetz (1992) focused. Here, however, we distinguish between net buyers and net sellers. Because net buyers and net sellers can be strictly ordered along the real line describing net sales ($S_t \equiv Q_t^{s*} - Q_t^{b*}$) positions, we can treat the $\{I_{rt}^{b*}, I_{rt}^{s*}\}$pair as an ordinal variable: $\{I_{rt}^{b*} = 1, I_{rt}^{s*} = 0; I_{rt}^{b*} = 0, I_{rt}^{s*} = 0;$ and $I_{rt}^{b*} = 0, I_{rt}^{s*} = 1\}$, equivalent to net buyers, autarkic households and net sellers, respectively.

In the sequential choice model, several things change. First, note that fixed costs should no longer have any effect on sales quantity decisions, only on the market participation choices, I_{rt}^{b*} and I_{rt}^{s*}. Conditional on finding that the data support the sequential formulation of the household marketing choice, tests of the exclusionary hypothesis that fixed costs are unrelated to quantities sold or purchased thus serve as tests of the prospective behavioral anomaly that households take fixed costs into account when microeconomic theory posits they really should not. Second, because households do not precommit to sales volumes prior to receiving full, current information on prices, one would expect greater price elasticity of demand under the sequential marketing decisions model than under the simultaneous decisions model. The rest of the predicted relations between sales or purchase quantities and the explanatory variables are as in the simultaneous choice case.

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The distinction between whether a household makes its market participation and purchase or sales volume decisions sequentially or simultaneously thus has significant implications for several relationships of interest in market participation studies. Most especially, if herders choose how much to sell or how much to buy at the same time they choose whether to sell or buy, i.e., before they get to market and know the prevailing market price, then they are more likely to exhibit price inelastic demand and supply for animals and to be more vulnerable to exploitation by traders. However, if herders first choose whether they will be buyers, sellers or non-participants, then, conditional on their choosing to be buyers or sellers, they go to market, uncover more details about the conditions under which they can transact, and subsequently decide how much to buy or sell, the sequential nature of household marketing choice reduces the likelihood of trader exploitation of poor herders. In the theoretical model above, if the information sets are identical in periods r=0 and r=1, then the sequential decision model collapses to the special case of simultaneous choice. We can exploit this generalization to determine whether livestock herders make *ex ante* or *ex post* marketing decisions. We now present an estimator that permits estimation of the discrete choice over I_{rt}^j as well as the continuous choice over Q_{rt}^j and allows one to test whether the sequential or simultaneous choice model fits the data better.

3 An Econometric Model of Market Participation

This section develops the ordered tobit model we implement in section 5. The idea behind the model comes from the assumed sequence and joint estimation of the household's marketing decisions, as just described. The key insight is that because a household's net sales (sales minus purchases) volume spans the real line[3], one can partition the continuous market participation outcome into three distinct categories: net buyer (households whose net sales are strictly negative), autarkic (households whose net sales are equal to zero) and net seller (households whose net sales are strictly positive) households. Because these categories are logically ordered, and since it is informative to distinguish between net buyers and net sellers rather than just lump them together as "market participants", we can first estimate an ordered probit participation decision, then estimate a censored model of net sales or net purchase volume. By comparing the results of the ordered tobit with those of a bivariate tobit, we can then test whether households decide sequentially or simultaneously.

Our ordered tobit[4] specification allows us to study fixed and variable transactions costs separately, as do Key et al. (2000), but using an estimator that we find converges more readily than does their somewhat more cumbersome likelihood function. This approach also allows for non-zero censoring points, as in Key et al.(2000) and Holloway et al. (2001).

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[3]In the presence of non-zero censoring points, regions between zero and the censoring point(s) may have zero density.

[4]Klein and Sherman (1997) also combine the ordered probit and tobit estimators into what they term an "orbit" estimator, but in the reverse order. They first estimate a censored regression and then use the parameters from that first stage to fit an ordered response model. Our approach thus differs significantly from theirs.

The specification of the ordered tobit model is as follows. Let y_{1i} denote the category — net buyer ($y_{1i} = 0$), autarkic ($y_{1i} = 1$), or net seller ($y_{1i} = 2$) — to which household i belongs. The specification of the first-stage decision is that of an ordered probit. The innovation comes at the second stage. Let $y_{2i} > 0$ be the total units of livestock purchased by household i and let $y_{3i} > 0$ be the total units of livestock sold by household i. Note that these two variables define clear, mutually exclusive subsets of the dataset. A household cannot simultaneously be a net buyer and a net seller.

