COMDISCO SIMULATION RESULTS FOR PCM/PM RECEIVERS IN NON-IDEA], CHANNELS

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Abstract

This paper studies, by computer simulations, the performance of a PCM/PM/NRZ receiver in the presence of two separate effects: unbalanced data stream and band-limited channel. The results obtained are then compared to the theoretical results presented in a previous report.

1 Introduction

A previous report, [1], was written to study the behavior of PCM/PM receivers in non-ideal channels. This paper verifies, by simulation and measurement, the theoretical results presented in [1].

The presence of an imperfect data stream produces undesirable spectral components which degrade the performance of the system. The objective of this paper is to study the separate effects of unbalanced data and InterSymbol Interference (ISI) caused by bandlimited channel on the performance of PCM/PM/NRZ receivers.

The Signal Processing Worksystem (SPW) was used for developing, testing and simulating the system. The Symbol Error Rate (SER) was measured for perfect and imperfect data streams, and the results obtained were compared to the theory presented in [1].

The organization of this paper is as follows: Section 2 describes perfect, unbalanced and bandlimited data streams for PCM/PM/NRZ. Section 3 gives a brief description of the PCM/PM receiver blocks which were used to build the system in SPW, section 4 shows and discusses the simulation results and compares them to theory, and finally, section 5 presents the conclusion of this paper.

2 Description of PCM/PM Data Streams

The deep space received telemetry signal, in the absence of a subcarrier, is given by:

$$s_r(t) = \sqrt{2P}\{\cos(m_T)\cos(\omega_c t + \theta_c) + d(t)\sin(m_T)\sin(\omega_c t + \theta_c)\} + n(t) \tag{1}$$

where P is the transmitted power, m_T is the telemetry modulation index in rad, $\omega_c = 2\pi f_c$ is the angular carrier center frequency in rad/sec, θ_c is the carrier phase, $n(t)$ is an additive white Gaussian noise (AWGN), and $d(t)$ is the data stream (NRZ) with transition density, P_t, less or equal to 0.5 and is defined by

$$d(t) = \sum_{k=-\infty}^{\infty} d_k p(t + kT) \tag{2}$$

where $d_k = \pm 1$ and $p(t)$ is the baseband pulse. The first and second terms of Eq. (1) are the residual carrier and data components, respectively.

2.1 Perfect Data Stream

In a perfect data stream, the probability of transmitting a +1 pulse, p, is equal to the probability of transmitting a - 1, q, with transition density, P_t, equal to 0.5 and given by

$$P_t = 2pq \tag{3}$$

where $q = 1 -$ j').

When the data is l.danced ($p = 0.5$), the carrier term of Eq. (1) generates a spike at f_c with power, P_c, given by

$$P_c = P\cos^2(m_T) \quad (4)$$

and the spectrum generated by the data component has power

$$P_D = P\sin^2(m_T) \quad (5)$$

Combining the carrier and data terms, the power spectrum of a 1 'CM/PM/NRZ perfect data stream is shown in Fig 1 .a. and is given by:

$$S(f) = S_D(f) + S_c(f) \quad (6)$$

2.2 Unbalanced Data Stream

'The imbalance between +1's and -1's in the data stream causes an additional corruption to the received signal in (1) generating undesirable spectral components which degrade the performance of the telemetry system. When p is not equal to 0.5 (and therefore, $P_t < 0,5$), the data component in (1) will be affected and the data spectrum can be written as

$$S_D(f) = P\sin^2(m_T)\{S_{dc}(f) + S_{con't}(f)\} \quad (7)$$

where $S_{dc}(f)$ is the dc (or harmonic) component caused by the imperfect data stream that falls on the RF carrier, and $S_{con't}(f)$ is the continuous data spectrum.

For PCM/PM/NRZ unbalanced data stream, the dc and continuous power spectral density components are found to be [1]

$$S_{dc}(f) = (1 - 2p)^2\delta(f) \quad (8)$$

$$S_{con't}(f) = 4Tp(1-p)\left\{\frac{\sin^2(\pi fT)}{(\pi fT)^2}\right\} \quad (9)$$

Therefore, in addition to the tone generated at f_c by the carrier tom].mnmt in (1) with power given by (3), the spectrum of the unbalanced NRZ data stream will have another tone at f_c generated by the inbal anew between +1's and -- 1's with power given by

$$P_{dc} = (1 - 2p)^2 P\sin^2(m_T) \quad (1o)$$

and the continuous data spectrum will have power given by

$$P_{con't} = 4p(2 - p)P\sin^2(m_T) \quad (11)$$

where

$$P_D = P_{dc} + P_{con't} \quad (12)$$

The spectrum of an unbalanced NRZ data stream is shown in Fig. 1.b for $p = 0.4$.

2.3 Bandlimited Channel

An additional impairment that contributes to the degradation of the overall performance of the system is the InterSymbol Interference (ISI) caused by the bandlimited channel. Bandlimiting causes interference between successive pulses producing the ISI effect which behaves like an additional random noise.

If we let $p(t)$ denote the pulse shape of the data and $h(t)$ denote the impulse response of the equivalent low-pass filter of the RF bandpass filter with bandwidth B, the received data can be expressed as [1]

$$d(t) = \sum_{k=-\infty}^{\infty} d_k g(t + kT_s) \tag{13}$$

where $d_k = \pm 1$ with $p = q = 0.5$, and $g(t)$ is given by

$$g(t) = p(t) * h(t) \tag{14}$$

where $*$ denotes convolution.

