Chapter 13, the first on cooling, of lectures to be given at Acc School in Vanderbilt in Jan 99, by R.B.Palmer.

13. Very High Energy & Advanced Cooling

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13.1 Luminoosity Scaling

13.1.1 Luminosity vs Energy, for a Given Ring

For a fixed collider lattice, operating at energies lower than the design value, the luminosity will fall as γ^3. One power comes from the γ in Eq.**??**; a second comes from n_{eff}, the effective number of turns, that is proportional to γ; the third factor comes from β^*, which must be increased proportional to γ in order to keep the beam size constant within the focusing magnets. The bunch length σ_z must also be increased proportional to γ so that the required longitudinal phase space is not decreased for the same dp/p. As a result, A $= \sigma_z/\beta^*$ remains constant.

13.1.2 Scaling for Rings at Different Energies

If a final focus multiplet is scaled keeping the relative component lengths constant, with maximum aperture a_{max} (proportional to all apertures in the multiplet), and quadrupole field at the maximum apertures B_{max}, and assuming $\beta_v \approx 1$, then, from eq. **??** θ^*, the rms angle of muons diverging from the focus:

$$\theta^* \ \propto \ \sqrt{\frac{a_{max}\ B_{max}}{\gamma}} \tag{1}$$

Since

$$\ell^* \ \propto \ \frac{a_{\max}}{B_{\max}}$$

$$\ell^* \ \propto \ \sqrt{\frac{a_{\max}\ \gamma}{B_{\max}}} \tag{2}$$

where ℓ^* is the free space from the target to the first quadrupole (proportional to all quadrupole lengths in the multiplet).

Returning to eq. 1 and noting that

$$\theta^* \approx \sqrt{\frac{\epsilon_n}{\beta\ \gamma}}$$

$$\beta^* \propto \frac{\epsilon_n}{B_{\max}\ a_{\max}} \tag{3}$$

The normalized emittance ϵ_n is constrained by the ionization cooling, but since one can exchange transverse and longitudinal emittance, it is, in principle, the six dimensional emittance ϵ_6 that is constrained.

$$\epsilon_6 = \epsilon_{nx}\ \epsilon_{nx}\ \delta\ \sigma_z\ \gamma\ \beta_v. \tag{4}$$

σ_z cannot be large compared with the focus parameter β^*, so, taking them to be proportional to one another, taking the $\beta_v \approx 1$, and taking $\epsilon_{nx} = \epsilon_{nx} = \epsilon_n$, then:

$$\epsilon_6 \propto (\epsilon_n)^2\ \ \delta\ \beta^*\ \gamma \tag{5}$$

Substituting for β^* from eq. 3:

$$(\epsilon_n)^3 \propto \frac{\epsilon_6\ a_{\max}\ B_{\max}}{\delta\ \gamma} \tag{6}$$

$$(\beta^*)^3 \propto \frac{\epsilon_6}{\delta\ \gamma\ \ a_{\max}^2\ B_{\max}^2} \tag{7}$$

Now

$$\mathcal{L} = \frac{\mathrm{N}_\mu^2\ \mathrm{f_{rep}}\ \mathrm{n_{eff}}}{\epsilon_\mathrm{n}\ \beta^*}$$

so substituting for ϵ_n and β^* from above:

$$\mathcal{L} \propto \mathrm{N}_\mu^2 \ \mathrm{f_{rep}} \ \mathrm{n_{eff}} \ \left(\frac{\gamma\ \delta}{\epsilon_6}\right)^{2/3} \ (\mathrm{B_{max}}\ \mathrm{a_{max}})^{1/3} \tag{8}$$

13.1.3 Scaling if Tune Shift Limited

If N_μ is limited by the beam beam tune shift:

$$N_\mu \propto \epsilon_n \ \Delta\nu \tag{9}$$

substituting this, we get

$$\mathcal{L} = \Delta\nu \ \frac{\mathrm{N}_\mu \ \mathrm{f_{rep}} \ \mathrm{n_{eff}}}{\beta^*}$$

so substituting for β^* from above:

$$\mathcal{L} \propto \Delta\nu \ \mathrm{N}_\mu \ \mathrm{f_{rep}} \ \mathrm{n_{eff}} \ (\mathrm{B_{max}}\ \mathrm{a_{max}})^{2/3} \ \left(\frac{\gamma\ \delta}{\epsilon_6}\right)^{1/3} \tag{10}$$

