IQI 04, Seminar 1

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- Seminar overview.
- Classical information units and processing.
- Information science: The big picture.
- Qubit state space.
- Simple qubit gates.
- Black box problems.

QUANTUM INFORMATION PROCESSING, SCIENCE OF - The theoretical, experimental and technological areas covering the use of quantum mechanics for communication and computation. Kluwer Enc. Math. III

E. "Manny" Knill: knill@boulder.nist.gov

Seminar overview

Goal: To learn the basic concepts and tools of quantum information, appreciate its power and limitations, and understand the issues involved in realizing it.

Prerequsites: Linear algebra, polynomials, binary logic, probability.

Structure: 15 seminars, each consisting of a 50min lecture, followed by discussions and/or problem solving.

Grading: Based on participation–see hand-out. Required meeting with me in the second half of the semester.

Assignments: Problems to be handed out. Errors in solutions handed in have no effect on grade.

Reading: References provided in handout, limited number of hard copies of LAScience issue.

Office hours: CU: Wednesdays after class, 1pm-3pm, S315 or by appointment. NIST: Thursdays after class, 2:15pm-3:15pm, Bldg 1, Rm 4049, or drop in any time I am there.

Sign-up: Please provide your email, if possible. Let me know if it is difficult for you to use PDF and PS attachments.

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- The classical information unit is the *bit*.
  The bit is a *system* with state space $\{0, 1\}$.

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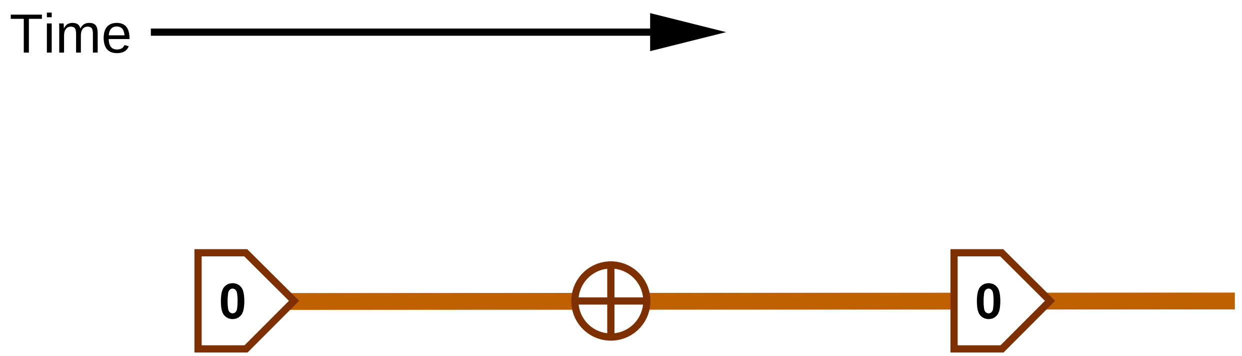
- Multiple units’ state space: By concatenation of states.
  - Two bits’ state space: $\{00, 01, 10, 11\}$.
  - How many states do n bits have? Answer: 2^n.

Classical Gate Networks

- A one-bit network.



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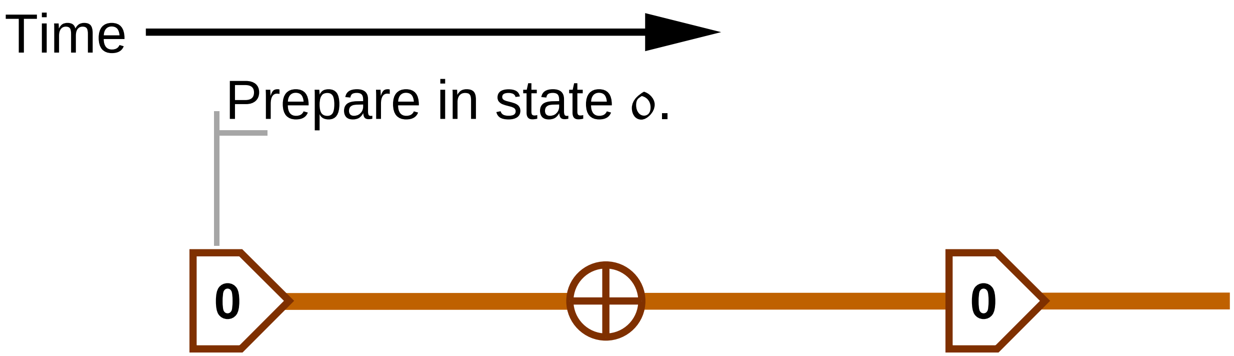

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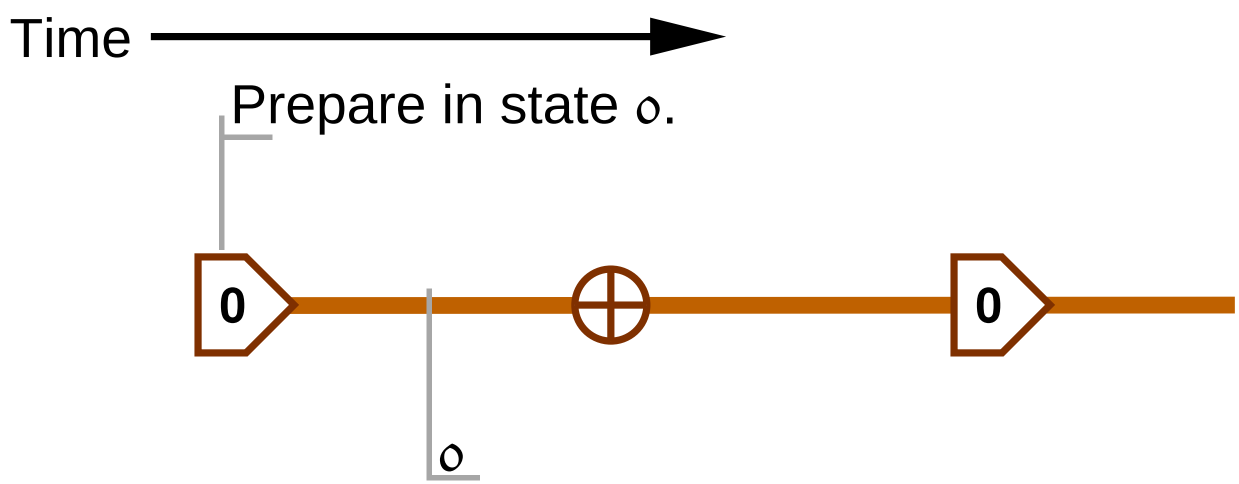

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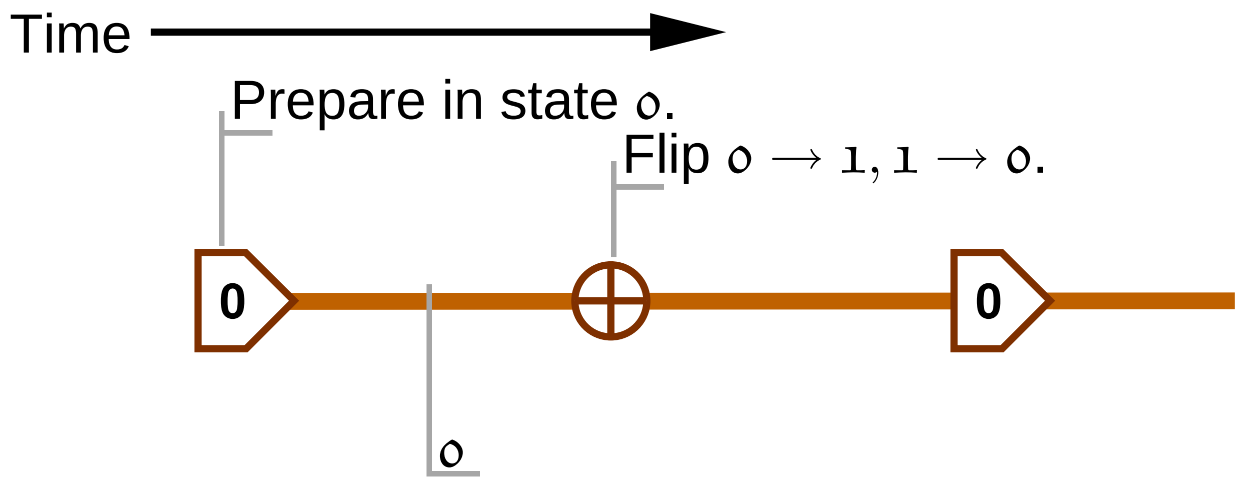

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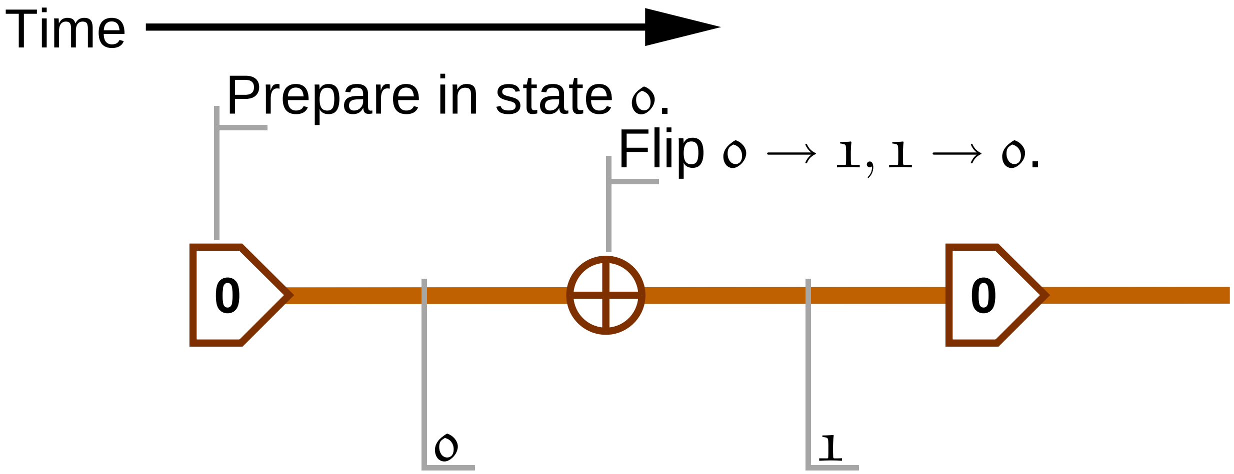

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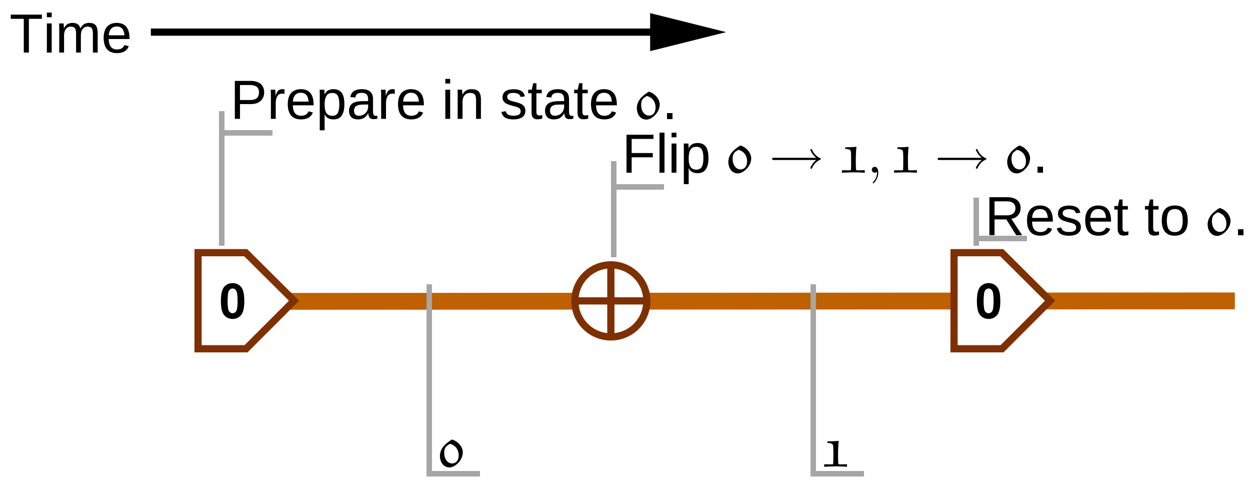

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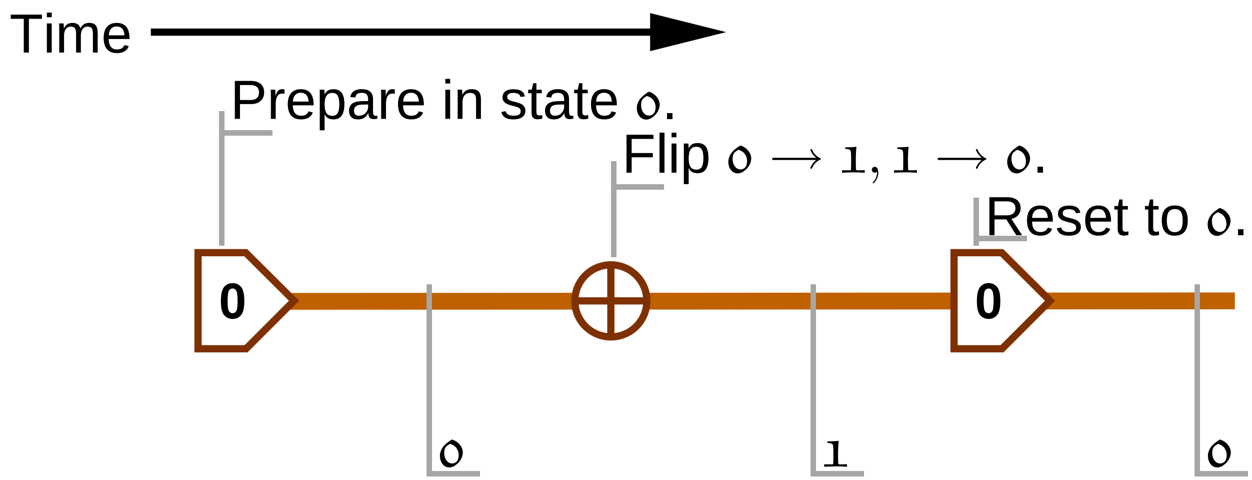

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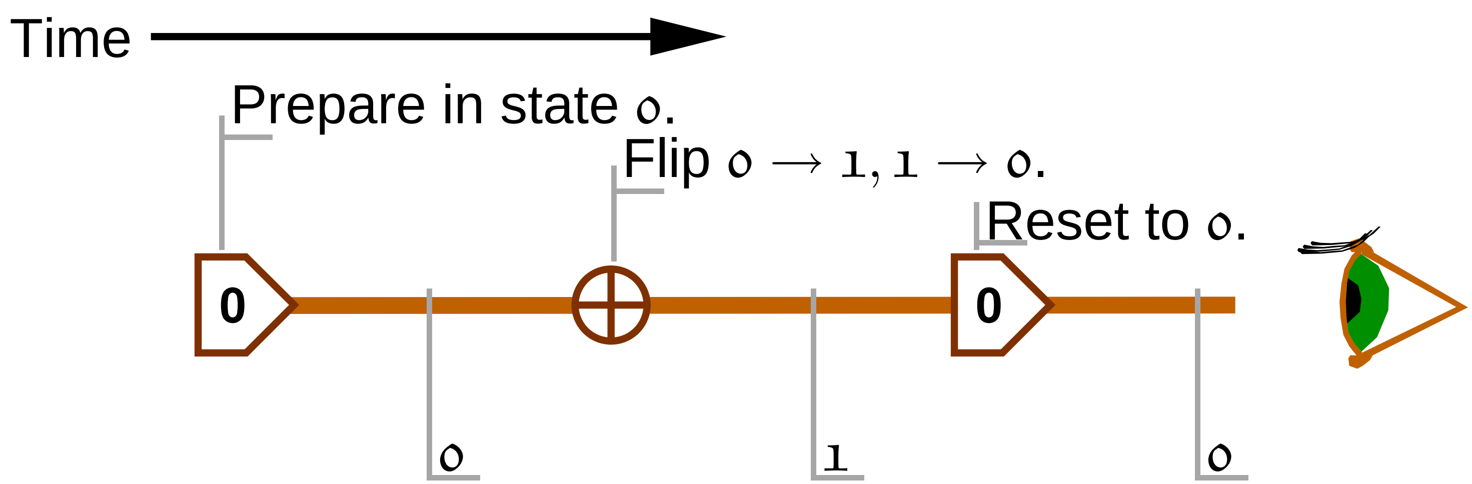

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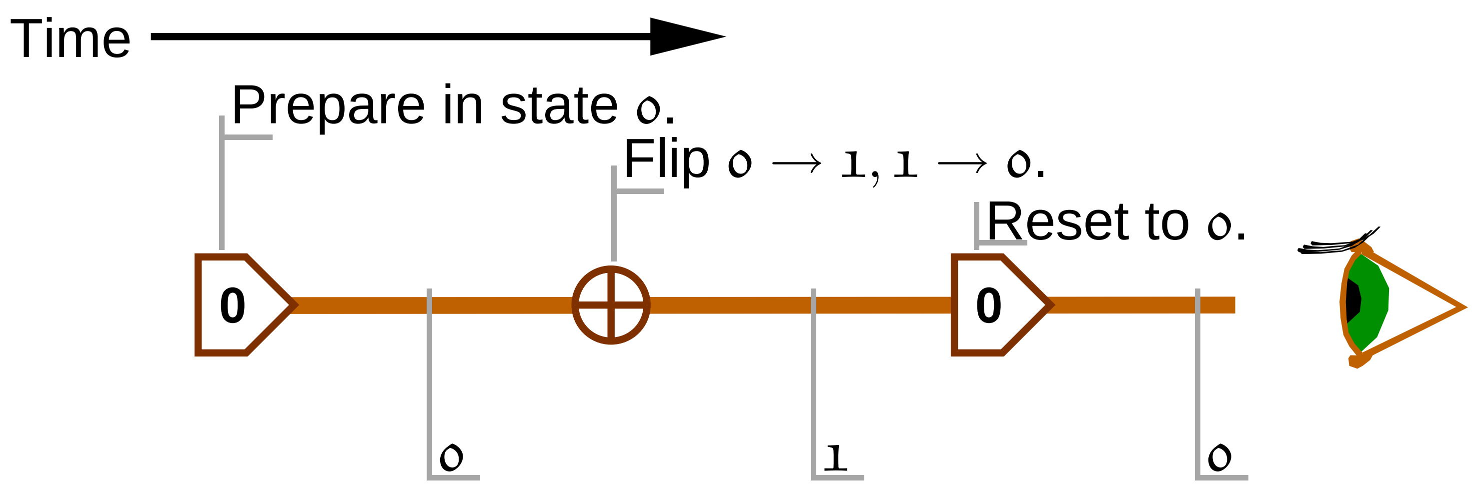