We could treat the full problem under the maintained hypothesis of simultaneous choice by estimating a bivariate tobit, with one equation for net buyer households and one for net seller households. Following the theoretical discussion of Section 2, however, one would prefer to allow for the possibility of sequential decision-making. It would therefore be better to estimate an ordered probit in the first stage and then append two linear regressions to the $y_1 = 0$ and $y_1 = 2$ categories: one for net buyers, and one for net sellers, respectively, and then test whether or not the ordered tobit specification is better supported by the data relative to the bivariate tobit. This effectively allows direct testing of the hypothesis that household market participation and volume decisions are made sequentially versus the null that they are made simultaneously.

In the following empirical analysis, $x_1 \neq x_2$ and $x_1 \neq x_3$, but $x_2 = x_3$, where x_1 is the vector of first-stage regressors, x_2 is the vector of second-stage regressors thought to affect the volume of purchases, and x_3 is the vector of second-stage regressors thought to affect the volume of sales. Thus, the end result is an ordered probit combined with two

of what Amemiya (1985) refers to as Type II tobit models. Therefore, what we estimate in section 5 is, more precisely an ordered Heckit, but this is just a special case of the more general ordered tobit. We also adapted the Heckman correction for standard errors to our model (a detailed appendix is available by request).

The log-likelihood for our ordered tobit estimator is

$$
\ell(\alpha', \beta', \sigma') = \sum_{i=1}^{N} \mathbb{I}(y_{1i} = 0) \Bigg\{ \Bigg[\ln \Phi \left(\frac{\alpha_1 - x_{1i}\beta_1 + (y_{2i} - x_{2i}\beta_2)\rho_{12}/\sigma_2}{\sqrt{1 - \rho_{12}^2}} \right) \\
- \frac{1}{2} \left(\frac{y_{2i} - x_{2i}\beta_2}{\sigma_2} \right)^2 - \ln \left(\sqrt{2\pi}\sigma_2 \right) \Bigg] \\
+ \mathbb{I}(y_{1i} = 1) \Bigg[\ln[\Phi(\alpha_2 - x_{1i}\beta_1) - \Phi(\alpha_1 - x_{1i}\beta_1)] \Bigg] \\
+ \mathbb{I}(y_{1i} = 2) \Bigg[\ln \Phi \left(\frac{x_{1i}\beta_1 - \alpha_2 + (y_{3i} - x_{3i}\beta_3)\rho_{13}/\sigma_3}{\sqrt{1 - \rho_{13}^2}} \right) \\
- \frac{1}{2} \left(\frac{y_{3i} - x_{3i}\beta_3}{\sigma_3} \right)^2 - \ln \left(\sqrt{2\pi}\sigma_3 \right) \Bigg] \Bigg\}
\tag{24}
$$

where α is a (2×1) vector of unknown threshold parameters, $\beta = (\beta_1, \beta_2, \beta_3)$ is a $([K + L + M] \times 1)$ vector of parameters, and σ is (2×1) vector of variance parameters, one for each linear component, i.e., net purchases and net sales. Thus, the model estimates $K + L + M + 4$ parameters by maximum likelihood.

The ordered tobit model has been the object of very little published work. Groot and van den Brink (1999) study overpayment and earnings satisfaction, developing a computationally similar but atheoretical model.[5] Ranasinghe and Hartog's (1997) unpublished

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[5]Groot and van den Brink use an estimator to ours, but incorporating a Type I rather than a Type II tobit.

working paper explores investment in post-compulsory education in Sri Lanka. Finally, Greene (2003) discusses a model similar to ours, i.e., first-stage ordered probit, second-stage linear regression, except his second stage only consists of one linear regression. Yet, the prospective applications of this model are many — as we discuss briefly in the concluding section — and it is rather easy to estimate with any statistical package that accommodates maximum likelihood.

4   Data and Descriptive Statistics

We study livestock market participation by pastoralists in a large, contiguous area of northern Kenya and southern Ethiopia. Observers have long been puzzled by the limited use of livestock markets by east African pastoralists who hold most of their wealth in the form of livestock, who face considerable income variability, and who regularly confront climatic shocks that plunge them into massive herd die-offs and loss of scarce wealth (Desta 1999, Little et al. 2001, Osterloh et al. 2003, Lybbert et al. 2004). It would seem that opportunistic use of markets would permit herders to increase their wealth by buying when prices are low and selling when prices are high and to smooth consumption through conversion between livestock and cash useful for purchasing food. Yet such behavior seems relatively rare (Osterloh et al. 2003, Lybbert et al. 2004, McPeak 2004).

The data come from a study of risk management among east African pastoralists and consist of a panel of 337 pastoralist households from eleven sites in the arid and semi-arid lands of northern Kenya and southern Ethiopia. Each household was observed quarterly between June 2000 and June 2002. We pool all nine time periods together and treat the

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dataset as a cross-section, first due to the inherent complexity that an extension of the ordered tobit model to a panel setting would involve, and second because of the highly unbalanced nature of our panel.[6] The descriptive statistics presented here thus treat household i in period t and household i in period s as two distinct observations for $s \neq t$. Further details on the surveys, sites and instruments are available in Barrett et al. (2004).

