Light-Front Realization of Chiral Symmetry Breaking

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based on

- Itakura & Maedan, Prog. Theor. Phys. 97 (1997) 635
- Itakura, Prog. Theor. Phys. 98 (1997) 527
- Itakura & Maedan, Phys. Rev. D61 (2000) 045009
- Itakura & Maedan, Phys. Rev. D62 (2000) 0850xx (in press)
- Itakura & Maedan, Invited paper for PTP (in preparation)

Plan of the talk

1. Introduction
2. Chiral Symmetry on the LF
3. Characteristic Constraints and Gap Eq.
4. Consequence of the coexistence of $\langle\bar{\psi}\psi\rangle \neq 0$ and the trivial vacuum
5. LC wavefunctions and Masses of π and σ
6. Conclusion

Introduction

- Light-Front Quantization

  Quantize systems with x^+ being "time"

$$
\begin{aligned}
x^+ &= (x^0 + x^3)/\sqrt{2} && \text{time} \\
x^- &= (x^0 - x^3)/\sqrt{2} && \text{space (longitudinal)} \\
x_\perp^i &= x^1,\ x^2 && \text{space (transverse)}
\end{aligned}
$$

  ex) equal x^+ commutator for a scalar field

$$[\phi(x^-, x_\perp; x^+), \pi(y^-, y_\perp; x^+)] = \frac{i}{2}\delta(x^- - y^-)\delta^{(2)}(x_\perp - y_\perp)$$

- Advantages

  – Boost transf. becomes kinematical
  $\Longrightarrow$ Relativistic Hamiltonian Formulation

  – Simplicity of the vacuum $|\text{vac}\rangle \simeq |0\rangle$

$$p^- = \frac{p_\perp^2 + M^2}{2p^+} \geq 0 \quad \Longrightarrow \quad p^+ \geq 0$$

  Particle modes ($p^+ \neq 0$) cannot form totally ZERO momentum states.

  – Valence picture for excited states (CQM from QCD?)

Big Question: Where is "Vacuum Physics"?

1. Symmetry Breaking
   - Spontaneous sym. br. $\langle \phi \rangle \neq 0$
   - Dynamical chiral sym. br. $\langle \bar{\Psi}\Psi \rangle \neq 0$
   - Higgs mechanism
2. θ Vacua $|\theta\rangle = \Sigma\, e^{in\theta}|n\rangle$
3. Gluon condensate $\langle FF \rangle \neq 0$
4. Confinement (?)

“Vacuum Physics” on the LF

$\Longrightarrow$ Physics around $P^+ = 0$ Modes

- The only point where the vacuum triviality may break down. (ZMs can mix with the Fock vacuum)

- How to deal with the $P^+ = 0$ modes?

  1. Exact Zero-Mode Approach:
     Struggle with them in the DLCQ approach

  2. Counter-Term Approach:
     Avoid the ZMs by infrared cutoff $P^+ > \epsilon$
     $\times$ Need nonperturbative renormalization

  3. Near-LC Approach:
     Define LFQ as a limit of near-LC coordinates
     $\times$ Lose the simple vacuum

This talk: 1. Exact ZM Approach and its variant

Chiral Symmetry on the Light Front

- On the LF, half of the fermion is a dependent dof.

$$\psi_\pm = \Lambda_\pm \Psi, \quad \Lambda_\pm = \gamma^\mp \gamma^\pm / 2$$

$$\bar{\Psi} i \partial\!\!\!/ \Psi = i\psi_+^\dagger \partial_+ \psi_+ + i\psi_-^\dagger \partial_- \psi_- + \cdots$$

$\Longrightarrow$ "Fermionic Constraint"

- LF Chiral transformation

$$\begin{array}{rl} \text{Definition :} & \psi_+ \longrightarrow e^{i\gamma_5\theta}\psi_+ \\ \text{ETquantization} & \Psi \longrightarrow e^{i\gamma_5\theta}\Psi \end{array} \qquad (1)$$

In general, "LF chiral sym."$\neq$ usual chiral sym.

cf) Massive free fermion is invariant under (1)!!

