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Untangling of Meshes in ALE Simulations

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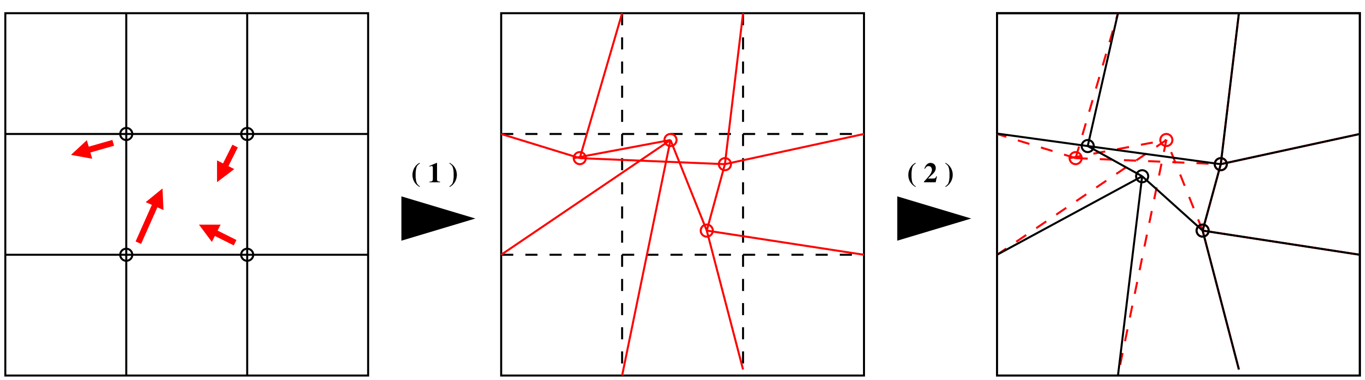
ALE Simulations

- **Eulerian Approach**
- **Lagrangian Approach**
- **ALE Algorithm (Arbitrary Lagrangian - Eulerian)**
  - **(1) Lagrangian step**
  - **(2) Mesh untangling / improvement (rezoning)**
  - **(3) Remapping**



Outline

- **simple methods**

  geometrical

  complete Feasible Set

  simplex-based Feasible Set

  optimization of objective function

  local (moving vertices separately)

  Feasible Set

  local optimization

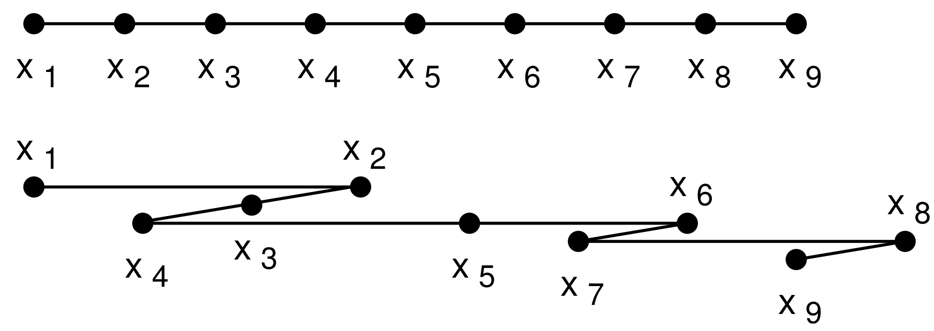
  global (moving all vertices at once)

  global optimization

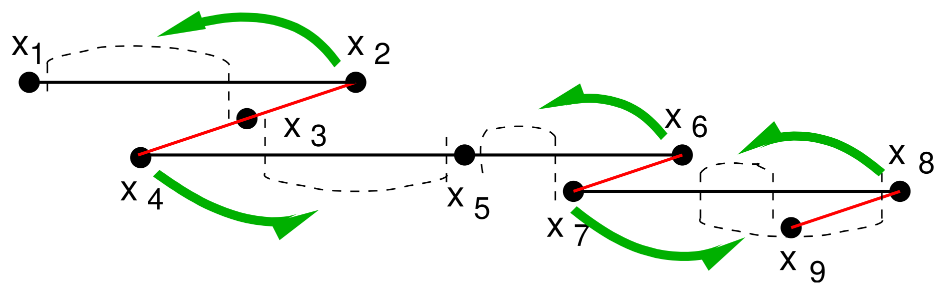
- **composite 3-step algorithm**

Feasible Set Method in 1D

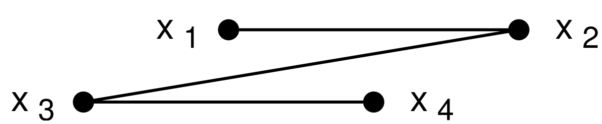
- **Notation**



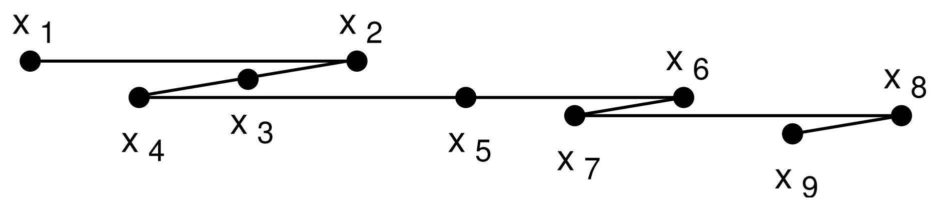
- **Algorithm**



- **FS may be empty**



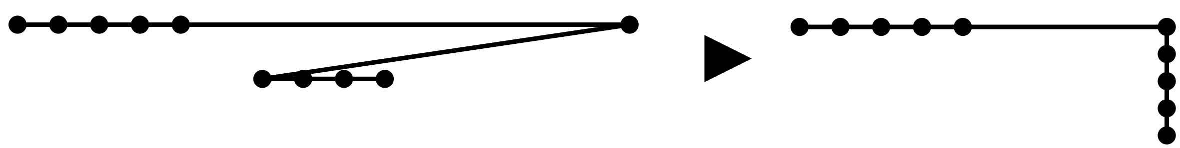
Global Optimization in 1D



- **Objective function** $$\mathcal{F} = \sum_{i=1}^{N-1} \underbrace{\left(|x_{i+1} - x_i| - (x_{i+1} - x_i)\right)^2}$$

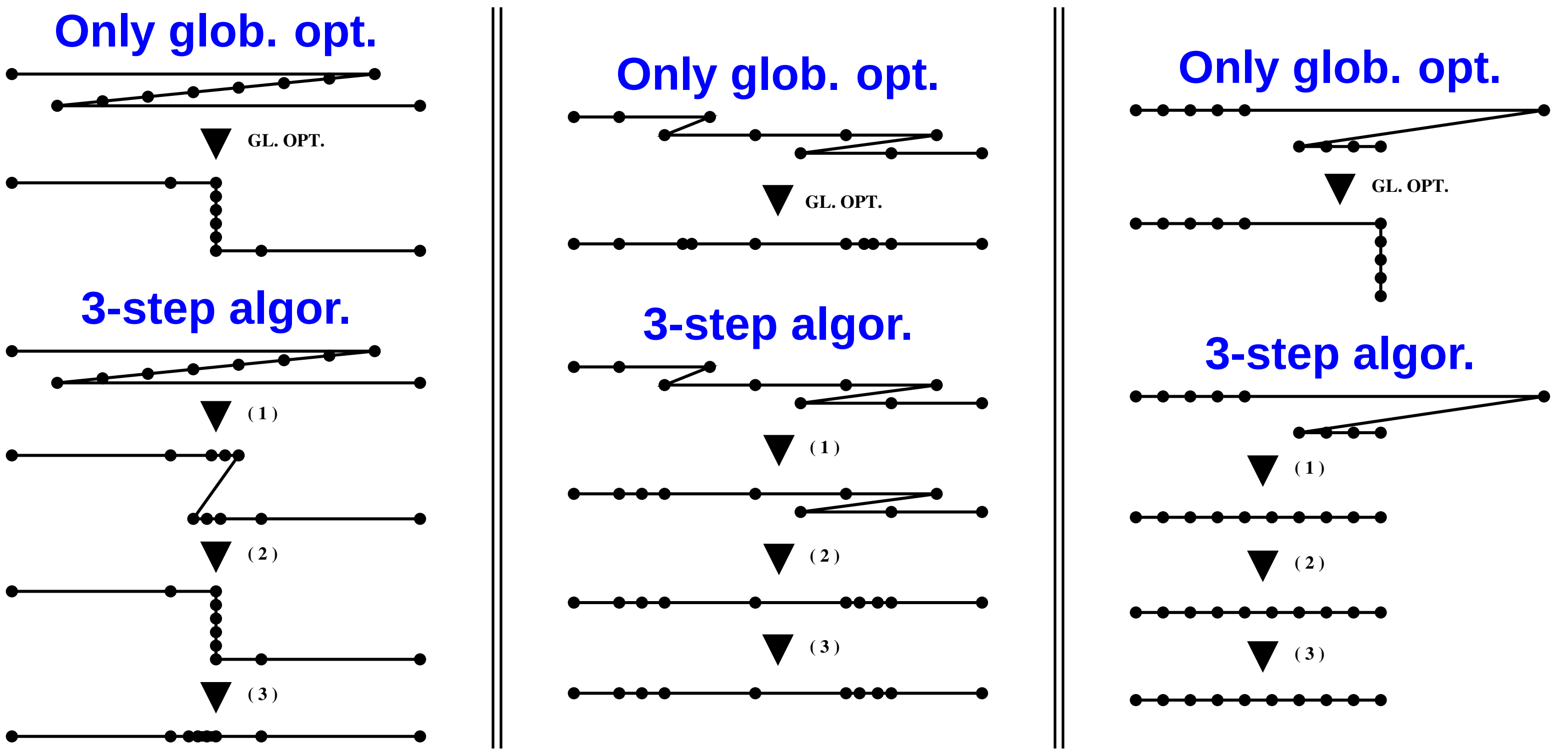
$$= \begin{cases} 0 & \textit{if cell valid} \\ 4\left(x_{i+1} - x_i\right)^2 & \textit{if inverted} \end{cases}$$

