Simulations of Cold Electroweak Baryogenesis

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Strong and Electroweak Matter 2006, BNL, 10.-13./5, 2006

The baryon asymmetry of the Universe

The Universe contains unequal amounts of matter and anti-matter. Some process (Baryogenesis) in the early Universe produced the asymmetry. From WMAP [Spergel et al.:2006]:

$$\eta = \frac{n_B}{n_\gamma} = 6.1 \times 10^{-10}.$$

n_B, n_γ are number densities of baryons, photons.

One such mechanism is Electroweak Baryogenesis, baryogenesis at the electroweak scale [Kuzmin, Rubakov, Shaposhnikov:1985].

Cold Electroweak Baryogenesis

Simulations of Baryogenesis, taking place after Electroweak-scale small-field Hybrid Inflation, during an inflaton-triggered, zero-temperature, Electroweak Symmetry Breaking transition.

- After inflation, Universe is cold; $T = 0$.
- Symmetry breaking transition is the reheating mechanism.
- Reheating temperature below electroweak scale: No sphaleron wash-out.
- Embedded in extension of Standard Model including an inflaton: Keep extension minimal.

Baryogenesis during electroweak symmetry breaking has been studied in: [Krauss & Trodden:1999,Garcia-Bellido et al.:1999,2003,2004, Copeland et al.:2001, Smit et al(AT).: 2002,2003,2004,2006].

Realisation

A baryon asymmetry can only be generated in the presence of baryon number violating, CP violating processes out of thermal equilibrium [Sakharov:1967].

$$\mathcal{L} = \mathcal{L}_{\text{inflaton}} + \mathcal{L}_{U(1)} + \mathcal{L}_{SU(2)} + \mathcal{L}_{SU(3)} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{fermions}} + \ldots$$

Restrict to minimal realisation of the scenario

$$S = -\int d^4x \left[\frac{1}{2g^2} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger D^\mu \phi + \mu_{\text{eff}}^2(t) \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 + \mathcal{L}_{CP} \right].$$

CP-violation

Include generic, lowest order CP-violating term of ϕ and A_μ,

$$\mathcal{L}_{CP} = \kappa\, \phi^\dagger \phi \mathrm{Tr}\, F^{\mu\nu} \tilde{F}_{\mu\nu} = \left(\frac{6\delta_{\rm cp}}{g^2}\right) \left(\frac{\phi^\dagger \phi}{v^2/2}\right) \quad \dot{n}_{\rm cs},$$

$$N_{\rm cs} = \int d^3x\, dt\, \dot{n}_{\rm cs}, \qquad \dot{n}_{\rm cs} = \frac{1}{16\pi^2} \mathrm{Tr}\, F^{\mu\nu} \tilde{F}_{\mu\nu}.$$

Symmetry breaking is triggered by the rolling inflaton, through the replacement

$$\mu^2_{\rm eff}(t) = \mu^2 - \lambda_{\sigma\phi} \sigma^2 \phi^\dagger \phi = \mu^2 \left(1 - \frac{2t}{t_Q}\right).$$

Parameter space

Leaves 3 free parameters:

$$\delta_{\rm cp}, \qquad \left(\frac{m_H}{m_W}\right)^2 = \frac{8\lambda}{g^2}, \qquad m_H t_Q,$$

Ideally, the dependence is separable (the real world ideal? Ha!):

$$\begin{aligned}\frac{n_B}{n_\gamma} \quad &= f\left(\delta_{\rm cp}, m_H t_Q, m_H/m_W\right) \\ &= f_1\left(\delta_{\rm cp}\right) \times f_2\left(m_H t_Q\right) \times f_3\left(\frac{m_H}{m_W}\right).\end{aligned}$$

