A CLASSICAL ODDERON IN HIGH ENEGY QCD

Raju Venugopalan
BNL

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Outline of talk:

- The ground state of a large nucleus at high energies
- Random walks & Path Integrals for SU(N) quarks
- A “classical” Odderon
- Results
- Summary and Outlook

Work in collaboration with Sangyong Jeon (McGill/RBRC)

- **A CLASSICAL ODDERON IN QCD AT HIGH ENERGIES.**
  Sangyong Jeon, Raju Venugopalan **Phys.Rev.D71:125003,2005**

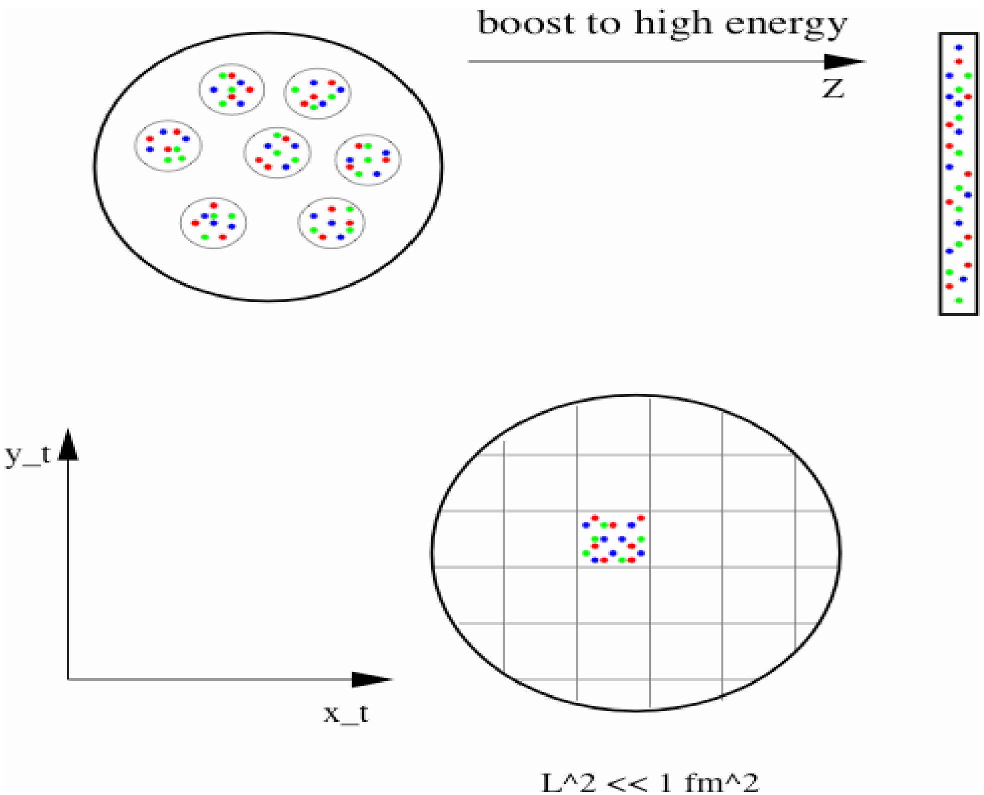
- **RANDOM WALKS OF PARTONS IN SU(N(C)) AND CLASSICAL REPRESENTATIONS OF COLOR CHARGES IN QCD AT SMALL X.**
  Sangyong Jeon, Raju Venugopalan **Phys.Rev.D70:105012,2004**

Odderon paper inspired by

- **ODDERON IN THE COLOR GLASS CONDENSATE.**
  Y. Hatta, E. Iancu, K. Itakura, L. McLerran
  Nucl.Phys.A760:172-207,2005