The impulse response of an ideal channel $h(t)$ is given by the inverse Fourier transform of the transfer function $H(f)$

$$H(f) = \begin{cases} 1 & -B < f < B \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

resulting in

$$h(t) = 2B\sin(2\pi Bt)/(2\pi Bt) \tag{16}$$

For an ideal filter and a perfect data stream, g_{NRZ} can be found to be [1]

$$g_{NRZ}(t + kT) = \frac{1}{\pi}\left\{Si\left\{2\pi B(t + T(k + 1/2))\right\} - Si\left\{2\pi B(t + T(k - 1/2))\right\}\right\} \tag{17}$$

where

$$Si(x) = \int_0^x \frac{\sin(u)}{u}\,du \tag{18}$$

Fig. 2 snows a plot of the output response of the ideal filter to NRZ pulse ($g_{NRZ}(t/T)$ vs. t/T) for various values of time-bandwidth product BT. Note that the shape of the output is dependent on BT. For $BT >> 1$, the filtering is nonexistent and the output signal is the same as the input. As BT gets closer to 1, the rise and fall times of the output are significant when compared to the input, and the output signal is more spread in time.

3 Description of PCM/PM Receiver Blocks

Fig. 3 shows the block diagram of a PCM/PM receiver. This receiver consists of the TSG (Test Signal Generator), the ARX (Advanced Receiver), and the Error Counter. The TSG, shown in Fig. 4, generates the deep-space spacecraft signal at an intermediate frequency (IF). The TSG's Random Data block controls the parameter p (probability of +1). The TSG parameters are:

sampling rate f_s:	500×10^3 Hz
carrier frequency f_c:	100×10^3 Hz
initial carrier phase UC:	0 deg
symbol rate R_s:	10×10^3 Hz
modulation index m_T:	71,619725 deg (corresponding to 1.25 rad)

and the power-to-noise ratio P/N_0 is calculated using

$$P/N_0 = \frac{E_s}{N_0} - 10\log_{10}(\sin^2 m_T) + 10\log_{10} R_s - 10\log_{10}(4p(1-p)) \qquad (19)$$

where E_s/N_0 is the Symbol SNR in dB. The last term in Eq. (19) is a factor generated by the unbalanced data stream and is equal to 0 dB when $p = 0.5$. This factor shows a reduction in the transmitting power when the data stream is unbalanced.

The ARX block, shown in Fig. 5, consists of the Carrier PLL (Phase Lock Loop) block which estimates the carrier phase, the Digital DTTL (Data Transition Tracking Loop) block which generates the symbol timing reference, the Sum Dump 1101(1 Symbol block which outputs the soft symbols, and a Butterworth Lowpass Filter. Finally, the Error Counter block compares the soft symbols of the ARX to the transmitted symbols and outputs the number of errors N.

4 Discussion and simulation results

Using SPW, simulations were performed at 7, 8, 9 and 10 dB Symbol SNR, E_s/N_0, and the corresponding P/N_0 was calculated. The result of each simulation was the number of errors N (produced by the Error Counter as a result of comparing the soft symbols to the transmitted ones). The average error probability P_e was then calculated using

$$P_e = \frac{N}{\text{Number of Iterations}/(f_s/R_s)} \qquad (20)$$

where f_s is the sampling frequency in Hz, the fraction $\frac{f_s}{R_s}$ is the number of samples per symbol, and

$$\text{Number of Iterations} = \frac{100\, f_s}{SER\, R_s} \qquad (21)$$

where SER is the Symbol Error Rate as given by the theory. Finally, P_e was plotted vs. Symbol SNR and the results were compared to the theory presented in [1].

4.1. Unbalanced Data

To verify the performance of the receiver in the presence of an unbalanced data stream, simulations were performed for $p = 0.5, 0.45$ and 0.4.

For PCM/PM/NRZ, the simulation results are shown in Table 1 and Fig. 6. When p is equal to 0.5 (balanced data stream), the simulation results are worse than, but close to the theoretical results (less than 0.125 dB difference). When p deviates from 0.5 (unbalanced data case), the simulation results were different than the theoretical results, and the difference ranged between 0.13 dB and 1 dB, except for $p = 0.4$ and symbol SNR > 9 dB where the simulation results are better than the theoretical.

The discrepancy between the theory and simulation may be explained by:

- It was assumed in the theory that the phase of the tone causal by the unbalanced data is coherent with the carrier phase. In reality, the phase of the tone may not.

- It was assumed in [1] that) the pdf of the carrier tracking phase error has Tikhonov distribution with modified variance. This assumption may breakdown for certain values of interference-to- signal power ratios, loop bandwidth and loop SNR.

4.2 Bandlimited Channel

In order to test the effect of the bandlimited channel on the overall performance of the system, simulations were performed for different values of time-bandwidth product (BT): 1, 2 and 3. As expected, the higher the value of the product BT, the better the performance of the system. The simulation results are shown in Table 2 and Fig. 7.

The theoretical and simulation results are in good agreement. However, the simulation results are a little worse than the theoretical results. This is because the theoretical results were obtained for the case when the ISI is caused by two adjacent pulses, that is, two pulses before and two pulses after the current pulse are considered in the SER calculation.

5 Conclusion

This report studied, by simulations, the separate effects of unbalanced data and bandlimited channel of the performance of a PCM/PM/NRZ receiver. The simulation results were in agreement with the theoretical results for the balanced data stream and bandlimited channel cases. However, there are some discrepancies between the theory and simulation for the unbalanced data stream case. It is recommended that the loop SNR and Symbol SNR are verified with the theoretical values for that case.

Similar testing as the one described in this report is currently being performed for PCM/PM/Bi-ϕ receivers. The Bi-ϕ data stream is generated by multiplying the NRZ data with a squarewave subcarrier with the subcarrier frequency being identical to the NRZ symbol rate. In addition, both PCM/PM/NRZ and PCM/PM/Bi-ϕ receivers' performances will be tested under the effect of data asymmetry which is due to rising and falling voltage transitions. It is recommended that the testing be continued to verify all the above mentioned cases for future work.