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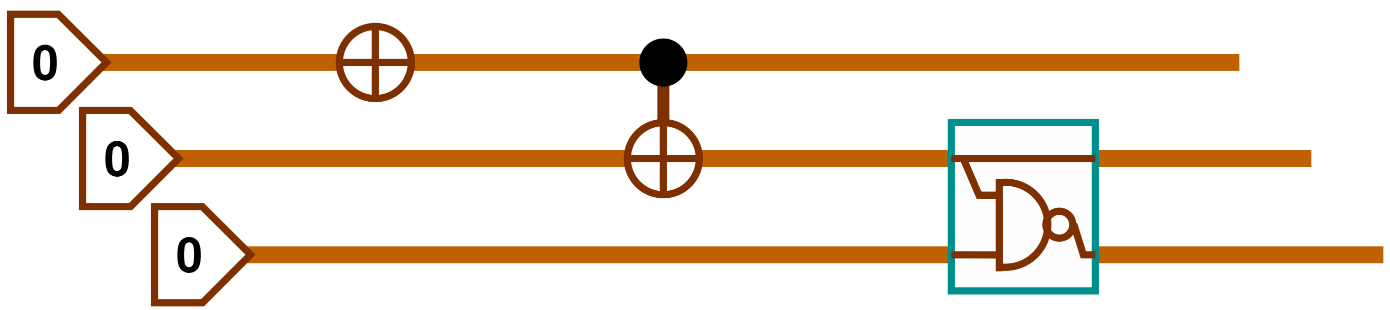

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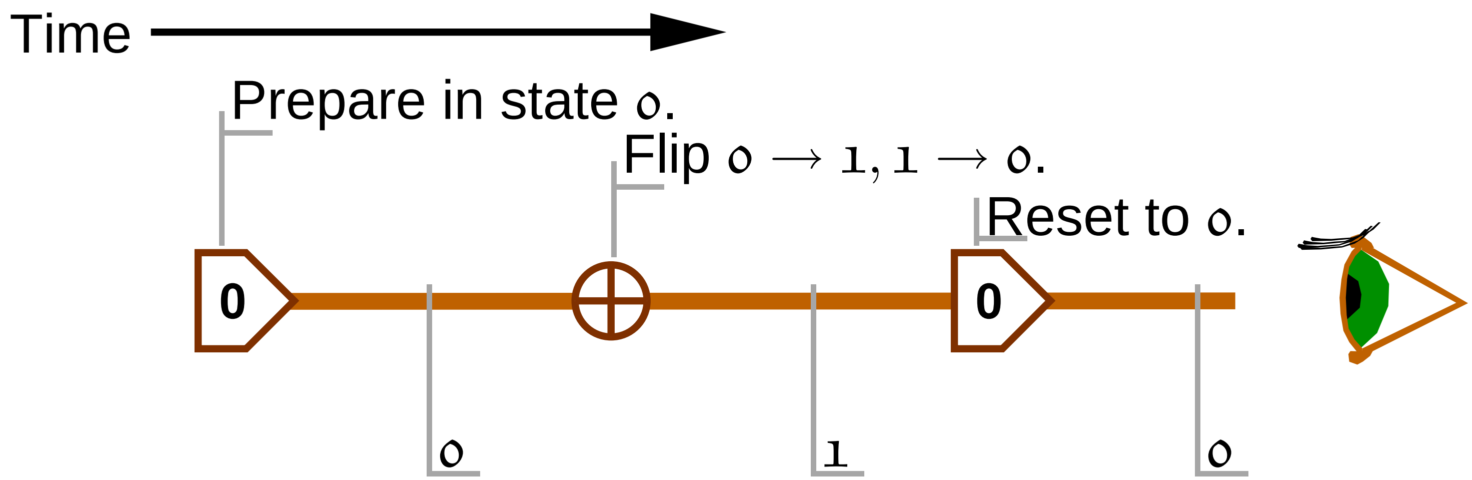

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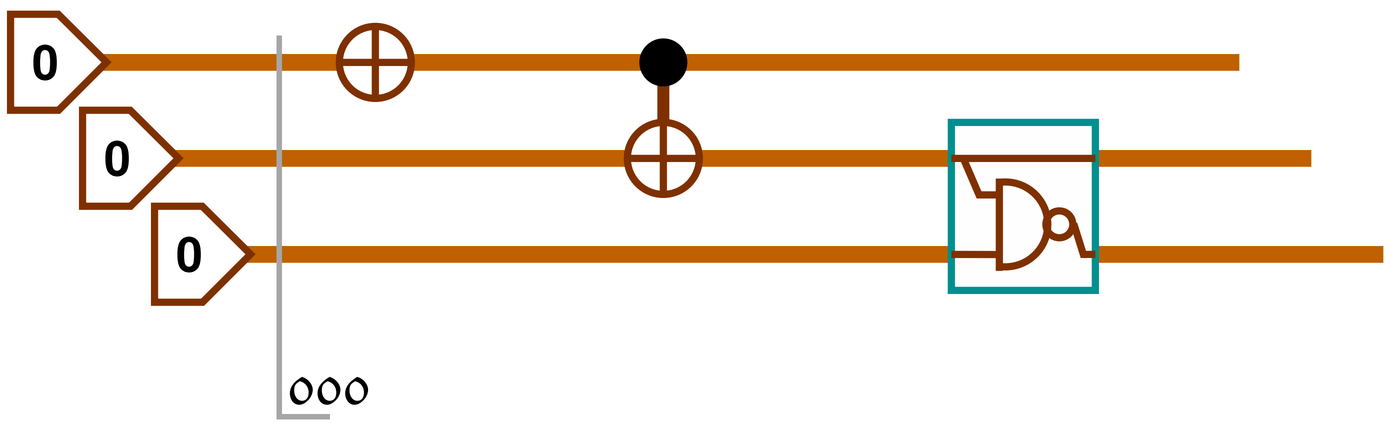

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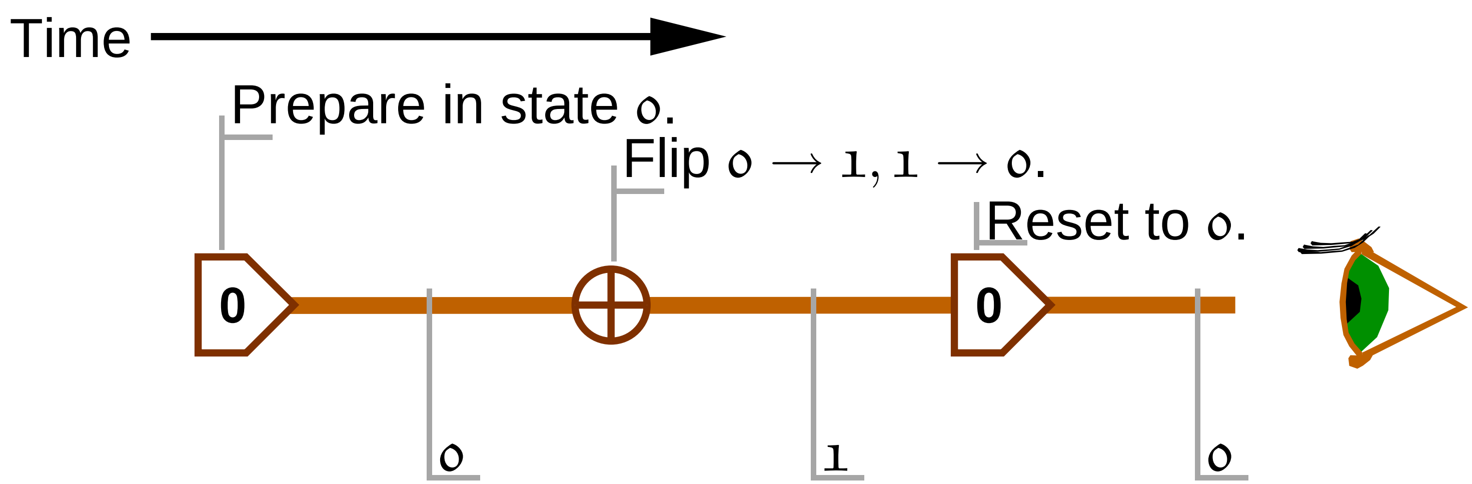

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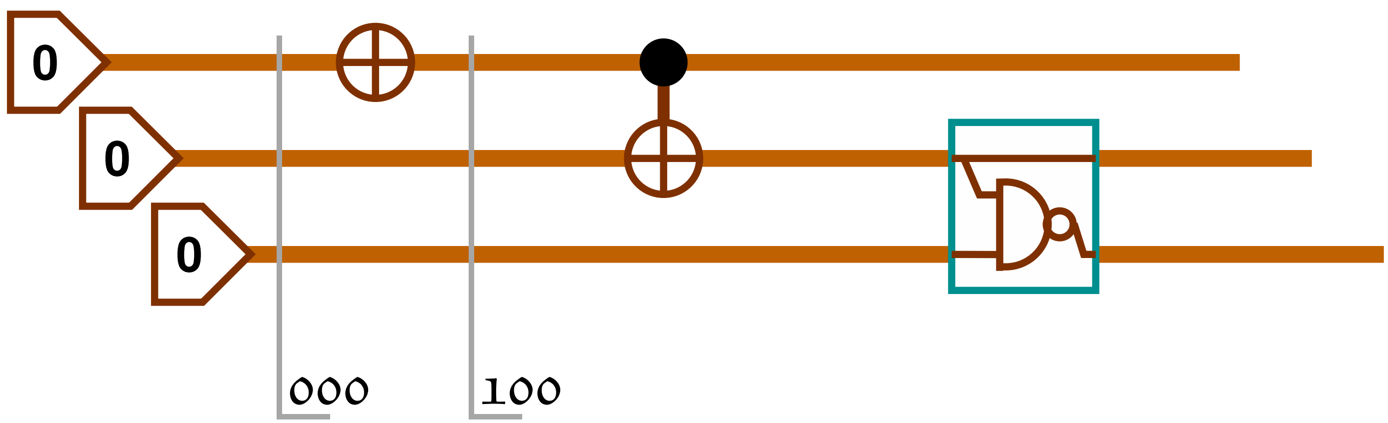

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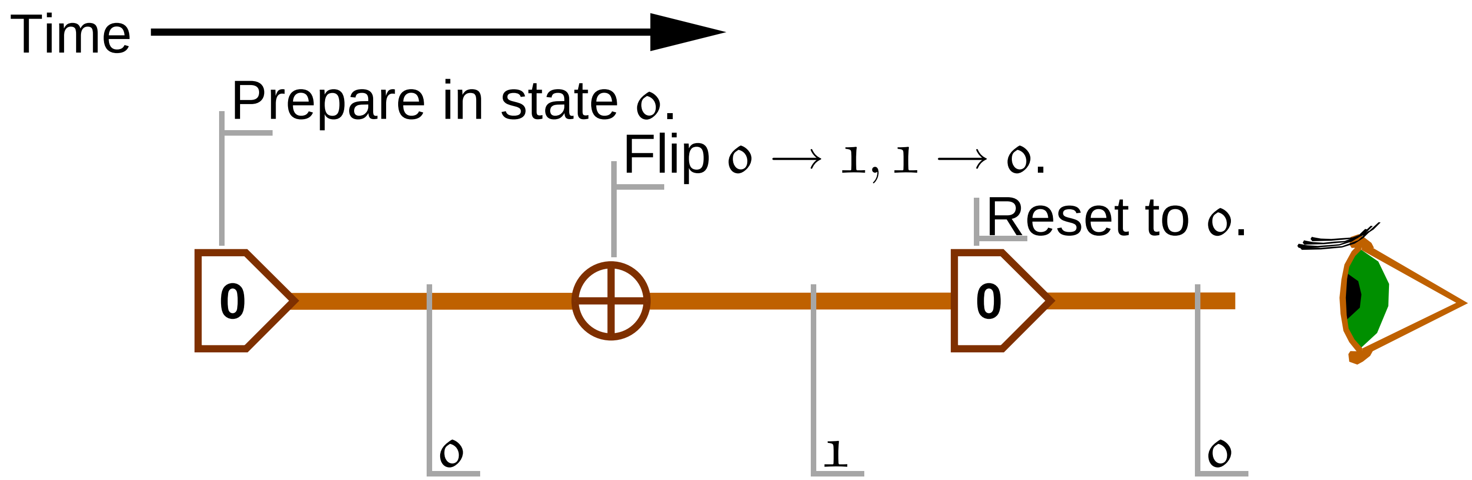

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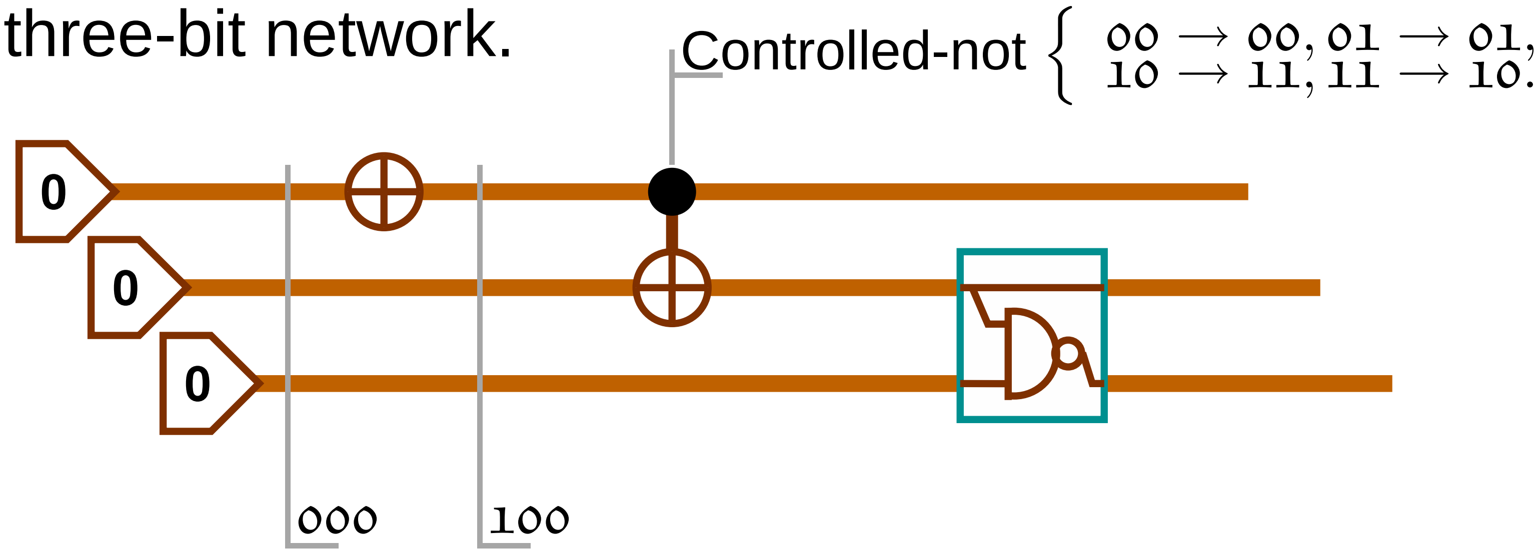

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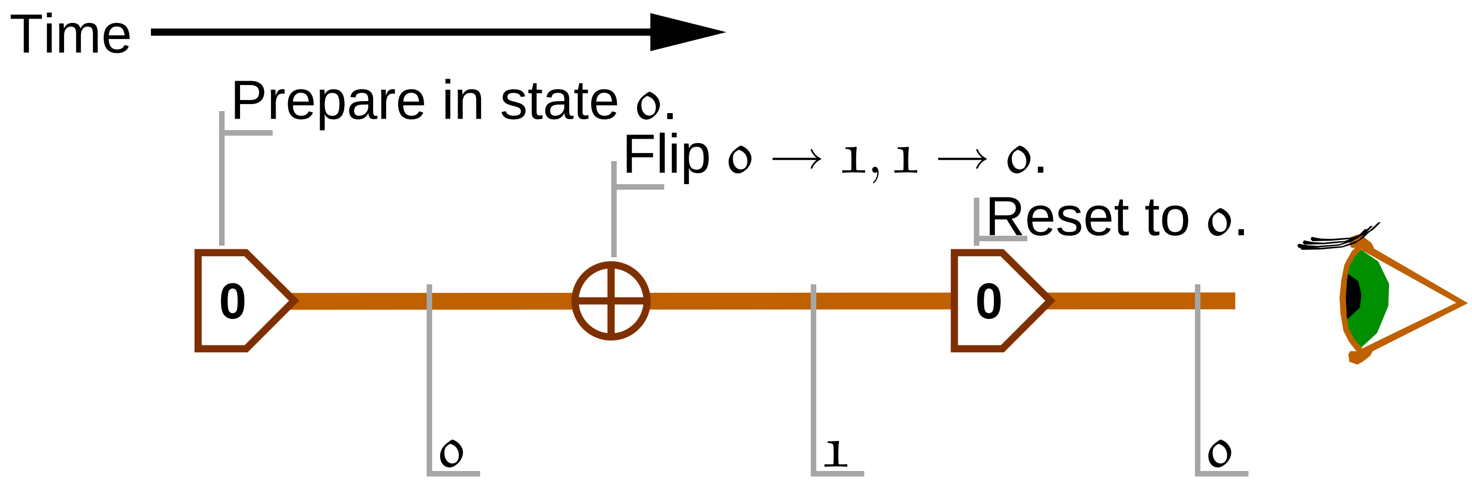