The Models

- The NJL model $\quad \Psi^a \ (a = 1, \ldots, N) : U(N)$ fermion

$$\mathcal{L}_{\rm NJL} = \bar{\Psi}(i\partial\!\!\!/ - m_0)\Psi + \frac{g^2}{2}\left[(\bar{\Psi}\Psi)^2 + (\bar{\Psi}i\gamma_5\Psi)^2\right]$$

- The Chiral Yukawa model $\quad (g^2 N = g_0^2)$

$$\begin{aligned}\mathcal{L}_{\rm Yukawa} &= \bar{\Psi}(i\partial\!\!\!/ - m_0)\Psi + \frac{N}{2\mu^2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\pi\partial^\mu\pi) \\ &\quad -\frac{N}{2g_0^2}(\sigma^2+\pi^2) - (\sigma\bar{\Psi}\Psi + \pi\bar{\Psi}i\gamma_5\Psi)\end{aligned}$$

Chiral Yukawa $\longrightarrow$ NJL $\quad (\mu \to \infty)$

- DEPENDENT degrees of freedom
  - NJL $\cdots\cdots$ bad component ψ_-
    $\to$ One (complicated) Fermionic Constraint
  - Chiral Yukawa (in the DLCQ approach)
    $\cdots$ bad component ψ_- and scalar ZMs $\sigma_0, \ \pi_0$.

$$\begin{pmatrix}\sigma_0(x_\perp) \\ \pi_0(x_\perp)\end{pmatrix} = \frac{1}{2L}\int_{-L}^{L} dx^- \ \begin{pmatrix}\sigma(x) \\ \pi(x)\end{pmatrix}$$

One (simple) Fermionic Constraint
Two Zero-Mode Constraints.

Characteristic Constraints

- Fermionic Constraint in the NJL model

  In two-component representation,

$$\text{good} \quad \psi_+ = 2^{-1/4}\begin{pmatrix}\psi\\0\end{pmatrix}, \quad \text{bad} \quad \psi_- = 2^{-1/4}\begin{pmatrix}0\\\chi\end{pmatrix}$$

$$\begin{aligned} i\partial_-\chi_a = & \frac{1}{\sqrt{2}}\Big(-\sigma^i\partial_i + m_0\Big)\psi_a \\ & -\frac{g^2}{2}\left[\Big(\psi_b^\dagger\chi_b + \chi_b^\dagger\psi_b\Big)\psi_a + \Big(\psi_b^\dagger\sigma_3\chi_b - \chi_b^\dagger\sigma_3\psi_b\Big)\sigma_3\psi_a\right] \end{aligned}$$

- Constraints in the chiral Yukawa model

  Fermionic Constraint

$$i\partial_-\chi = \frac{1}{\sqrt{2}}\Big(-\sigma^i\partial_i + m_0 + \sigma(x) - i\pi(x)\sigma_3\Big)\psi$$

  Zero-Mode Constraints

$$\left(\frac{\mu^2}{g_0^2} - \partial_\perp^2\right)\sigma_0(x_\perp) + \frac{\mu^2}{N}\int_{-L}^{L}\frac{dx^-}{2L}\,\frac{1}{\sqrt{2}}\left(\psi^\dagger\chi + \chi^\dagger\psi\right) \approx 0$$

$$\left(\frac{\mu^2}{g_0^2} - \partial_\perp^2\right)\pi_0(x_\perp) + \frac{\mu^2}{N}\int_{-L}^{L}\frac{dx^-}{2L}\,\frac{i}{\sqrt{2}}\left(\chi^\dagger\sigma_3\psi - \psi^\dagger\sigma_3\chi\right) \approx 0$$

Quantum and Nonperturbative Solution

Various problems are resolved by solving the ZM/fermionic constraints.