ACKNOWLEDGMENTS

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REFERENCES

1. T. M. Nguyen, "Behavior of PCM/PM Receivers in Non-Ideal Channels. Part 1: Separate Effects of Imperfect Data Streams and Bandlimiting Channels on Performances," *Jet Propulsion Laboratory*, Pasadena, CA.

PCM/PM/NRZ

Unbalanced Data

(m=1.25 rad, Rs=10000 Hz, fs=500000 Hz, BL=5 Hz. 2. BL/Rs=0.001)

Prob. of Mark	Eb/No	Pt/No	# of Iterations	Nbar	Pe	Pe theory
0.5	7	47.455	6,600,000	:19.333333	9.040E-04	7.727E-04
0.5	8	48.455	28,200,000	127	2.252E-04	1.909E-04
0.5	9	49.455	150,000,000	133.666667	4.456E-05	3.363E-05
0.5	10	50.455	1,300,000,000	132	5.077E-06	3.872E-05
0.45	7	47.498	6.600.000	292	2.212E-03	1.100E-03
0.45	8	48.498	25,200,000	427	8.472E-04	3.1 OW-O4
0.45	9	49.498	80,200,000	440	2.743E-04	6.600E-05
0.45	10	50.498	500,200,000	605	6.048E-05	1.100E-05
0.4	7	47.632	6,600,000	1.27E+03	9.621E-03	5.400E-03
0.4	8	48.632	10,000,000	1.02E+03	5.100E-03	3.250E-03
0.4	9	49.632	15,000,000	625	2.083E-03	2.000E-03
0.4	10	50.632	25,000,000	404	8.080E-04	1.500E-03

TABLE 1

Simulation Data and **Results** for **PCM/PM/NRZ** Unbalanced Data

PCM/PM/NRZ

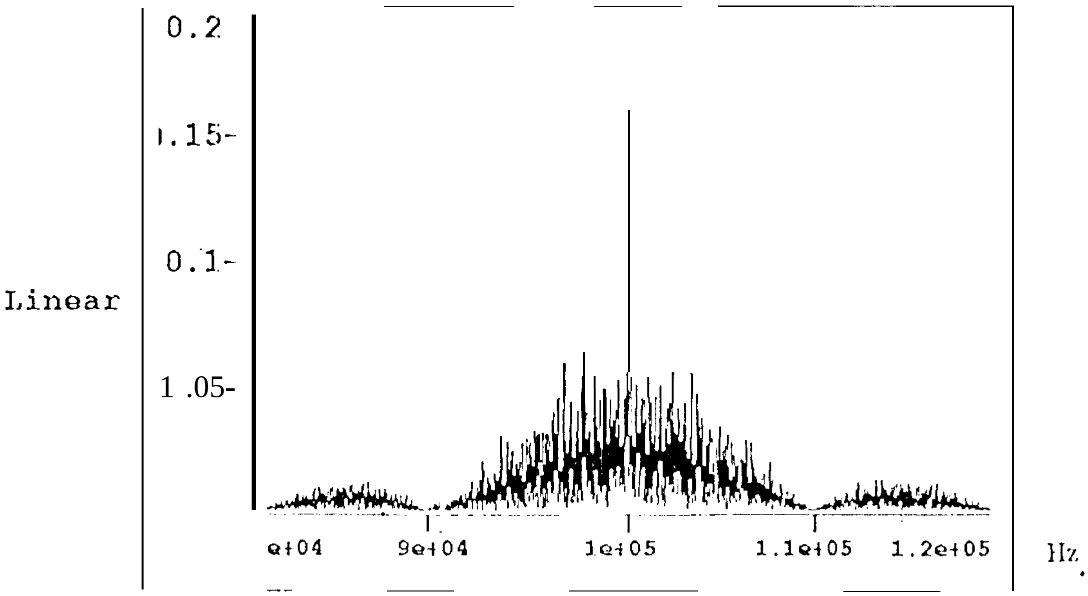
Band-Limited Channel

(m=1.25 rad, Probability of Mark=0.5, Rs=10000 Hz, fs=500000 Hz, BL=5 Hz, 2.BL/Rs=0.001)