GROUND STATE OF A NUCLEUS AT HIGH ENERGIES

Consider large nucleus in the IMF frame: $P^+ \to \infty$

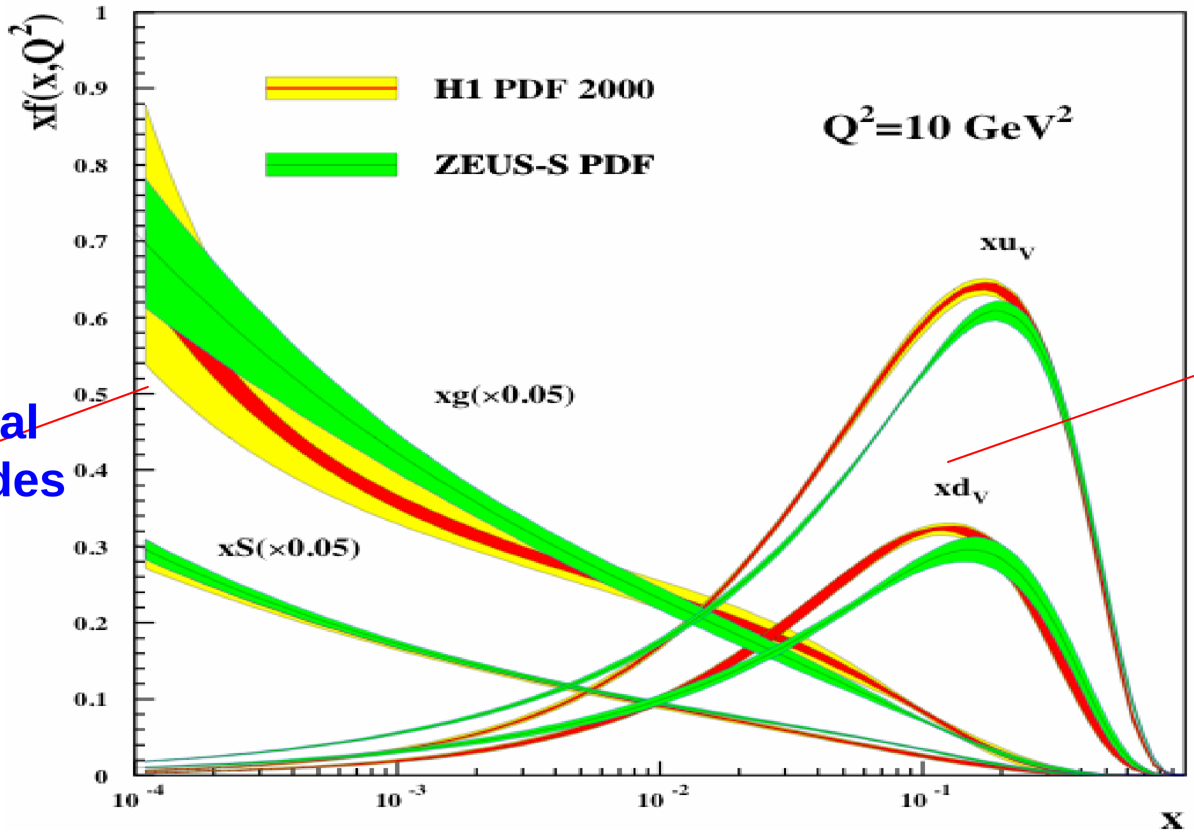


$$A^+ = 0$$

$$\mathcal{Z} = < P | e^{i x^+ P^-_{\rm QCD}} | P > = \lim_{x^+ \to i\infty} \sum_{N,Q} < N, Q | e^{i x^+ P^-_{\rm QCD}} | N, Q >$$

Born-Oppenheimer: separation of large x and small x modes

Dynamical Wee modes



Valence modes-are static sources for wee modes

$$\tau_{\text{wee}} \sim \frac{1}{k^-} = \frac{2k^+}{k_\perp^2} \equiv \frac{2x\,P^+}{k_\perp^2}$$

$$\tau_{\text{valence}} = \frac{2P^+}{k_\perp^2} >> \tau_{\text{wee}} \text{ for } x << 1$$