$\Longrightarrow$ QUANTUM and NONPERTURBATIVE $(1/N)$ solutions

- Nonzero condensate.
- Properties of the LF Chiral Transformation
- Reconcilation between the nonzero condensate and the trivial vacuum

Nonzero Condensate

Nonzero Condensate is obtained by nontrivial solution of the constraints

ZM should be expressed by non-zero modes.

$$\sigma_0 = \sigma_0^{(\mathrm{c})} + \sigma_0^{(\mathrm{op})}(\varphi_\sigma, \varphi_\pi)$$

$\sigma_0^{(\mathrm{c})} \cdots\cdots$ c-number part
$\sigma_0^{(\mathrm{op})} \cdots\cdots$ operator part

If we had a solution with NONZERO $\sigma_0^{(\mathrm{c})}$, then

$$\langle 0|\, \sigma \,|0\rangle = \langle 0|\, \sigma_0 \,|0\rangle = \sigma_0^{(\mathrm{c})} \neq 0 \Longrightarrow \text{SSB!!}$$

even with the Fock vacuum!

Gap Equation

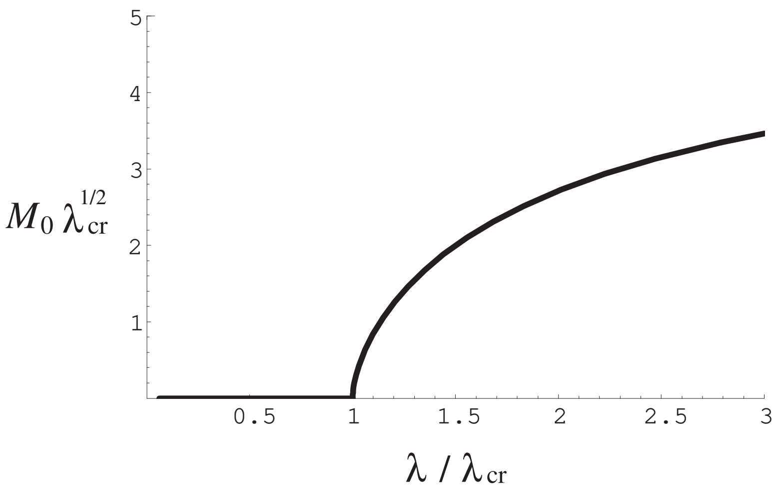
Constraints goes to the gap equation in quantum and nonperturbative treatment.

Fermionic and zero-mode constraints in the leading order of $1/N$

$$\frac{M - m_0}{M} = \frac{g^2 N \Lambda^2}{4\pi^2}\left\{2 - \frac{M^2}{\Lambda^2}\left(1 + \ln\frac{2\Lambda^2}{M^2}\right)\right\}$$

Dynamical mass $\quad M = m_0 - g^2\langle\bar{\Psi}\Psi\rangle.$

Chiral limit



Chiral Transformation in the Broken Phase

1. Classical solutions of the constraints tell that the LF chiral transformation is equivalent to the usual one.
2. Quantum solutions show modified transformation law for the dependent variables:

   Solution of the gap equation (constraints) $\cdots$ two choices

   - symmetric solution $\chi = \chi_{\rm sym}$
     Everything is equivalent to the usual chiral transf.
   - broken solution $\chi = \chi_{\rm br}$
     In LF-NJL with the broken solution, the transformation law itself changes.

   $$\left[\bar{\Psi} i\gamma_5 \Psi, Q_5^{\rm LF}\right] = 2i\bar{\Psi}\Psi + \Delta, \quad \Delta \propto M$$

3. Multiple Hamiltonians with a single trivial vacuum

Symmetric Hamiltonian	$H_{\rm sym} = H(\psi, \chi = \chi_{\rm sym})$
Broken Hamiltonian	$H_{\rm br} = H(\psi, \chi = \chi_{\rm br})$