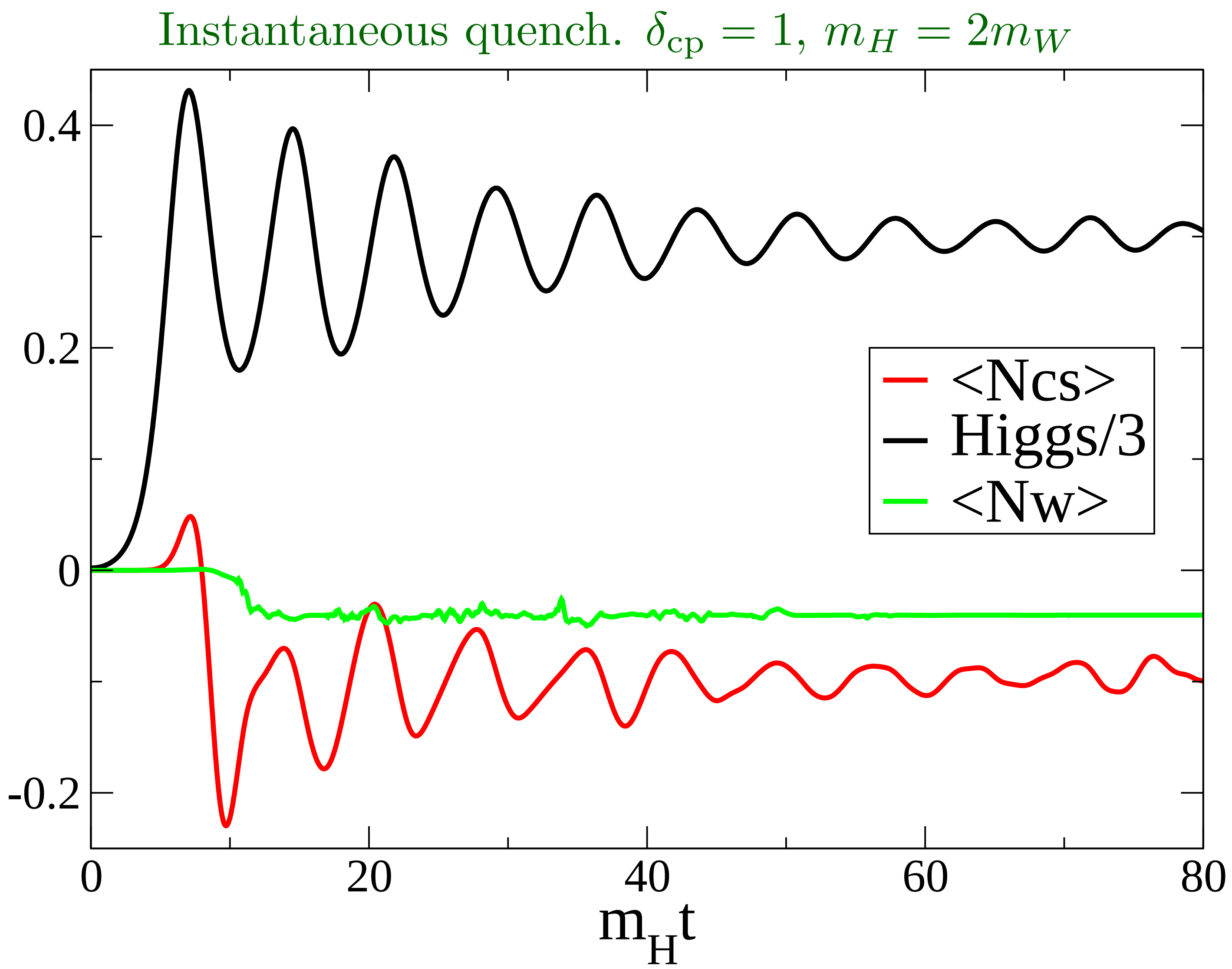
Baryon number non-conservation

- Baryon number is not conserved in the SM.
- A quantum anomaly relates changes in the baryon and lepton numbers B, L of fermions coupled axially to a background ($SU(2)$) gauge field to changes in the Chern-Simons number $N_{\rm cs}$ of that gauge field ['t Hooft:1976]:

$$\begin{aligned}\langle B(t)-B(0)\rangle &= 3\langle[N_{\rm cs}(t)-N_{\rm cs}(0)]\rangle \\ &= \frac{3}{16\pi^2}\int_0^t dt\int d^3x\langle \mathrm{Tr}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\right]\rangle.\end{aligned}$$

- The vacua of the SU(2)-Higgs model have integer Chern-Simons number. Higgs winding number $N_{\rm w}$ is integer and in the vacua $N_{\rm w}=N_{\rm cs}$. $N_{\rm w}$ settles first (in the simulations), and it is useful to use:

$$\langle B(t)-B(0)\rangle = 3\langle[N_{\rm w}(t)-N_{\rm w}(0)]\rangle.$$



Instantaneous quench. $\delta_{cp} = 1$, $m_H = 2m_W$
0.4
0.2
0
-0.2
0
20
40
60
80
$m_H t$
<Ncs>
Higgs/3
<Nw>

Stage 1: Chern-Simons number chemical potential

Think of the CP-violation as a chemical potential for Chern-Simons number,

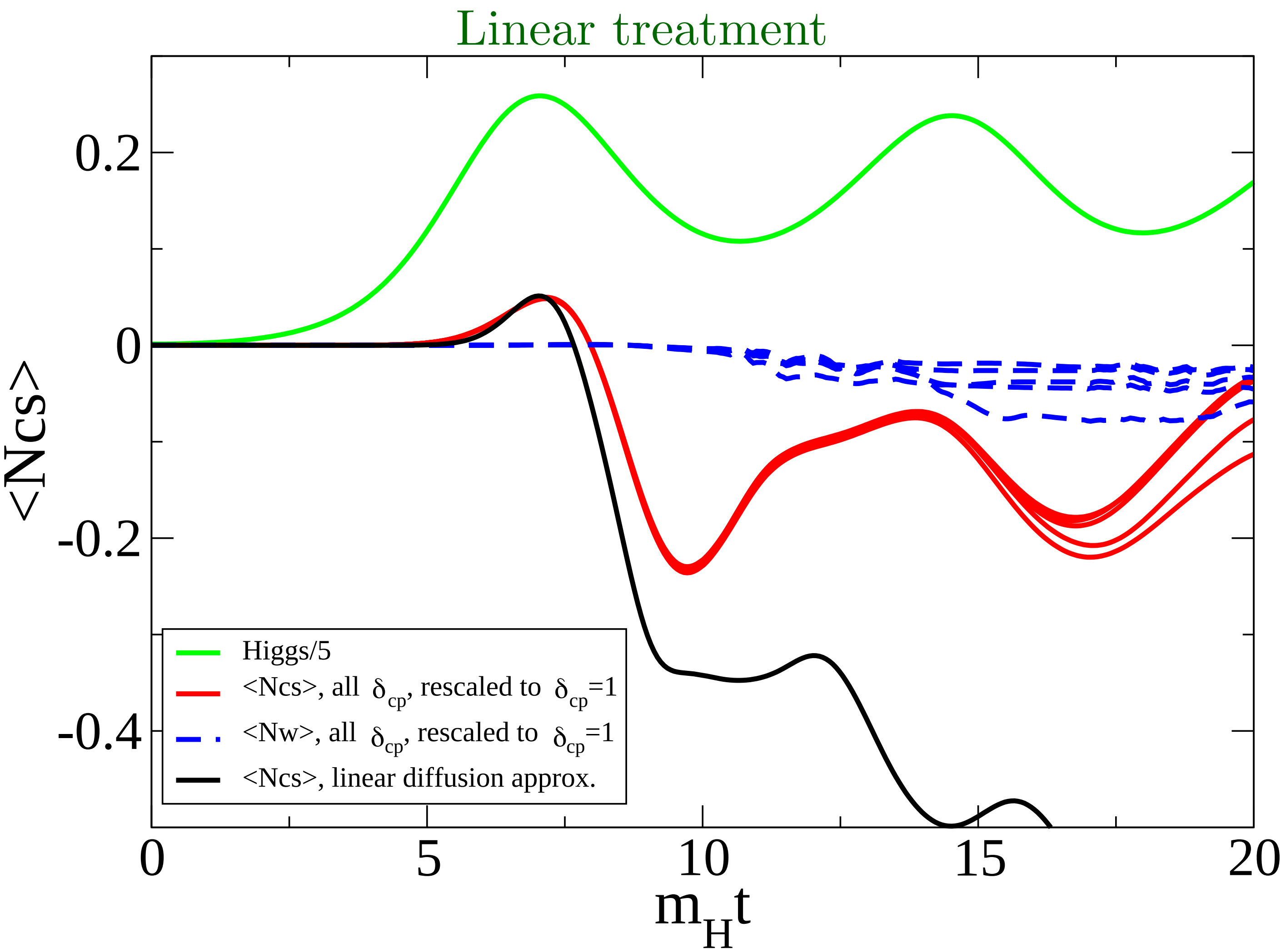
$$\int dt\, \mathcal{L}_{CP} = \int dt \left(\frac{6\delta_{\rm cp}}{g^2}\right)\left(\frac{\phi^\dagger\phi}{v^2/2}\right) \dot{n}_{\rm cs},$$

$$\simeq \int dt\, \mu_{\rm ch} n_{\rm cs}, \quad \mu_{\rm ch}(t) = -\frac{6\delta_{\rm cp}}{g^2}\frac{d}{dt}\frac{\phi^\dagger\phi}{v^2/2}.$$