The effective action

Scale separating sources and fields

Generating functional:

$$\mathcal{Z}[j] = \int [d\rho]\, W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho] - \int j\cdot A}}{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho]}} \right\}$$

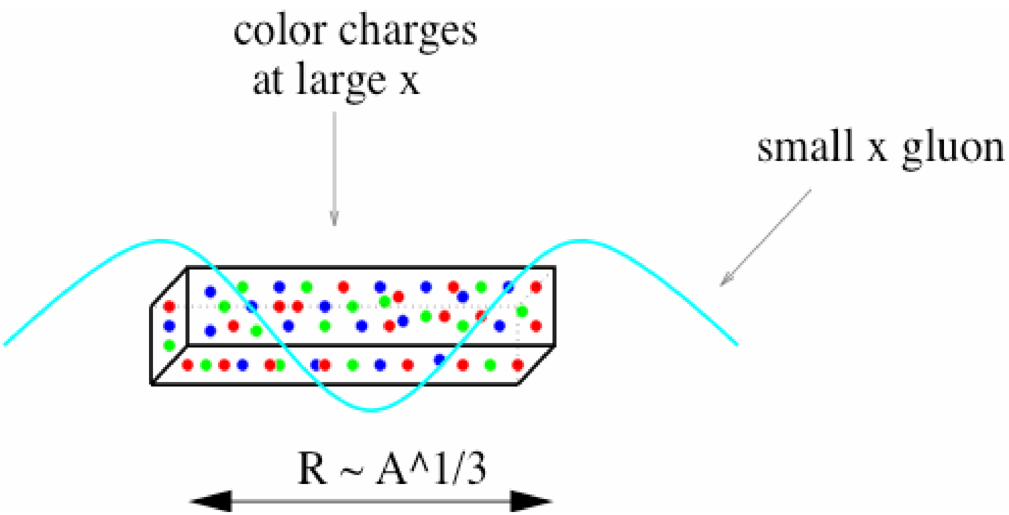
Gauge invariant weight functional describing distribution of the sources- W obeys RG (B-JIMWLK) equations with changing scale

$$S[A,\rho] = \frac{-1}{4}\int d^4x\, F_{\mu\nu}^2 + \frac{i}{N_c}\int d^2x_\perp dx^- \delta(x^-) \mathrm{Tr}\left(\rho(x_\perp)\, U_{-\infty,\infty}[A^-]\right)$$

with $$U_{-\infty,\infty}[A^-] = \mathcal{P}\exp\left(ig\int dx^+ A^{-,a}T^a\right)$$

Focus in this talk on weight functional - for large nuclei

$$\lambda_{\text{wee}} >> \lambda_{\text{valence}} \Rightarrow x << A^{-1/3}$$



Coarse grained field theory:

of random quarks in box of size $\Delta x_\perp \sim \frac{1}{p_\perp}\ ;\ p_\perp >> \Lambda_{\text{QCD}}$

$$k_{\Delta x_\perp} = \frac{N_{\text{val}}}{\pi R^2} (\Delta x_\perp)^2$$

Well defined math problem:

Given k non-interacting quarks belonging the the fundamental SU(N) representation:

a) What is the distribution of degenerate irreducible representations?

b) What is the most likely representation?

c) Is it a classical representation ? N_c = Infinity is classical even for k=1

Random walk problem in space spanned by the N_c -1 Casimirs of SU(N)