4. Nonconservation of the Chiral Charge

   $H_{\rm br}$ is not invariant under the LF chiral transformation.

   $$\left[H_{\rm br.}, Q_5^{\rm LF}\right] \propto M \neq 0$$

LC Wavefunctions of σ and π

Solve the bound-state equation for $q\bar{q}$ state at $P_\perp = 0$:

$$H_{\rm br} \mid \text{meson}; P\,\rangle = \frac{M^2_{\rm meson}}{2P^+} \mid \text{meson}; P\,\rangle$$

where

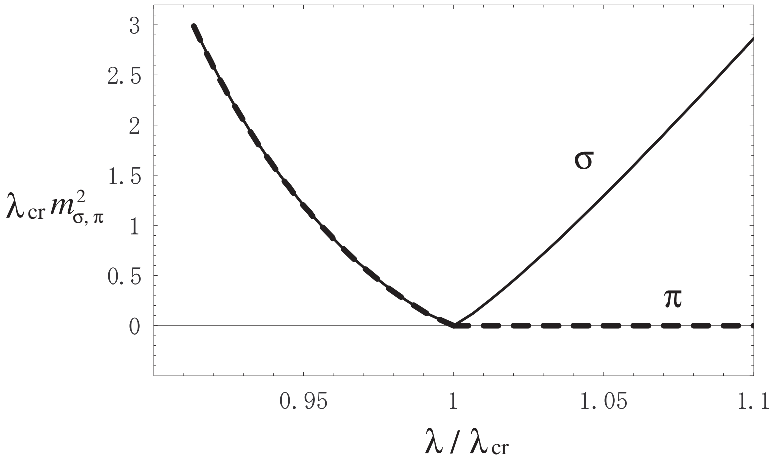
$$\mid \text{meson}; P\,\rangle = \frac{1}{\sqrt{N}} \int [d^3k]\ \Phi^{\alpha\beta}(\boldsymbol{k}) b^{a\dagger}_{\alpha}(\boldsymbol{k}) d^{a\dagger}_{\beta}(\boldsymbol{P}-\boldsymbol{k})\ |0\,\rangle$$

LC wavefunctions

$$\begin{aligned}
\Phi^{\alpha\beta}_{\pi}(x,k_\perp) &= -C_\pi \left\{ \left(ik^i_\perp \sigma^i + M \right) \sigma^3 \right\}_{\alpha\beta} \\
&\quad \times \frac{g_0^2}{(2\pi)^3} \frac{M}{m_0} \frac{1}{x(1-x) - (k_\perp^2 + M^2)/m_\pi^2} \,, \\
\Phi^{\alpha\beta}_{\sigma}(x,k_\perp) &= -C_\sigma \left\{ ik^i_\perp \sigma^i + (1-2x)M \right\}_{\alpha\beta} \\
&\quad \times \frac{g_0^2}{(2\pi)^3} \frac{M}{m_0} \frac{m_\sigma^2 - 4M^2}{m_\sigma^2} \frac{1}{x(1-x) - (k_\perp^2 + M^2)/m_\sigma^2} \,,
\end{aligned}$$

Masses of σ and π in the NJL

Chiral limit ($m_0 = 0$)



reproduce the reasonable behavior of m_π and m_σ

Conclusion

1. To find a broken Hamiltonian is essential instead of finding a broken vacuum. Vacuum effects are converted into Hamiltonians.
2. To solve the FC and ZMC was important. Especially, the ZM projection of FC is identified with the gap equation.
3. For $m_0 = 0$, LF chiral transf. $=$ ET chiral transf.
4. Chiral Transformation in the broken phase gets modified through the (nontrivial) bad component
5. Nonconservation of the Chiral Charge, or equivalently, the non-invariant Hamiltonian under the LF chiral transformation
6. Masses and LC Wavefunctions of σ and π are calculated in the NJL model.