# Influence of inter-electrode coupling on BPM performance.

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# Influence of inter-electrode coupling on BPM performance.

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**1. Introduction.** Parasitic coupling between BPM electrodes is considered as an important factor in BPM design. Demand to provide coupling in the range of several percent (at operating frequency) limits maximum subtended angle of the electrodes thus reducing the BPM sensitivity or leads to complicated BPM design with separators [1]. The aim of this paper is to calculate effect of electrode coupling on BPM accuracy and find criteria for choice of acceptable coupling. It is shown that effect of coupling on the accuracy of coordinate reconstruction depends on the processing algorithm. Inter – electrode coupling can be also responsible for accuracy of phase measurements and it may the main reason for strong dependence of measured phase upon beam position observed in numeric simulation [1] and not understood yet.

**2. Coupling and beam position measurement accuracy.** Consider BPM with four electrodes named as T, L, R, B (top, left, right and bottom) as shown in fig. 1. Voltage measured on each of the electrodes is:

$$
\begin{aligned}
u_T &= k_{TT} \cdot I_T + k_{TR} \cdot I_R + k_{TB} \cdot I_B + k_{TL} \cdot I_L \\
u_R &= k_{RT} \cdot I_T + k_{RR} \cdot I_R + k_{RB} \cdot I_B + k_{RL} \cdot I_L \\
u_B &= k_{BT} \cdot I_T + k_{BR} \cdot I_R + k_{BB} \cdot I_B + k_{BL} \cdot I_L \\
u_L &= k_{LT} \cdot I_T + k_{LR} \cdot I_R + k_{LB} \cdot I_B + k_{LL} \cdot I_L
\end{aligned}
$$

where $u_i$ is voltage on i-th electrode and $k_{i,j}$ is coupling impedance between voltage on i-th electrode and current $I_j$ through j-th electrode.

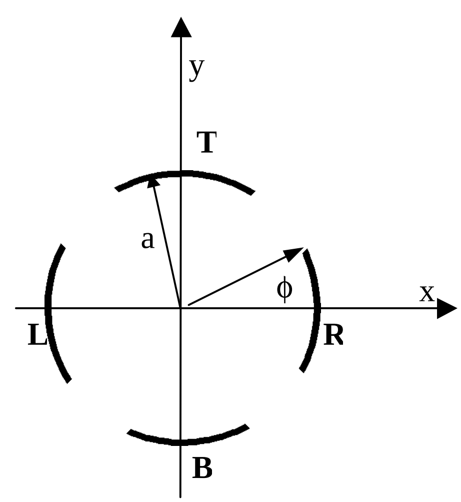


Figure 1. A simplified model of a 4-lobe strip line BPM.

If BPM has four fold symmetry, then

$$\begin{aligned}
&k_{TT} = k_{RR} = k_{BB} = k_{LL} = k_0 \\
&k_{TR} = k_{RT} = k_{LT} = k_{TL} = k_{LB} = k_{BL} = k_{RB} = k_{BR} = k_1 \\
&k_{TB} = k_{BT} = k_{LR} = k_{RL} = k_2
\end{aligned}$$

Beam position can be calculated using "difference over sum" algorithm, then vertical difference $D_v$ , sum S and its ratio are:

$$D_V = u_T - u_B = k_0 \cdot I_T + k_1 \cdot I_R + k_2 \cdot I_B + k_1 \cdot I_L - k_0 \cdot I_B - k_1 \cdot I_R - k_2 \cdot I_T - k_1 \cdot I_L =$$

$$= (k_0 - k_2)(I_T - I_B)$$

$$S = u_T + u_B + u_L + u_R = (I_T + I_B + I_L + I_R)\,(k_0 + 2\,k_1 + k_2)$$

$$R_V = \frac{D_V}{S} = \frac{(I_T - I_B)(k_0 - k_2)}{(I_T + I_B + I_L + I_R)\,(k_0 + 2\,k_1 + k_2)} = R_{V0} \cdot K\,,$$

where $R_{v0}$ is difference over sum ratio in absence of coupling and K is a correction coefficient due to coupling. As one can see this coefficient

$$K = \frac{1 - \frac{k_2}{k_0}}{1 + 2\,\frac{k_1}{k_0} + \frac{k_2}{k_0}}$$

is completely defined by BPM geometry through coupling impedances and it doesn't depend on beam position. Therefore inter-electrode coupling doesn't affect BPM accuracy and linearity directly but reduces its sensitivity. Note that reduction of sensitivity can be overcome by increasing of electrode subtended angle. For example, in Ref.1, p.5 coupling coefficients are numerically calculated for electrodes with 45° and 60° subtended angles. In that case larger electrode increases signal power by 2.3dB while larger coupling reduce sensitivity by .4dB only. More important may be increase of BPM linearity with larger electrodes.