- **Fixing of a local problem may move even distant vertices**



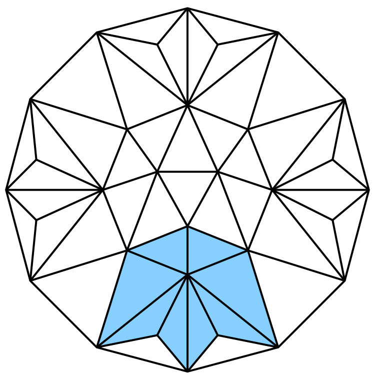
3-Step Algorithm in 1D

(1) Feasible Set Method
(2) Global Optimization
(3) Feasible Set Method

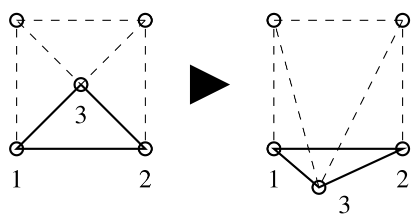


Feasible Set Method in 2D

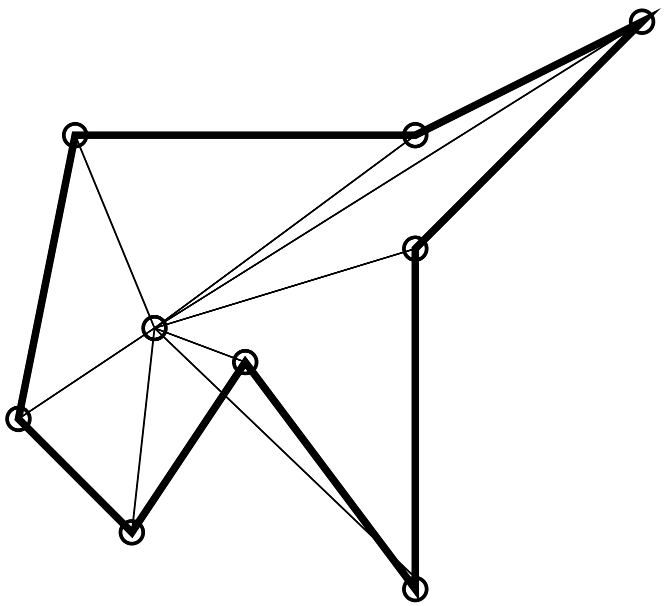
Triangular mesh



Cell validity

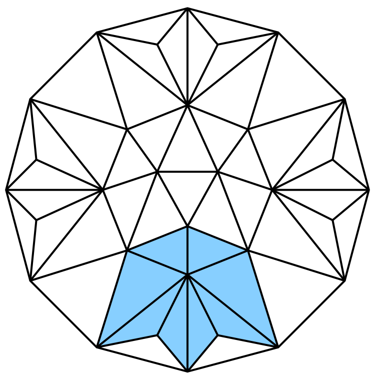


Tangled patch

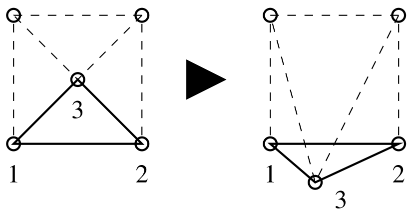


Feasible Set Method in 2D

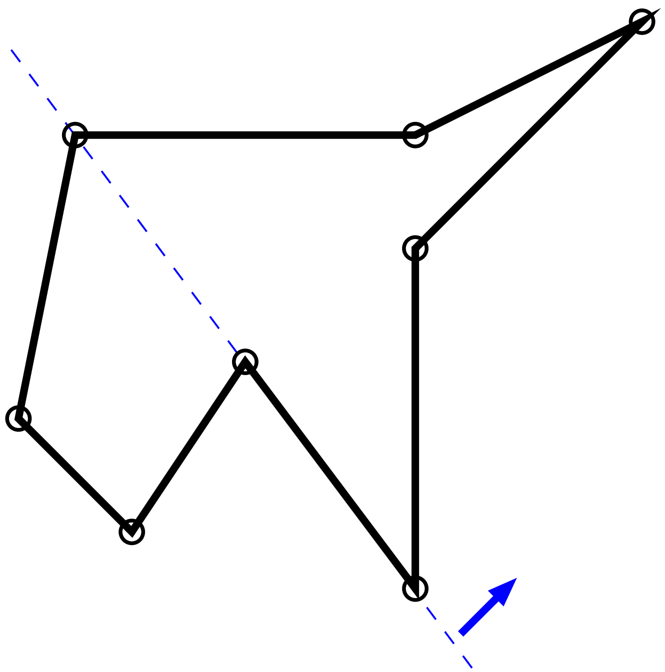
Triangular mesh



Cell validity

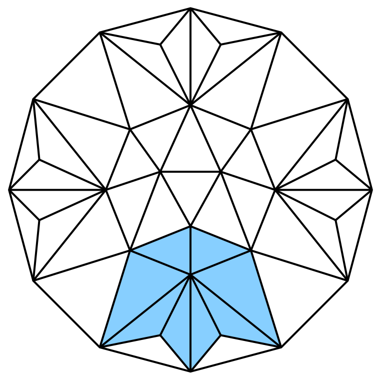


Imposing linear constraints

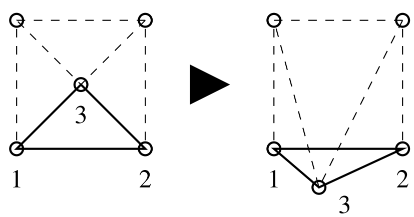


Feasible Set Method in 2D

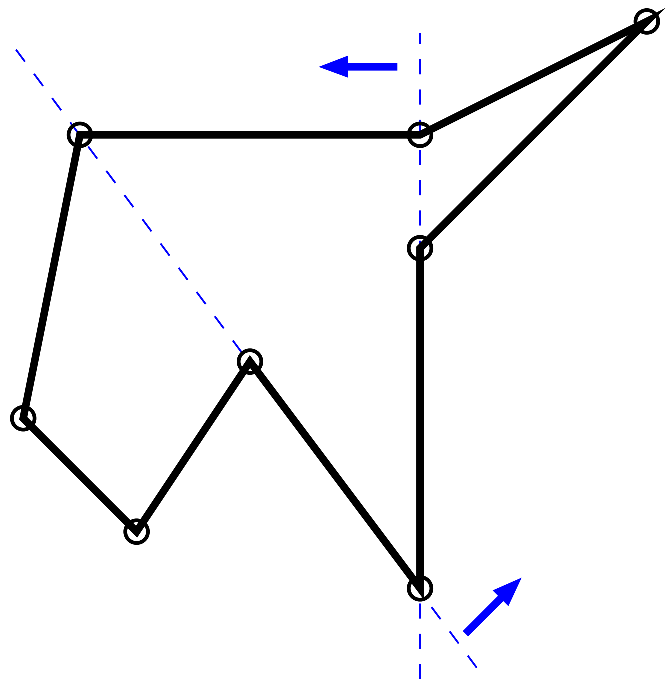
Triangular mesh



Cell validity

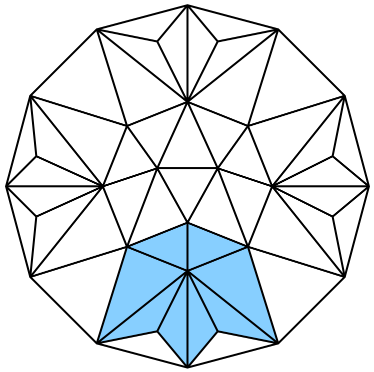


Imposing linear constraints

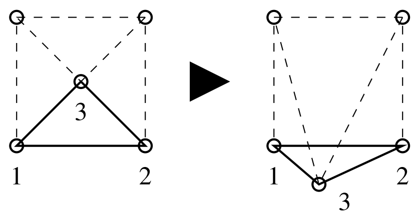


Feasible Set Method in 2D