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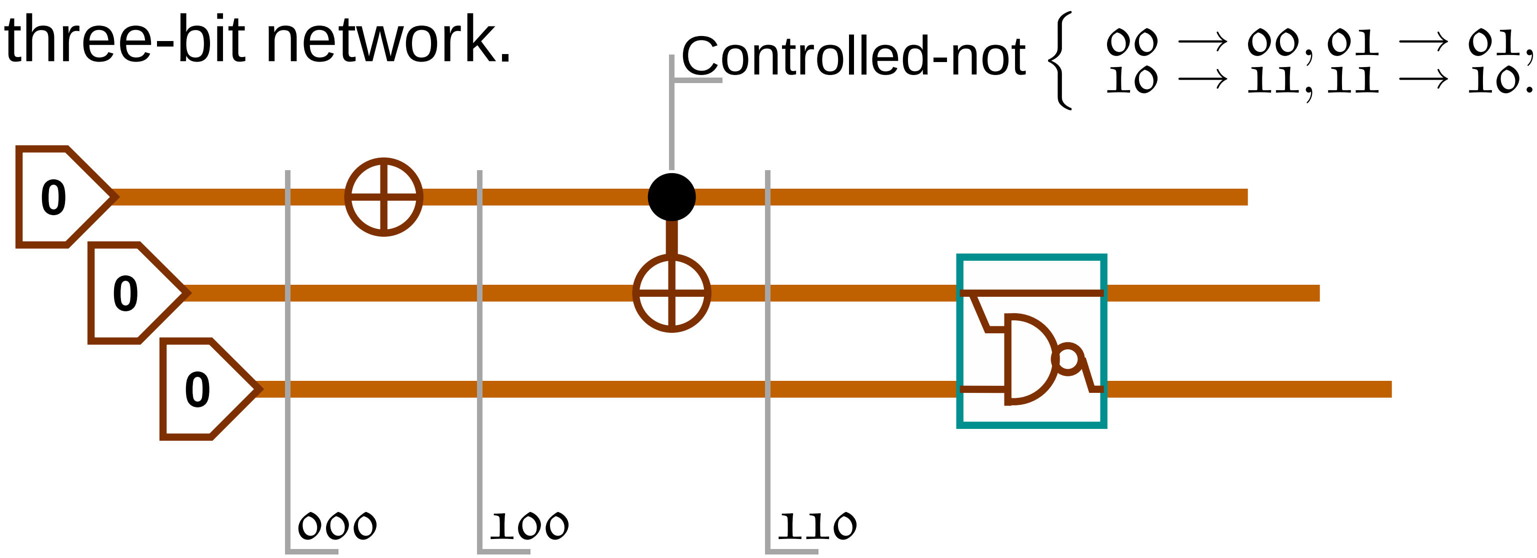

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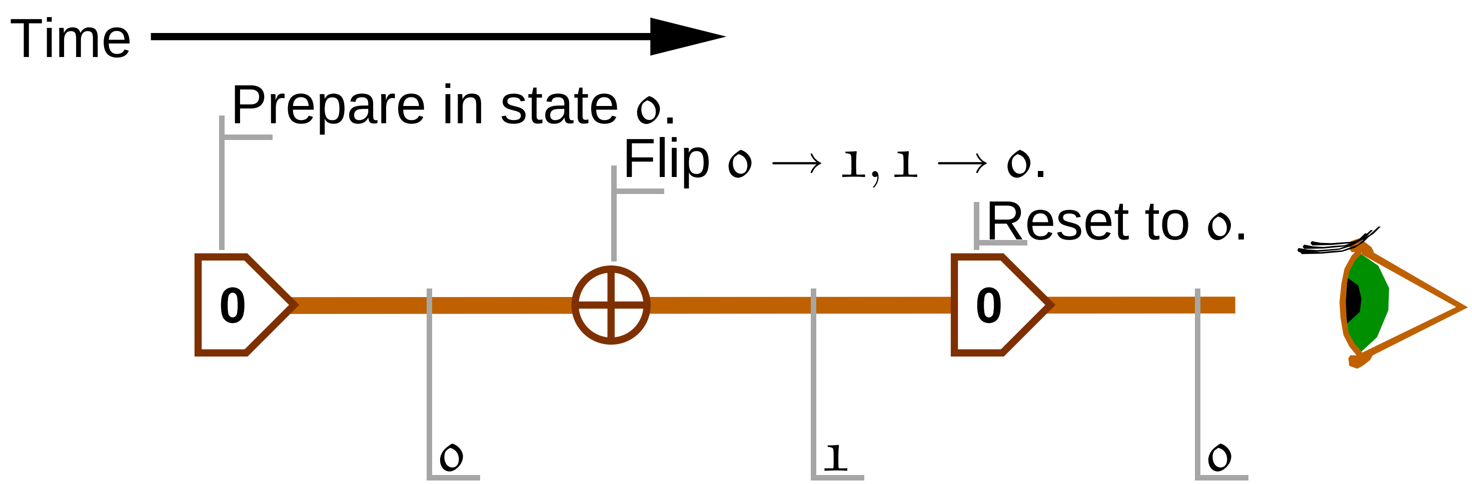

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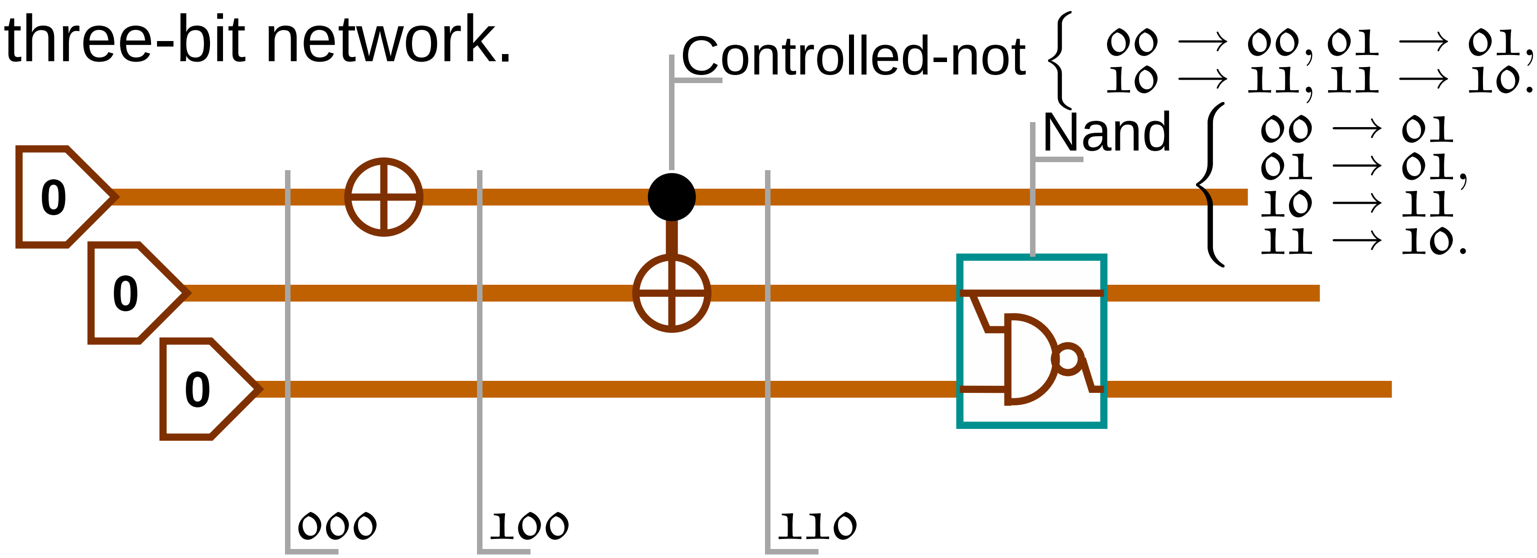

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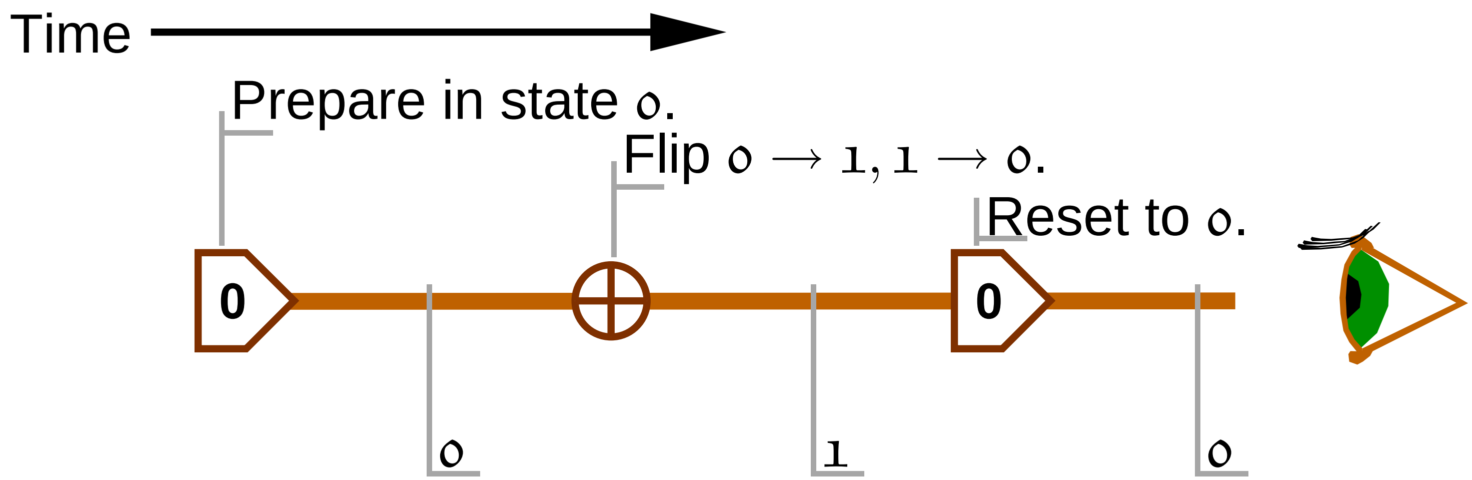

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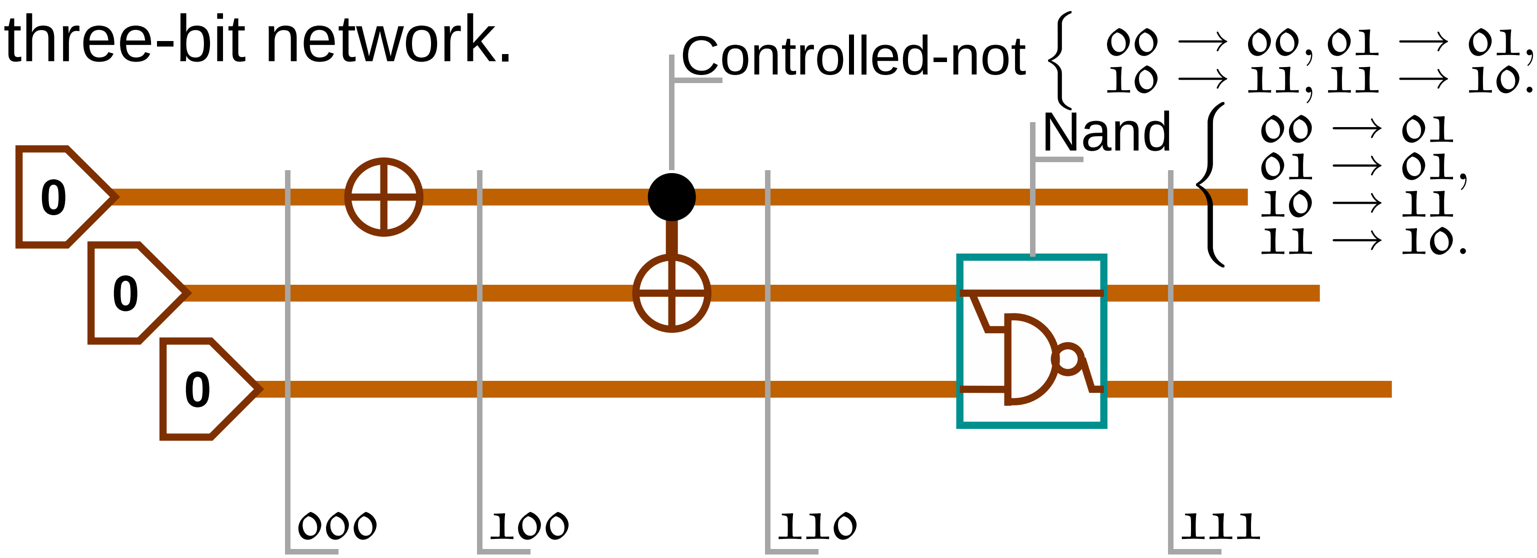

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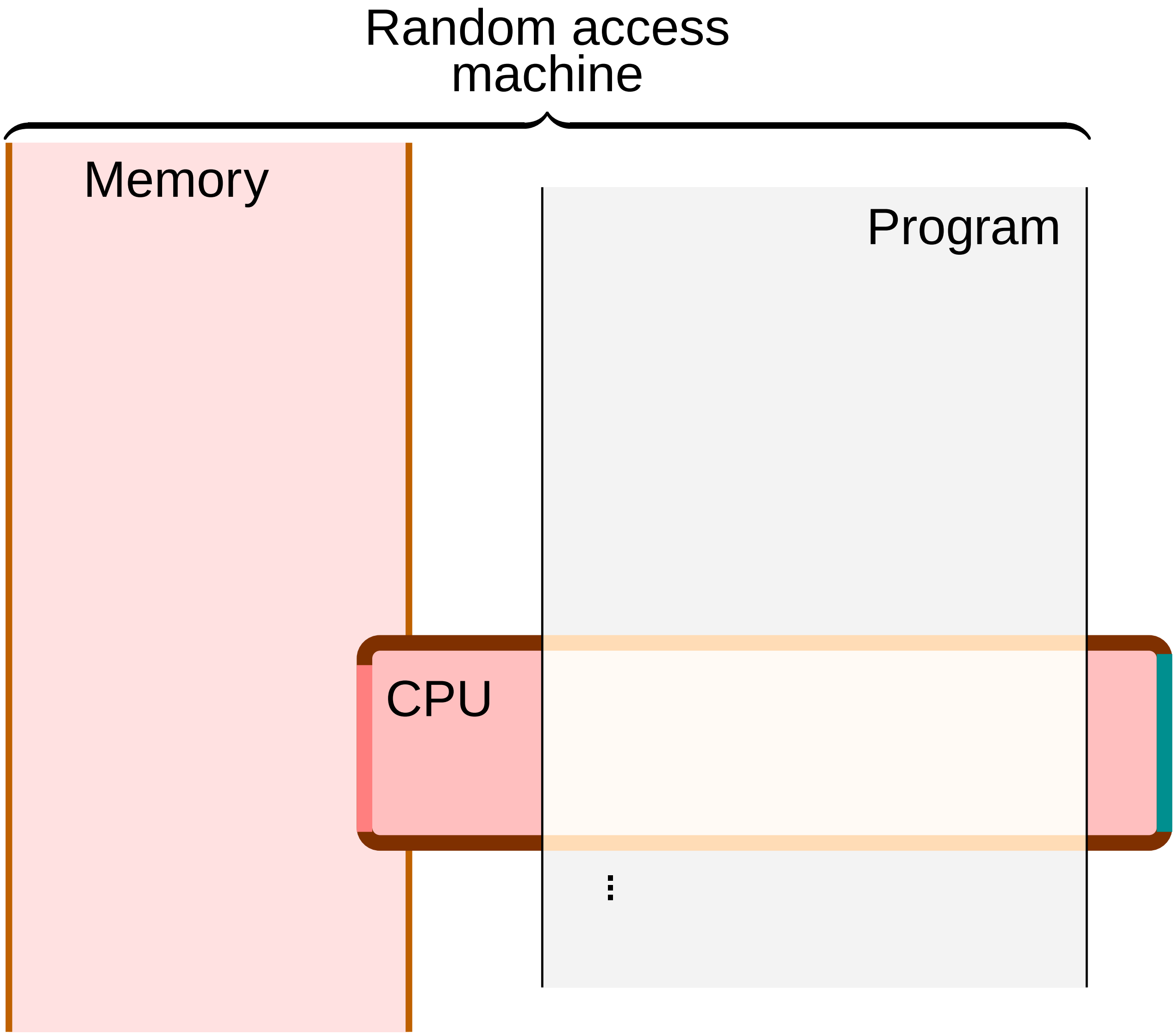