Table 1 provides descriptive statistics. Almost 70 percent of the households are male-headed, with an average size of 7.3 people and a dependency ratio of nearly 0.5.[7] Most households own livestock, with an average herd size of about 20 tropical livestock units (TLU), a standard measure for aggregating across ruminant species such as camels, cattle, goats and sheep.[8] Herds reproduce, on average, at a rate of about 6.5 percent annually (animal births/total herd size). Pastoralists have a strong preference for holding cows for milk and calves, so herds are more than two-thirds female, on average.

Property rights in livestock can be complex. Households often give or lend animals to one another without surrendering all rights in the animal. For example, it is common for a household to "own" an animal given to it by a relative, yet the household is not permitted to sell or slaughter the animal nor to give it to anyone outside of the clan or village. While these encumbered or restricted property rights may matter to marketing decisions, especially with respect to purchasing cows (for which restricted gifts may be

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[6]The number of observations per time period ranged from 233 to 255, reflecting a mixture of attrition and interruption.

[7]A household's dependency ratio is calculated by dividing the number of individuals under 15 years of age plus the number of individuals over 64 years of age by the total number of individuals in the household.

[8]One TLU equals 0.7 camel, 1 cattle, 10 goats or 11 sheep.

a substitute) or selling bulls, they affect less than ten percent of a household's herd, on average. Mean land holdings are small, at about 1.4 hectares, much of which goes uncultivated any given year due to insufficient rainfall. Other assets owned by the household include bicycles, radios, wooden beds, tables and other furniture, watches, lanterns, ploughs, small shops or other businesses, non-local breed animals, vehicles and urban property, all valued in Kenyan shillings (Ksh).[9] The value of these assets amounts to a bit more than US$35 per capita, while household income (the sum of the market value of milk and crop production, sales of firewood, charcoal, crafts and hides and skins, and wage and salary earnings) over the preceding quarter averaged around $1.75 per day, or less than $0.24 daily per capita income, underscoring the poverty these herders suffer.

Fixed and variable cost expenditures on market participation represent a surprisingly modest share of price.[10] Variable costs related to per animal transport costs and market fees add (for buyers, subtract for sellers) only about 12 percent to the small stock (goat or sheep) price and less than 2 percent to the large stock (camel and cattle) price. Fixed costs associated with transport and lodging expenditures of the individual who sells or buys animals and any market fees unrelated to volumes sold or purchased are about 30 percent larger than variable costs per TLU.

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[9]For Ethiopian households, we use 1 Ethiopian birr = 8.75 Ksh. Note that US$1.00 $\cong$ Ksh75.

[10]In our analysis, fixed fees include accommodations, food and transportation for the herder as well as bribery, security expenditures and medications. Variable fees are fees per animal paid to county or municipal authorities as well as District Veterinary Officer (DVO) inspection and other veterinary fees.

5 Results and Analysis

This section first presents estimation results for a bivariate tobit model consistent with the model outlined earlier when household market participation and volume choices are made simultaneously. Then we present estimation results from the ordered tobit model that allows for the possibility of sequential household choice. Finally, we test the null hypothesis that a simultaneous choice model suffices for describing the livestock marketing behavior of our sample households using Davidson and MacKinnon's (1993) J-test of non-nested hypotheses.

5.1 Bivariate Tobit Results

We estimate the bivariate tobit model[11] — i.e., one tobit for net buyer households, another for net seller households — under the maintained hypothesis that discrete participation and continuous volume decisions are made simultaneously. The two tobits share the autarkic observations in common, and Table 2 reports the estimated bivariate tobit coefficients. Note that we omit the coefficient estimates for the quarterly seasonal (March to June, June to September, and September to December) and location dummies included to account for climatic, range, security conditions and other unobserved spatial or temporal characteristics common to all households in the sample.

Given limitations of space, and because the bivariate tobit is only instrumental in our approach, we turn directly to a discussion of the variables of interest: prices and transaction costs.[12] Variable costs are positively associated with the number of animals bought,

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[11]We thank Daniel Lawson for sharing his Stata code for estimating a bivariate tobit.