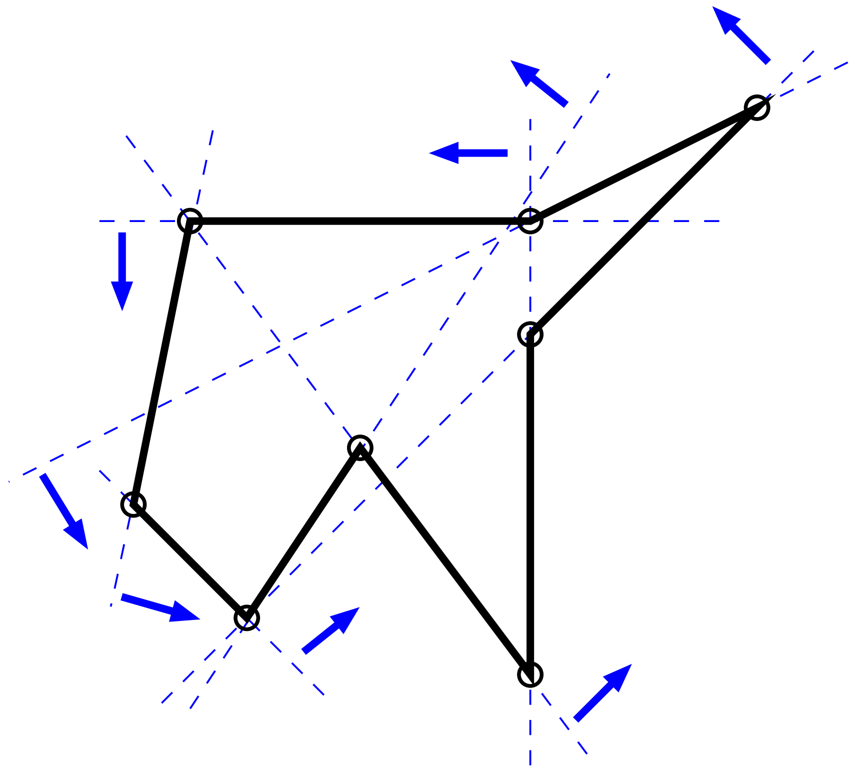
Triangular mesh



Cell validity

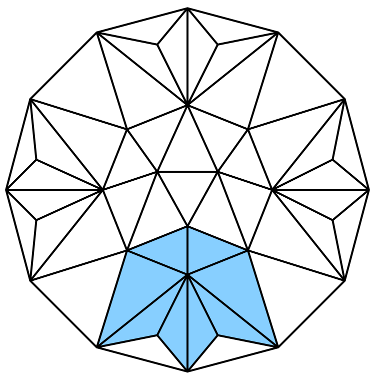


Imposing linear constraints

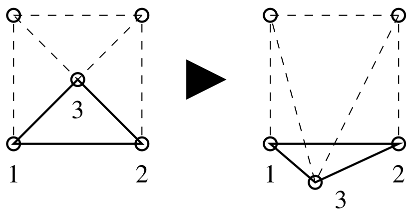


Feasible Set Method in 2D

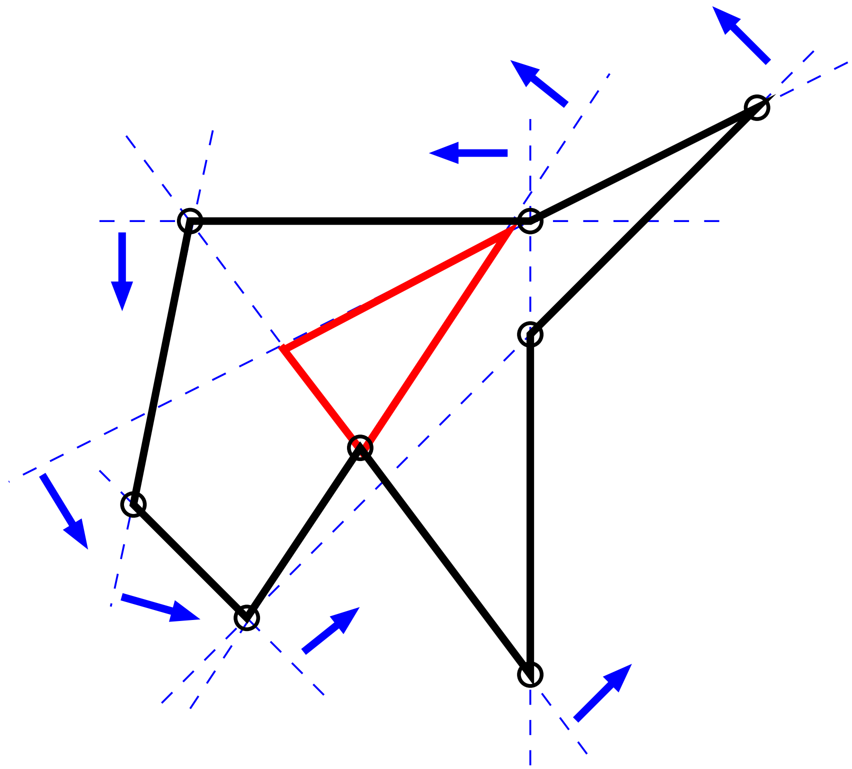
Triangular mesh



Cell validity

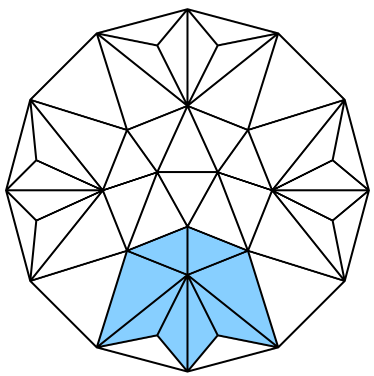


Intersection of halfplanes

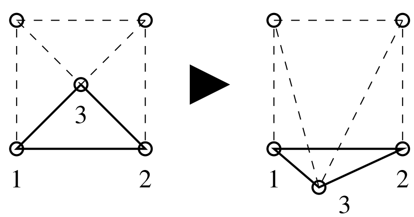


Feasible Set Method in 2D

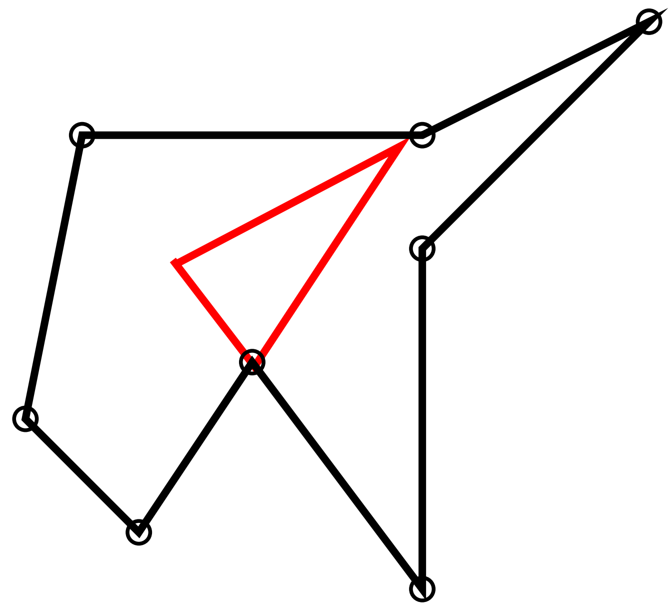
Triangular mesh



Cell validity

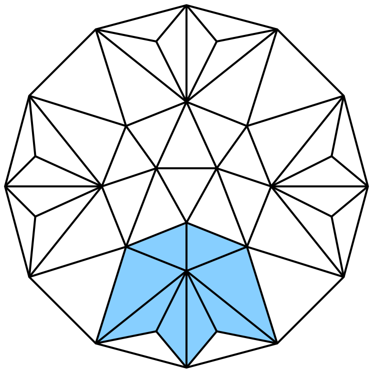


Intersection is the feasible set

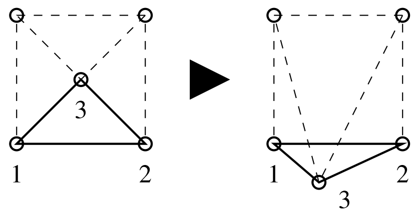


Feasible Set Method in 2D

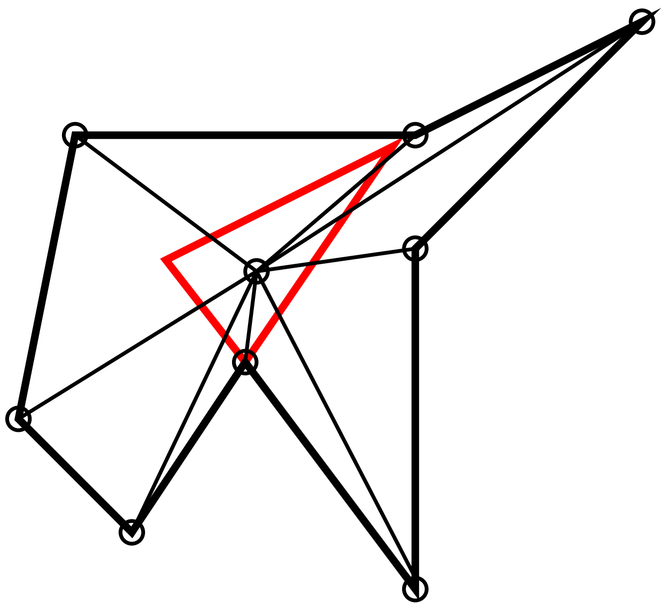
Triangular mesh



Cell validity

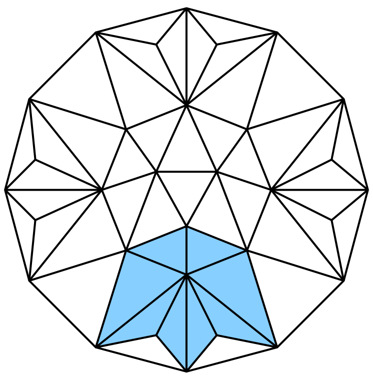


Placing the central vertex

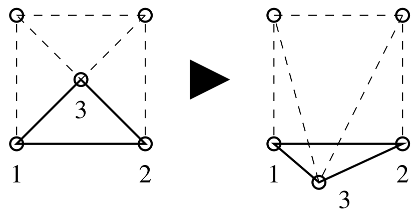


Feasible Set Method in 2D