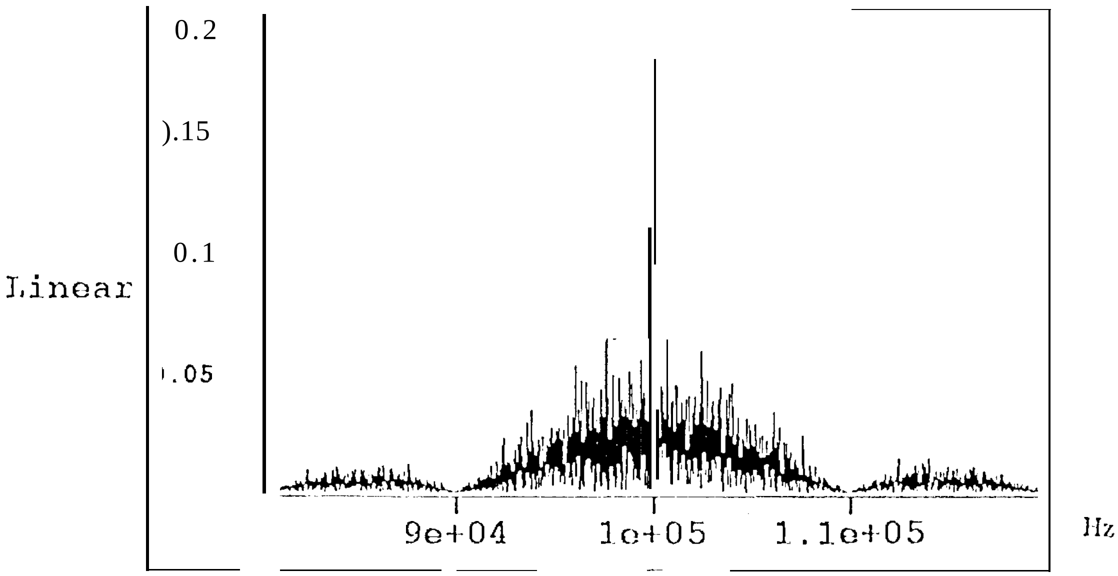
BT	Eb/No	Pt/No	# of Iterations	Nbar	Pe	Pe theory
1	7	47.455	6,600,000	387	2.932E-03	1.800E-03
1	8	48.455	11,000,000	206.333333	9.379E-04	5.800E-04
1	9	49.455	30,000,000	126.5	2.108E-04	1.700E-04
1	10	50.455	160,000,000	104.5	3.266E-05	3.300E-05
2	7	47.455	6,600,000	172.333333	1.306E-03	9.200E-04
2	8	48.455	21,000,000	146.5	3.488E-04	2.500E-04
2	9	49.455	105,000,000	157	7.476E-05	4.830E-05
2	10	50.455	800,000,000	137	8.563E-06	6.500E-06
3	7	47.455	6,600,000	146.333333	1.109E-03	8.600E-04
3	8	48.455	22,600,003	131	2.898E-04	2.300E-04
3	9	49.455	114,000,000	138	6.053E-05	4.400E-05
3	10	50.455	800,000,000	106	6.625E-06	5.600E-06

TABLE 3

Simulation Data and Results for PCM/PM/NRZ Band-Limited Channel



(a) p = 0.5 (Balanced data stream)



(1)) p :0.4 (Unbalanced data stream)

FIGURE 1

Spectrum of PCM/PM/NRZ for Different Values of p

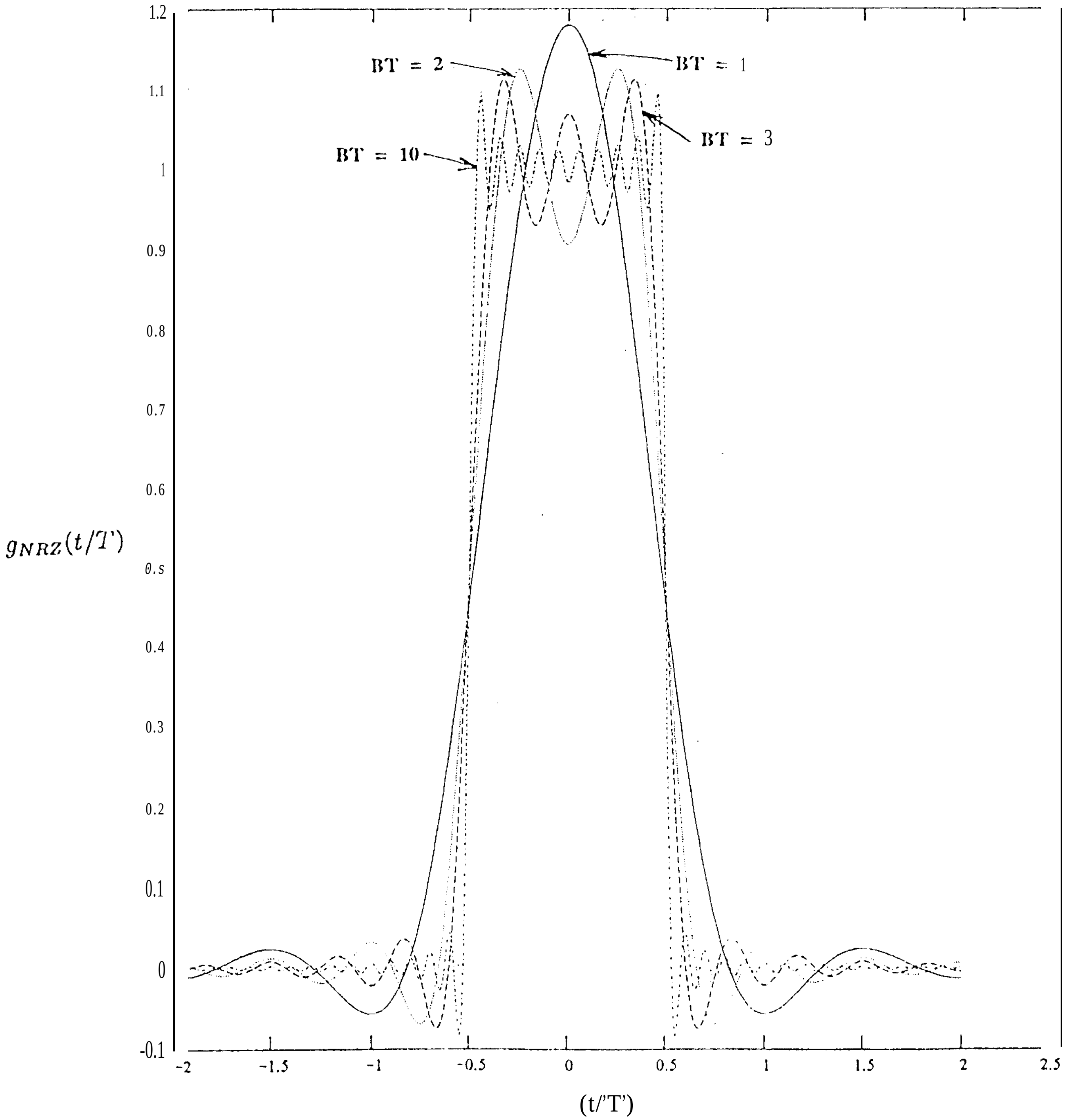


FIGURE 2

Output Response of the **Ideal** Filter to **NRZ** Puke for various **values of BT**