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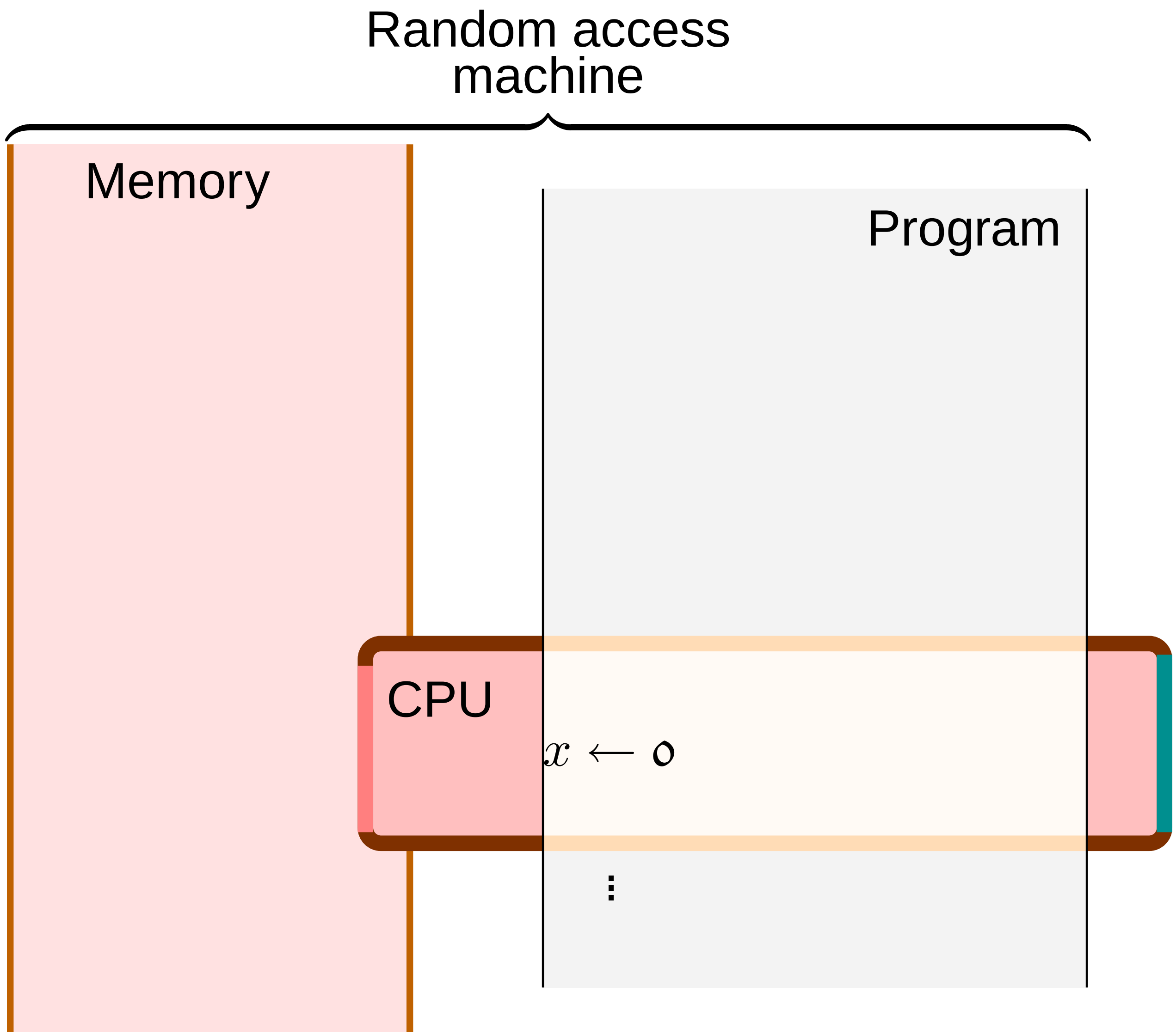

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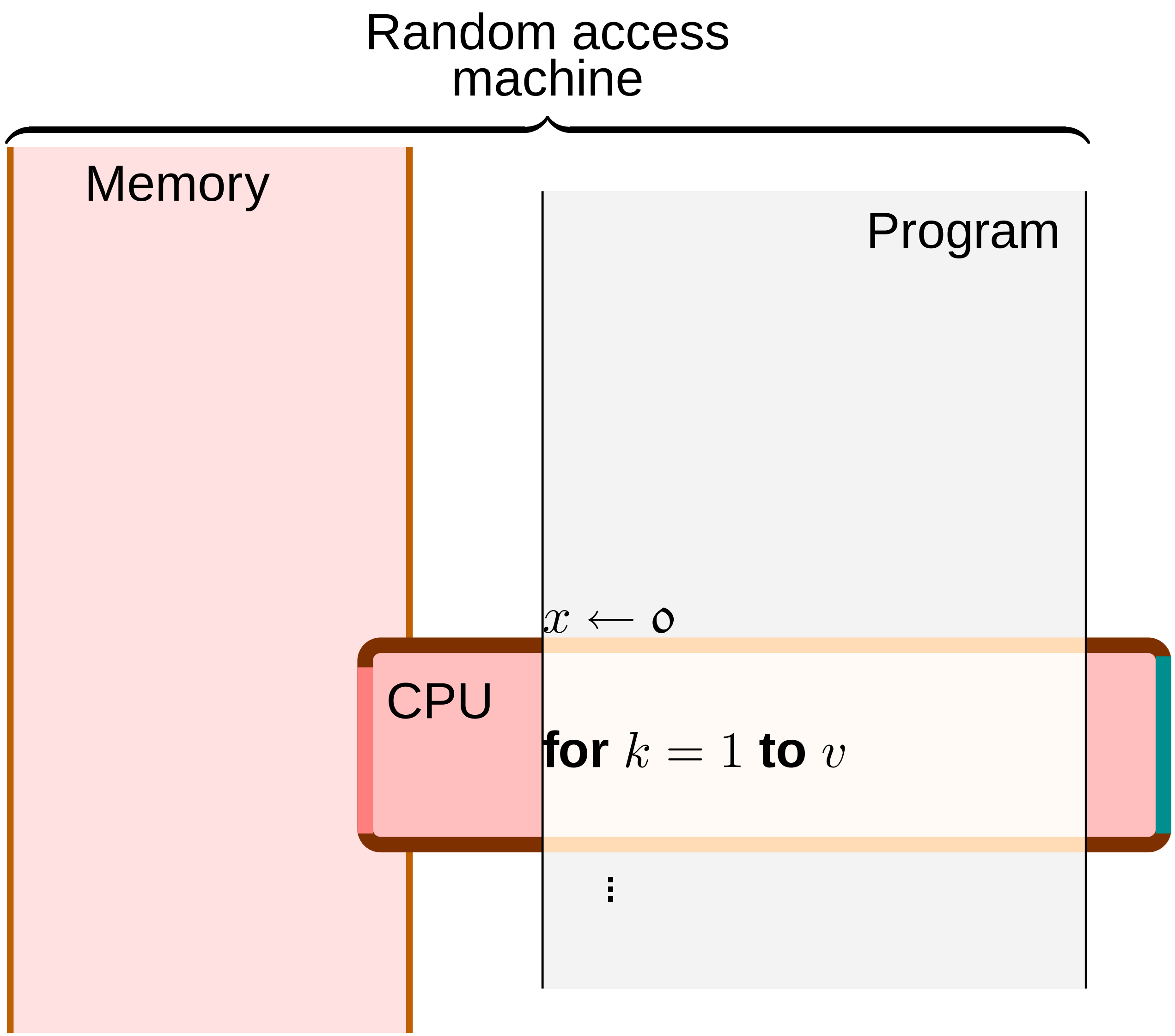

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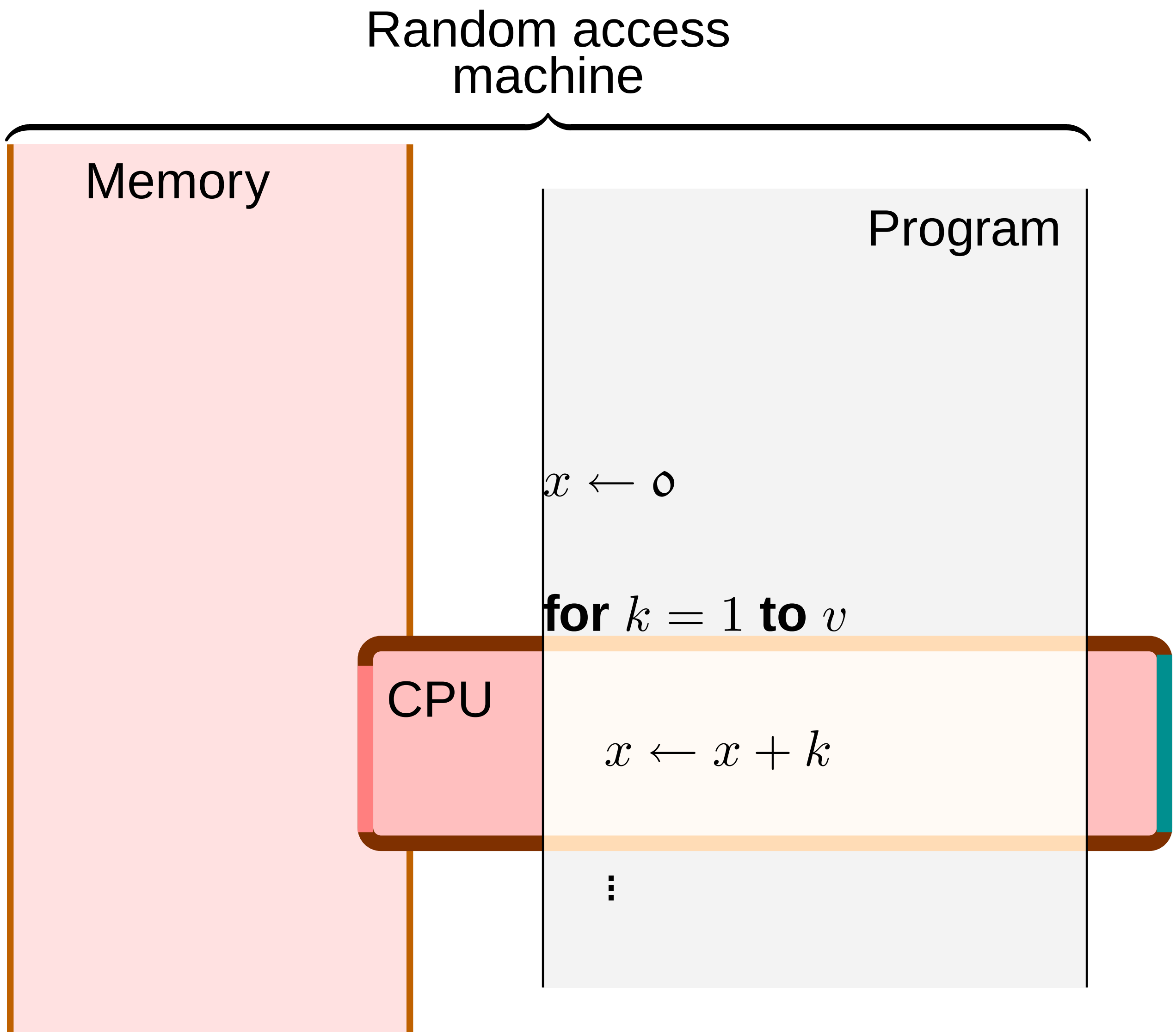

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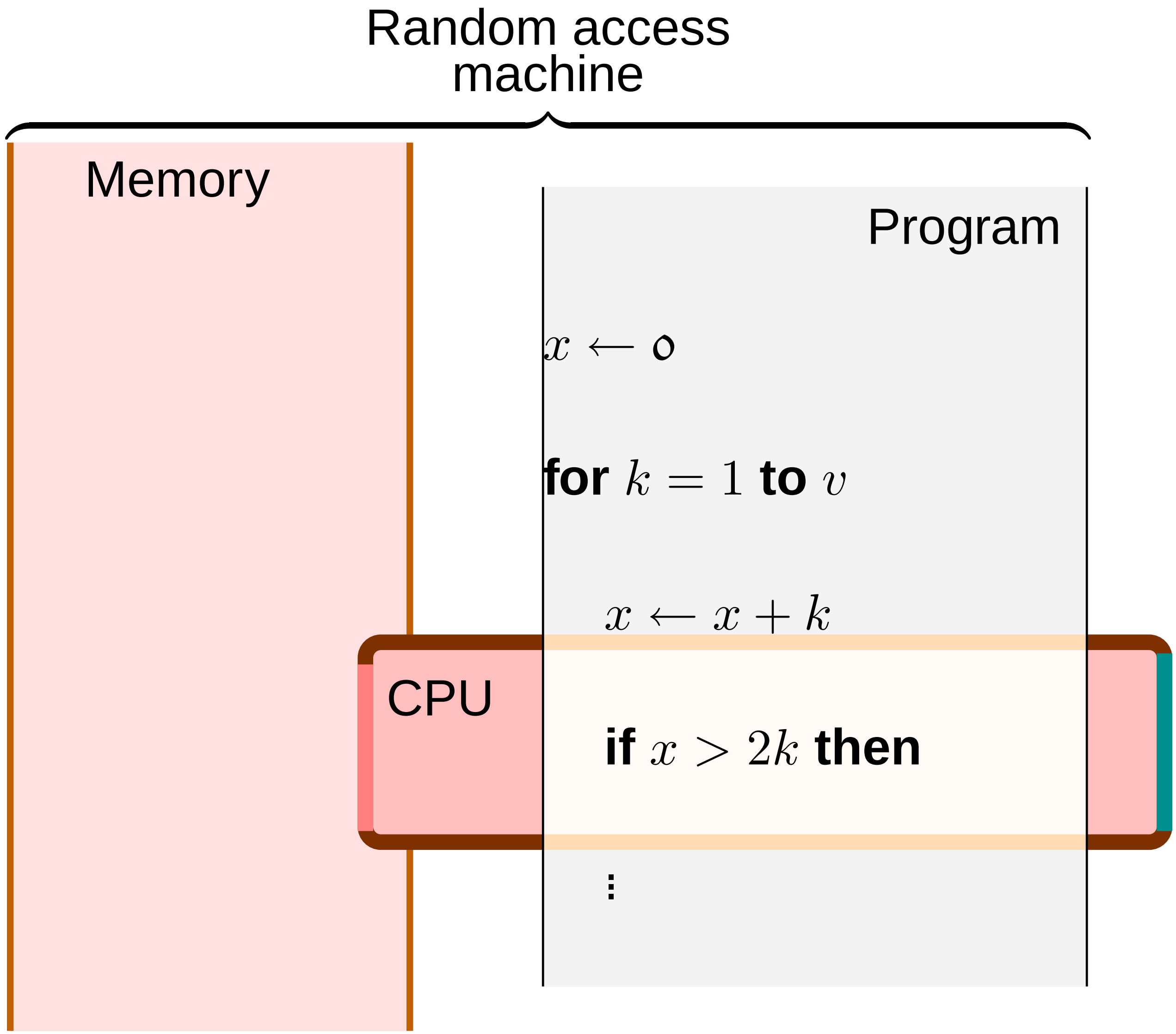

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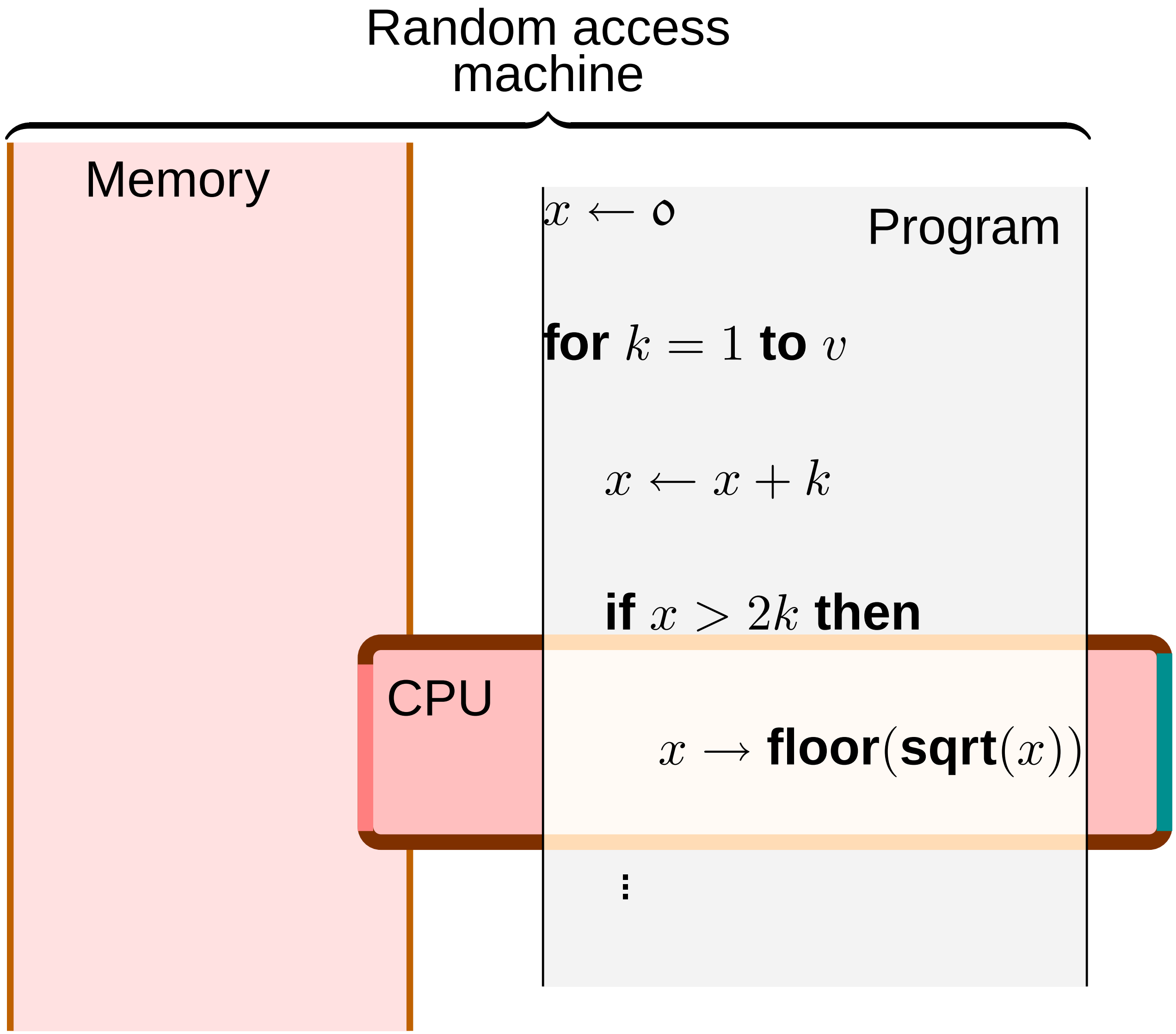