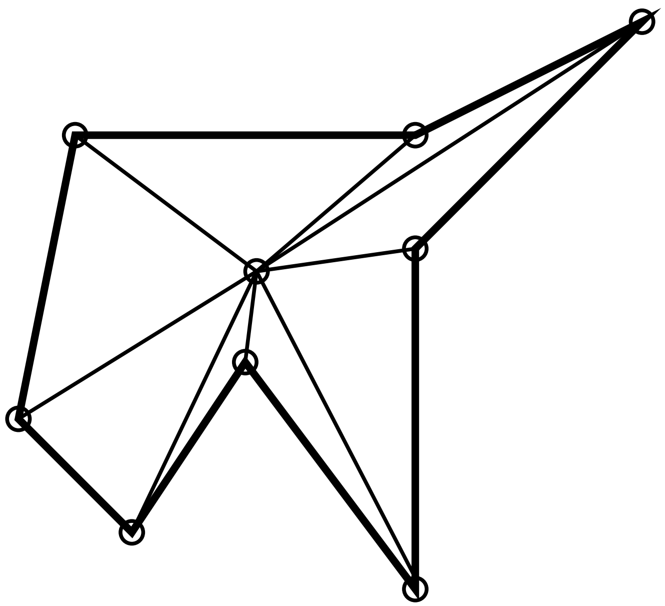
Triangular mesh



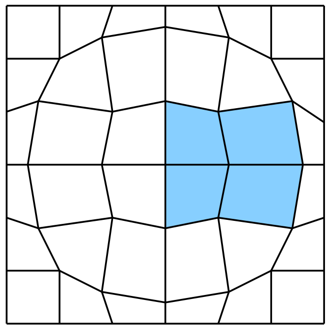
Cell validity

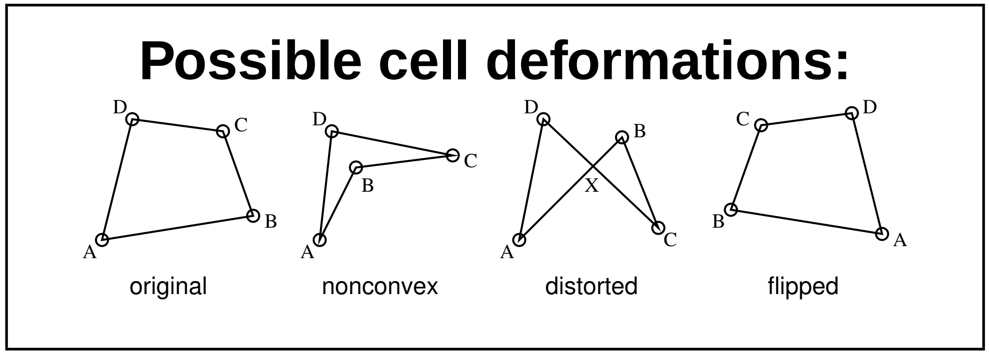


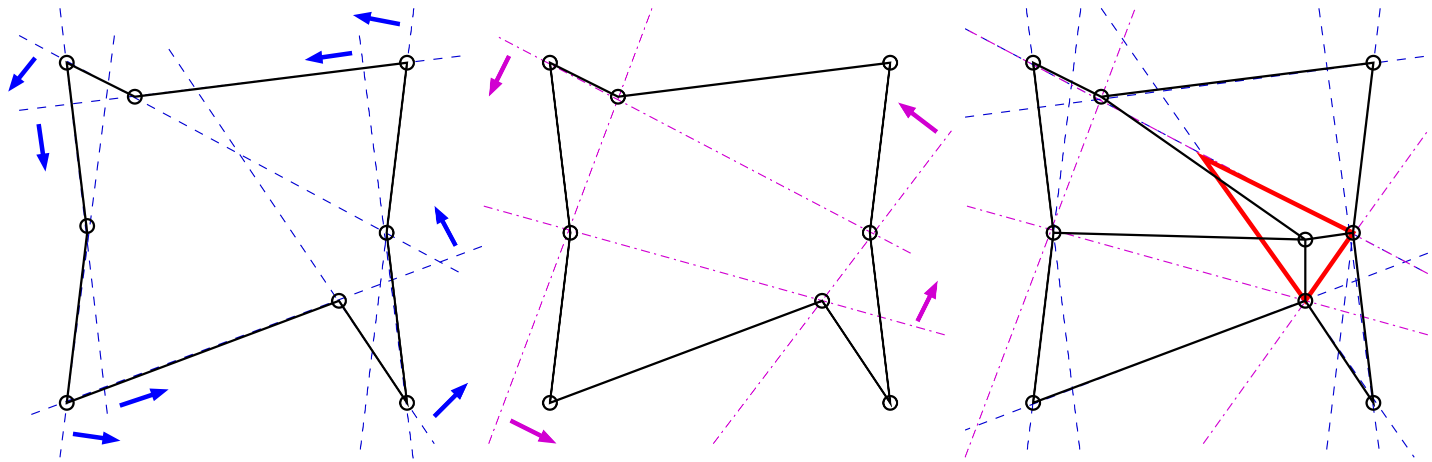
Untangled patch



Feasible Set Method in 2D

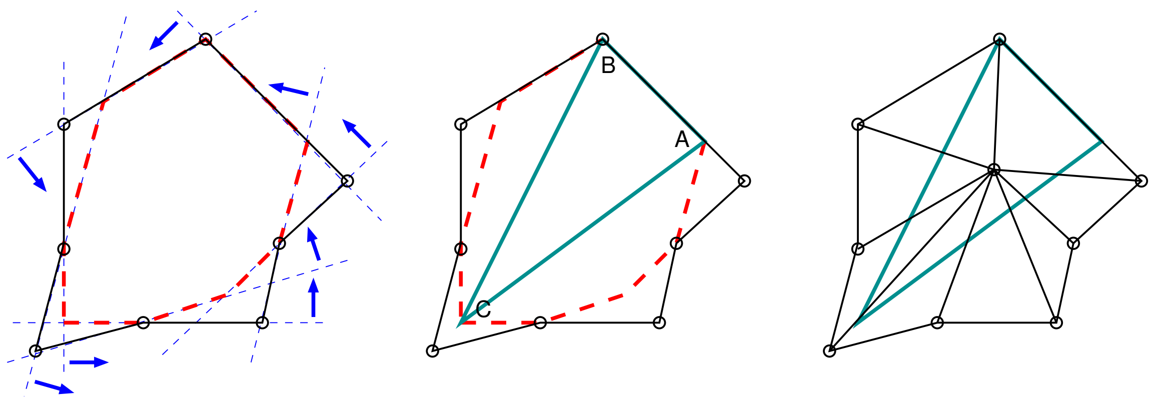






Feasible Set in 2D Using the Simplex Method

- **Analytical search of Feasible Set**
  - ◇ **Intersections of linear constraints - expensive esp. in 3D**
  - ◇ **We need only one point inside FS**
- **Simplex Method**
  - ◇ **We maximize simple linear functions $f = x$, $f = -x$, $f = y$ and $f = -y$, using the same constraints as for FS method**
  - ◇ **We always find a subset of the Feasible Set (if FS exists)**
  - ◇ **No need of intersections**



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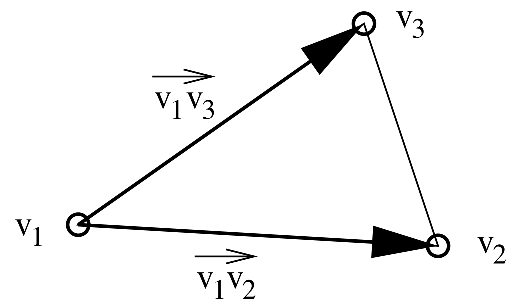
Optimization in 2D

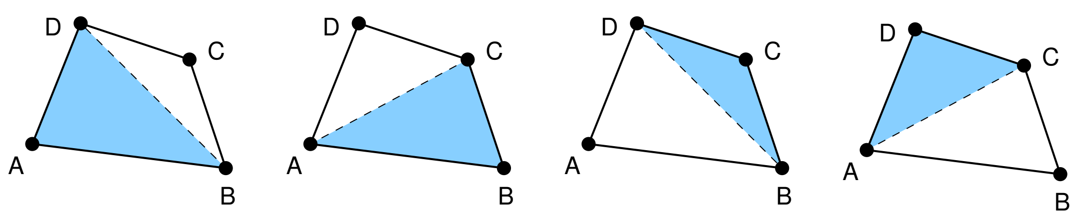