[12]Since household income is likely endogenous, we instrumented for it using the household head's

a puzzling result since one would expect the volume of trade to be strictly decreasing in variable costs.[13] Fixed market participation costs decrease the number of animals bought but increase the number of animals sold. The former effect is consistent with the existence of binding liquidity constraints that reduce the number of animals a herder can afford to purchase the more she must spend on fixed costs *ex ante*. The latter effect is consistent with the walking bank hypothesis, that the herder sells as many animals as are needed to meet immediate cash needs, and that number increases with the fixed costs the herder must incur. Alternatively, these results could reflect the well-known behavioral anomaly that people take sunk costs into consideration at the margin even when, in theory, they should not. Under this hypothesis, buyers seek to limit their expenditures and sellers try to recover them by considering sunk costs at the margin. Pastoralists appear highly responsive to prices on the demand side, with demand for both large and small stock decreasing in prices (albeit not significantly so for small stock). Under the maintained assumption of simultaneous choice, however, herders appear nonresponsive to price on the supply side. Since herders sell far more often than they buy, the implication of the estimation results under the assumption of simultaneous choice is that herders can be exploited by traders.

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education, time-and-location interaction terms, beginning period herd size and land assets. Detailed results on the instrumenting equation are available from the authors by request. The instrumenting regression had an adjusted R^2 of 0.2630, so we can rule out overfitting.

[13]Given motorized transport and inspection bottlenecks in the region, it is possible that variable transport costs and inspection and certification fees are endogenous to aggregate market demand, increasing at those times when households most want to restock. Our data, however, do not include information on aggregate market transactions, making it infeasible to control for this possibility, which could explain the anomalous result.

5.2   Ordered Tobit Results

We now explore whether these estimation results change if we relax the assumption that households make simultaneous marketing decisions by employing the ordered tobit model to estimate the more general, sequential choice model.[14] Given that both linear components include a selection term — the usual inverse Mills ratio (IMR) — we apply Heckman's correction to the variance-covariance matrices for each of the second-stage regressions. The only difference between our method and that of Heckman comes from the first-stage, and in that sense, our model offers a modest extension to Heckman (1979). This ultimately allows us to run a likelihood ratio-based J-test of simultaneous versus sequential choice.

The ordered probit model of discrete market participation yields intuitive results (Table 3). The non-zero censoring points are of opposite signs, with the lower censoring threshold at 1.59 TLU net purchases and the upper threshold at 0.95 TLU net sales, each statistically significantly different from zero. These estimates suggest that purchases or sales of less than one TLU are generally uneconomical, given the monetary and nonmonetary costs of market participation in this region. People are more willing to enter the market for smaller volume sales than purchases, likely reflecting the fact that sales of livestock are essentially means by which households meet immediate cash needs related to payment of school fees, food purchases and ceremonial or emergency health expenses.

---

[14]We estimate the ordered tobit using limited information maximum likelihood (LIML) rather than full information maximum likelihood (FIML), i.e., we use Heckman's two-step estimator rather than a simultaneous estimator following the recommendations of Puhani (2000) given collinearity in our sample data.

Female-headed households are more likely to be autarkic than to be net sellers and are more likely to be net buyers than to be autarkic, *ceteris paribus.* Human births positively affect the categorical outcome, i.e., it makes net buyer households more likely to be autarkic and autarkic households more likely to be net sellers, again consistent with the notion that exogenous demographic shocks associated with culturally mandated expenditures affect livestock marketing patterns. Animal births likewise exert a positive effect on the ordered market participation variable. The more animal births a household herd enjoys in a period, the more likely it is to be autarkic instead of being a net buyer and the more likely it is to be a net seller instead of being autarkic. Wealth and income have no statistically significant effect on the discrete market participation decision in the first stage of the ordered tobit.

The fixed costs of market participation exert an increasing, concave effect on market participation up through almost the 75th percentile of the data, at which point the effect turns negative. This implies that over most of the range of fixed costs observed in these data, the marginal effect is greatest with respect to purchase decisions, moving households from net purchases to autarky. However, when high fixed costs are extremely high — beyond about Ksh415 — this encourages households to move from net seller positions to autarky.

The second stage net purchase and net sales volume choices conditional on expected market participation likewise make sense, repeating many of the more intuitive results from the bivariate tobit model (Table 4). Pronounced and intuitive life cycle effects emerge, as households buy more and sell less up through about age 50 — roughly the mean in these

data — and then switch to selling more and buying less. Livestock sales (purchases) are decreasing (increasing) in household non-livestock income. When income is high, they sell fewer animals and when income is low, they sell more, *ceteris paribus*. Sales and purchases are both increasing in households' non-livestock assets as wealthier people buy and sell in larger volumes than poorer households with equal probability of market participation. Household land holdings are positively related to sales because pastoralists who own land have effectively sedentarized themselves, reducing the herd sizes they can manage within a fixed space subject to considerable intertemporal variability in forage and water availability. Herd size matters to livestock marketing patterns. Households with larger herds sell slightly more animals, although this effect, while statistically significant, is small in magnitude, indicating that marketing is not used to regulate herd sizes (Lybbert et al., 2004).

The data do not support our hypothesis that complex indigenous livestock gifting and loaning institutions that encumber some animals in many households' herds impede livestock marketing behavior. Nor does it appear that the gender mix of a household's herd matters to market participation or transactions volumes.