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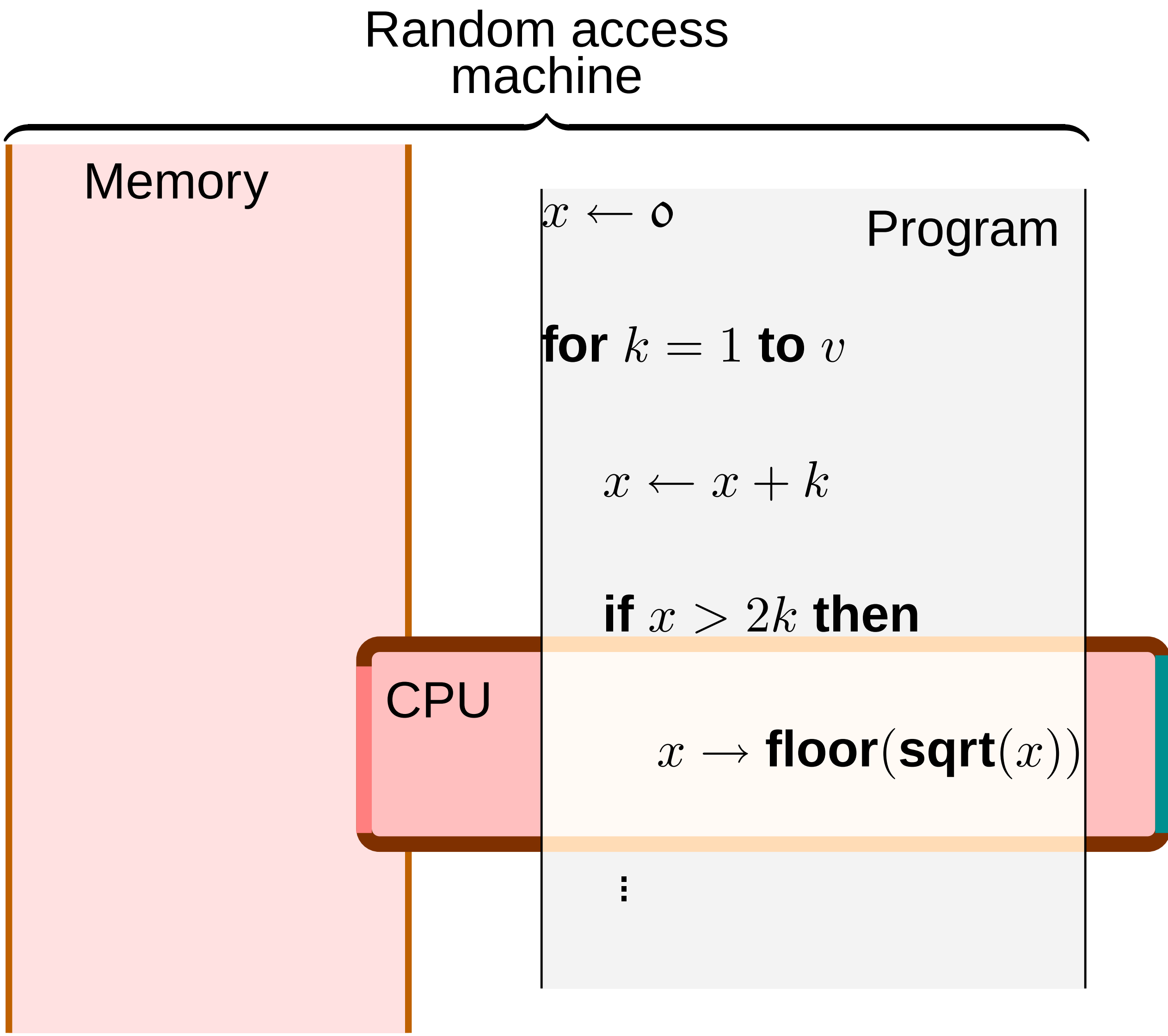


Capabilities added:

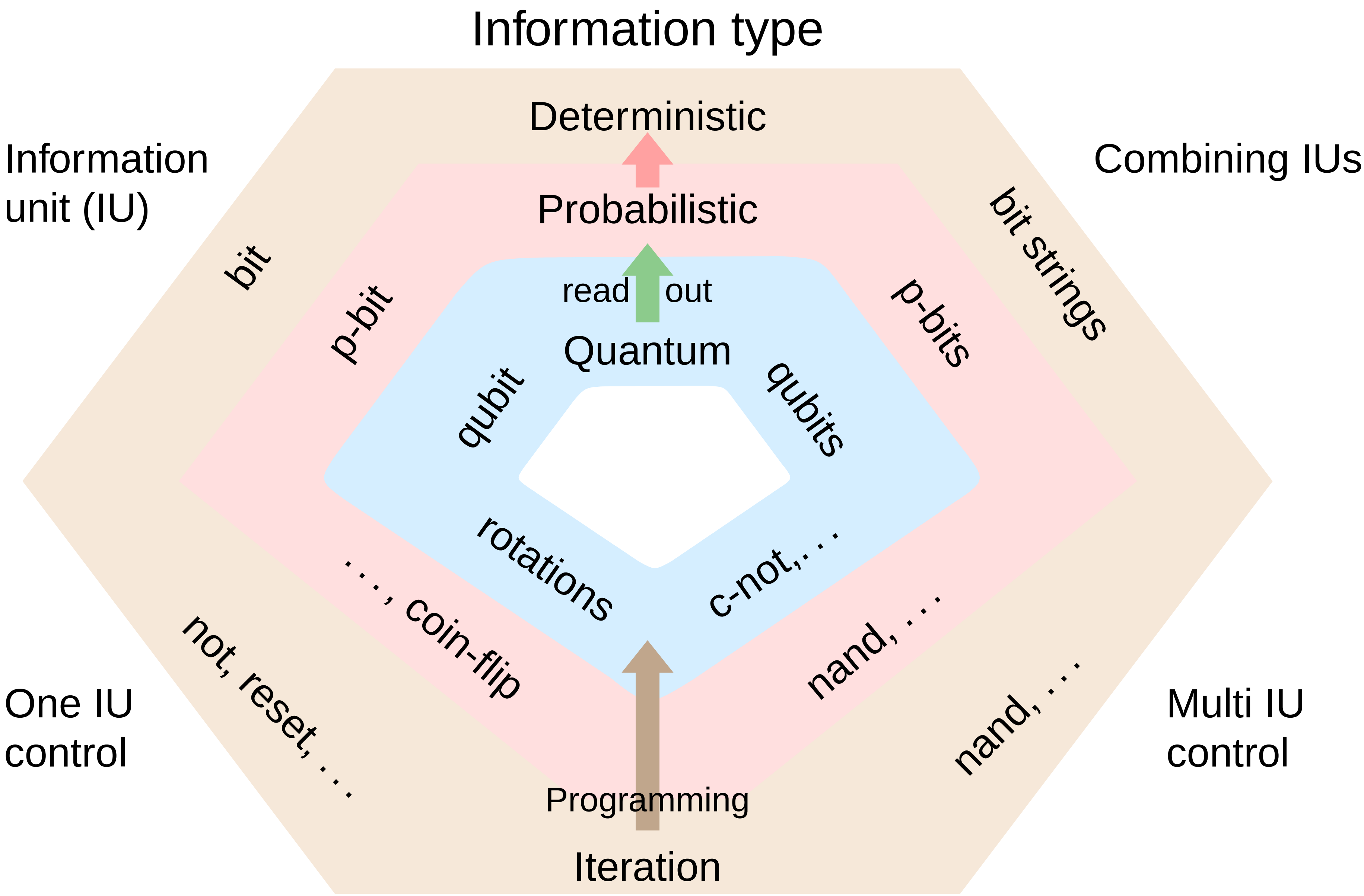
⋮

- Loops, iteration, recursion.

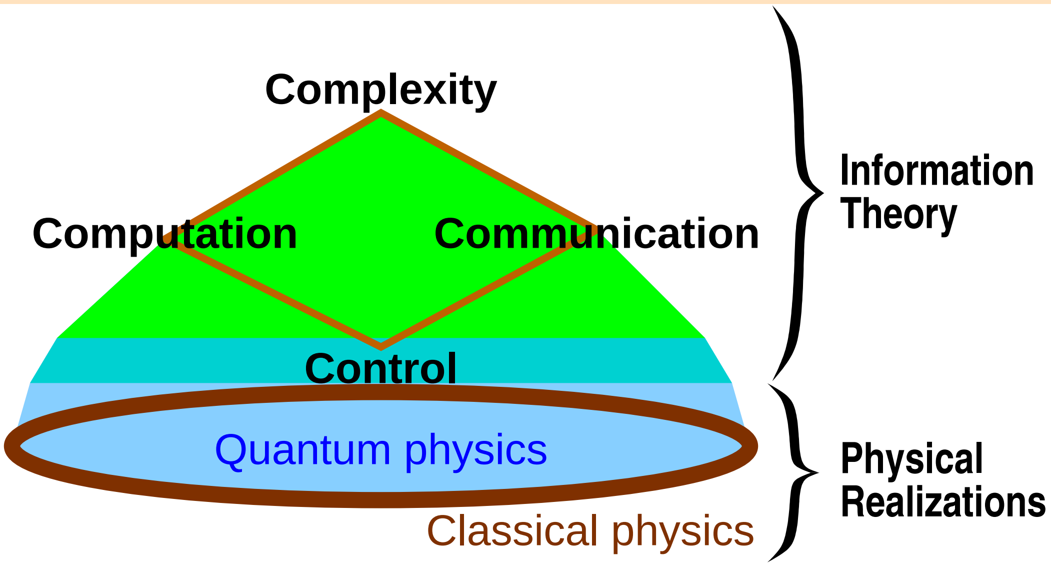
Universality:

- Anything “effective” can be computed by a RAM.

Guide to Information Processing



Quantum Information Science



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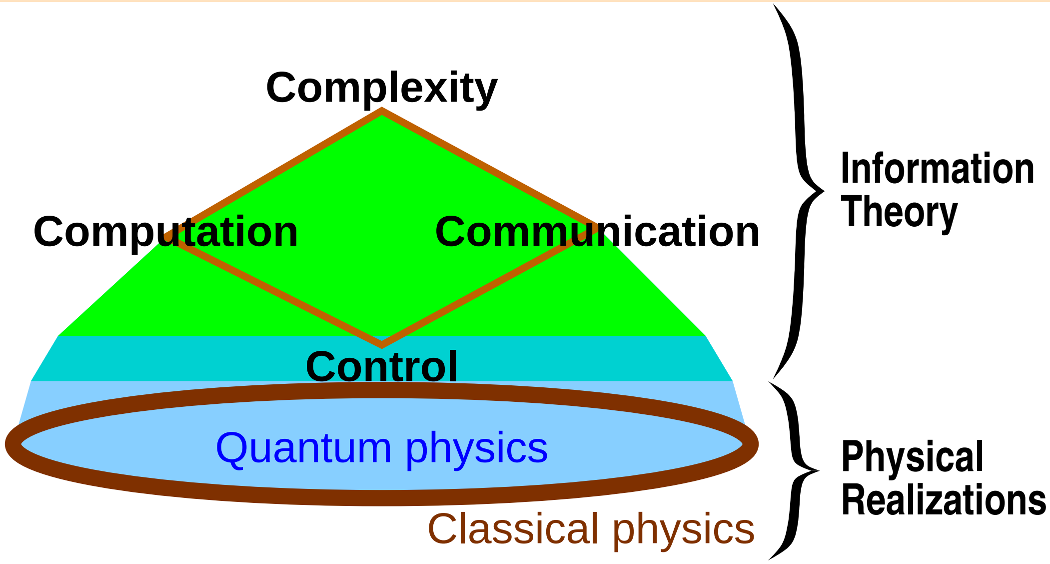

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  - Quantum physics simulation.
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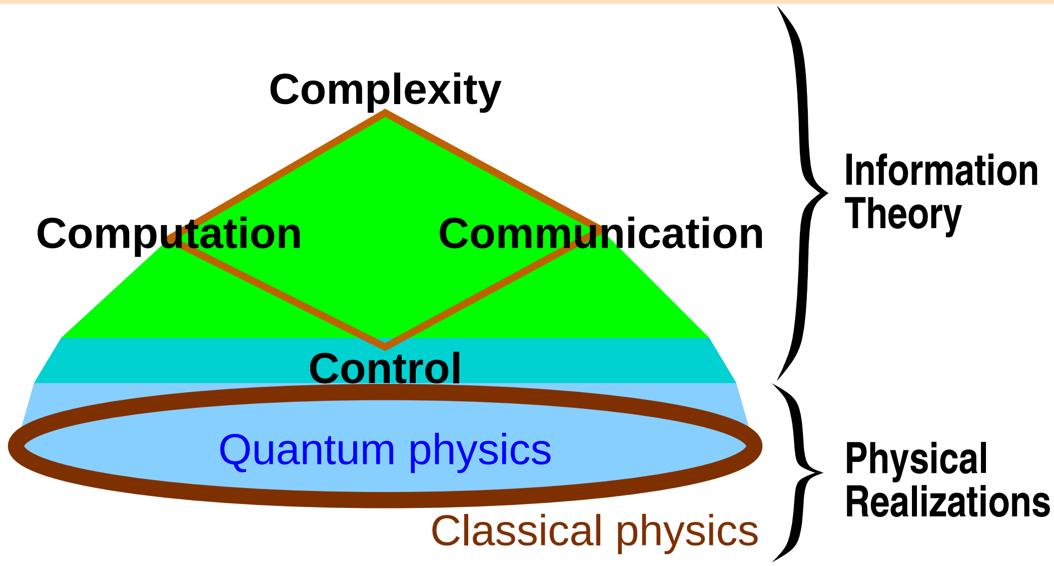

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- Practical relevance.

> - QIP is physically realizable in principle:
>
> **Accuracy Threshold Theorem:** *If the error rate is sufficiently low, then it is possible to efficiently process quantum information arbitrarily accurately.*

The Quantum Bit

- The *qubit*: A system with (pure) state space all superpositions of two *logical* states $|0\rangle$ and $|1\rangle$:

$$\{\ \alpha|0\rangle + \beta|1\rangle \text{ with } |\alpha|^2 + |\beta|^2 = 1\ \}$$

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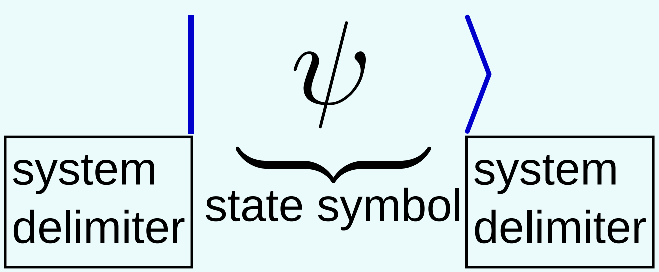

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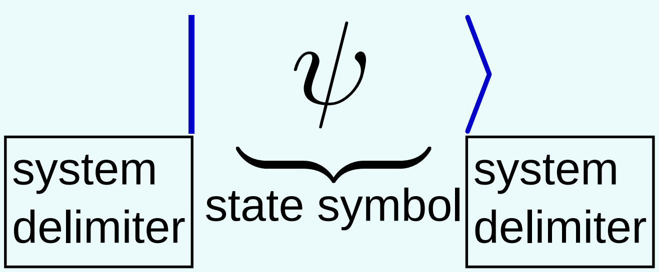

For example: $|\psi\rangle = \frac{3}{5}|0\rangle + \frac{4i}{5}|1\rangle$

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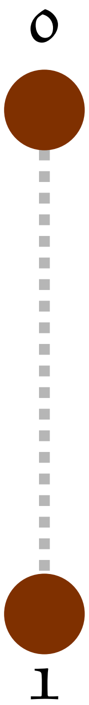

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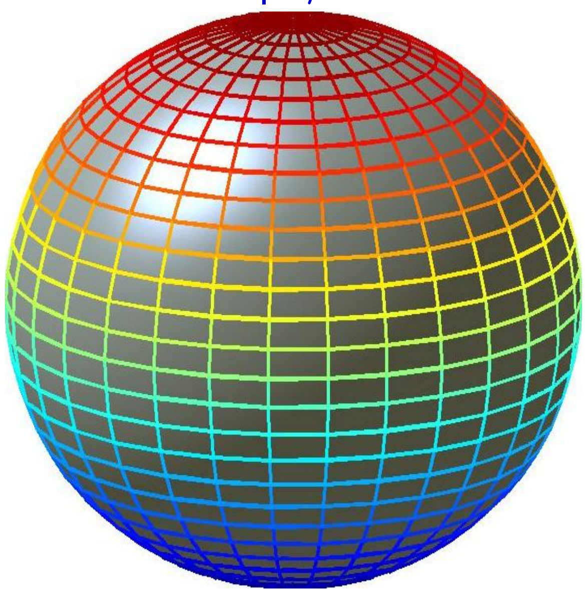

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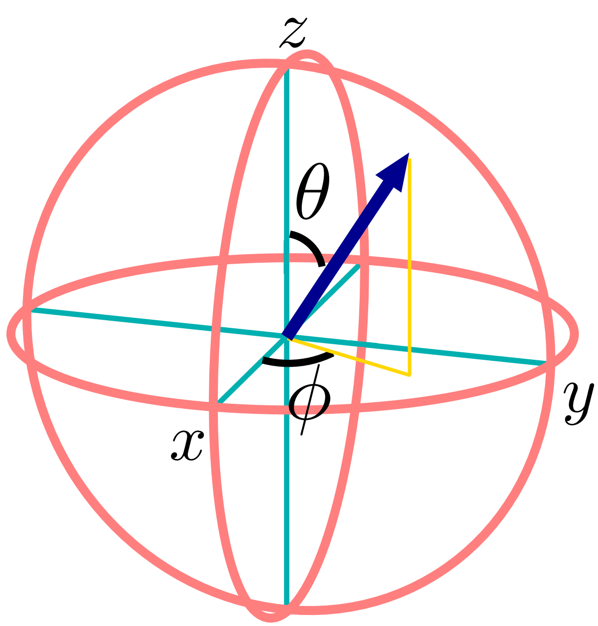