Objective function

$$\mathcal{F} = \sum_{i=1}^{N} (A_i - |A_i|)^2$$



$$A \;=\; \frac{1}{2}\, \overrightarrow{v_1 v_2} \times \overrightarrow{v_1 v_3}$$

- **Triangular mesh: take functional A for the whole cell**
- **Other Meshes: virtually subdivide cells and compute functional A for each triangle**



Optimization in 2D

- **Linear:** $$\mathcal{F} = \sum_{i=1}^{N} (A_i - |A_i|)$$
  - ◇ **jumps in gradients $\Rightarrow$ can get stuck**
- **Quadratic:** $$\mathcal{F} = \sum_{i=1}^{N} (A_i - |A_i|)^2$$
  - ◇ **smooth**
  - ◇ **faster than linear when using conjugate gradient methods**
- **With parameter:** $$\mathcal{F} = \sum_{i=1}^{N} ((A_i - \beta) - |A_i - \beta|)^2$$
  - ◇ **suitable size of parameter β is problem dependent**

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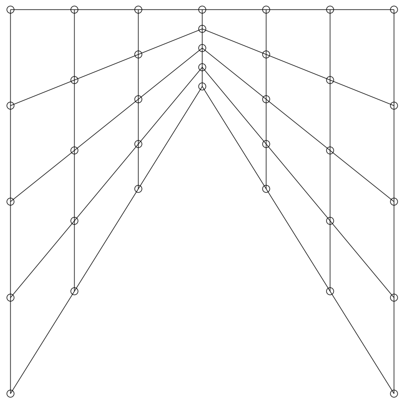
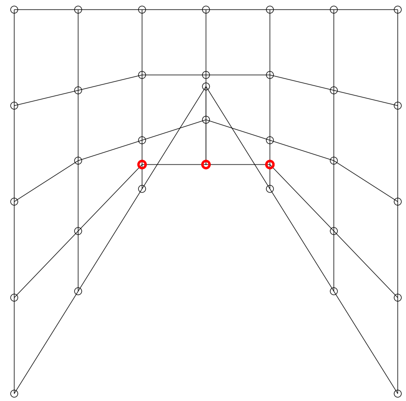
3-step Algorithm in 2D