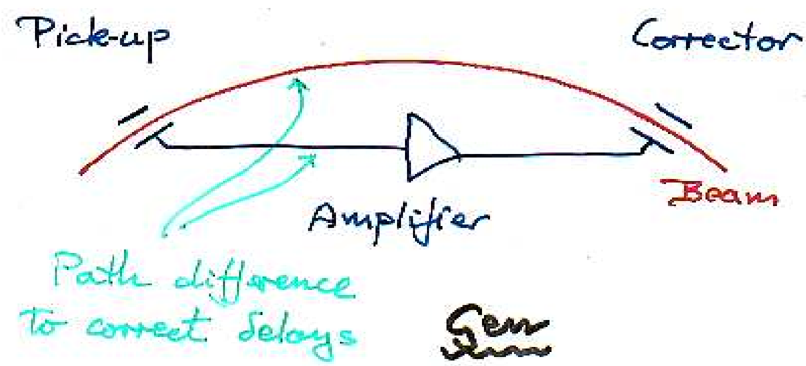
13.2 Stochastic Cooling

Refs

Van de Meer
Thorndal
Palmer BNL-18395 (73)

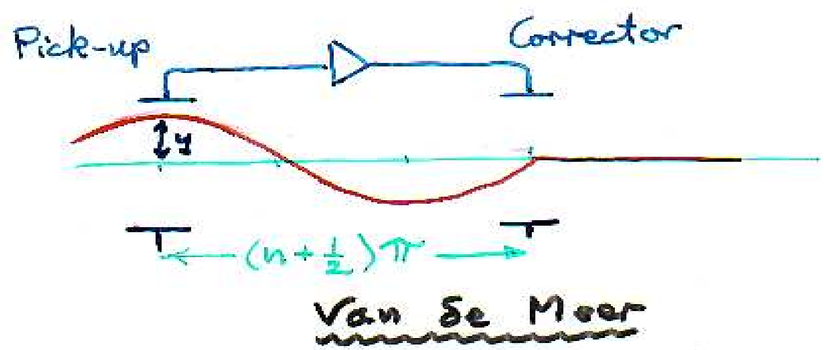
13.2.1 Introduction

Stochastic cooling works by measuring the offset of particles and, at a later time, correcting them. It is stochastic, in that the pickup sees many particles at a time, corrects only the average error, but decreases all errors if repeated many times.

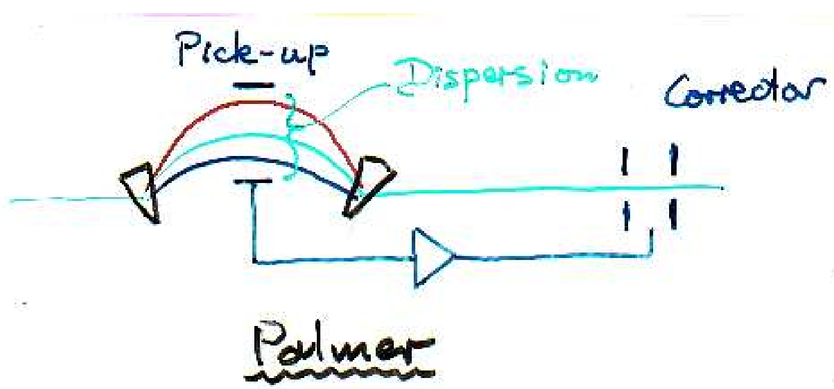


Consider first the correction of a single particle. I will denote the offset of the particle as y. y can be a transverse or longitudinal offset, and could, in principle represent either the offset itself, or the rate of change of that offset. In practice we cool:

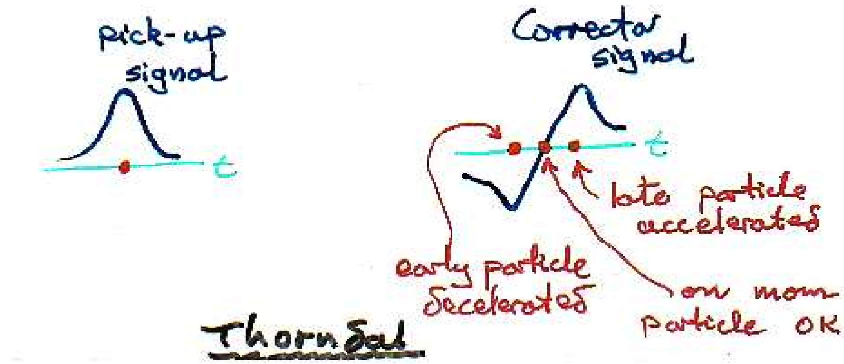
- Transverse displacements; correcting them by transverse angular deflections at a position with betatron phase $(n + 1/2)\,\pi$ from the pickup (Van de Meer Method).



- Momentum displacements; corrected by subsequent acceleration. The momentum dependent signal can be obtained in two ways:
  - By measuring transverse positions where there is dispersion (Palmer Method); or



  - By using the time of flight between pickup and corrector. The pickup identifies the initial time, the signal is differentiated (notch filter) and fed to the later acceleration corrector. The phases are arranged so that an on momentum particle arrives at the corrector at the zero crossing, a low momentum particle arrives later, where the signal is positive and accelerates it, and a high momentum article arrives early where the signal is negative and decelerates it (Thorndal Method).