QCD IN THE LARGE A ASYMPTOTICS:

$$x << A^{-1/3} \;\; => \;\; Y \equiv \ln(1/x) >> \ln(A)$$

Evolution effects small =>

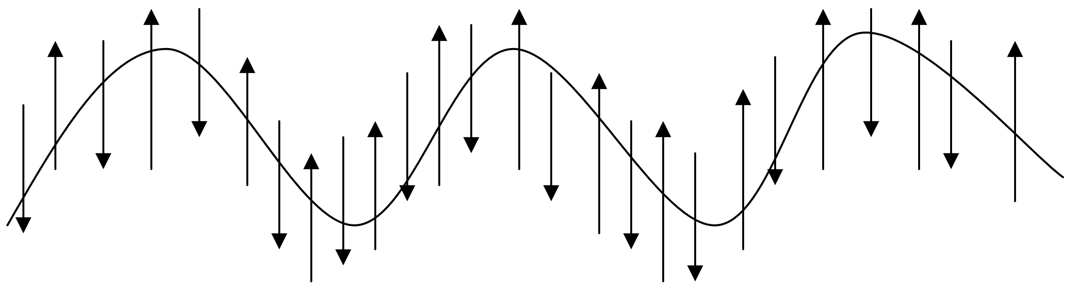
$$\alpha_S \ln(1/x) << 1 \;\; => \;\; Y << A^{1/6}$$

Kinematic region of applicability:

$$\ln(A) << Y << A^{1/6}$$

Kovchegov: Extend discussion to $Y >> 1/\alpha_S$

An SU(2) Random walk



Random walk of spin 1/2 partons:

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{3}$$

$$(\mathbf{2} \otimes \mathbf{2}) \otimes \mathbf{2} = (\mathbf{1} \oplus \mathbf{3}) \otimes \mathbf{2} = \mathbf{2} \oplus \mathbf{2} + \mathbf{4}$$

In general, $\mathbf{2} \otimes \mathbf{s} = \mathbf{s} - \mathbf{1} \oplus \mathbf{s} + \mathbf{1}$

$v_s^{(k)}$ = **Multiplicity of representation s when k fundamental reps. are multiplied**

$$v_s^{(k)} = v_{s-1}^{(k-1)} + v_{s+1}^{(k-1)}$$

Binomial Coefficients satisfy:

$$\binom{k}{s} = \binom{k-1}{s-1} + \binom{k-1}{s}$$

Multiplicity satisfies: $v_s^{(k)} = G_{k:s-1} - G_{k:s+1}$

with $$G_{k:s} = \frac{k!}{\left(\frac{k+s}{2}\right)!\ \left(\frac{k-s}{2}\right)!}$$

Using Stirling's formula, k >> s >> 1:

$$v_s^{(k)} \approx \frac{2^{k+1/2}}{k\sqrt{k\pi}}\, s\, e^{-s^2/2k}$$

Probability:

$$\int ds\, s\, v_s^{(k)} = 2^k$$

degeneracy of state (→ s)

Mult. of representation (→ $v_s^{(k)}$)

degrees of freedom (→ k)

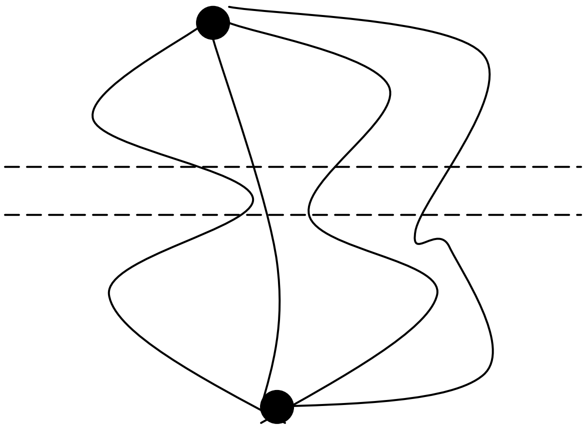
$$P_k(l) = \left(\frac{2}{k\pi}\right)^{3/2} e^{-2\,l^2/k}$$

Analogous to Maxwell-Boltzmann distribution

Casimir: $D_2 = l(l+1) \approx l^2$

Average value: $\bar{D}_2 = \frac{k}{4}$

PATH INTEGRAL:



Coarse graining -> Box of size 1/p_t in transverse plane

Sum over spins in box:

$$\sum_l v_l^{(k)} \sum_{m=-l}^{l} |l,m><l,m| \to \int d^3l \, e^{-2\,l^2/k}$$

Classical color/spin density:

$$l^a = (\Delta x_\perp)^2 \frac{1}{g} \rho^a(x_\perp)$$ =>

$$2\,\frac{l^2}{k} = \frac{\pi R^2}{g^2 A} (\Delta x_\perp)^2 \rho^a \rho^a$$

McLerran, RV
Kovchegov

Summing over all boxes -> Classical path integral over SU(2) color charge density

$$\int [d\rho] \exp\left(- \int d^2 x_\perp \frac{\rho^a \rho^a}{2\mu_A^2} \right)$$

$$\mu_A^2 = \frac{g^2 A}{2\pi R^2} \propto A^{1/3}\, \mathrm{fm}^{-2}$$

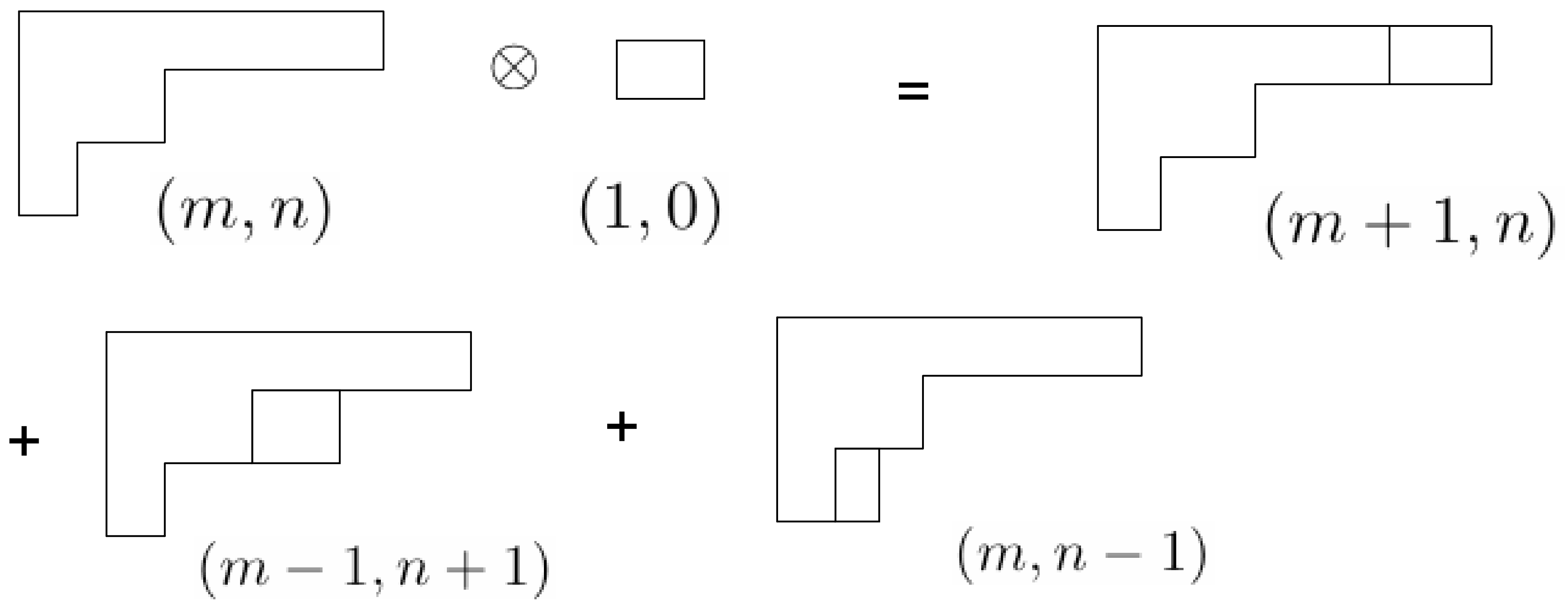
Color charge squared per unit area- closely related to saturation scale.