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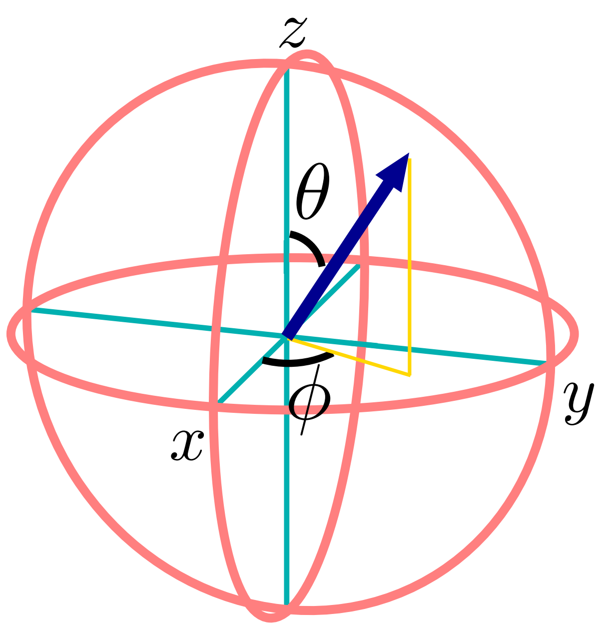

$$\alpha|0\rangle + \beta|1\rangle \cong e^{-i\phi/2}\cos(\theta/2)|0\rangle + e^{i\phi/2}\sin(\theta/2)|1\rangle$$

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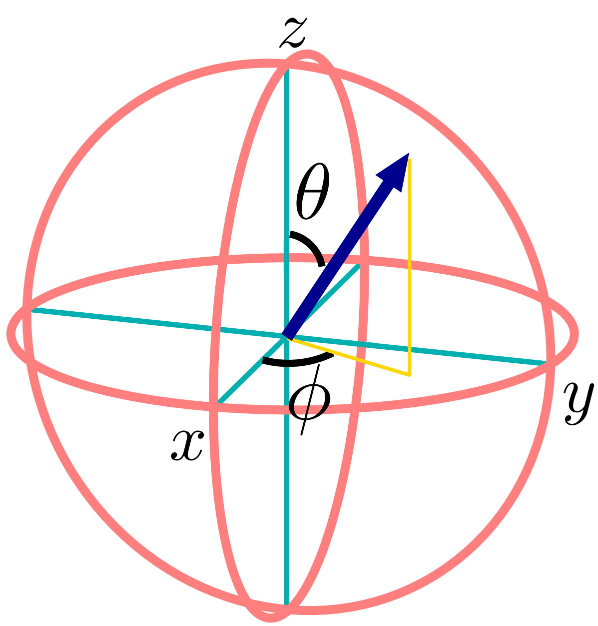

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  - Global phase:
    $\alpha|0\rangle + \beta|1\rangle$ and $e^{i\varphi}\alpha|0\rangle + e^{i\varphi}\beta|1\rangle$ are the same state.

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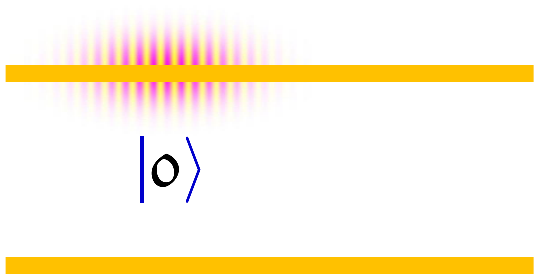

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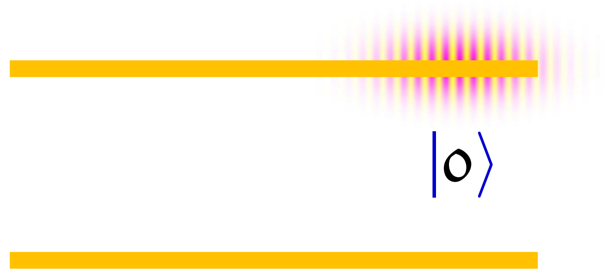

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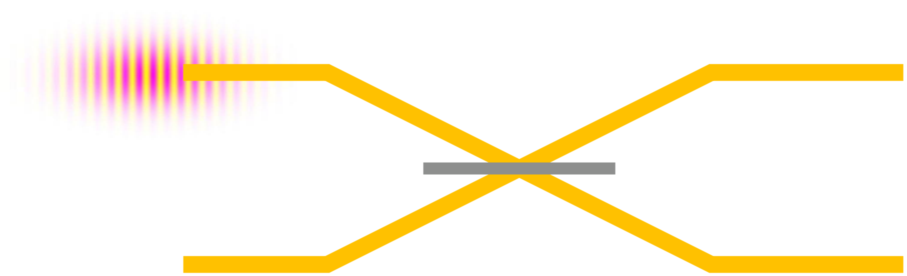

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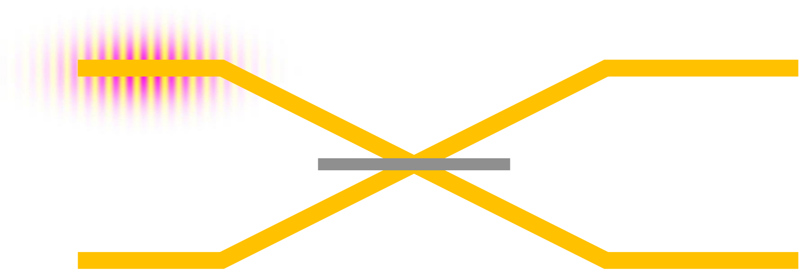

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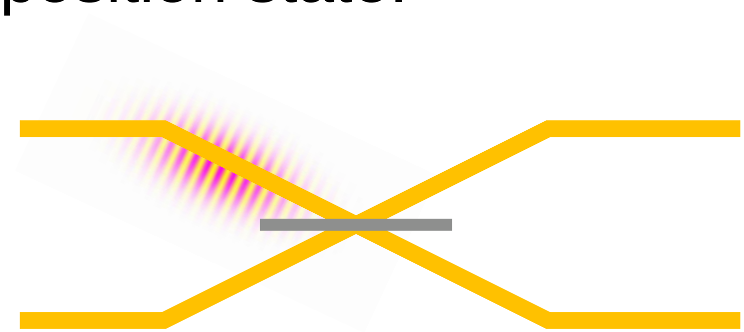

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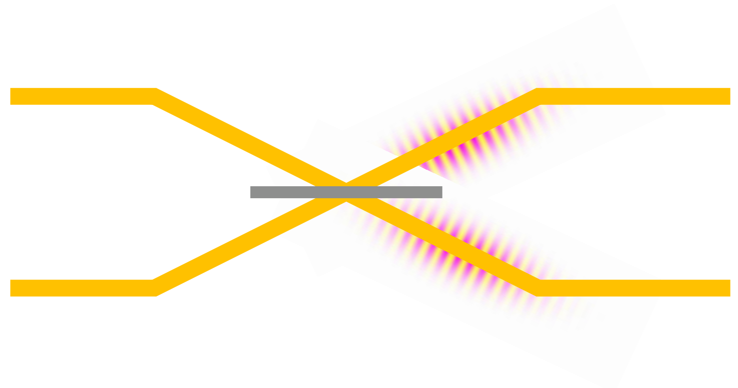
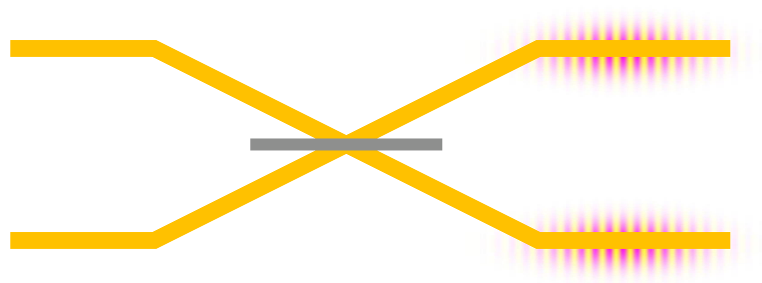


Photonic Qubit

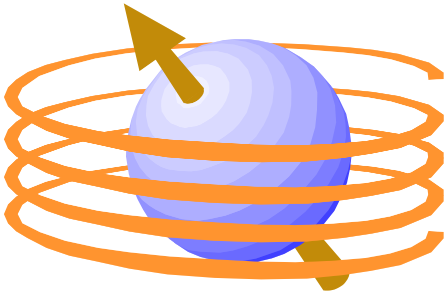
- Photonic qubit: One photon in a superposition of two modes.

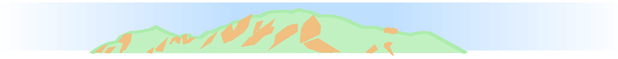
  - Photonic qubits are usually "flying" qubits.
  - Making a superposition state:



Spin $1/2$ Qubit

- Spin $1/2$ in oriented space: One particle in a superposition of the states “up” ($|\uparrow\rangle$) and “down” ($|\downarrow\rangle$).





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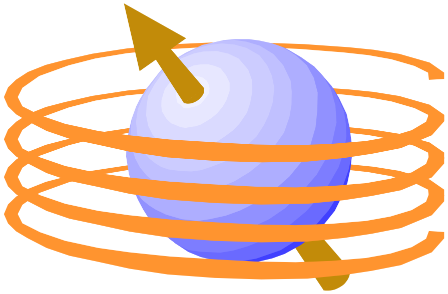
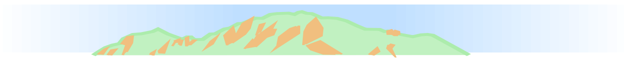


  - Orientation of magnetic moment (average) corresponds to the state in the Bloch sphere.



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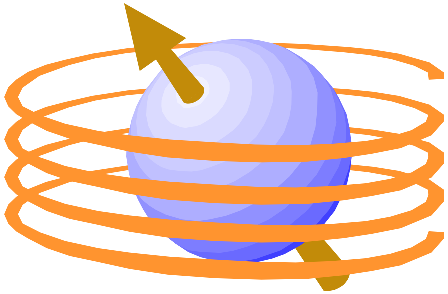

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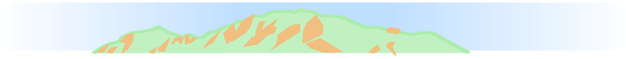
  - Examples include nuclear spins of ^{13}C and ^{1}H.
    These are observable by nuclear magnetic resonance.



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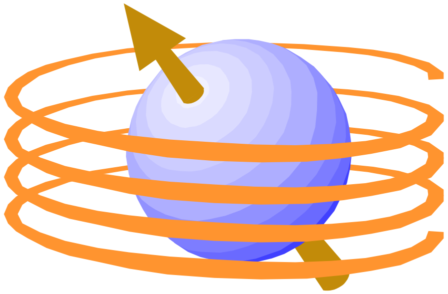

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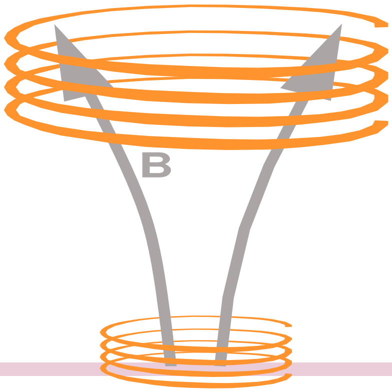

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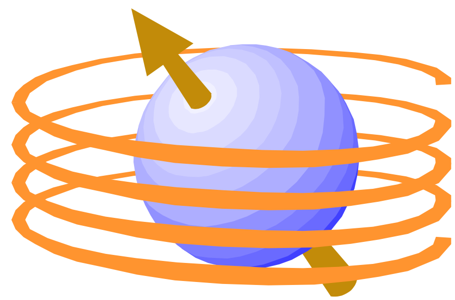

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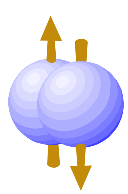

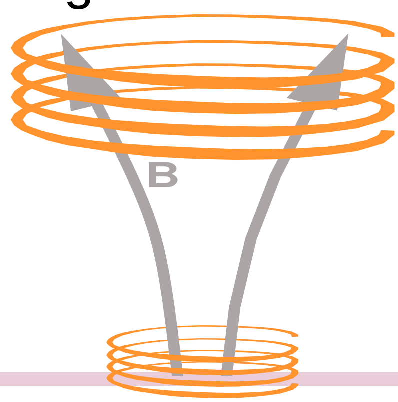

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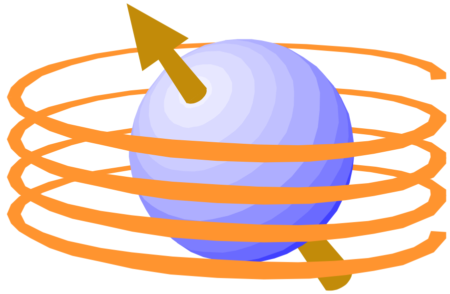

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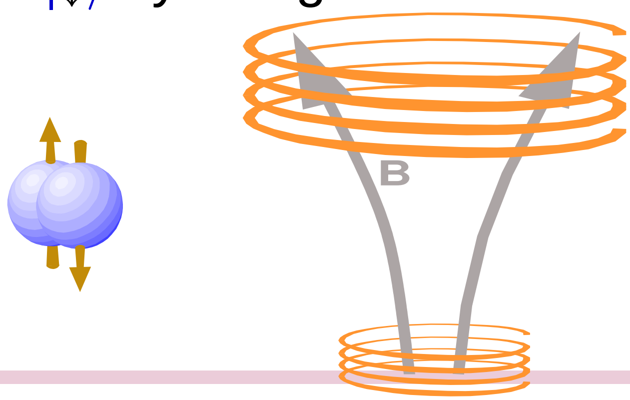

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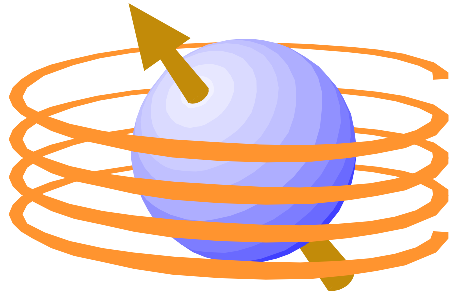

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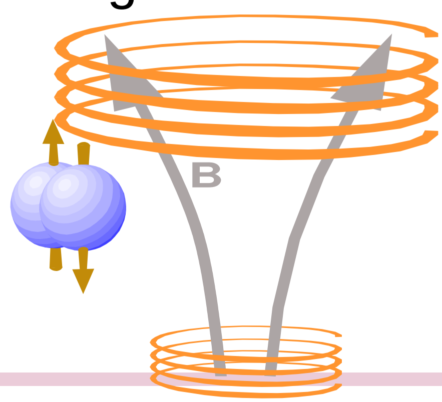

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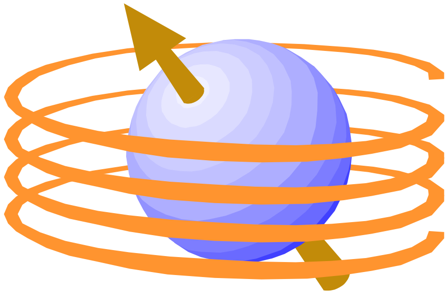

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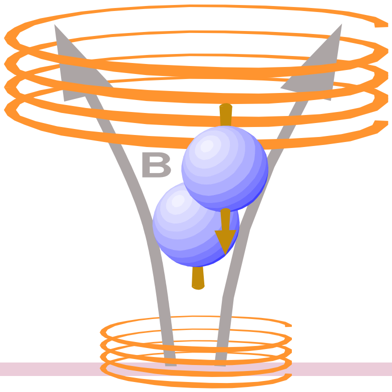

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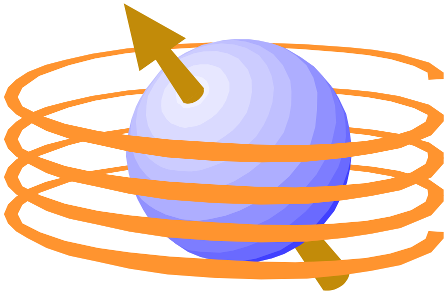

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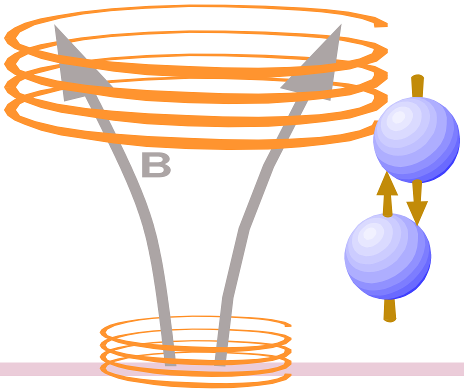

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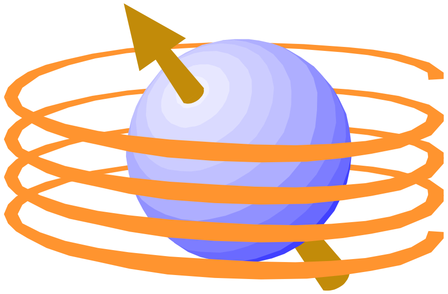

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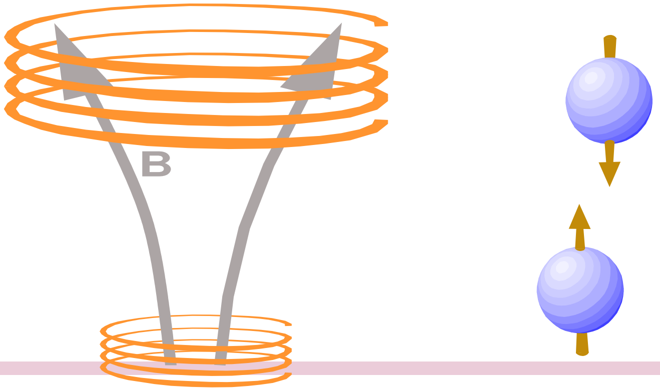