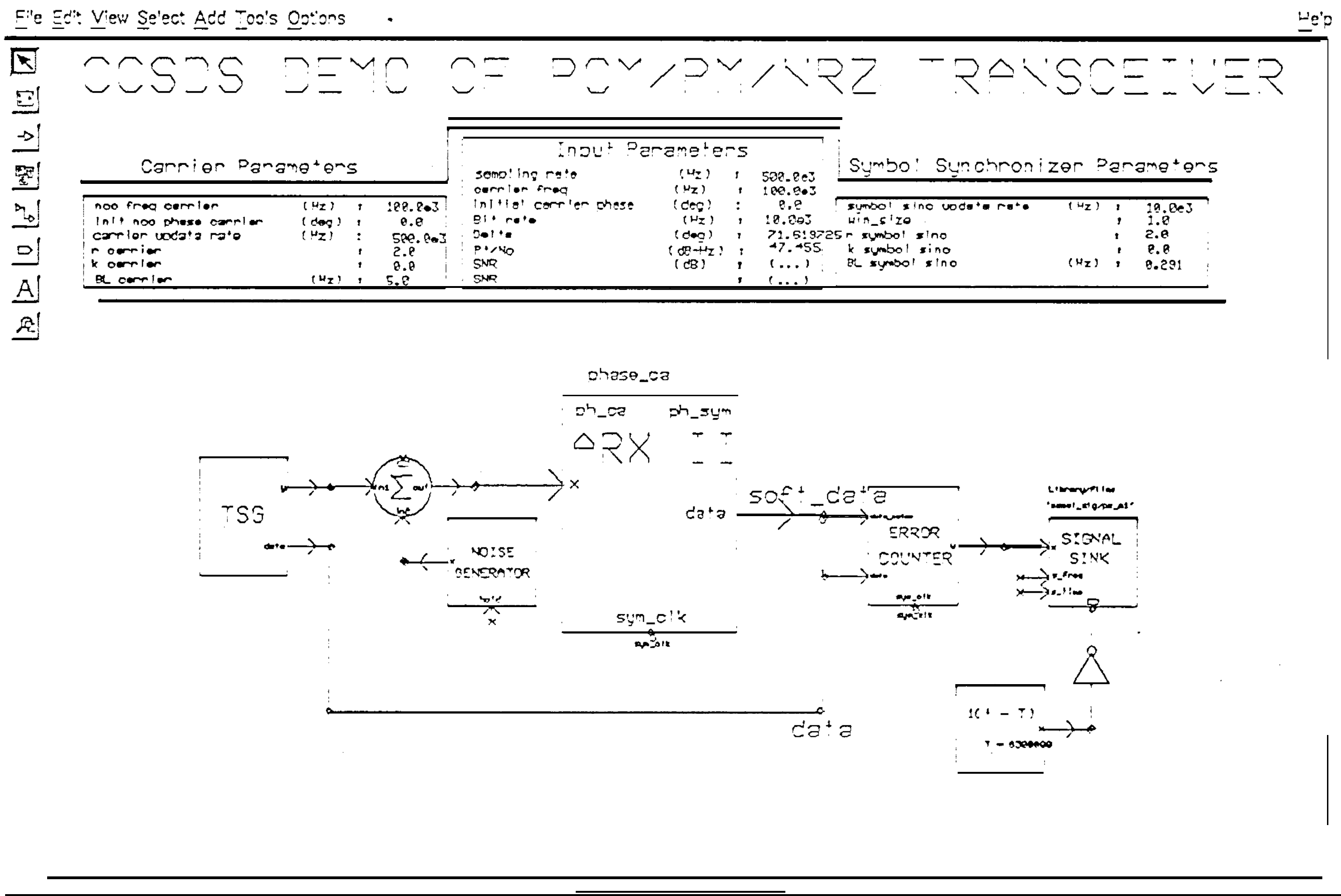


FIGURE 3

Block Diagram of PCM/PM/NRZ Receiver

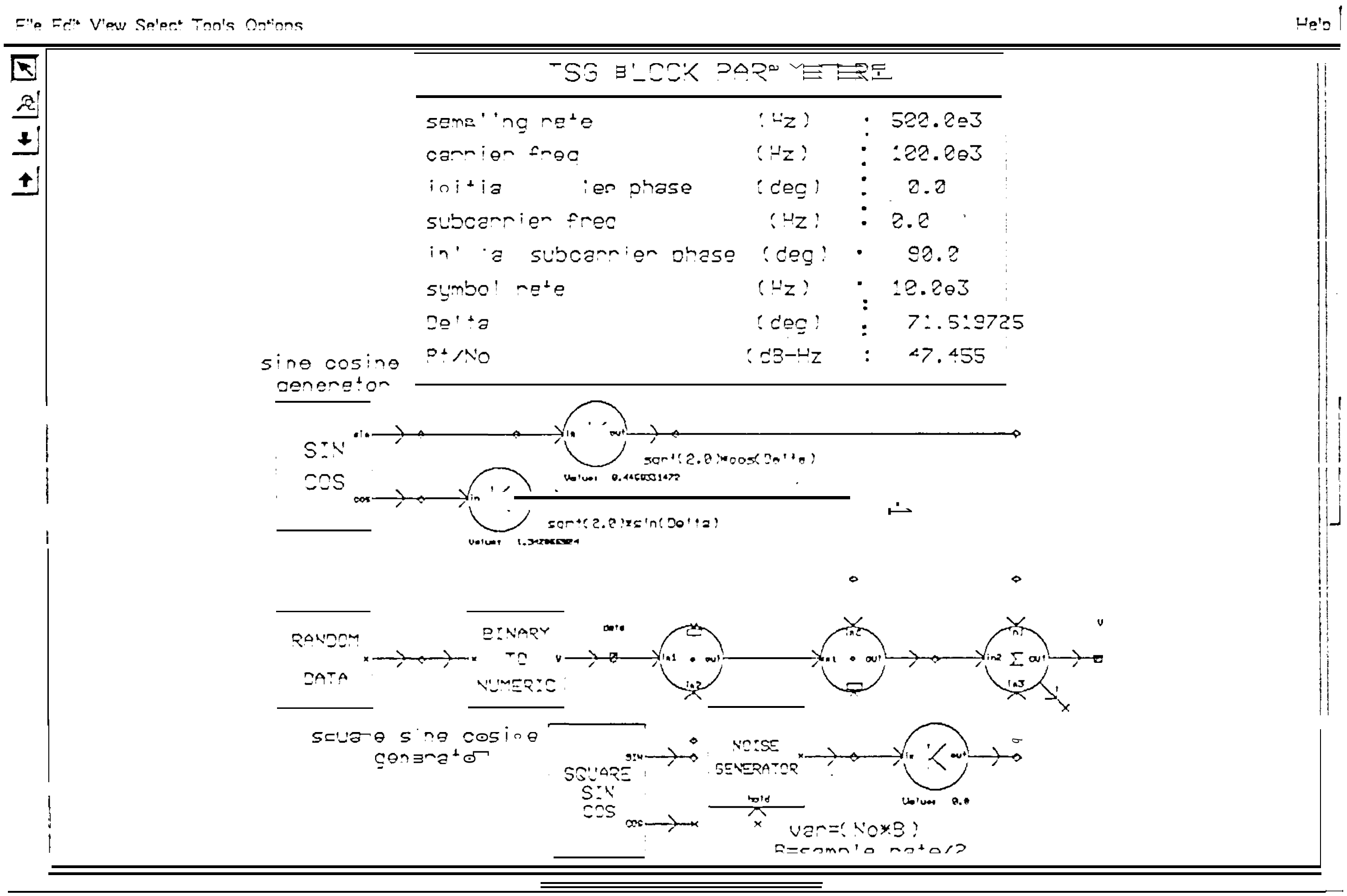


FIGURE 4

Test Signal Generator (TSG) Block Diagram

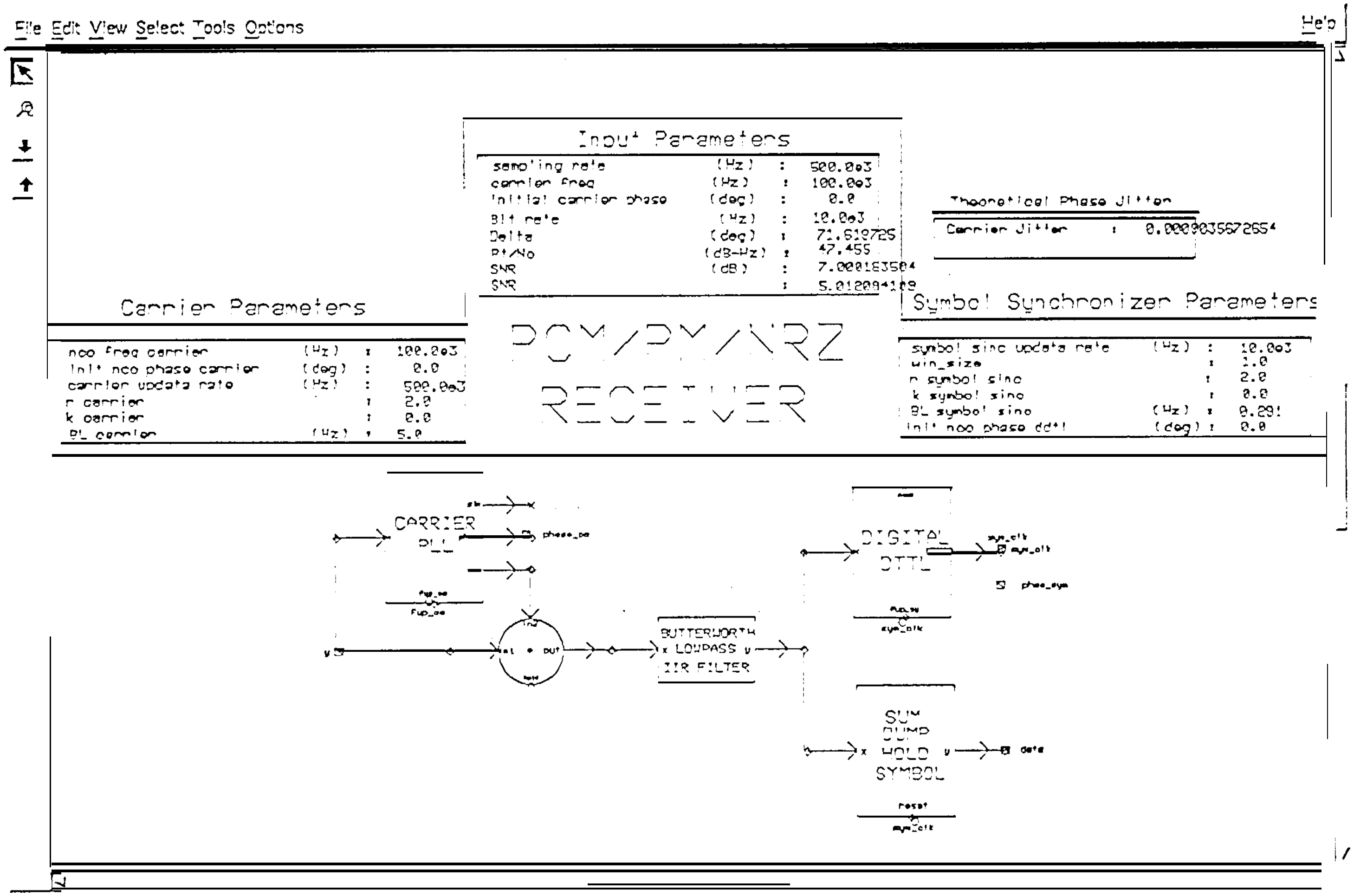


FIGURE 5

Advanced Receiver (ARX) Block Diagram

PCM/PM/NRZ

Unbalanced Data

(m=1.25 rad, Rs=10000 Hz. fs=500000 Hz, 3>51-12, 2.BL/Rs=0.001)

Prob. of Mark	Eb/No	Pt/No	# of Iterations	Nbar	Pe	Pe theory