For A >> 1, coupling runs as a function of this scale

Random walk of SU(3) Color charges

Denote SU(3) representations by (m, n) :

Recursion relation for SU(3) :



$N_{m,n}^{(k)}$ = **multiplicity of (m, n) state in the kth iteration**

$$N_{m,n}^{(k+1)} = N_{m-1,n}^{(k)} + N_{m+1,n-1}^{(k)} + N_{m,n-1}^{(k)}$$

Trinomial coefficients:

$$G_{k;m,n} = \frac{k!}{\left(\frac{k+2m+n}{3}\right)!\left(\frac{k-m+n}{3}\right)!\left(\frac{k-m-2n}{3}\right)!}$$

Solution:

$$N_{m,n}^{(k)} = G_{k:m,n} + G_{k:m+3,n} + G_{k:m,n+3}$$
$$- G_{k:m+2,n-1} - G_{k:m-1,n+2} - G_{k:m+2,n+2}$$

Again, use Stirling's formula…

$$D_2^{m,n} = \frac{(m^2 + mn + n^2)}{3} + (m+n)$$ Quadratic Casimir

$$D_3^{m,n} = \frac{1}{18}(m+2n+3)(n+2m+3)(m-n)$$ Cubic Casimir

$$N_{m,n}^{(k)} \approx \frac{27mn(m+n)}{k^3} \frac{3^{3/2+k}}{2k\pi} \exp\left(-3\,D_2^{m,n}\right)\left(1 + 3\,D_3^{m,n}/k^2\right)$$

Probability:

$$\mathcal{N} \int dm\,dn\,d_{mn}\,N_{m,n}^{(k)} = 1$$

Dimension of representation

$$d_{mn} = \frac{1}{2}(m+1)(n+1)(m+n+2) \approx \frac{mn(m+n)}{2}$$

Note:

$$d_{mn}\,N_{m,n}^{(k)} \propto m^2 n^2\,(m+n)^2$$

Prove:

$$1 \approx \left(\frac{N_c}{k\pi}\right)^4 \int d^8\mathbf{Q}\, e^{-N_c\mathbf{Q}^2/k+3\,D_3(\mathbf{Q})/k^2}$$

Classical color charge:

$$\mathbf{Q} = (Q_1, Q_2, \cdots, Q_8)\ ;\ |\mathbf{Q}| = \sqrt{Q^a Q^a} = \sqrt{D_2^{m,n}}$$

$$D_3(\mathbf{Q}) = d_{abc}\, Q^a Q^b Q^c$$

Proof:

For any SU(3) representation,

$$d^8Q = d\phi_1\, d\phi_2\, d\phi_3\, d\pi_1\, d\pi_2\, d\pi_3\, dm\, dn \left(m\, n\, (m+n)\, \frac{\sqrt{3}}{48}\right)$$

Canonically conjugate "Darboux" variables

Canonical phase space volume of SU(3):

Johnson;
Marinov;
Alexeev,Fadeev,
Shatashvilli

$$\int \prod_{i=1}^{3} d\phi_i d\pi_i = \frac{(2\pi)^3}{2}\, m\, n\, (m+n)$$

Hence, $$\int d^8Q = \frac{(2\pi)^3}{32\sqrt{3}} \int dm\, dn\ \left(m^2\, n^2\, (m+n)^2\right)$$

Measure of probability integral has identical argument in m & n to RHS- hence can express in terms of LHS.

End of Proof.

Path Integrals:

Generates Odderon excitations!