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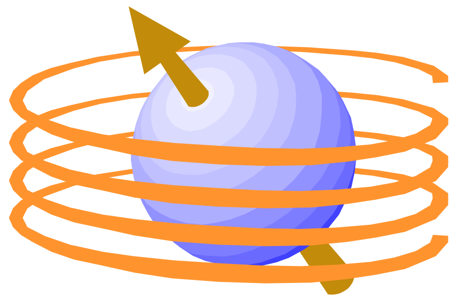

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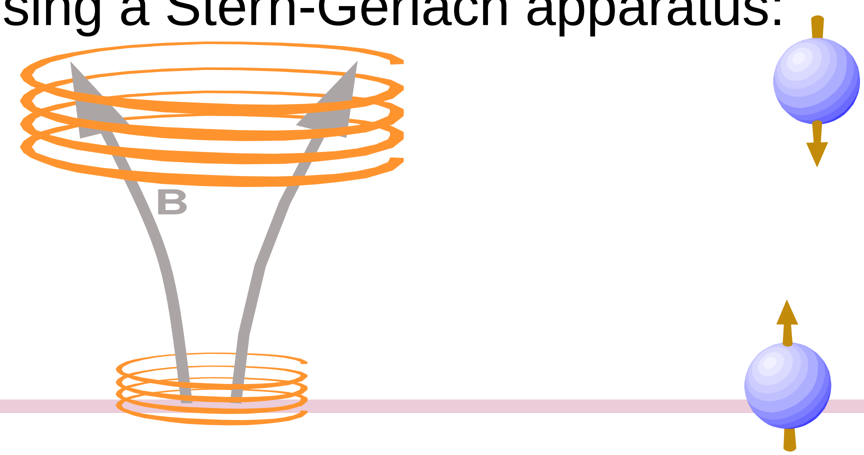


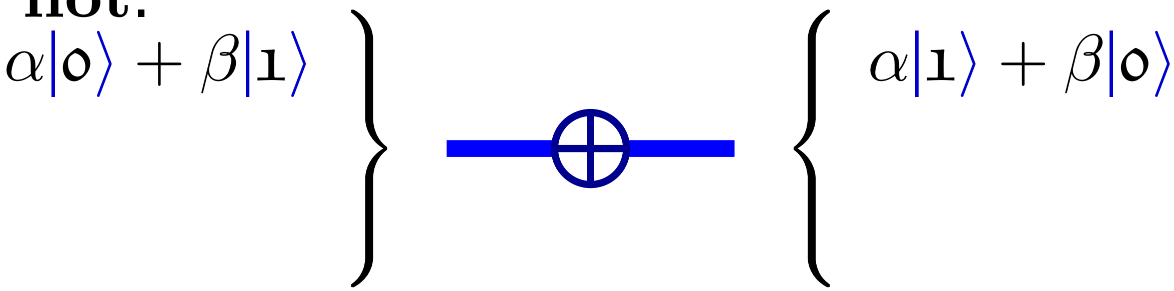
One-Qubit Gates I

- State preparation, $\mathbf{prep}(0), \mathbf{prep}(1)$.

One-Qubit Gates I

- State preparation, $\mathbf{prep}(0), \mathbf{prep}(1)$.

- Bit flip, $\mathbf{not}$.



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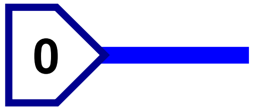
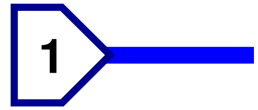


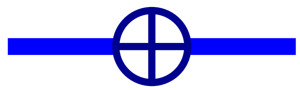
$|0\rangle$



$|1\rangle$

- Bit flip, $\mathbf{not}$.

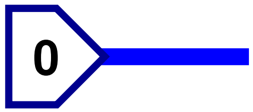
$$\left.\begin{array}{l} \alpha|0\rangle + \beta|1\rangle \\ \begin{pmatrix}\alpha \\ \beta\end{pmatrix} \end{array}\right\}$$



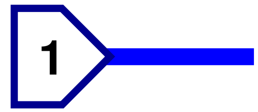
$$\left\{\begin{array}{l} \alpha|1\rangle + \beta|0\rangle \\ \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix}\beta \\ \alpha\end{pmatrix} \end{array}\right.$$

One-Qubit Gates I

- State preparation, $\mathbf{prep}(0), \mathbf{prep}(1)$.



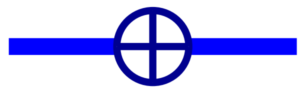
$|0\rangle$



$|1\rangle$

- Bit flip, $\mathbf{not}$.

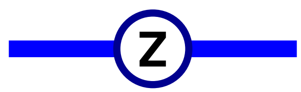
$$\left.\begin{array}{l} \alpha|0\rangle + \beta|1\rangle \\ \begin{pmatrix}\alpha \\ \beta\end{pmatrix} \end{array}\right\}$$



$$\left\{\begin{array}{l} \alpha|1\rangle + \beta|0\rangle \\ \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix}\beta \\ \alpha\end{pmatrix} \end{array}\right.$$

- Sign flip, $\mathbf{sgn}$.

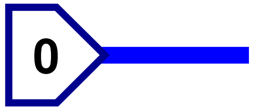
$$\left.\alpha|0\rangle + \beta|1\rangle\right\}$$



$$\left\{\alpha|0\rangle - \beta|1\rangle\right.$$

One-Qubit Gates I

- State preparation, $\mathbf{prep}(0), \mathbf{prep}(1)$.



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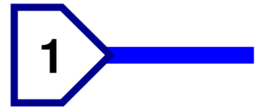
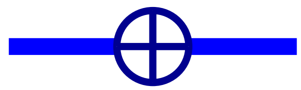


$|1\rangle$

- Bit flip, $\mathbf{not}$.

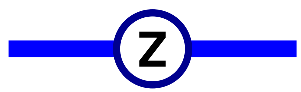
$$\left.\begin{matrix} \alpha|0\rangle + \beta|1\rangle \\ \begin{pmatrix}\alpha \\ \beta\end{pmatrix} \end{matrix}\right\}$$



$$\left\{\begin{matrix} \alpha|1\rangle + \beta|0\rangle \\ \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix}\beta \\ \alpha\end{pmatrix} \end{matrix}\right.$$

- Sign flip, $\mathbf{sgn}$.

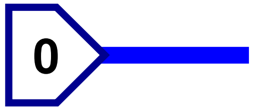
$$\left.\begin{matrix} \alpha|0\rangle + \beta|1\rangle \\ \begin{pmatrix}\alpha \\ \beta\end{pmatrix} \end{matrix}\right\}$$



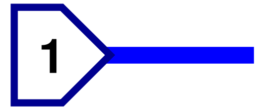
$$\left\{\begin{matrix} \alpha|0\rangle - \beta|1\rangle \\ \begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix}\alpha \\ -\beta\end{pmatrix} \end{matrix}\right.$$

One-Qubit Gates I

- State preparation, $\mathbf{prep}(0), \mathbf{prep}(1)$.



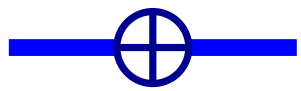
$|0\rangle$



$|1\rangle$

- Bit flip, $\mathbf{not}$.

$$\left.\begin{array}{l} \alpha|0\rangle + \beta|1\rangle \\ \begin{pmatrix}\alpha \\ \beta\end{pmatrix} \end{array}\right\}$$



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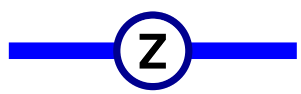


$$\left\{\begin{array}{l} \alpha|0\rangle - \beta|1\rangle \\ \begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix}\alpha \\ -\beta\end{pmatrix} \end{array}\right.$$

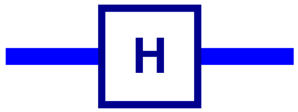
  - So far: Cannot generate proper superpositions.

One-Qubit Gates II

One-Qubit Gates II

- Hadamard.

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

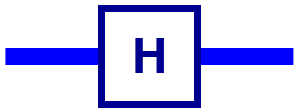


$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\alpha+\beta\\ \alpha-\beta\end{pmatrix}$$

One-Qubit Gates II

- Hadamard.

$$\left.\begin{array}{l}\alpha|0\rangle + \beta|1\rangle \\ \begin{pmatrix}\alpha \\ \beta\end{pmatrix}\end{array}\right\}$$



$$\left\{\begin{array}{l}\frac{1}{\sqrt{2}}(\alpha+\beta)|0\rangle + \frac{1}{\sqrt{2}}(\alpha-\beta)|1\rangle \\ \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\alpha+\beta \\ \alpha-\beta\end{pmatrix}\end{array}\right.$$

One-Qubit Gates II

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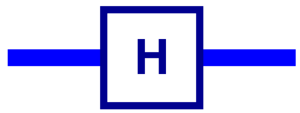

$$\left\{\begin{array}{l}\frac{1}{\sqrt{2}}(\alpha+\beta)|0\rangle + \frac{1}{\sqrt{2}}(\alpha-\beta)|1\rangle \\ \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\alpha+\beta \\ \alpha-\beta\end{pmatrix}\end{array}\right.$$

- Example: Prepare the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.

One-Qubit Gates II

- Hadamard.

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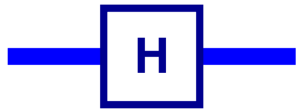

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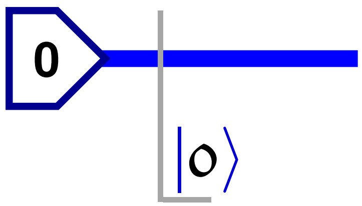
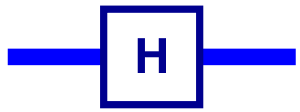


One-Qubit Gates II

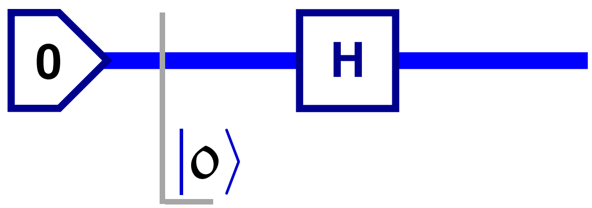
- Hadamard.

$$\left.\begin{array}{l}\alpha|0\rangle + \beta|1\rangle \\ \begin{pmatrix}\alpha\\ \beta\end{pmatrix}\end{array}\right\}$$



$$\left\{\begin{array}{l}\frac{1}{\sqrt{2}}(\alpha+\beta)|0\rangle + \frac{1}{\sqrt{2}}(\alpha-\beta)|1\rangle \\ \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\alpha+\beta\\ \alpha-\beta\end{pmatrix}\end{array}\right.$$

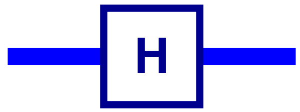
- Example: Prepare the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.



One-Qubit Gates II

- Hadamard.

$$\left.\begin{matrix} \alpha|0\rangle + \beta|1\rangle \\ \begin{pmatrix}\alpha \\ \beta\end{pmatrix} \end{matrix}\right\}$$



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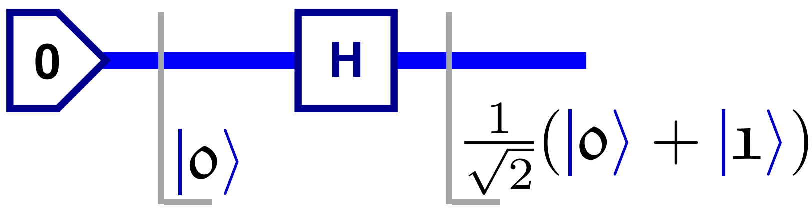

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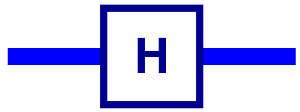

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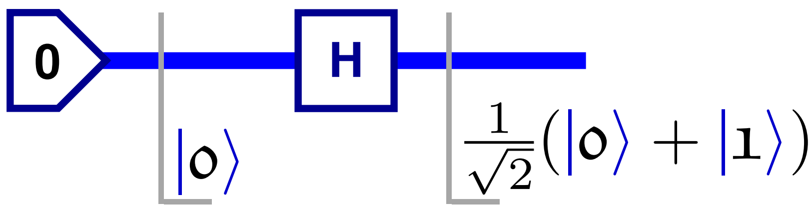

prep(0) . had

One-Qubit Gates II

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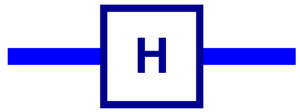

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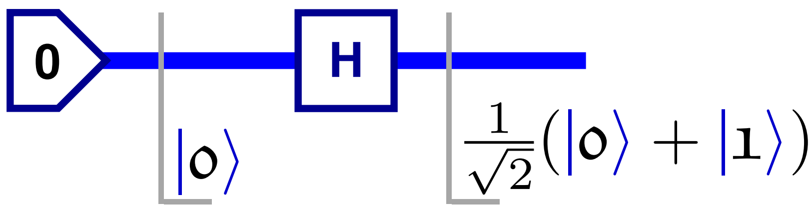
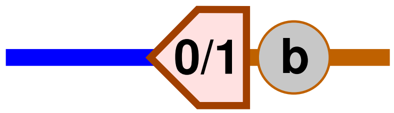


$$\mathbf{prep}(0) \quad . \quad \mathbf{had}$$

- With the gates so far, can we prepare $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$?

Read-out

- Read-out reduces a state destructively to classical information.



Read-out

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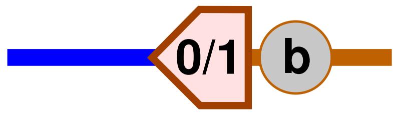
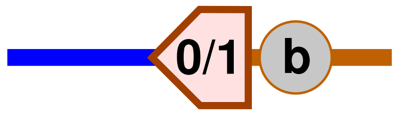
$|0\rangle$



$\{ \; b = 0$

Read-out

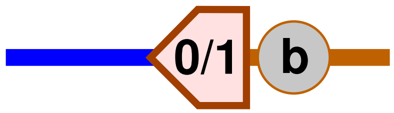
- Read-out reduces a state destructively to classical information.

$|1\rangle$  $\{\ b = 1$

Read-out

- Read-out reduces a state destructively to classical information.

$$\alpha|0\rangle + \beta|1\rangle$$

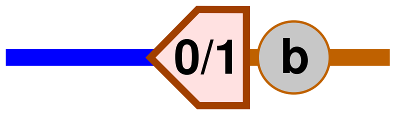


$$\begin{cases} \mathsf{b} = 0 & \text{with probability } |\alpha|^2, \\ \mathsf{b} = 1 & \text{with probability } |\beta|^2. \end{cases}$$

Read-out

- Read-out reduces a state destructively to classical information.

$\alpha|0\rangle + \beta|1\rangle$



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- Example:

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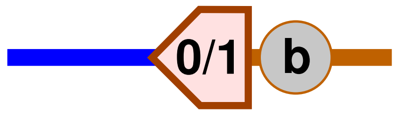

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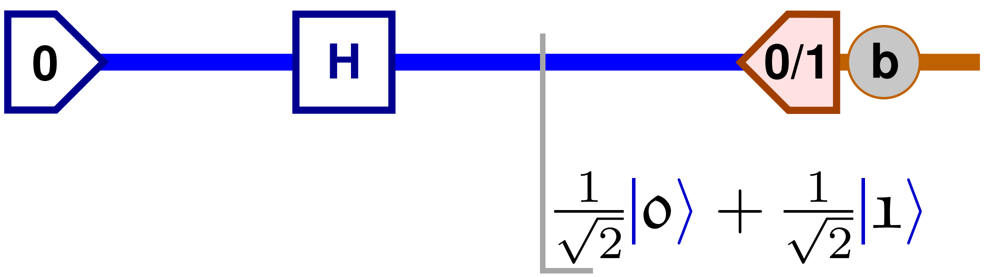

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Read-out

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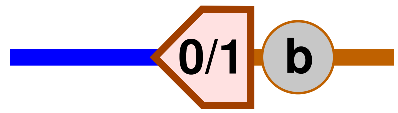
$\alpha|0\rangle + \beta|1\rangle$



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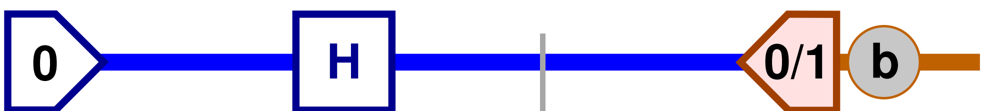

$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$$\begin{cases} 0 & \text{with probability } \left(\frac{1}{\sqrt{2}}\right)^2 = .5, \\ 1 & \text{with probability } \left(\frac{1}{\sqrt{2}}\right)^2 = .5. \end{cases}$$

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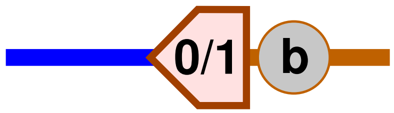

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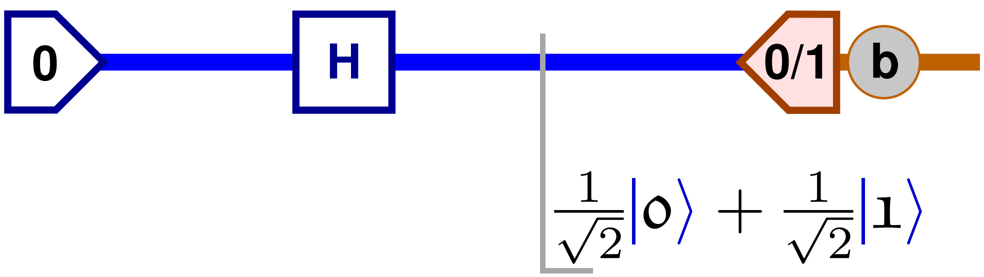

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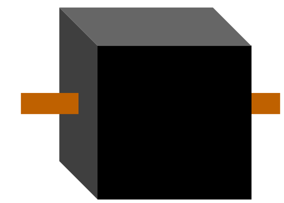
$\mathbf{prep}(0)\ .\ \mathbf{had}\ .\ \mathbf{meas}(Z \mapsto b)$

“Black Box” Problems

Classical:

- Given: Unknown one-bit device, a “black box”.