A linear treatment, using the Chern-Simons number diffusion rate gives [Khlebnikov & Shaposhnikov:1988],

$$\Gamma(t) = \frac{d\left(\langle N_{\rm cs}^2\rangle - \langle N_{\rm cs}\rangle^2\right)}{dt}, \quad \langle N_{\rm cs}(t)\rangle = \frac{1}{T_{\rm eff}}\int_0^t dt'\,\Gamma(t')\mu_{\rm ch}(t'),$$

and reproduces early behaviour well.



Linear treatment
0.2
0
-0.2
-0.4
0
5
10
15
20
<Ncs>
$m_H t$
Higgs/5
<Ncs>, all δ_{cp}, rescaled to δ_{cp}=1
<Nw>, all δ_{cp}, rescaled to δ_{cp}=1
<Ncs>, linear diffusion approx.

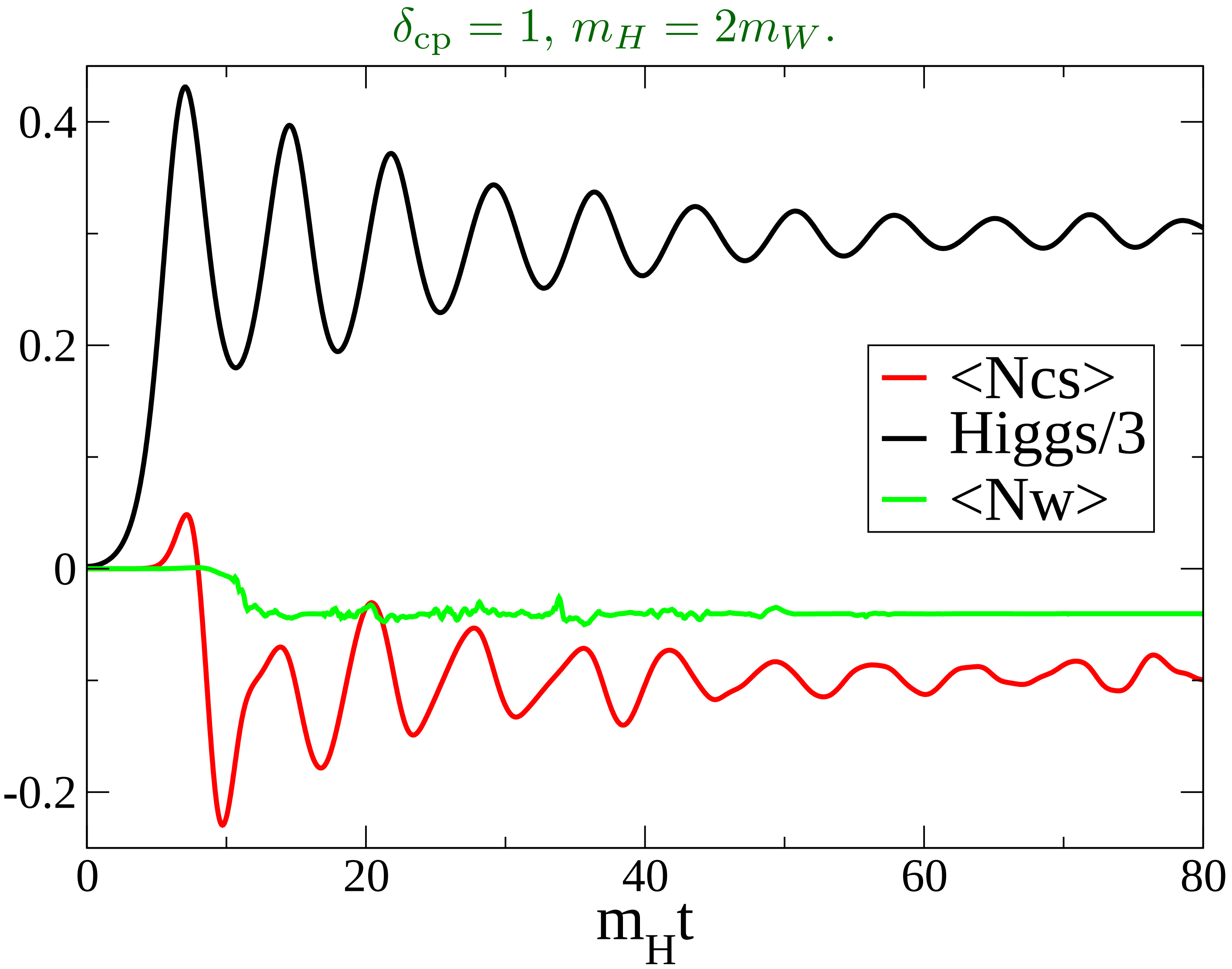
Stage 2: Relaxation of winding number

The minimal gradient energy configurations are pure gauge (vacuum),