The multifunctional nature of livestock holding in pastoralist regions again becomes evident when we consider the estimated effect of livestock prices on net sales and purchases. Larger stock (camels and cattle) are productive assets held for long-term equity growth. Net sales decrease modestly with price while net purchases decrease sharply as prices rise, with price elasticities of supply and demand of $\eta_s = -0.10$ and $\eta_b = -2.73$, respectively, at the sample means. Herders are highly price responsive on the buyer side, but

much less price responsive on the seller side. Moreover, the backward-bending estimated supply curve is consistent with the "walking bank" model of livestock management. Herders tend to liquidate animals, as needed, to meet immediate cash needs (Osterloh et al. 2004), thus the number they sell falls as price increases. Note that the estimated price effects under the ordered tobit model are statistically significantly different from zero and of larger magnitude than under the bivariate tobit model, consistent with the basic point made earlier that sequential decision-making implies greater price elasticity of herder demand and supply and thus less opportunity for traders to exercise market power.

The estimated effects of transactions costs are qualitatively unchanged from those under the bivariate tobit model. Variable costs appear to exert a small, significant negative effect on sales volumes (this effect was statistically insignificant in the bivariate tobit model), as one would expect, but an anomalous positive effect on purchase volumes. Meanwhile, fixed costs appear to affect purchase (sales) volumes negatively (postively) and significantly. Recall that the theoretical model based on sequential choice predicts that fixed costs should have no effect whatsoever on the continuous volume decision, only on the discrete participation decision, which was indeed affected by fixed costs. This thus seems to offer a bit more evidence in support of the behavioral anomaly hypothesis, although we still cannot identify that effect separately from a liquidity constraint effect.

One concern in the ordered tobit model estimates is the large standard errors for the net buyer component. This arises from lack of effective identification available in the data. We only have two variables (the number of children in the household as well as the num-

ber of animals born in the last quarter) to identify whether households are net buyers, autarkic or net sellers. Moreover, although our sample includes 1394 autarkic households and 565 net seller households, it only includes 78 net buyer households. Thus, both weak identification and multicollinearity likely come into play here. We therefore include the standard errors without the Heckman correction for the net buyer results in the third column of Table 4. Selection into the 3 percent of households who are net buyers proves difficult to explain with the variables in our dataset.

The results of the ordered tobit differ from those of the bivariate tobit model under the assumption of simultaneous choice. Many of the more intuitive results only emerge from the more general estimation method we introduce here. For example, fixed costs of marketing and the responsiveness of livestock sales to prices are statistically significant only in the more general, two-stage model. These qualitative differences suggest that the estimator we introduce more accurately reflects livestock marketing behavior among these households.

Having established that the simultaneous and sequential model specifications yield different results and that the ordered tobit results appear intuitively more plausible, we now turn to the question of which model better fits the data statistically. One method is to check whether the IMR variables are statistically significant in either of the second-stage linear components of the ordered tobit model. The weakness of that approach is that it depends fundamentally on the instruments used to identify the first-stage choice. As already discussed, the data set offers few good instruments for that purpose. Weak identification causes imprecise estimation of the effect of the IMR on second-stage sales or

purchase volumes. As a consequence, the IMR coefficient estimates are not statistically significantly different from zero in either regression reported in Table 4.

An alternative and much better method relies on a J-test (cf. Davidson and MacKinnon, 1993) to discriminate between our two non-nested models. We first obtained the predicted values for net buyers and net sellers from the ordered tobit model and included them as regressors in their respective bivariate tobit components. We then obtained the predicted values for net buyers and net sellers from the bivariate tobit model and included them as regressors in their respective ordered tobit components. Our null hypotheses are as follows: (i) the estimated coefficients for the predicted values of the ordered tobit model are jointly not statistically significantly different from zero in the bivariate tobit model; and (ii) the estimated coefficients for the predicted values of the bivariate tobit model are jointly not statistically significantly different from zero in the ordered tobit model. Thus, our hypotheses respectively test that (i) the ordered tobit model has no explanatory power with respect to the bivariate tobit model; and (ii) the bivariate tobit model has no explanatory power with respect to the ordered tobit model. Rejection of null hypothesis (i) coupled with failure to reject null hypothesis (ii) favors the ordered tobit model over the bivariate tobit model, i.e., favors the hypothesis that households make livestock marketing decisions sequentially and not simultaneously. The test statistics, each distributed $\chi^2(2)$, were 7.20 and 2.64 for hypotheses (i) and (ii), respectively. Thus, not only do we reject the hypothesis that the ordered tobit does not have explanatory power with respect to the bivariate tobit, we cannot reject the hypothesis that the bivariate tobit does not have explanatory power with respect to the ordered tobit. This is strong evidence in favor of the sequential theoretical formulation of herder marketing

behavior and the resulting ordered tobit empirical specification.