0.5	7	47.455	6,600,000	119.333333	9.040E-04	7.727E-04
0.5	8	48.455	28,200,000	127	2.2525-04	1.909E-04
0.5	9	49.455	150,000,000	133.666667	4.456E-05	3.363E-05
0.5	10	50.455	1,300,000,000	132	5.077E-06	3.872E-05
0.45	7	47.498	6,600,000	292	2.212E-03	1.100E-03
0.45	8	48.498	25,200,000	427	8.472E-04	3.100E-04
0.45	9	49.498	80,200,000	440	2.743E-04	6.600E-05
0.45	10	50.498	500,200,000	605	6.048E-05	1.100E-05
0.4	7	47.632	6,600,000	1.27E+03	9.621E-03	5.400s-03
0.4	8	48.632	10,000,000	1.02E+03	5.100E-03	3.250E-03
0.4	9	49.632	15,000,000	625	2.083E-03	2.000E-03
0.4	10	50.632	25,090,000	404	8.080E-04	1.500E-03

TABLE 1

Simulation Data and Results for PCM/PM/NRZ Unbalanced Data

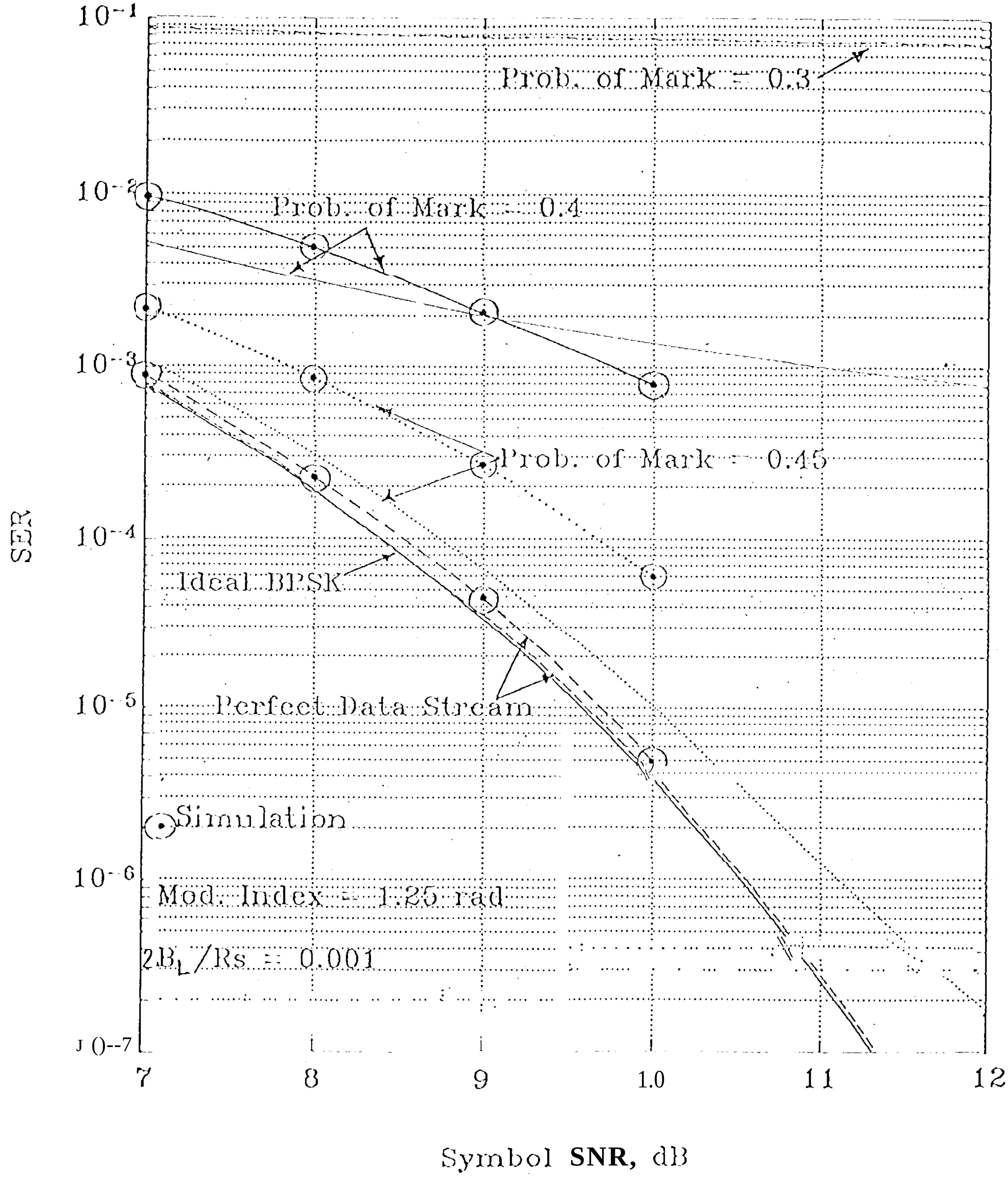


FIGURE 6

Theory and Simulation SER vs. Symbol SNR for PCM/PM/NRZ Unbalanced Data

PCM/PM/NRZ

Band-Limited Channel

(m=1.25 rad, Probability of Mark=0.5, Rs=10000 Hz, fs=500000 Hz, BL=5 Hz, 2.B L/Rs=0.001)