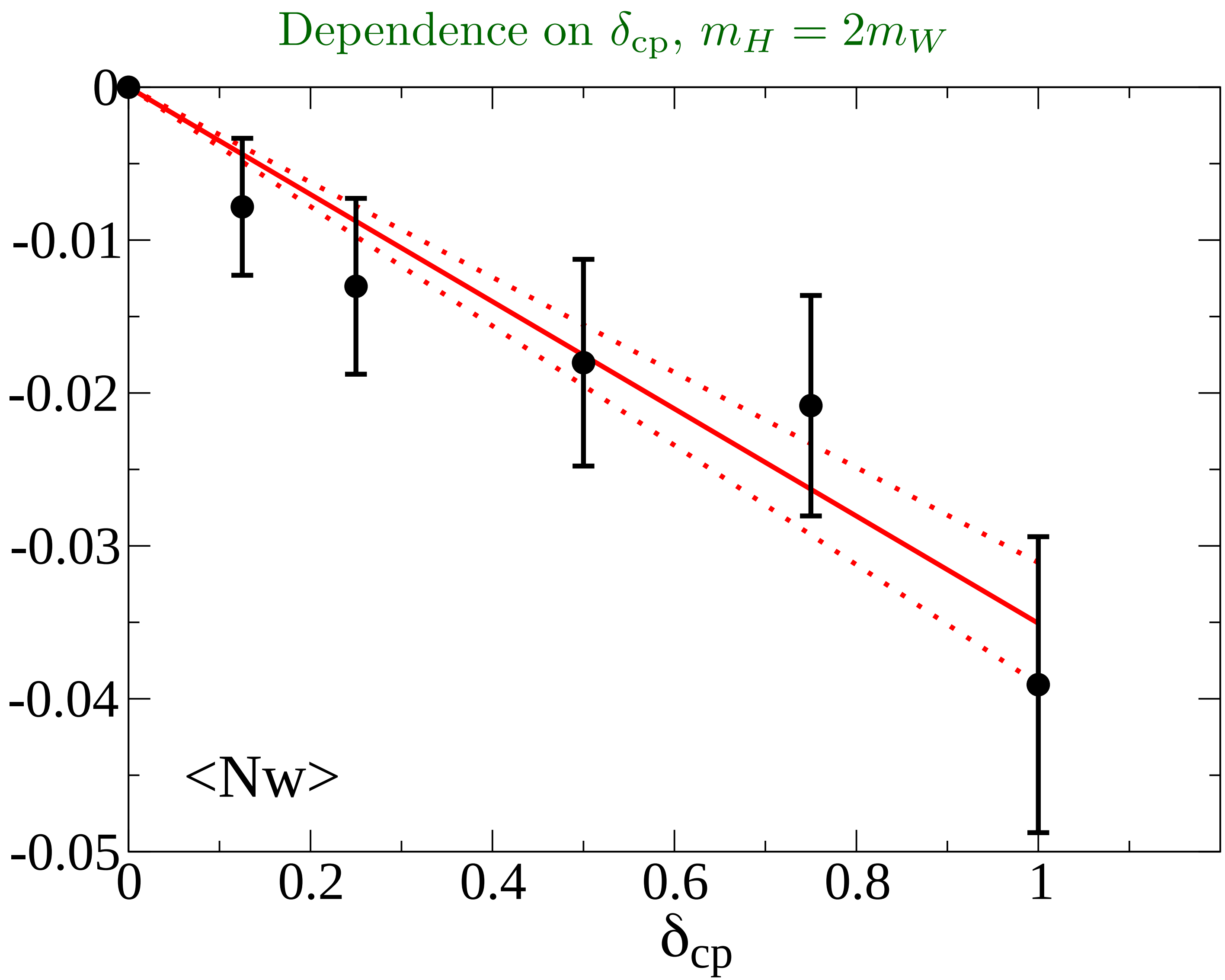
$$\Phi = \frac{v}{\sqrt{2}} U, \quad A_j = -i\partial_j U U^\dagger.$$

$$N_\mathrm{w} = \frac{1}{24\pi^2} \int d^3x \, \epsilon_{ijk} \mathrm{Tr} \left[(\partial_i U) U^\dagger (\partial_j U) U^\dagger (\partial_k U) U^\dagger \right],$$

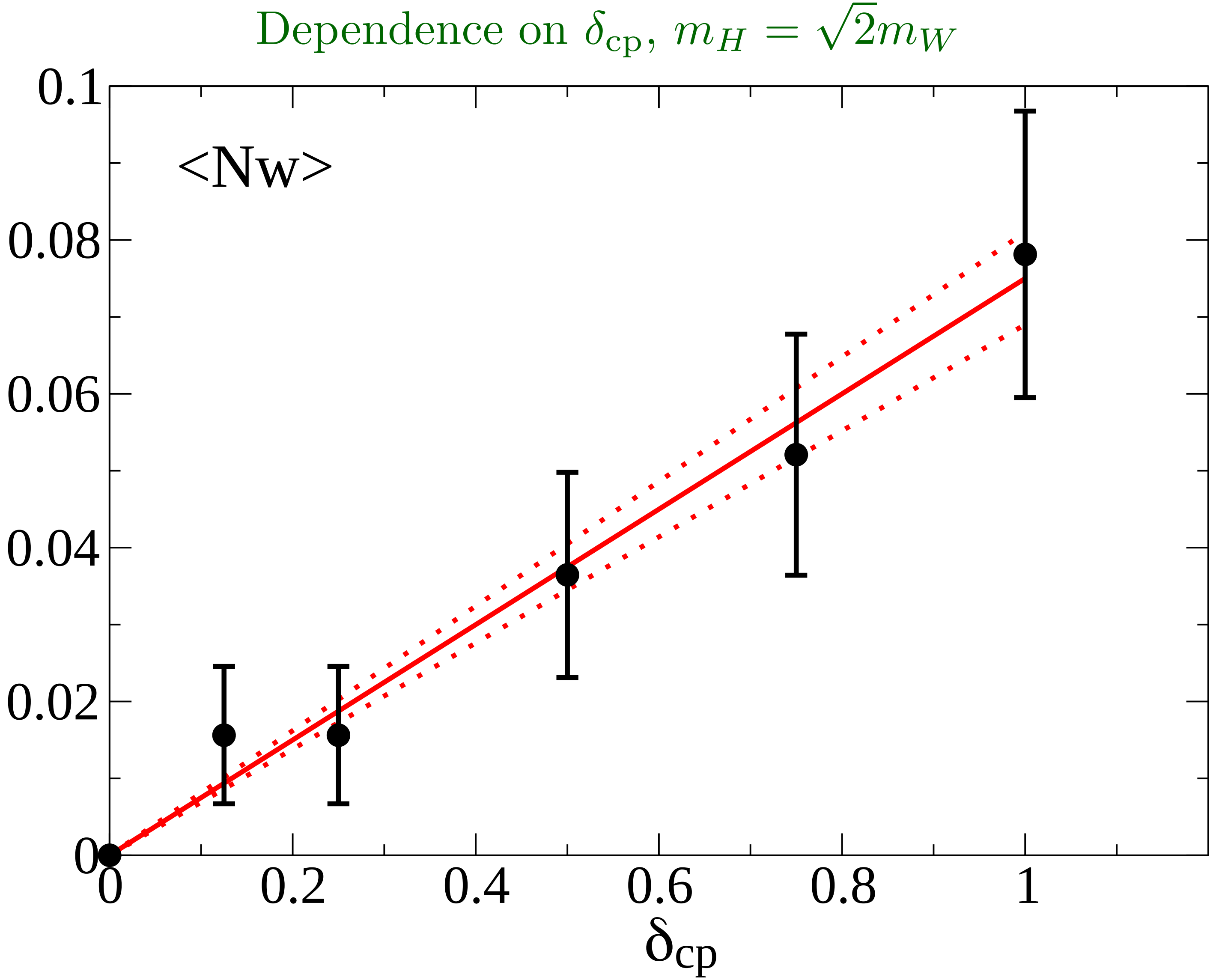
Then $N_\mathrm{w} = N_\mathrm{cs}$. Relaxing from $N_\mathrm{w} \neq N_\mathrm{cs}$ requires change of N_cs or N_w. Change of N_w can only take place through a zero of the Higgs field. The process is local, depending on size of “blobs” [Turok & Zadrozny:1990,1991] and availability of Higgs zeros [van der Meulen et al(AT):2006].