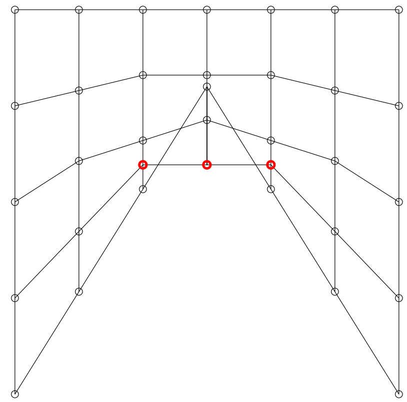
original



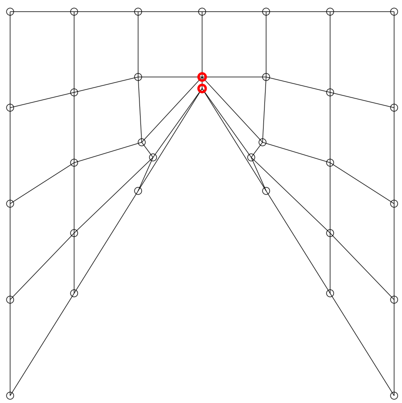
tangled



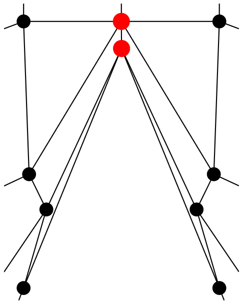
FS



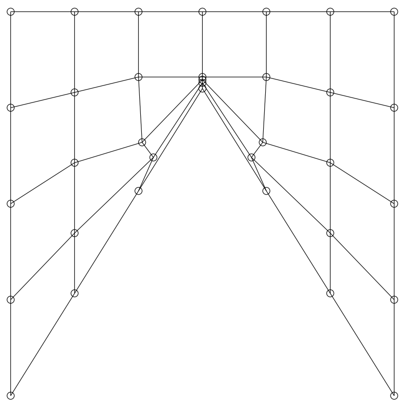
FS ⇛ Global Optimization



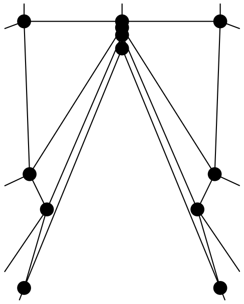
detail



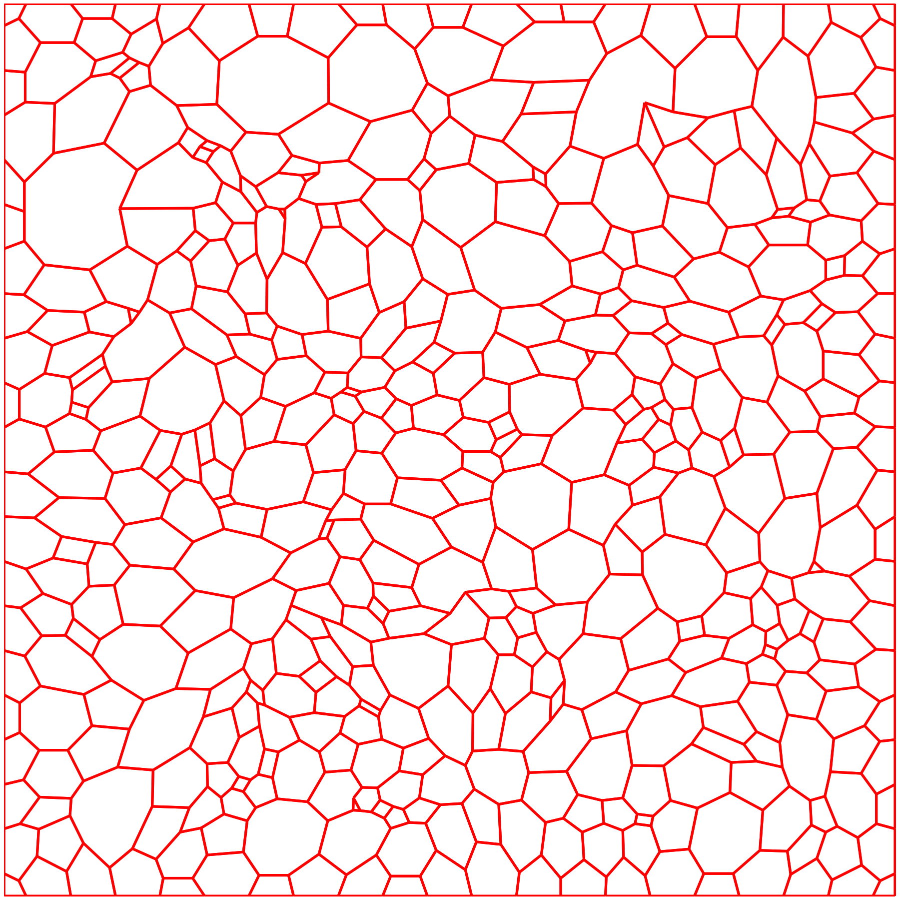
FS ⇛ Glob. Opt. ⇛ FS



detail

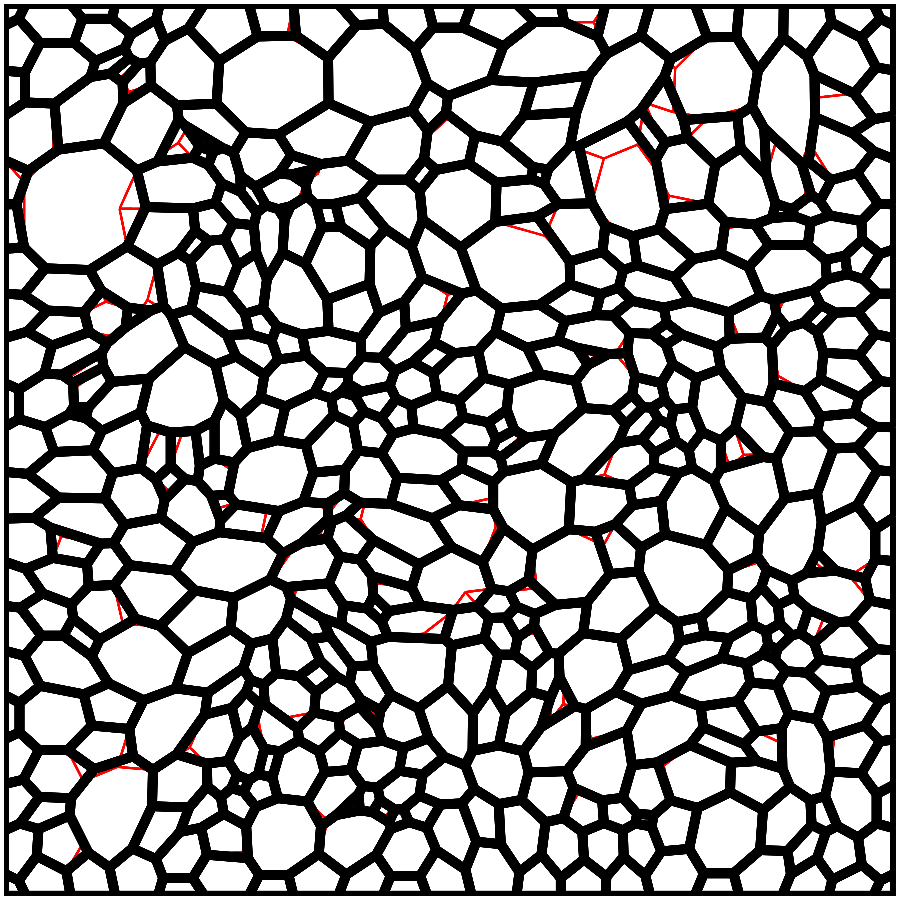


2D mesh of arbitrary polygons



tangled

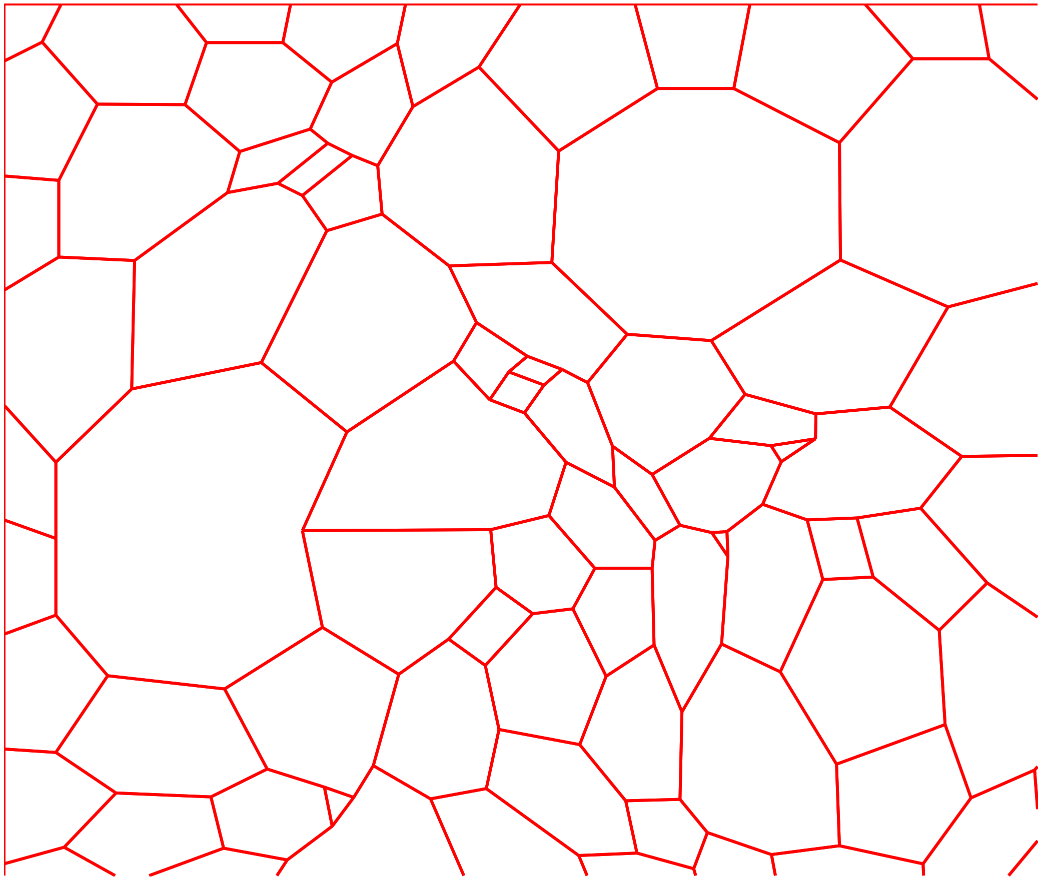
2D mesh of arbitrary polygons



tangled

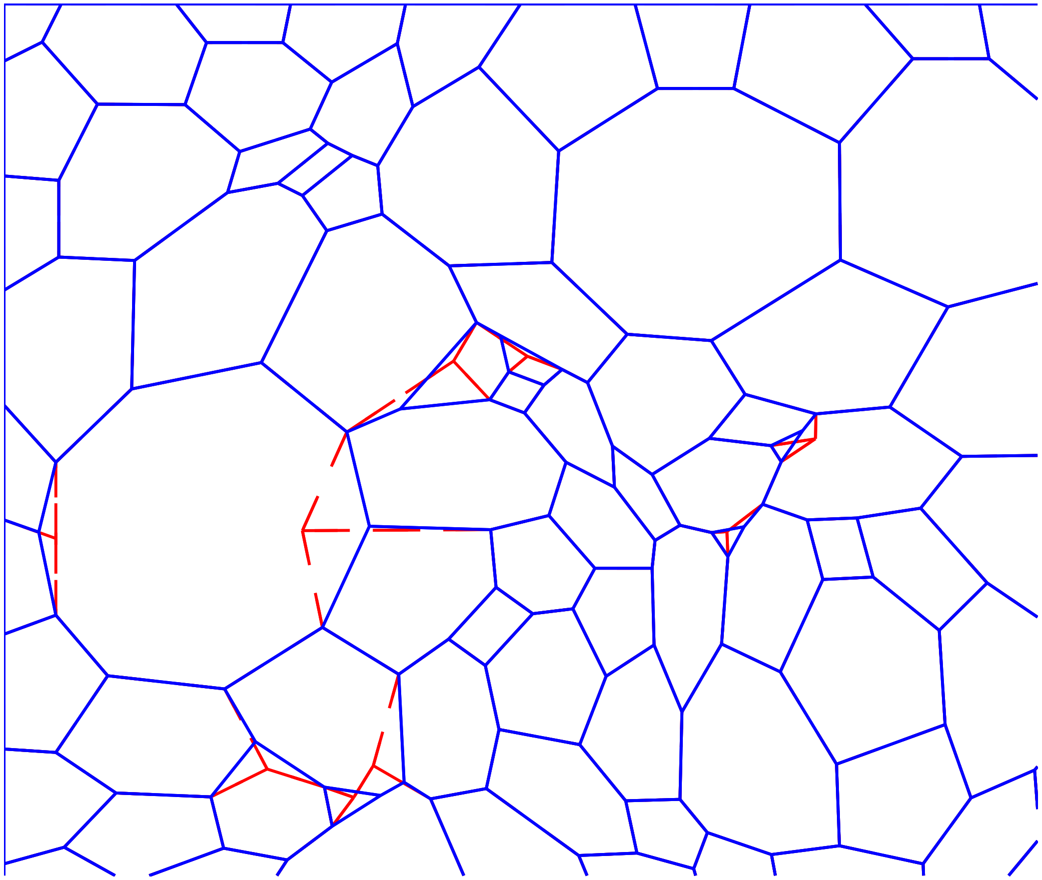
untangled

2D mesh of arbitrary polygons



tangled