If beam position calculated using "log of ratio" algorithm then

$$R_V = \log\frac{u_T}{u_B} = \log\frac{k_0\cdot I_T + k_1\cdot I_R + k_2\cdot I_B + k_1\cdot I_L}{k_0\cdot I_B + k_1\cdot I_R + k_2\cdot I_T + k_1\cdot I_L} = \log\frac{I_T}{I_B} + \log\frac{1+\frac{k_1}{k_0}\cdot\frac{I_R+I_L}{I_T}+\frac{k_2}{k_0}\cdot\frac{I_B}{I_T}}{1+\frac{k_1}{k_0}\cdot\frac{I_R+I_L}{I_B}+\frac{k_2}{k_0}\cdot\frac{I_T}{I_B}} =$$

$$= R_{V0} + dR_V\,.$$

Correction

$$dR_V = \log\frac{1+\frac{k_1}{k_0}\cdot\frac{I_R+I_L}{I_T}+\frac{k_2}{k_0}\cdot\frac{I_B}{I_T}}{1+\frac{k_1}{k_0}\cdot\frac{I_R+I_L}{I_B}+\frac{k_2}{k_0}\cdot\frac{I_T}{I_B}}$$

is not as simple as in the previous case and depends on beam position, therefore BPM linearity will be affected by coupling. In the case of weak coupling, assuming $k_1 << 1, \quad k_2 << 1$,

$$dR_V = \log\frac{1+\frac{k_1}{k_0}\cdot\frac{I_R+I_L}{I_T}+\frac{k_2}{k_0}\cdot\frac{I_B}{I_T}}{1+\frac{k_1}{k_0}\cdot\frac{I_R+I_L}{I_B}+\frac{k_2}{k_0}\cdot\frac{I_T}{I_B}} = \log(1+\frac{k_1}{k_0}\cdot\frac{I_R+I_L}{I_T}+\frac{k_2}{k_0}\cdot\frac{I_B}{I_T}) - \log(1+\frac{k_1}{k_0}\cdot\frac{I_R+I_L}{I_B}+\frac{k_2}{k_0}\cdot\frac{I_T}{I_B})$$

$$dR_V \approx \frac{k_1}{k_0}\cdot\frac{I_R+I_L}{I_T}+\frac{k_2}{k_0}\cdot\frac{I_B}{I_T}-\frac{k_1}{k_0}\cdot\frac{I_R+I_L}{I_B}-\frac{k_2}{k_0}\cdot\frac{I_T}{I_B} = \frac{I_B-I_T}{I_T I_B}(k_1(I_R+I_L)+k_2(I_B+I_T))$$

then, assuming $I_R + I_L \approx I_T + I_B = S$, $I_T - I_B = D$, we have finally:

$$R_V \approx R_{V0} - \frac{2D}{S}\cdot\frac{k_1+k_2}{1-\frac{D^2}{S^2}} \approx R_{V0}\left(1-\frac{k_1+k_2}{1-\frac{R_{V0}^2}{4}}\right) \approx R_{V0}(1-k_1-k_2)\,.$$

In the first order coupling leads to decreasing of sensitivity to beam position. Effect of coupling on BPM non-linearity have to be calculated numerically.

**3. Effect of coupling on phase measurements.** In order to investigate how inter–electrode coupling affects phase measurements consider simplified case of BPM with two electrodes. Its equivalent scheme is shown in fig.3 where current source represents beam, $Z_0$ is impedance of strip line and Z is coupling impedance. Output voltage can be found by simple circuit analysis:

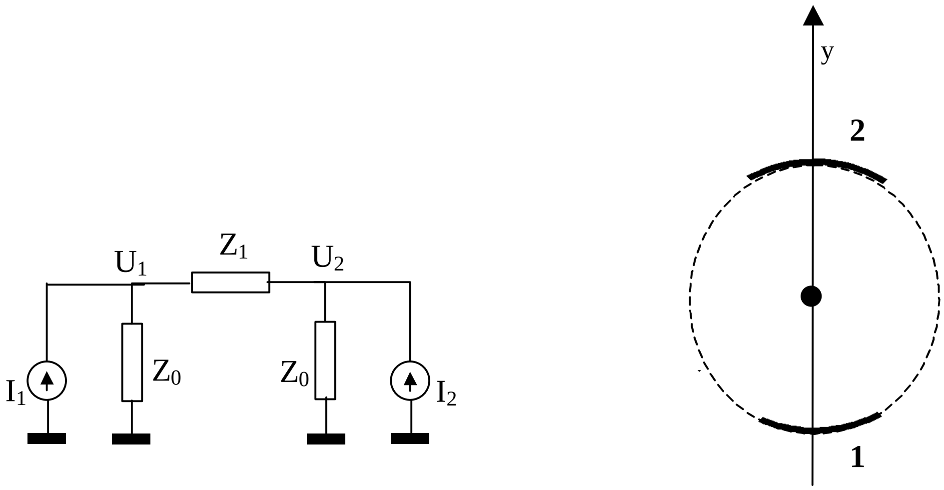


Figure 3. Equivalent scheme of two-electrode pickup with coupling.