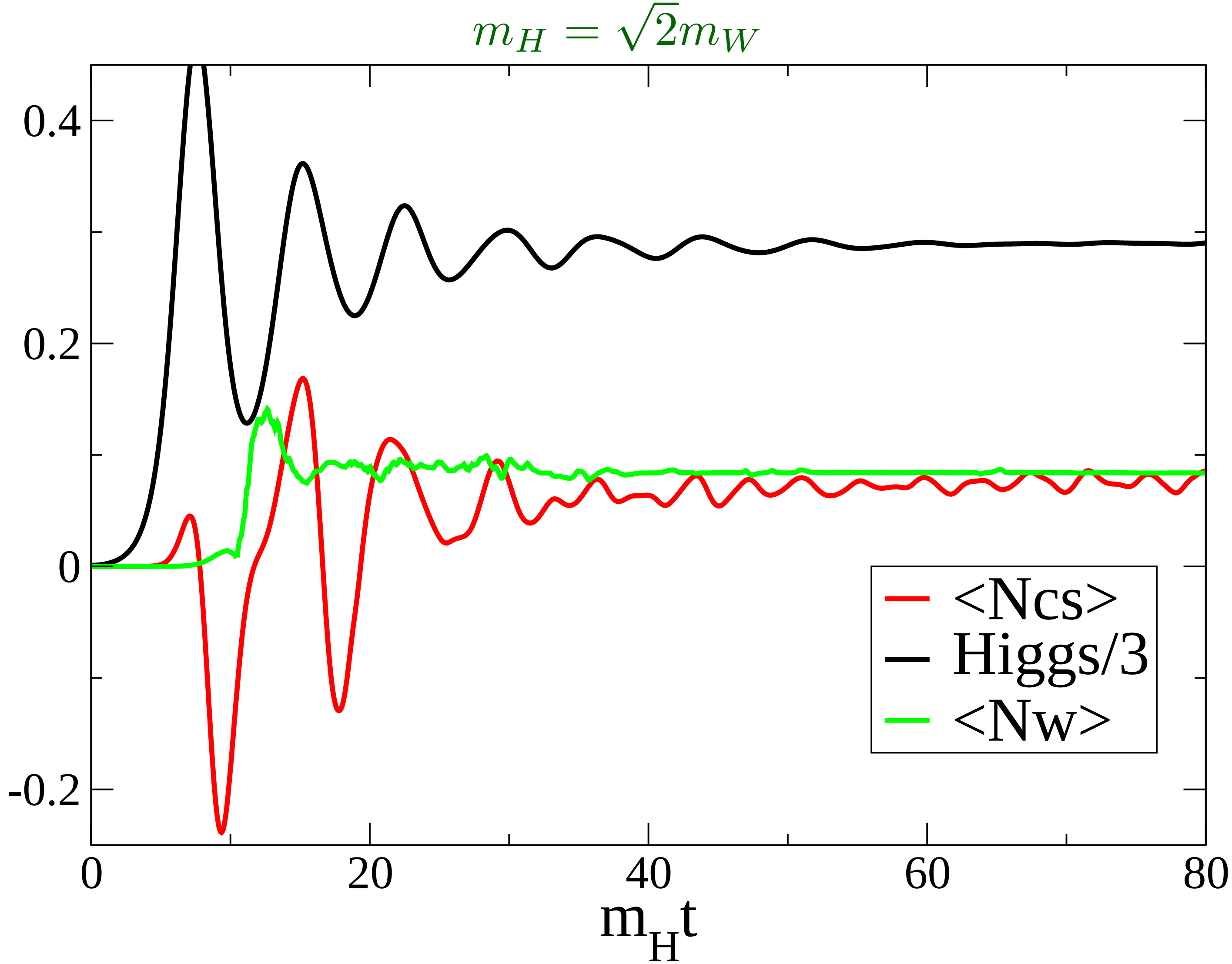
$\delta_{\rm cp} = 1,\, m_H = 2m_W.$
0.4
0.2
0
-0.2
0
20
40
60
80
$m_H t$
<Ncs>
Higgs/3
<Nw>