6   Conclusion

In this article, we highlighted the important differences in behavior depending on whether households make (discrete) market participation and (continuous) sales or purchase volumes choices sequentially or simultaneously. We then developed — and found strong empirical support for — a two-stage econometric model that permits direct testing between these competing ways of understanding household level marketing behavior. From a policy perspective, the most important implication of our results is that households that make sequential marketing decisions are more price responsive and less likely to be vulnerable to trader exploitation. This is consistent with recent price analysis in the region that finds little support for the hypothesis that traders are able to vary prices locally to take advantage of poor herders (Barrett and Luseno 2004).

Our empirical results shed some light on the contemporary puzzle of why pastoralist households in the arid and semi-arid lands of east Africa make relatively little use of livestock markets. Households follow strong life cycles of accumulation, steadily building their herds over most of their adult lives. Fixed costs of market participation also impede market participation. Mainly, however, households in this region keep livestock as a sort of walking bank, adjusting sales and purchase volumes to fixed costs and non-livestock income, as well as to prices, in a manner suggesting that they are used to meet immediate cash needs when cash is not otherwise available but that livestock are the preferred form in which to hold assets when cash is available to meet immediate expenditure needs. It

appears that east African pastoralists are less drawn to the commercialization of livestock than to accumulating substantial herds.

The ordered tobit method should be applicable to a range of other economic problems similarly characterized by an ordered first-stage and a continuous second. Examples include financial investments — e.g., ranking risk tolerance and then estimating the share of alternative instruments in a portfolio, or modeling market integration in domestic and international trade by first establishing whether markets are segmented, competitively integrated or non-competitively integrated and then estimating trade volumes. One could likewise adapt this basic approach to cover multinomial or count data, instead of ordinal data, in the first stage estimation. For now, we leave such topics to future research.

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Table 1 – Full-Sample Descriptive Statistics.

Variable	$N = 2037$	
	Mean	**Std. Dev.**
Female Household Head Dummy	.3068238	.4612888
Household Head Age (Years)	48.80511	14.70051
Dependency Ratio	.4827757	.1998441
Household Size (Persons)	7.260187	3.748484
Land (Hectares Owned)	1.412248	2.569049
Assets (Ksh)	19760.9	196087
Births (Persons)	.0618557	.2409524
Deaths (Persons)	.0166912	.1319204
Income (Ksh, Including Food Aid)	11818.03	22395.67
Herd Size (TLUs)	19.2374	29.29284
% Female TLUs	.6766991	.2448751
Encumbered Males (TLUs)	.4650221	2.928111
Encumbered Females (TLUs)	.8929848	4.501852
Avg. Price of Large Stock (Ksh)	5558.806	2664.602
Avg. Price of Small Stock (Ksh)	790.6819	424.0051
Animal Births (TLUs)	1.194113	3.294443
Net Buyer Dummy	.0382916	.1919465
Autarkic Dummy	.6843397	.4648924
Net Seller Dummy	.2773687	.4478099
Net Sales (TLUs)	.2229602	1.457145
Fixed Costs (Ksh)	126.4936	245.4636
Variable Costs (Ksh/TLU)	96.01313	135.4184
Sales (Net Sellers)	.9925133	1.828466
Purchases (Net Buyers)	1.366667	4.883537

Table 2 – Bivariate Tobit Estimation Results[15]

Variable	Quantity Bought		Quantity Sold	
	Coefficient	**Standard Error**	**Coefficient**	**Standard Error**
Household Head Gender	1.526364	1.001167	−.1930418	.146936
Household Head Age	.2963331**	.1515074	−.0737349***	.0258084
Household Head Age Squared	−.0029066**	.0014053	.0006811***	.000239
Individuals in Household	−.0424101	.1271381	.0093567	.0190557
Dependency Ratio	−3.527704*	2.000703	.243366	.3242443
Births	.9750697	1.342157	.0311499	.2021928
Deaths	2.152439	2.717678	.6644136*	.3773919
Household Assets	$7.21e-06$***	$2.61e-06$	$3.68e-06$***	1.94e-07
Land	.2239154	.1550544	.0659076**	.0266245
Income	.0001149*	.0000647	−.0000165*	8.44e-06
Herd Size	.0431043	.0317213	.0056346*	.0032272
% Female TLUs	4.052207**	1.651769	.0064873	.2972863
Encumbered Males	−.0581539	.4372356	.0133173	.0187927
Encumbered Females	−.1561055	.2140918	−.0043805	.0124091
Fixed Costs	−.0050478**	.002457	.0005547**	.0002498
Variable Costs	.0101344**	.0052924	−.0005345	.0006172
Log Avg. Price Large Stock	−1.894086**	.85668	−.0863504	.1475813
Log Avg. Price Small Stock	−1.710362	1.074863	−.0728132	.1343425
Constant	22.23479**	10.36375	4.120857**	1.648611
$\rho(\hat{\epsilon}_b, \hat{\epsilon}_s)$	−.5931155***			

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[15]The superscripts *, ** and *** indicate a coefficient significant at the 90, 95 and 99% levels of confidence, respectively.