As in SU(2), with $l^a \longrightarrow Q^a$

MV Path integral measure for SU(3):

$$1 \approx \int [d\rho] \exp\left(- \int d^2 x_\perp \left[\frac{\rho^a \rho^a}{2\mu_A^2} - \frac{d_{abc}\rho^a \rho^b \rho^c}{\kappa_A} \right] \right)$$

$$\boxed{\kappa_A = \frac{g^3 A^2 N_c}{\pi^2 R^4}}$$

Path integral approach reproduces diagrammatic computations of dipole and baryon C= -1 operators

To summarize…

- Representations of order $\sqrt{k}$ dominate for k >> 1

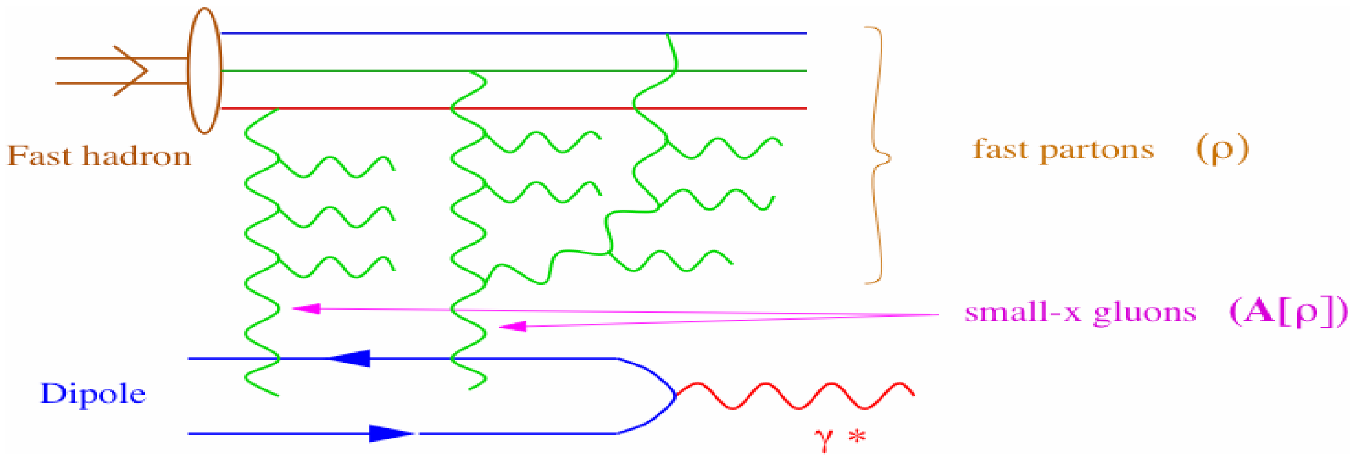
- These representations are classical - can be represented by an SU(3) classical path integral

- Can repeat analysis for gluon and quark-anti-quark pairs - only quadratic Casimir contributes…

- Can add glue representations to quarks-result as for valence quarks - with larger weight.

On to the Odderon...

DIS:



$$\sigma^{\gamma^* p}(x, Q^2) = \int_0^1 dz \int d^2r |\psi(z, r; Q^2)|^2 \sigma_{\rm dipole}(x, r)$$

$$\text{where } \sigma_{\rm dipole}(x, r) = 2 \int d^2b\,(1 - S(x, r, b))$$

IN CGC:

$$S(x, r, b) = \frac{1}{N_c} < {\rm Tr} V^\dagger(x) V(y) >_Y \equiv 1 - \mathcal{N}_Y(r, b)$$

$$V^\dagger(x) = \mathcal{P} \exp\left(ig \int dx^- \alpha_a(x^-, x) T^a \right)$$

Hatta et al.

Dipole Odderon operator:

$$\mathcal{O}(\mathbf{x},\mathbf{y}) = \frac{1}{2iN_c}\mathrm{Tr}\,\left(V_x^\dagger V_y - V_y^\dagger V_x\right)$$