$$U_1 = I_1 Z_0 \frac{1+X}{1+2X} + I_2 Z_0 \frac{X}{1+2X} \quad , \qquad (1)$$

$$U_2 = I_1 Z_0 \frac{X}{1+2X} + I_2 Z_0 \frac{1+X}{1+2X} \quad ,$$

where $X = \frac{Z_0}{Z}$ is normalized coupling conductivity. Defining $R = \frac{I_2 - I_1}{I_2 + I_1}$ and $S = I_2 + I_1$ we have for $I_1$ and $I_2$:

$$I_1 = \frac{S(1-R)}{2}, \quad I_2 = \frac{S(1+R)}{2}$$

Substituting it in (1) we have:

$$U_1 = \frac{Z_0 S}{2}\left(1 - \frac{R}{1+2X}\right),$$

$$U_2 = \frac{Z_0 S}{2}\left(1 + \frac{R}{1+2X}\right).$$

Phase difference is:

$$\Delta j = \arg U_2 - \arg U_1 \ .$$

If we assume that currents $I_1$ and $I_2$ have equal phases, which should be true at least in ultrarelativistic limit, then phase difference between $U_1$ and $U_2$ is completely defined by imaginary part of coupling conductivity, which is combination of capacitive and inductive coupling only and therefore pure imaginary. In this case

$$\Delta j = arctg\frac{2|X|R}{1+4|X|^2-R} + arctg\frac{2|X|R}{1+4|X|^2+R}$$

for small phase shifts $\Delta j << 1$ we can assume $arctg(x) \approx x$ , then

$$\Delta j = \frac{2|X|R}{1+4|X|^2-R} + \frac{2|X|R}{1+4|X|^2+R} = 4|X|R\frac{1+4|X|^2}{(1+4|X|^2)^2-R^2} \approx \frac{4|X|R}{1-R^2}$$

$R$ can be derived from beam displacement $dy$ in linear approximation as $R = \frac{dy}{k}$, where $k$ is sensitivity to transverse position in “difference over sum” algorithm. Then phase dependence of phase difference upon beam position is

$$\Delta j \approx \frac{4|X|}{k}\frac{dy}{1-\frac{dy^2}{k^2}}$$

Taking for example $k = 20$, $|X| = .01$, which is close to design linac’s BPM parameters we obtain the dependence of phase shift between electrodes shown in fig. 4. When beam is displaced half of aperture then phase difference is about 1.5°, which is not negligible.

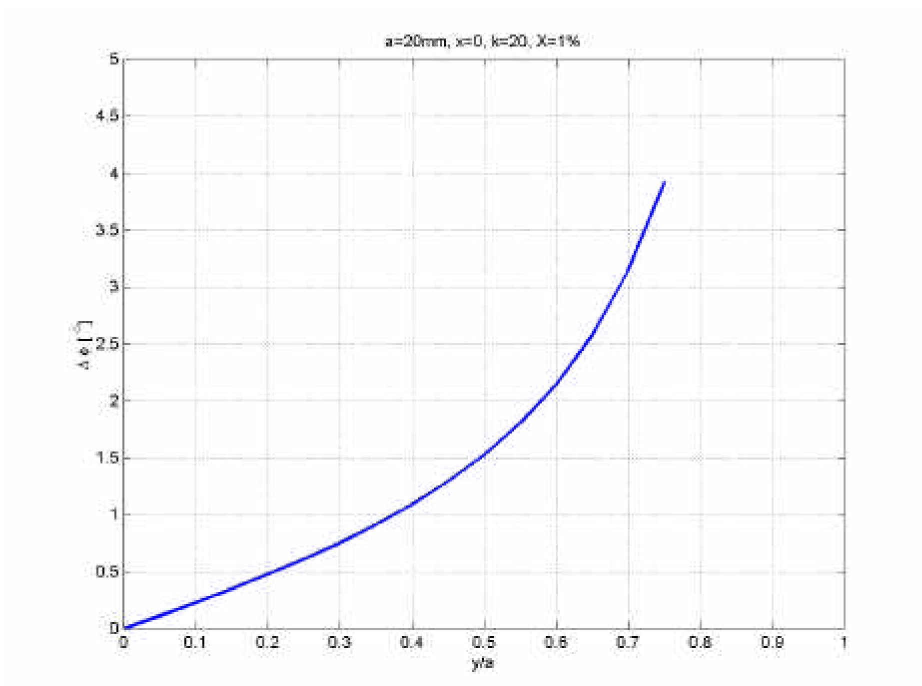


Figure 4. Dependence of phase shift upon beam displacement.

**4. Conclusion.** It is shown that effect of coupling on the accuracy of coordinate reconstruction depends on the processing algorithm. If "difference over sum" algorithm is used then coupling leads to decreasing of sensitivity but doesn't affect BPM linearity. For "log of ratio" algorithm both sensitivity and linearity are affected. Inter – electrode coupling affects accuracy of phase measurements as well. Even in two-electrode configuration it leads to considerable phase difference between electrodes. In dual plane BPM the effect can be bigger due to stronger coupling and it can be evaluated in the same way.

**5. Acknowledgements.** I wish to thank Sergey Kurennoy for useful discussions.

**6. References.**

[1] S. Kurennoy, Beam position monitors for SNS linac. SNS 2000-011 .