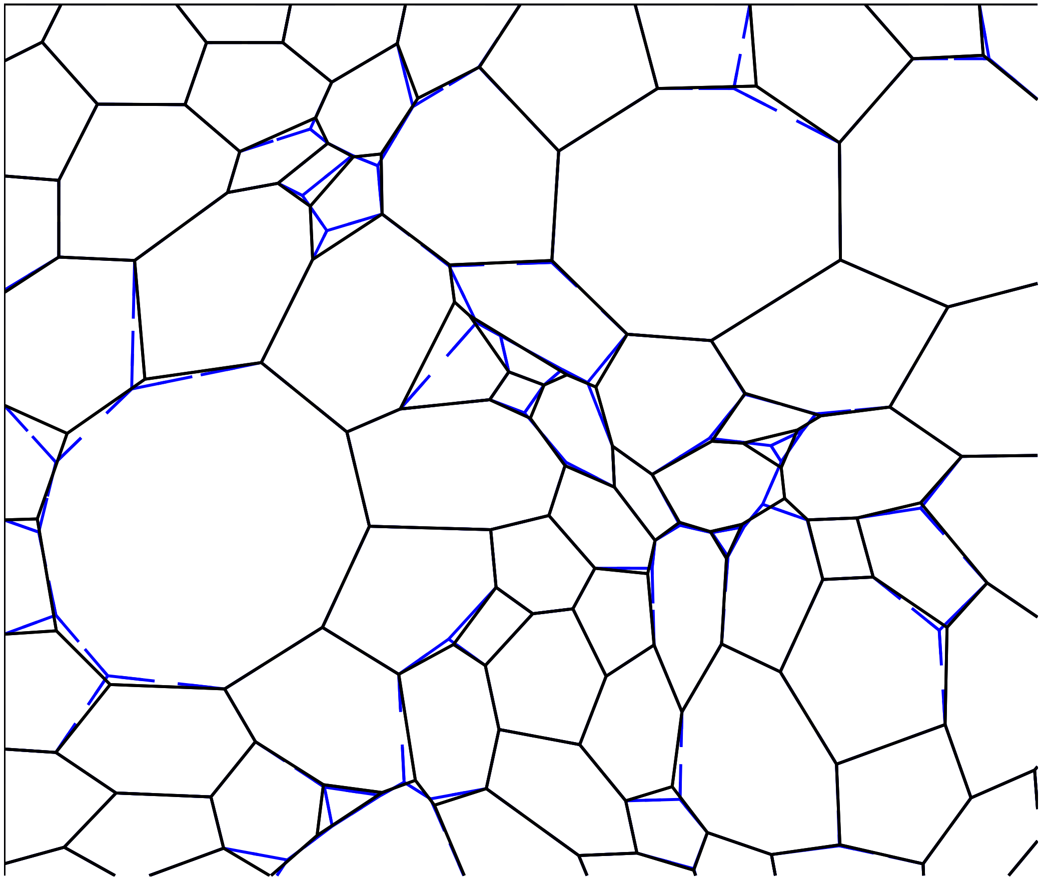
2D mesh of arbitrary polygons



tangled

FS

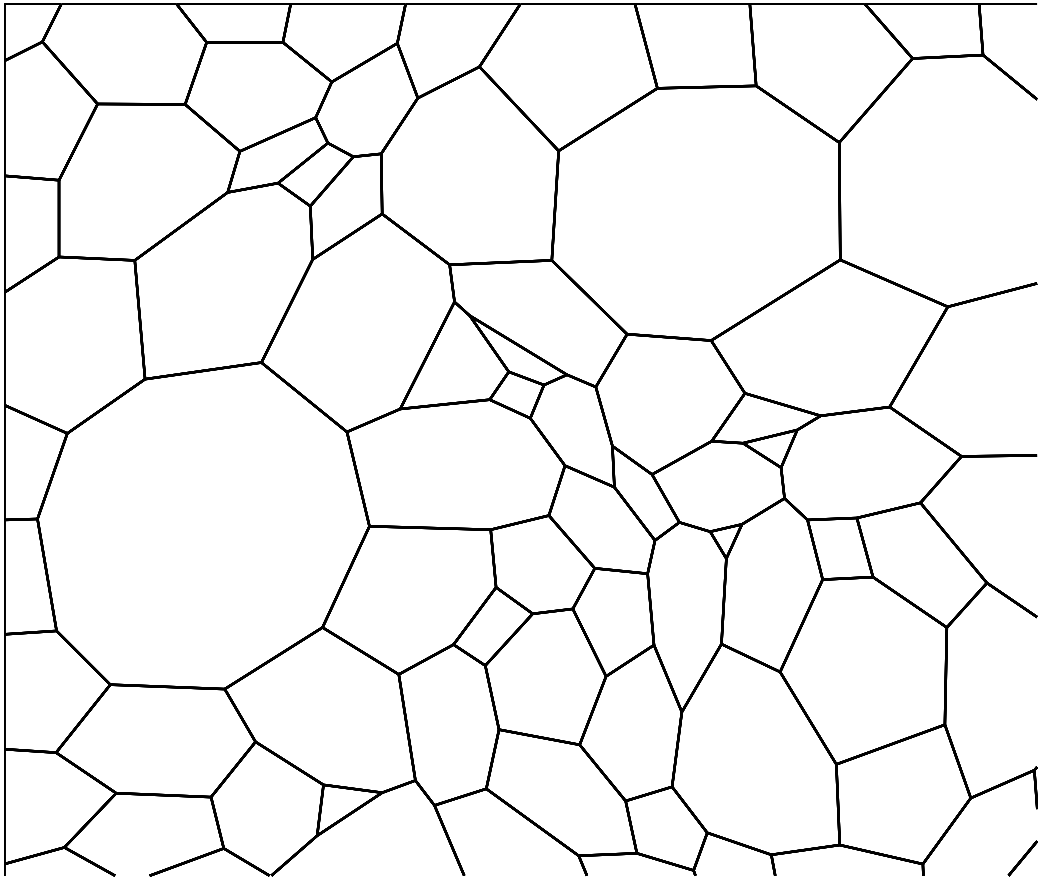
2D mesh of arbitrary polygons



FS

FS → Gl. Opt.

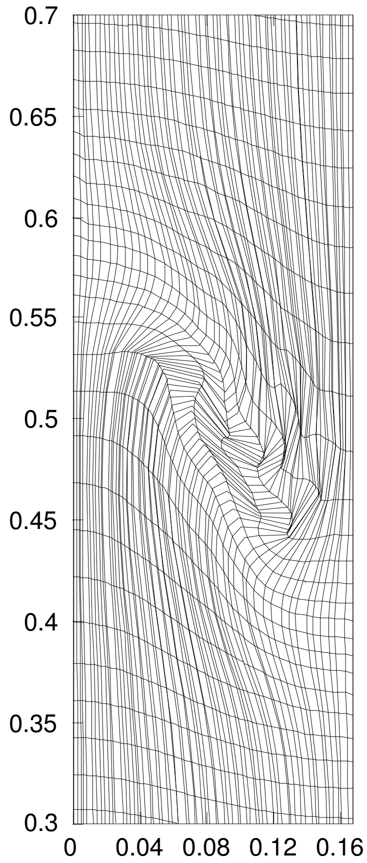
2D mesh of arbitrary polygons



untangled

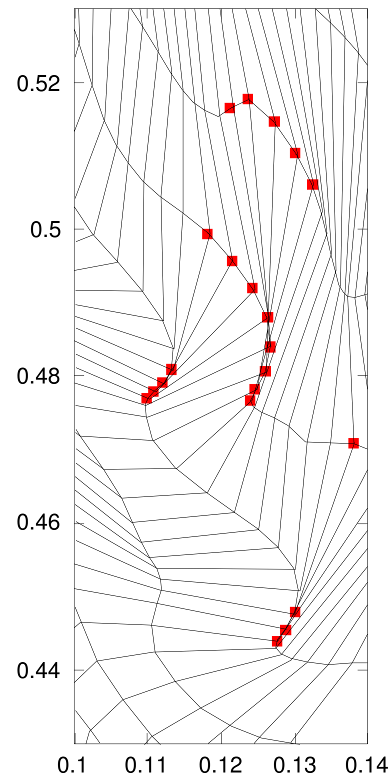
2D Rayleigh - Taylor Instability

WHOLE MESH

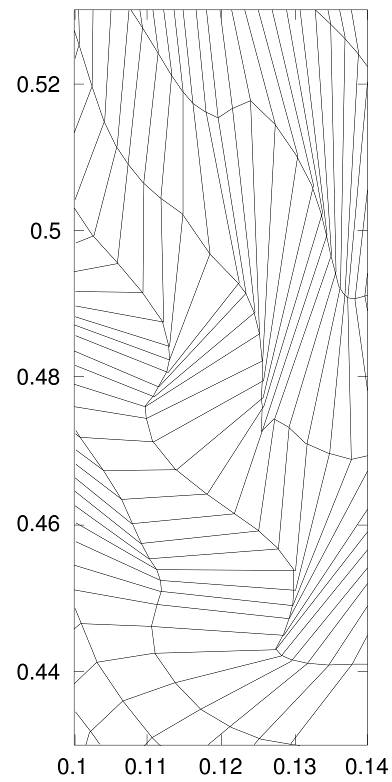


DETAIL

tangled



untangled



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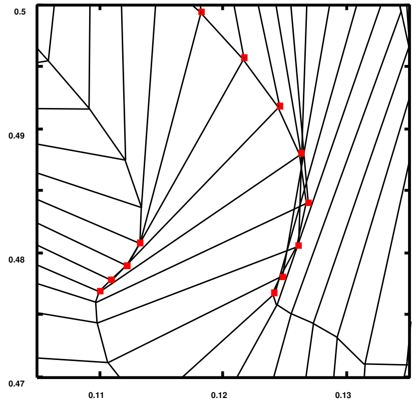
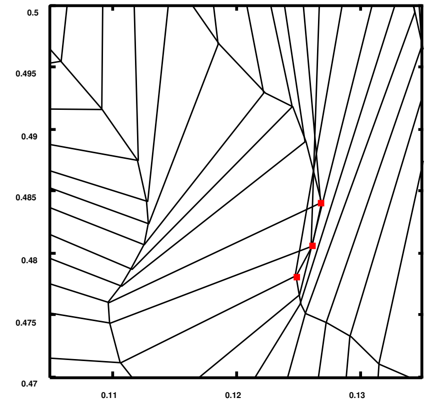
2D Rayleigh - Taylor Instability

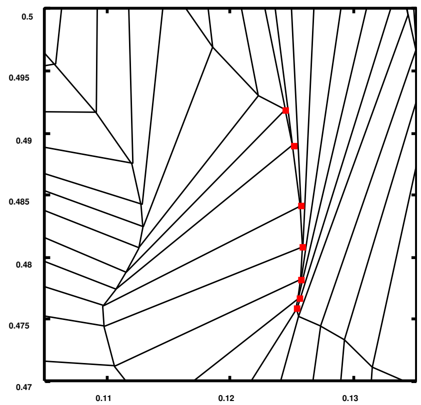
tangled



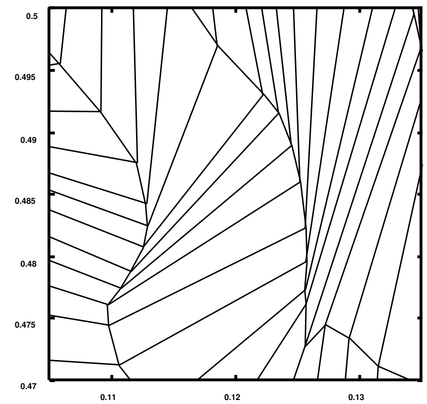
FS



FS $\Rrightarrow$ Gl. Opt.



FS $\Rrightarrow$ Gl. Opt. $\Rrightarrow$ FS



Conclusion

Universal 3-step algorithm for untangling of lagrangian meshes

- **implemented for arbitrary 2D and tetrahedral 3D**
- **extension for arbitrary 3D straightforward**
- **combines simple local and global methods**
- **works for all meshes we tried (unlike simple Feasible Set)**
- **treats local problems locally $\Rightarrow$ moves only what is necessary (unlike simple global optimization)**

Acknowledgments

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