To lowest order

$$\mathcal{O}(\mathrm{x},\mathrm{y}) \approx \frac{-\mathrm{g}^3}{24\mathrm{N_c}}\mathrm{d}^{\mathrm{abc}}\left(\alpha_{\mathrm{x}}^{\mathrm{a}} - \alpha_{\mathrm{y}}^{\mathrm{a}}\right)\left(\alpha_{\mathrm{x}}^{\mathrm{b}} - \alpha_{\mathrm{y}}^{\mathrm{b}}\right)\left(\alpha_{\mathrm{x}}^{\mathrm{c}} - \alpha_{\mathrm{y}}^{\mathrm{c}}\right)$$

$$\alpha_x^a = \frac{1}{4\pi}\int d^2\mathbf{z}\,\ln\left(\frac{1}{(\mathbf{x}-\mathbf{z})^2\Lambda^2}\right)\,\rho^a(\mathbf{z})$$

Can compute < O > with "SU(3) measure"

To lowest order,

$$\langle \mathcal{O}(\mathbf{x},\mathbf{y})\rangle = \alpha_S^3 \, \frac{(N_c^2-4)(N_c^2-1)}{4\,\pi\, r_0^2\, N_c^3} A^{1/3} \int d^2\mathbf{u} \ln^3 \frac{|\mathbf{x}-\mathbf{u}|}{|\mathbf{y}-\mathbf{u}|}$$

To all orders (in the parton density)

$$\langle \mathcal{O}\rangle = \langle \mathcal{O}\rangle_{\mathrm{l.o.}} \times \mathrm{Tr}\,\exp\left(-\frac{g^2\,\mu_A^2\,(N_c^2-1)}{16\pi^2\,N_c}\int d^2\mathbf{z}\,\ln^2\left(\frac{(\mathbf{y}-\mathbf{z})^2}{(\mathbf{x}-\mathbf{z})^2}\right)\right) .$$

$$\propto \alpha_S^3 A^{1/3}\, e^{-C\,\alpha_S^2 A^{1/3}}$$

Kovchegov, Szymanowski, Wallon

Dosch, Ewerz, Schatz

3-quark Baryon state scattering off CGC

$$\mathcal{B}(\mathbf{x},\mathbf{y},\mathbf{z}) = \frac{1}{3!\,2i}\left(\epsilon^{ijk}\,\epsilon^{lmn}\,V^{\dagger}_{il}(\mathbf{x})\,V^{\dagger}_{jm}(\mathbf{y})\,V^{\dagger}_{kn}(\mathbf{z}) - h.c.\right)$$

To lowest order,

$$\approx \frac{g^3}{144}\,d^{abc}\left\{\left(\alpha^a_x + \alpha^a_y - 2\alpha^a_z\right)\left(\alpha^b_y + \alpha^b_z - 2\alpha^b_x\right)\left(\alpha^c_z + \alpha^c_x - 2\alpha^c_y\right)\right\}$$

$$\langle\mathcal{B}\rangle = \frac{\alpha_S^3}{24\,\pi\,r_0^2}\,\frac{(N_c^2-4)(N_c^2-1)}{N_c^2}\,A^{1/3}\;\times\;\mathrm{logs}.$$

All order result feasible, but tedious…

Conclusions

- The ground state of a large nucleus contains configurations that generate Odderon excitations - these can be traced to the random walk of valence partons in color space.

- These results represent a “rigorous” proof in the large A asymptotics of QCD at high energies

- Dipole Odderon and Baryon Odderon operators are computed

- Phenomenological consequences - to be investigated further

Why are higher dim. Reps. classical?

Invariance grou $N_c \to \infty$ or higher dim $=> \hbar \to 0$

Yaffe

For a system prepared in this state, uncertainity in mom & position vanishes in this limit -> Coherent States

$$\Delta D_2 =< \psi | \sum_a Q_a^2 | \psi > - \sum_a < \psi | Q_a | \psi >^2$$

$$\frac{\Delta D_2}{D_2} = \frac{N_c}{N_c + k} \quad for\ k >> N_c$$

$$k \to \infty => \hbar \to 0$$

Gitman, Shelepin

SU(N)

Can follow same recursion procedure…

Quadratic Casimir dominates:
successive N-2 Casimirs parametrically suppressed by

$$\left(\frac{1}{\sqrt{k}}\right)^{N_c-2}$$