Table 3 – Estimation Results for the First Stage of the Ordered Tobit.

Variable	Coefficient	Standard Error
Household Head Gender	$-.1199407^{***}$	.0641554
Household Head Age	.0071154	.0115469
Household Head Age Squared	$-.0000848$	.0001079
Individuals in Household	.0180623	.0213252
Children	.0091998	.0395143
Dependency Ratio	.1057024	.2600769
Births	$.2543608^{**}$	.1138981
Deaths	.1213393	.2086002
Household Assets	$-3.91e-08$	$1.98e-07$
Land	.010639	.0141761
Income	$2.95e-06$	$4.71e-06$
Herd Size	.0028195	.0018374
% Female TLUs	.1717806	.1297437
Encumbered Males	.0161459	.0144535
Encumbered Females	$-.0059672$	.0083052
Fixed Costs	$.0005797^{**}$	.0002375
Fixed Costs Squared	$-2.23e-07^{*}$	$1.34e-07$
Animal Births	$.0268813^{**}$	.0121053
$\widehat{\alpha}_1$	-1.586686^{***}	.331772
$\widehat{\alpha}_2$	$.951289^{***}$	.3310051

Table 4 – Estimation Results for the Second Stage of the Ordered Tobit.

Variable	Quantity Bought			Quantity Sold	
	Coefficient	Std. Err. (Heckman)	Std. Err.	Coefficient	Std. Err. (Heckman)
Household Head Gender	1.285796	92.742948	1.769402	−.0364986	.39662522
Household Head Age	.3882739	1.7234782	.1999706	−.0774811***	.00725739
Household Head Age Squared	−.003572***	.00014832	.0019399	.0007297***	$6.282e-07$
Individuals in Household	−.8324656	2.7430249	.2501537	−.0112788	.01122807
Dependency Ratio	−16.89952	351.27406	2.831598	.1481089	1.4769656
Births	−3.723545	333.92481	2.065978	−.1305168	1.3862985
Deaths	2.052634	730.83915	3.946894	.6257654	3.0944864
Household Assets	.0000355***	$9.303e-10$	.0000116	$3.53e-06$***	$3.904e-12$
Land	−.3305331	3.3383848	.2524294	.0732192***	.01445112
Income	.0001777***	$2.958e-07$	.0001129	−.0000124***	$1.239e-09$
Herd Size	−.0579812	.11704397	.069558	.0026422***	.00042646
% Female TLUs	4.324919	279.42664	2.471809	−.0216494	1.1846152
Encumbered Males	.1230207	6.3491891	.8751133	−.0032226	.02502689
Encumbered Females	−.1867259	1.6364485	.42887	−.0013632	.00653026
Fixed Costs	−.0263224***	.00056822	.0056752	.0001716***	$2.035e-06$
Variable Costs	.0456513***	.00006896	.0102437	−.0000807***	$5.852e-07$
Log Avg. Price Large Stock	−3.732145***	.84147319	1.121651	−.1002418***	.00929865
Log Avg. Price Small Stock	.3518346	1.6675088	1.578528	−.1261232***	.01777674
Inverted Mills Ratio	19.86553	3598.7754	6.277778	−1.277819	20.417661
Constant	-13.17884	10202.617	17.87172	6.321585	66.95251
Price Elasticity	-2.730837	−	−	-0.100998	−

A Heckman-Corrected Variance Matrix

**(NOTE: THIS PART IS INTENDED FOR THE EDITOR AND THE REF-
EREES ONLY AND, UNLESS DEEMED NECESSARY, IS NOT INTENDED
FOR PUBLICATION.)**

In this section, we present the Heckman correction used to get the right standard errors
at the second stage of the ordered tobit model. The first step is to consider the ordered
probit model presented in equations (17) to (20) in section 3 above. From this first stage,
we derive that

$$
y_{1i} = \begin{cases}
0 & \text{if } \mathbf{x_1}\beta_1 + \epsilon_1 \leq \alpha_1 \\
1 & \text{if } \alpha_1 < \mathbf{x_1}\beta_1 + \epsilon_1 \leq \alpha_2 \\
2 & \text{if } \mathbf{x_1}\beta_1 + \epsilon_1 > \alpha_2
\end{cases}
\tag{25}
$$

so that

$$
y_{1i} = \begin{cases}
0 & \text{if } \epsilon_1 \in (-\infty, \alpha_1 - \mathbf{x_1}\beta_1] \\
1 & \text{if } \epsilon_1 \in [\alpha_1 - \mathbf{x_1}\beta_1, \alpha_2 - \mathbf{x_1}\beta_1) \\
2 & \text{if } \epsilon_1 \in (\alpha_2 - \mathbf{x_1}\beta_1, \infty)
\end{cases}
\tag{26}
$$