BT	Eb/No	Pt/No	# of Iterations	Nbar	Pe	Pe theory
1	7	47.455	6,600,000	387	2.932E-03	1.800E-03
1	8	48.455	11,000,000	206.333333	9.379E-04	5.800E-04
1	9	49.455	30,000,000	126.5	2.108E-04	1.700E-04
1	10	50.455	160,000,000	104.5	3.266E-05	3.300E-05
2	7	47.455	6,600,000	172.333333	1.306E-03	9.200E-04
2	8	48.455	21,000,000	146.5	3.488E-04	2.500E-04
2	9	49.455	105,000,000	157	7.476E-05	4.830E-05
2	10	50.455	800,000,000	137	8.563E-06	6.500E-06
3	7	47.455	6,600,000	146.333333	1.109E-03	8.600E-04
3	8	48.455	22,600,000	131	2.898E-04	2.300E-04
3	9	49.455	114,000,000	138	6.053E-05	4.400E-05
3	10	50.455	800,000,000	106	6.625E-06	5.600E-06

TABLE 2

Simulation Data and Results for PCM/PM/NRZ Band-Limited Channel

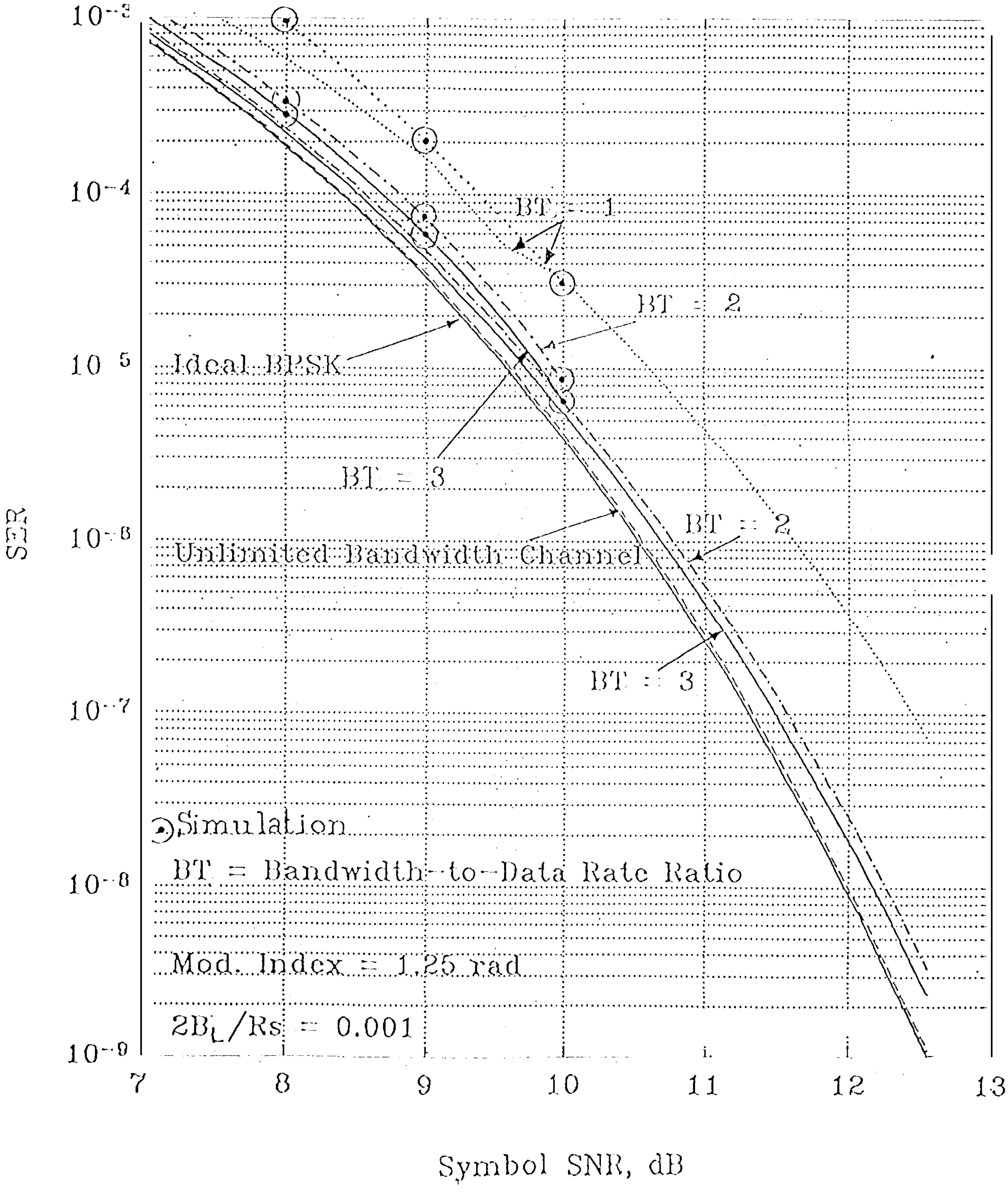


FIGURE 7

Theory and Simulation SER vs. Symbol SNR for PCM/PM/NRZ Bandlimited Channel