In conventional stochastic cooling, the pickup and amplifier are high frequency microwave devices, but they can also be optical, as will be discussed in more detail below.

The following mathematics applies to all the above methods.

13.2.2 Simplest Calculation

Consider a ring of some large circumference S containing N_T particles traveling near the velocity of light. Now suppose that we have some scheme that detects the mean lateral position ($\bar{y}$) of an element of the beam at a "pick up station" of length ℓ. The number of particles in that element we call N.

$$N = \frac{\ell}{S} N_T \tag{11}$$

If the signals corresponding to such displacements ($\bar{y}$) are amplified and used to correct these displacements at a "corrector station" at a point further round the ring, then the mean squared displacement a 2 will be slowly reduced. First we assume:

1. Negligible noise in the amplifier;
2. Adequate band width, ie band width F

   $$F \geq \frac{2\pi\beta\, c}{\ell} \qquad cycles/sec \tag{12}$$

   Where βc is the velocity of the particles;
3. Particle transit time between pickup and corrector station constant to a time small compared with $\frac{\ell}{\beta c}$.
4. Transit time for a complete rotation around the machine must have random fluctuations large compared with $\frac{\ell}{\beta c}$.

Given these conditions we can now calculate what might be called the ideal Stochastic Fooling rate. Let

y_i be the displacement of the ith particle. The mean square displacement will be

$$\sigma^2 = \frac{\sum_{1_T^N} y_i}{N_T} \tag{13}$$

The mean positions detected

$$\bar{y} \ = \ \frac{\sum_1^N y_i}{N} \tag{14}$$

will themselves have a distribution and will have a mean square value

$$< \bar{y}^2 > = \ \frac{\sigma^2}{N} \tag{15}$$

If the signals corresponding to these mean positions are amplified by a factor (a) and used to correct the trajectories, then the y position of each particle after correction will be $y_i - a\bar{y}$ and the resultant mean square displacement σ' will be given by:

$$(\sigma')^2 \ = \ \sum_N \ \frac{(y_i \ - \ a \ \bar{y})^2}{N}$$

$$= \ \frac{\Sigma_N \ (y_i) \ \ - \ \ 2 \ a \ \bar{y} \ \ \Sigma_N(y_i)}{N}$$

$$= \ \sigma^2 \ - \ 2 \ a \ \bar{y}^2 \ + \ a^2 \ \bar{y}^2$$

On average:

$$< \ (\sigma')^2 \ = \ \sigma^2 \ \ - \ \ (2a \ - \ a^2) \ < \bar{y}^2 \ >$$

$$= \sigma^2 \left(1 - \frac{2a - a^2}{N}\right)$$

ie on average the mean square deviation is reduced by the fraction

$$f = \frac{2a - a^2}{N} \tag{16}$$

Clearly maximum cooling is obtained with a = 1 and

$$f_{max} = \frac{1}{N} \tag{17}$$

If, as at the ISR, $N_T = 4\ 10^{14}$, $\ell = 3cm$, $F = 1\,GHz$, then $N = 10^{10}$ and $f = 10^{-10}$. The time τ reduce σ^2 to about 1/e is then $10^{10} \times 2$ μsec $\approx$ 6 hours. The CERN expected time τ is 24 hours but it turns out that at CERN our assump tion (3) is not satisfied, i.e. there is significant transit time spread between pick-up and corrector stations.

13.2.3 Transit time spread between pick-up and corrector

If on average a fraction ϵ of the particles in the length ℓ at the pick-up station remain in a length ℓ at the correction station then equation (16) must be modified (see Appendix I):

$$f \;=\; \frac{2a\,\epsilon \;\;-\;\; a^2}{N} \tag{18}$$

Maximum reduction is obtained for a=e and for this value

$$f_{max} \;=\; \frac{\epsilon^2}{N} \tag{19}$$

At CERN $\epsilon \approx 1/2$ and thus the cooling time τ is increased from 6 to 24 hours.

At CERN the transit time fluctuations for a complete cycle are very large compared with $\frac{\ell}{\beta c}$ and thus no correction is needed for this. Only in a machine near transition or a machine with very small momentum spread would this become a problem. If then the fluctuations in transit time are Δt and we define

$$\delta \;=\; \frac{\ell}{\beta\; c\; \Delta t}$$

then we find that the maximum cooling fraction is given by

$$f_{max} \;=\; \frac{\epsilon^2\;(1 \;\;-\;\; 2\delta)}{N}$$

13.2.4 Noise

At CERN the noise is less than 10% of the pick-up signal and has a negligible effect. The situation is very different, however, in the case of a $\bar{p}$ storage ring. In a $\bar{p}$ storage ring the total number of particles N_T is unlikely to be more than 10^{12} finally and not more than 10^7 would be made per pulse. If these were in a storage ring like the ISR the number N in a length of 3 cm would be only 200. Under such circumstances, without noise, the cooling time is only 400 μs. Noise in the pick-up system however makes this unattainable.