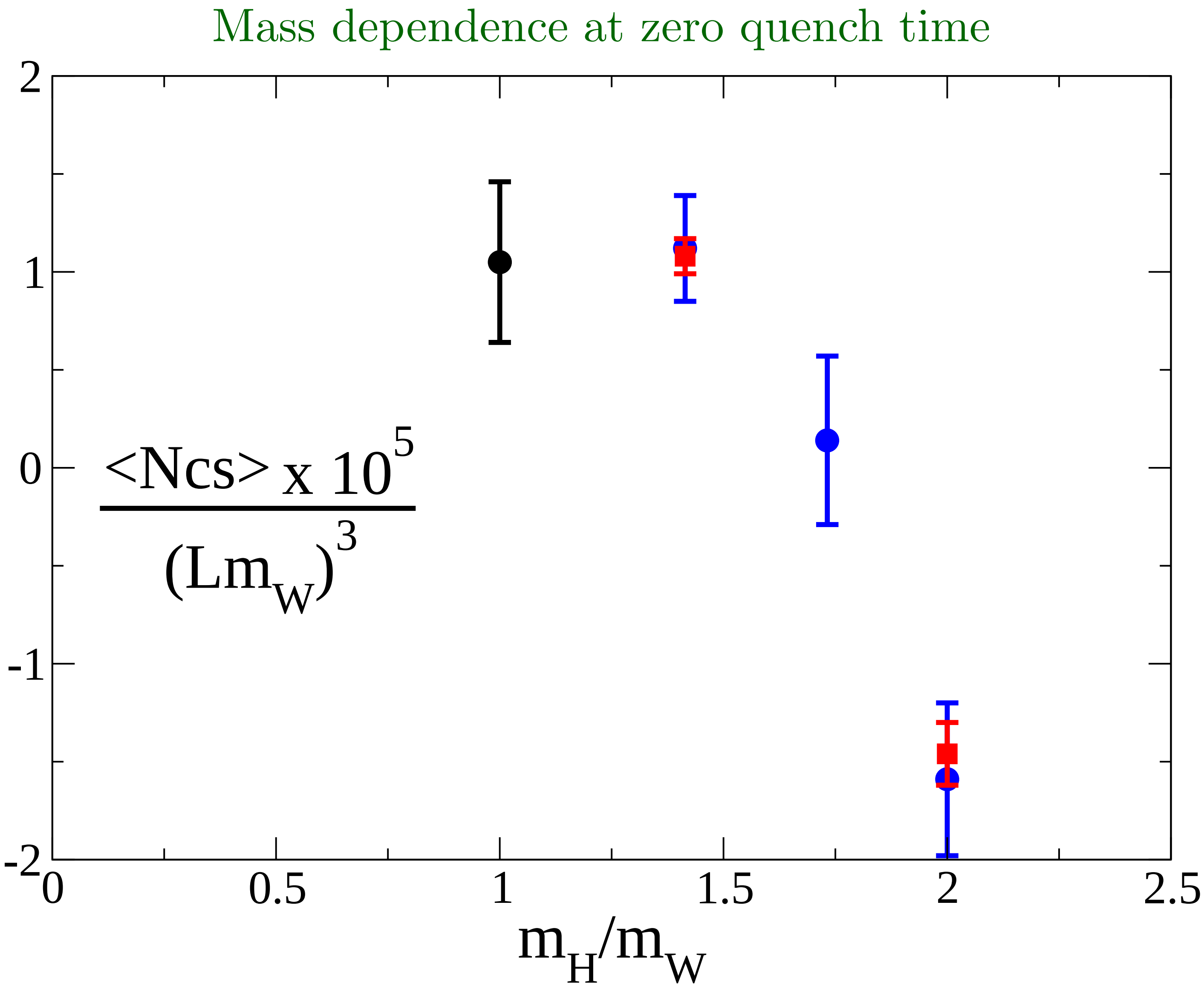
Dependence on δ_{cp}, $m_H = 2m_W$
0
-0.01
-0.02
-0.03
-0.04
-0.05
0
0.2
0.4
0.6
0.8
1
<Nw>
δ_{cp}