“Black Box” Problems

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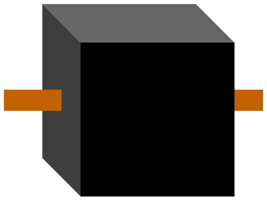
  Promise: It either flips the bit or does nothing.



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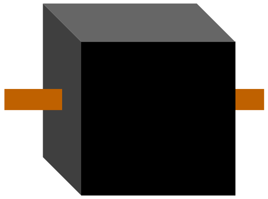

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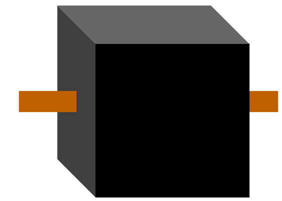

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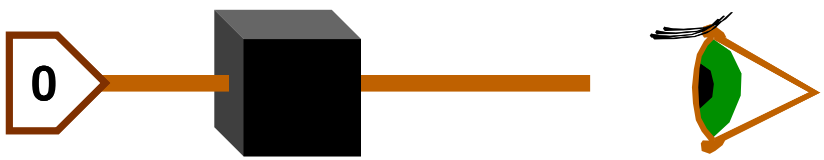

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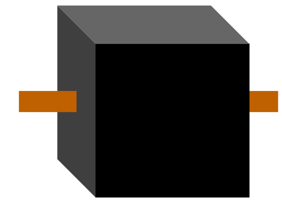

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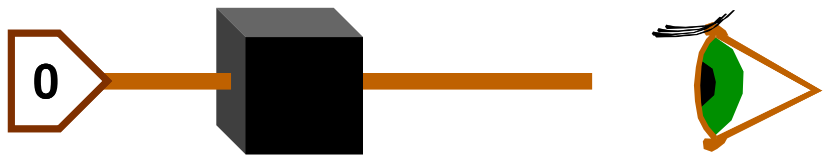

$$\begin{cases} 0: & \text{doesn't flip,} \\ 1: & \text{flips.} \end{cases}$$

“Black Box” Problems

Quantum:

“Black Box” Problems

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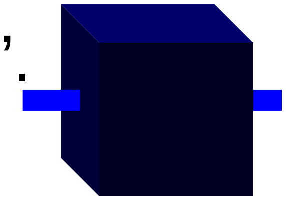

- Given: Unknown one-qubit device, a “black box”.
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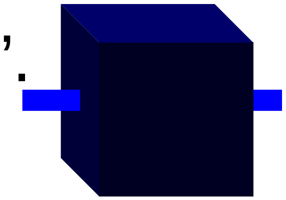

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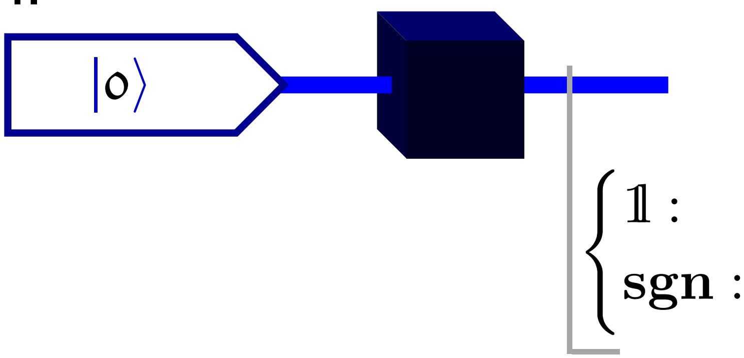

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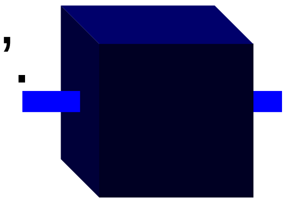

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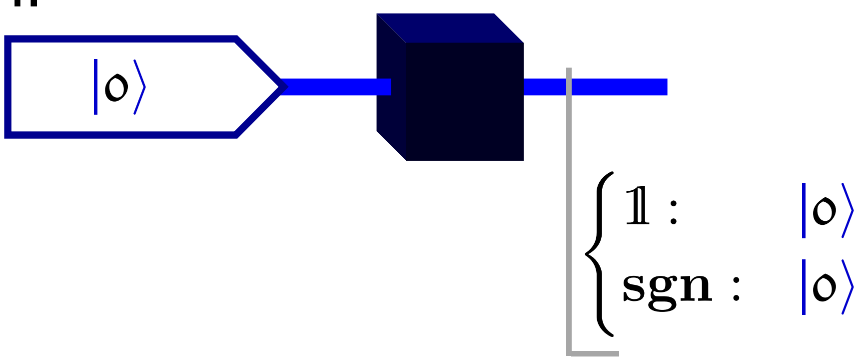

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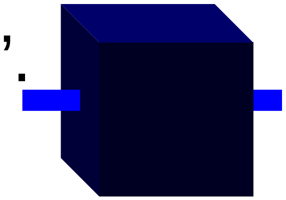

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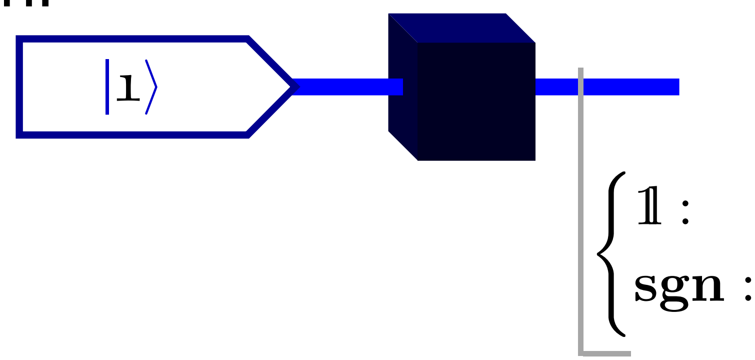

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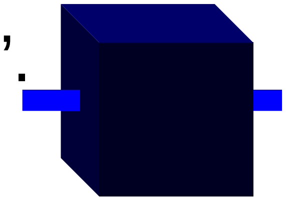

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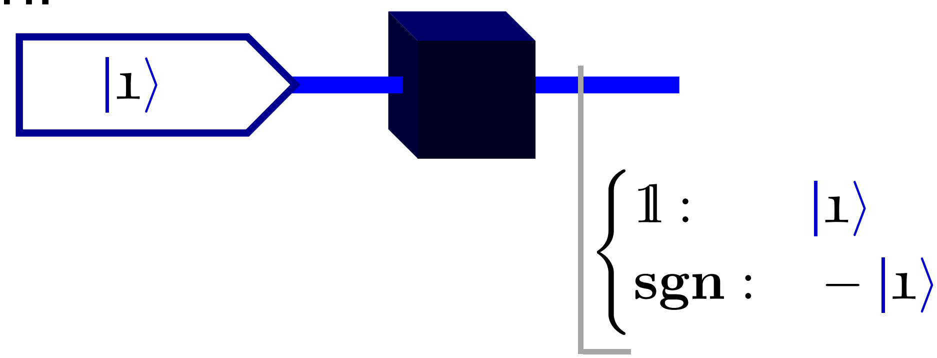

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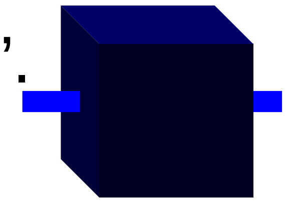

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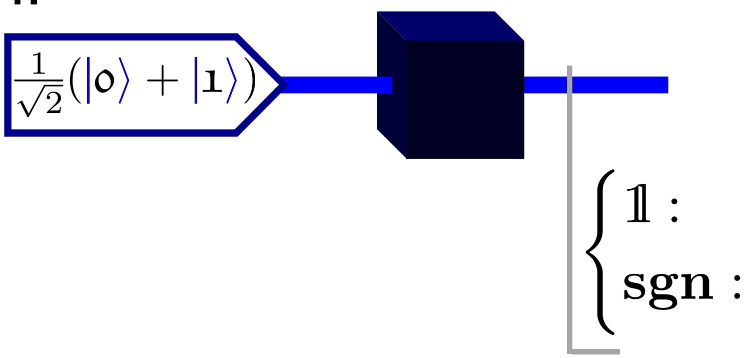

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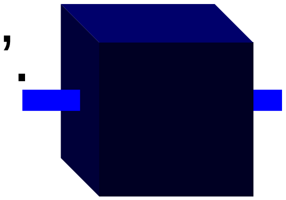

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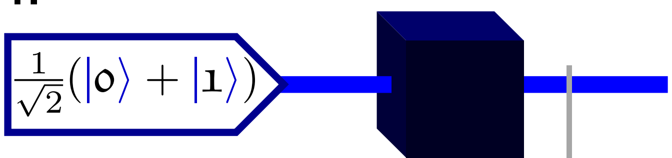

$$\begin{cases} \mathbb{1}: & \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\ \mathbf{sgn}: & \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \end{cases}$$

"Black Box" Problems

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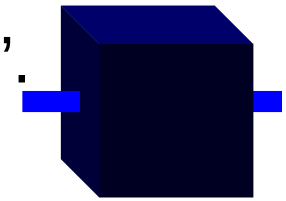

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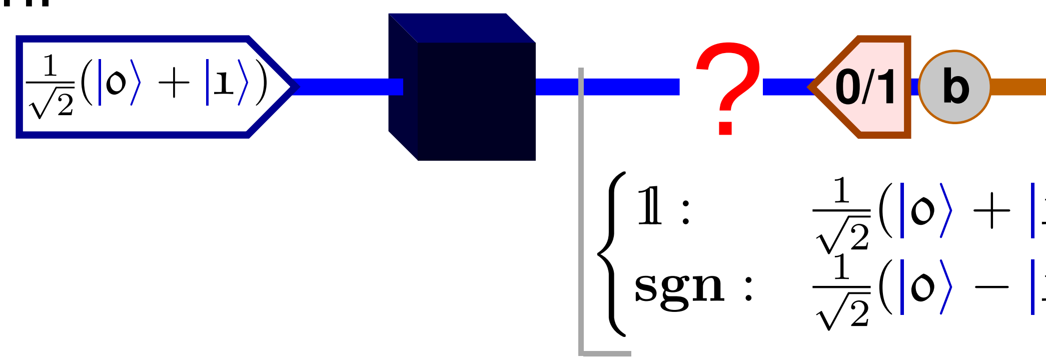

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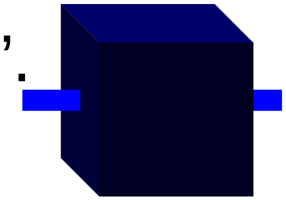

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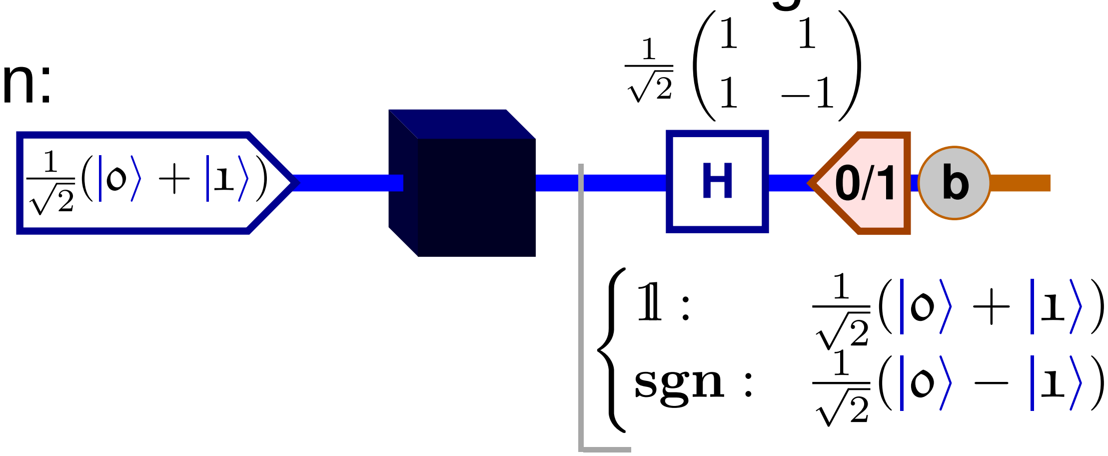

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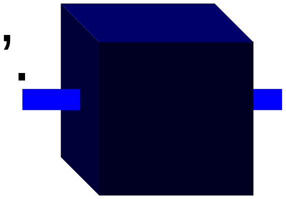

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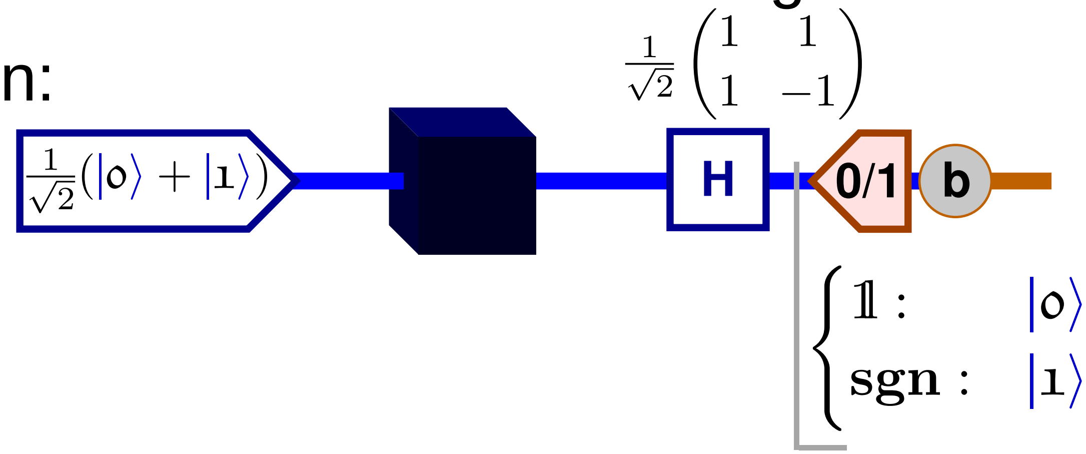

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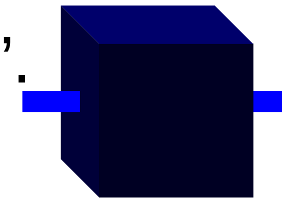

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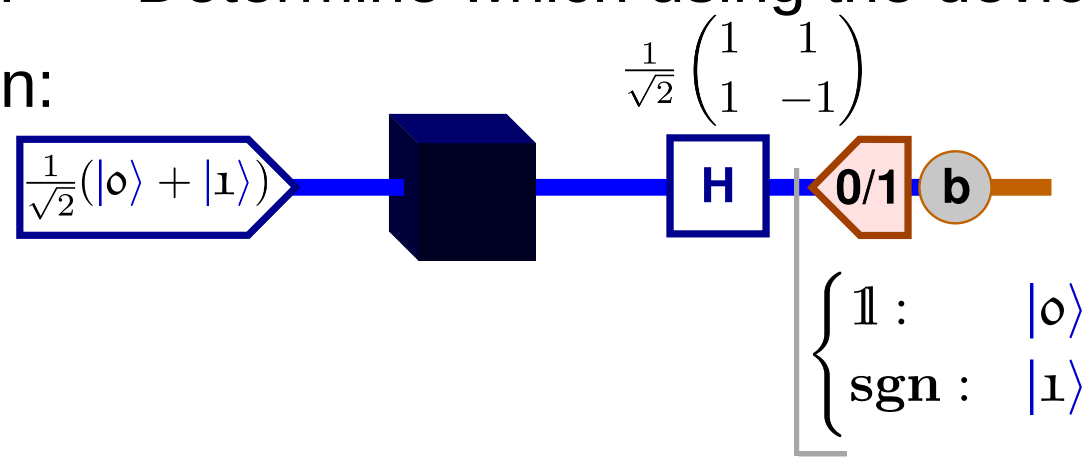

- Given: Unknown one-qubit device, a "black box".
  Promise: It either applies $\mathbf{not}$, $\mathbf{sgn}$, $\mathbf{sgn.not}$ or does nothing.
  Problem: Determine which using the device once.

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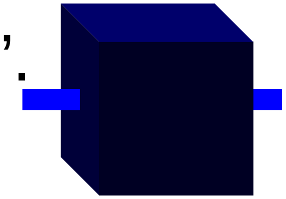
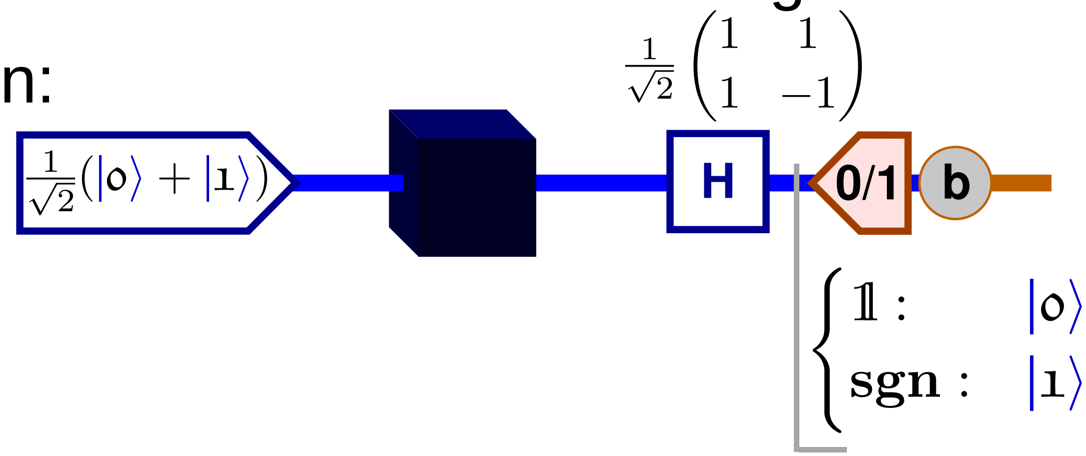


  - Solution:



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  Problem: Determine which using the device once.
  - Is this possible?

Contents

References

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