Thus, by symmetry of $\phi(\cdot)$, the standard normal pdf, we have that $P(y=0) = \Phi(\alpha_1 - \mathbf{x_1}\beta_1)$ and $P(y=2) = \Phi(\mathbf{x_1}\beta_1 - \alpha_2)$. We can then obtain the inverted Mills ratios (IMRs)
for net buyer and net seller households, respectively:

$$
\lambda_b = \frac{\phi(\alpha_1 - \mathbf{x_1}\beta_1)}{\Phi(\alpha_1 - \mathbf{x_1}\beta_1)},
\tag{27}
$$

and

$$
\lambda_s = \frac{\phi(\mathbf{x_1}\beta_1 - \alpha_2)}{\Phi(\mathbf{x_1}\beta_1 - \alpha_2)}.
\tag{28}
$$

Using these, we can fully describe the two-step estimator used in section 5. Our description closely follows that of Heckman's two-step estimator by Greene (2003). The first step is to estimate the first-stage ordered probit by maximum likelihood in order to obtain estimates for (α', β'_1). Then, for each observation $i \in \{1, ..., N\}$, the IMRs $\widehat{\lambda}_{bi}$ and $\widehat{\lambda}_{si}$ must be computed, but one must also compute:

$$\widehat{\delta}_{bi} = \widehat{\lambda}_{bi}(\widehat{\lambda}_{bi} - \widehat{\alpha}_1 - \mathbf{x_{1i}}\widehat{\beta_1}) \tag{29}$$

and

$$\widehat{\delta}_{si} = \widehat{\lambda}_{si}(\widehat{\lambda}_{si} - \widehat{\alpha}_2 - \mathbf{x_{1i}}\widehat{\beta_1}). \tag{30}$$

The second step is to estimate $(\beta_2, \beta_{b\lambda})$ and $(\beta_3, \beta_{s\lambda})$ by a regression of net purchases (net sales) on the set of covariates thought to affect net purchases (net sales) and on $\widehat{\lambda}_b$ $(\widehat{\lambda}_s)$.

Letting $j \in \{b, s\}$ denote net buyer and net seller households, respectively, the estimated residual variance of each second-stage linear component is such that

$$\widehat{\sigma}^2_{\epsilon j} = \frac{1}{n}\widehat{\epsilon}'_j\widehat{\epsilon}_j + \widehat{\overline{\delta}}_j\widehat{\beta}^2_{j\lambda}, \tag{31}$$

where

$$\widehat{\overline{\delta}}_j = \text{plim}\frac{1}{n}\sum_{i=1}^{n}\widehat{\delta_{ji}}, \tag{32}$$

and where $\widehat{\beta}^2_{j\lambda}$ is the square of the estimated coefficient for the jth IMR.

Moreover, we have that

$$\widehat{\rho}_j^2 = \frac{\widehat{\beta}_{j\lambda}^2}{\widehat{\sigma}_{\epsilon j}^2}. \tag{33}$$

Once we have obtained the above values, we can finally compute the Heckman-corrected variance-covariance matrices for our ordered (type II) tobit model, which are equal to:

$$\mathrm{Var}(\widehat{\beta}_j, \widehat{\beta}_{j\lambda}) = \widehat{\sigma}_{\epsilon j}^2 [X'X]^{-1}[X'(I - \widehat{\rho}_j^2\widehat{\Delta}_j)X + Q_j][X'X]^{-1}, \tag{34}$$

where the Q_j matrices are such that

$$Q_j = \widehat{\rho}_j^2 (X'\widehat{\Delta}_j X_1)\mathrm{Var}(\widehat{\beta}_1)(X_1'\widehat{\Delta}_j X), \tag{35}$$

and X is the data matrix of the second stage, which is identical for net buyer and net seller households, i.e., $X \equiv X_b = X_s$, $I - \widehat{\rho}_j^2\widehat{\Delta}_j$ is a diagonal matrix whose diagonal terms are equal to $1 - \widehat{\rho}_j^2\widehat{\delta}_{ji}$, X_1 is the data matrix of the first-stage ordered probit, and $\mathrm{Var}(\widehat{\beta}_1)$ is the variance matrix of the first-stage coefficients. Performing these computations thus yields efficient estimates for the second-stage parameters of our ordered (type II) tobit and offers an "ordered", modest extension to Heckman's (1979) method.