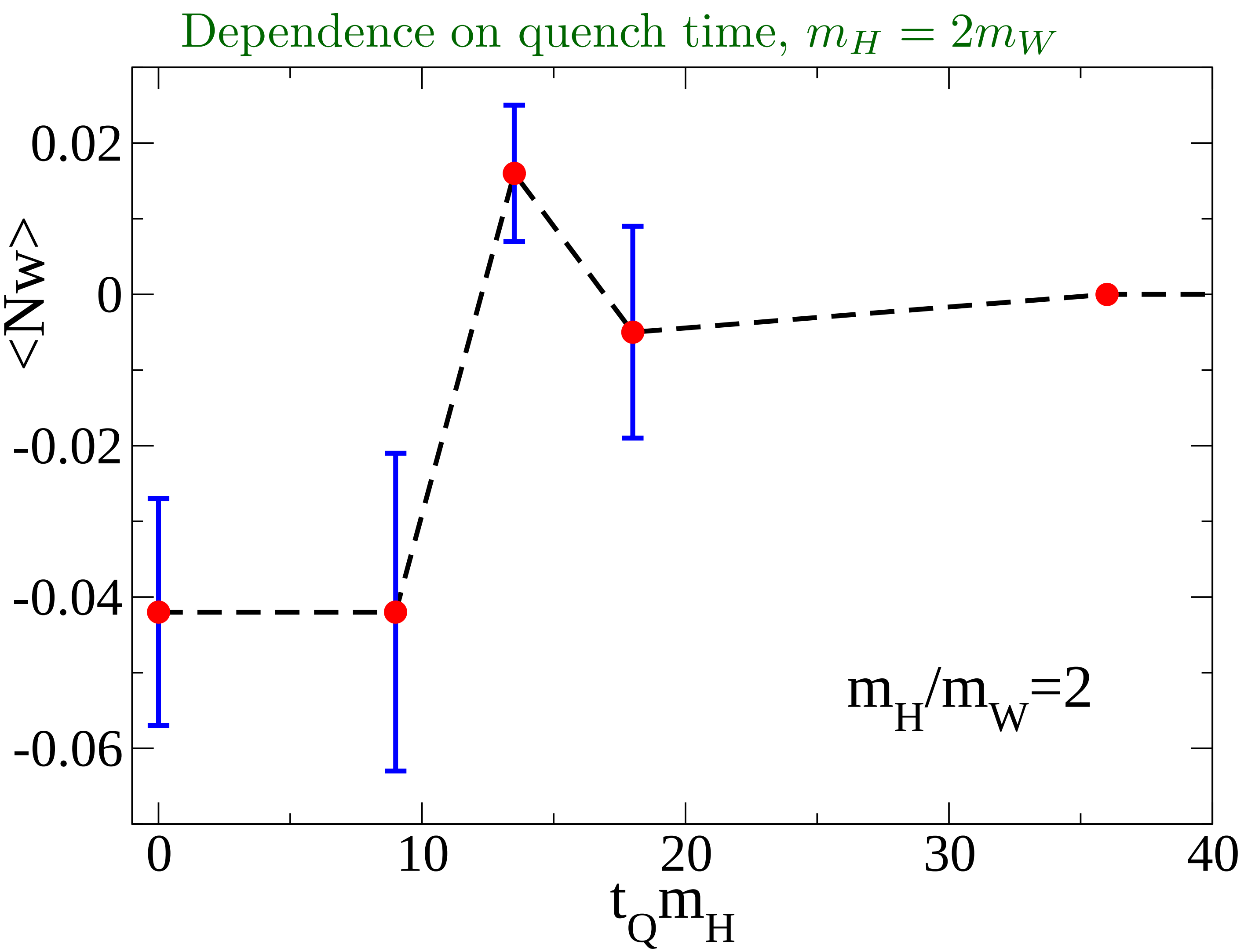


Dependence on $\delta_{\rm cp}$, $m_H = \sqrt{2} m_W$
<Nw>
0.1
0.08
0.06
0.04
0.02
0
0
0.2
0.4
0.6
0.8
1
$\delta_{\rm cp}$

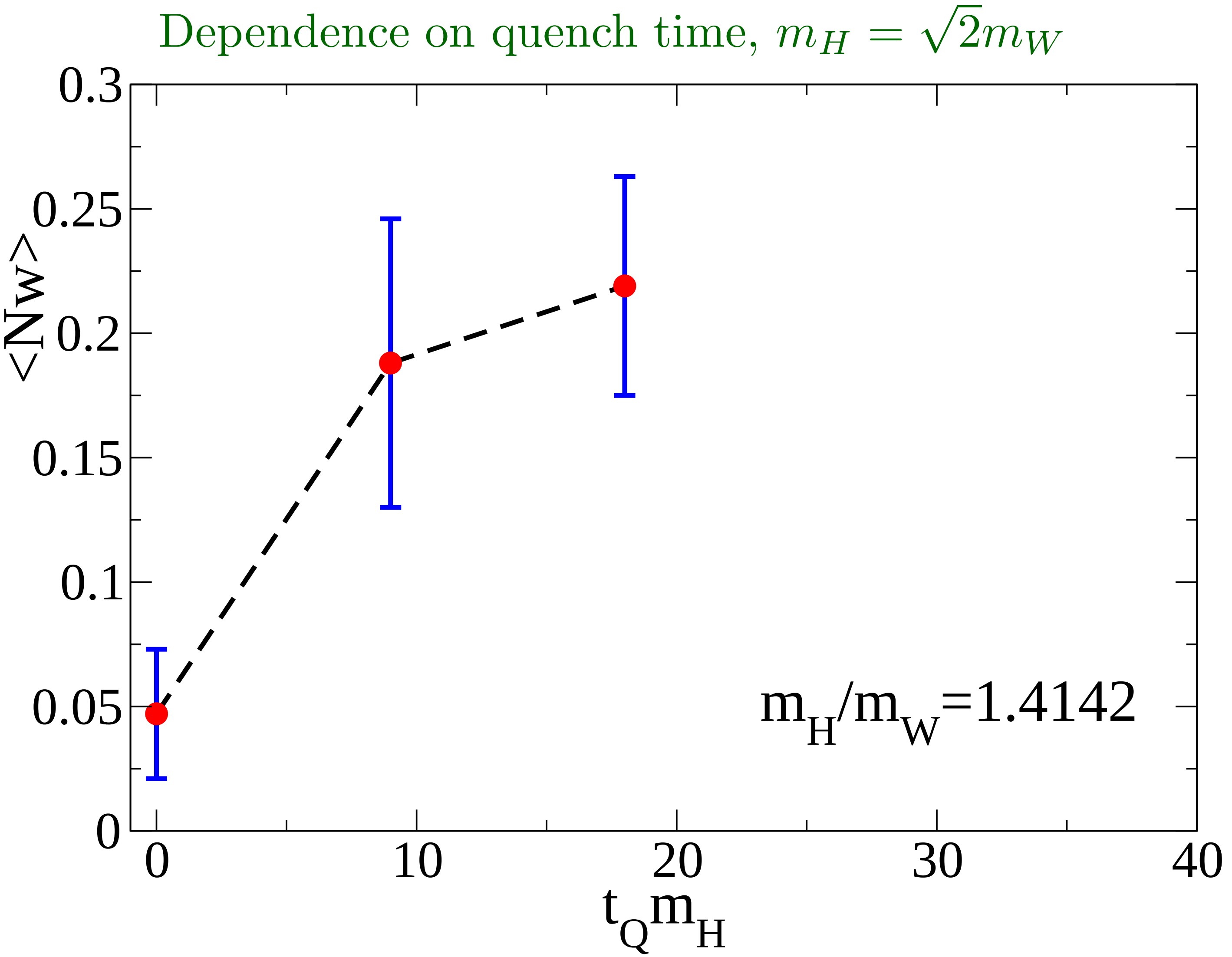


$m_H = \sqrt{2} m_W$
0.4
0.2
0
-0.2
0
20
40
60
80
$m_H t$
<Ncs>
Higgs/3
<Nw>


Mass dependence at zero quench time
2
1
0
-1
-2
0
0.5
1
1.5
2
2.5
$\frac{\langle Ncs \rangle \times 10^5}{(Lm_W)^3}$
m_H/m_W

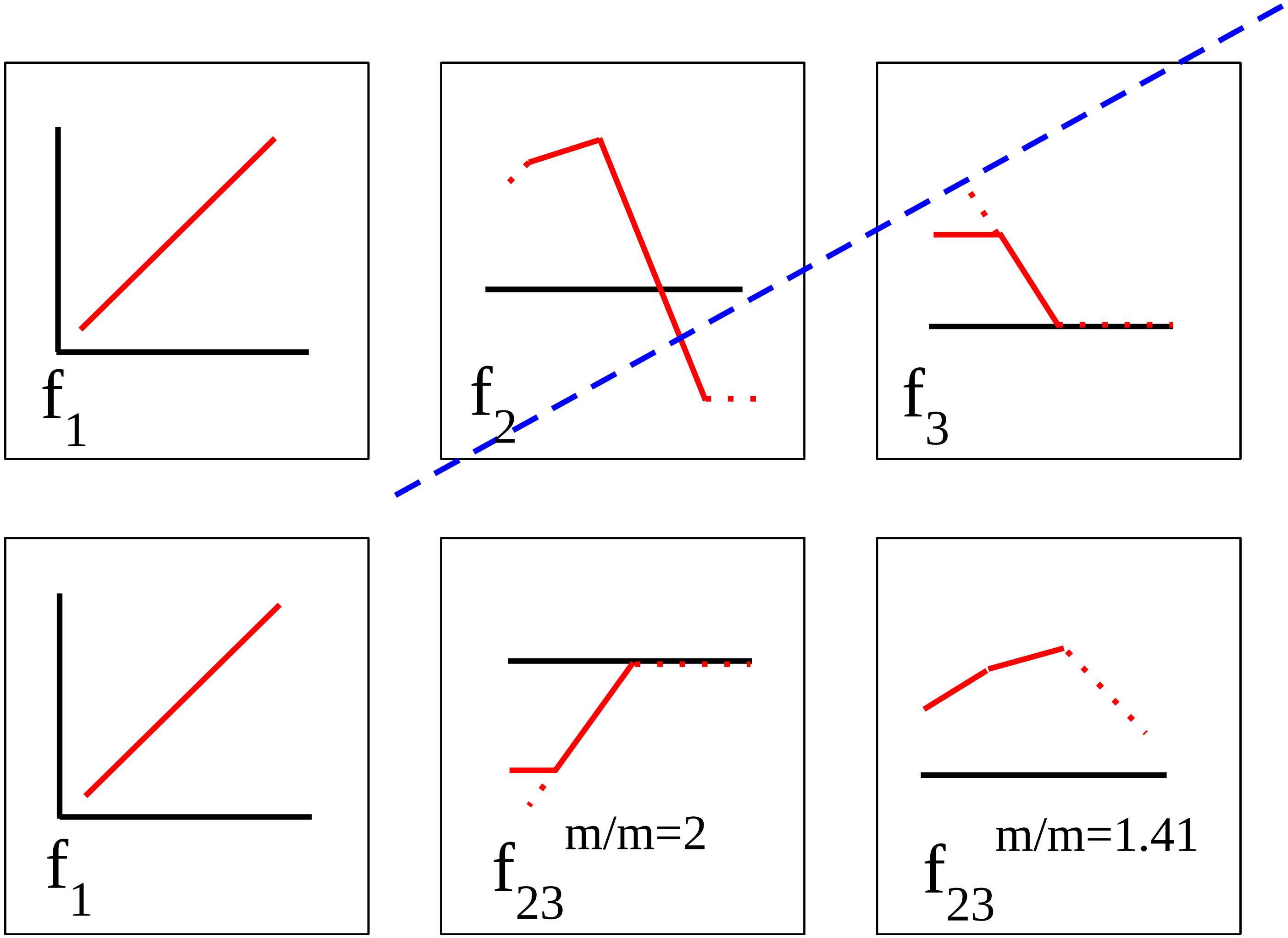


Dependence on quench time, $m_H = 2m_W$
0.02
0
-0.02
-0.04
-0.06
<Nw>
0
10
20
30
40
$t_Q m_H$
$m_H/m_W=2$



Dependence on quench time, $m_H = \sqrt{2} m_W$
0.3
0.25
0.2
0.15
0.1
0.05
0
<Nw>
0
10
20
30
40
$t_Q m_H$
m_H/m_W=1.4142

Factorisation



Final asymmetry

$$\langle B(t) - B(0)\rangle = 3\langle N_{\rm cs}(t) - N_{\rm cs}(0)\rangle, \;\; n_B = \frac{\langle B(t) - B(0)\rangle}{V}.$$

$$\frac{n_B}{n_\gamma} = 7.04\,\frac{n_B}{s}, \;\; s = \frac{2\pi^2}{45} g^* T^3, \;\; \frac{\pi^2}{30} g^* T^4 = V_0 = \frac{m_H^4}{16\lambda}.$$

$$\begin{aligned} \frac{n_B}{n_\gamma} &= -(0.46 \pm 0.08) \times 10^{-4}\,\delta_{\rm cp}, \;\; (m_H = 2m_W, t_Q = 0), \\ &= (0.40 \pm 0.03) \times 10^{-4}\,\delta_{\rm cp}, \;\; (m_H = \sqrt{2} m_W, t_Q = 0). \end{aligned}$$

To reproduce the observed asymmetry, we require

$$\begin{aligned} \delta_{\rm cp} &\simeq -1.5 \times 10^{-5}, \;\; (m_H = 2m_W, t_Q = 0), \\ &\simeq 1.6 \times 10^{-5}, \;\; (m_H = \sqrt{2} m_W, t_Q = 0). \end{aligned}$$

Conclusion

- Including CP-violation in the gauge-Higgs equations of motion results in a net asymmetry in Chern-Simons number.
- $\delta_{\rm cp}$-dependence is linear for small enough $\delta_{\rm cp}$. $f_1(\delta_{\rm cp})$ ok!
- The dependence on quench time is not monotonic but qualitatively understood(?) Dependence does not separate from...
- ...the dependence on the Higgs mass. Is also not monotonic; the overall sign depends on it.

• Viable CEB requires fast quenches; $t_Q < 18\, m_H^{-1}$.
• Necessary $\delta_{\rm cp} \simeq 10^{-5}$ can be amply accomodated in generic SUSY (or just add a second Higgs field). But probably not SM [Shaposhnikov:1987] (lepton sector?).
• Sensitiveness to Higgs mass and quench time means corrections from including all SM fields and dynamical inflaton may be crucial.

Workshop on

Classical Field Theory and Solitons,

Center for Mathematical Sciences,
University of Cambridge,
3.-6. July, 2006

www.damtp.cam.ac.uk/raid/gr/CFT