Let b be the noise quoted in terms of the apparent RMS y position of one particle in cm's. At CERN $b = 10^4$ cm, ie unless we have more than 10^4 particles, one could not find where they were to better than 1 cm!

Given this definition of b we get instead of equation (18)

$$f \;=\; \frac{2a\epsilon \;-\; a^2}{N} \;-\; \left(\frac{a\ b}{N\ \sigma}\right)^2 \tag{20}$$

The maximum cooling is obtained for

$$a \;=\; \frac{\epsilon}{1 \;+\; \frac{b^2}{N\ \sigma^2}} \tag{21}$$

If b is large then the reduction factor is given by:

$$f \;=\; \frac{\sigma^2\ \epsilon^2}{b^2} \tag{22}$$

and this is not a function of N!

If we again use the CERN signal to noise ratio we can then derive this maximum cooling rate.

$$\frac{b}{\sigma} \;=\; \frac{noise}{signal}\,\sqrt{N} \;=\; 1/10\;10^5 \;=\; 10^4$$

$$f \;=\; 10^{-8}\,\epsilon^2$$

for $\epsilon \approx 1$, $f = 10^{-8}$, and $\tau = 2(\mu s) \;\times\; 10^8 \;=\; 200\;(sec)$.

Two ways present themselves for further improving this rate:

1. lower the noise.
2. increase the rate of corrections per second.

The former can possibly be achieved when it is noted that the band width is not specified and need not be nearly as large as that used at CERN. Theoretically this does not lower the noise ratio since the plate lengths have to be lengthened as the band width is lowered. Despite this, the freedom of choosing the band width could probably allow lower noise amplifiers.

The second solution suggests the use of a small ring. Although the number of particles per unit length will be larger this does not affect the rate. The increased number of corrections per sec, however, will be directly useful. For instance, if we capture $\bar{p}$'s at 3 GeV we can have a ring 1/10 the size of the ISR and get a cooling time of only 20 seconds. This consideration must, however, be weighed against the smaller total phase space capacity of the smaller ring and thus the smaller number of $\bar{p}$'s that can be captured prior to cooling.

13.2.5 Appendix on Transit Time Fluctuations

We will consider the sample of particles N_o between the pick up electrodes, and the sample of particles N_4 that lie between the corrector plates at a later time when the correction signal derived from the first sample is applied to the corrector plates. Ideally these two samples are identical and the time delay is obviously set to maximize the number of particles that are present in both samples. Let us now subdivide each of the two samples derived above N_o and N_4)into subsets:

Let N_1 be the number of particles found in both samples N_o and N_4.

Let N_2 be those in the sample N_o that are not in N_1: $N_2 = N_o$ - N_1 Let N_3 be those in the sample N_4 that are not in N1: $N_3 = N_4$ - N_1 We will denote sums over the samples N_k by $\sum_k$ and define $S_k = \sum_k y_i$

The correction signal is now given by the amplification factor a times the mean position of the particles between the pick up electrodes; i.e. correction signal = $aS_o/N_o = a(S_1 + S_2)/N_o$. This signal is then used to correct the positions of all the particles between the corrector plates. Positions y_i of the particles i in the sample N_4 are then changed to $y_i - a(S_1 + S_2)/N_o$. The mean square deviation after correction $(\sigma')^2$ is then given by:

$$(\sigma')^2 \ = \ \sum_4 \frac{\left(y_i \ - \ \frac{a \ (S_1+S_2)}{N_o}\right)^2}{N_4}$$

$$= \ \frac{\sum_4 y_i^2}{N_4} \ - \ \frac{2a(S_1+S_2)}{N_o \ N_4} \ \sum_4 y_i \ + \ \frac{a^2 \ (S_1+S_s)^2}{N_o^2}$$

Now $\sum_4 y_i/N_4 \ = \ \sigma^2$, and $\sum_4 y_i \ = \ S_4 \ = \ S_1 \ + \ S_3$, so

$$(\sigma')^2 \ = \ \sigma^2 - \ \frac{2a(S_1+S_2)(S_1+S_3)}{N_o \ N_4} \ + \ \frac{a^2(S_1+S_2)^2}{N_o^2}$$

Now we must consider what the average or expectation values of the Sums S are when averaged over some number of cycles. The mean square expectation value of any sum S_k is given by

$$< \ S_k^2 \ \ = < \ 9\bar{y}_k \ N_k)^2 \ > = \ \sigma^2 \ N_k$$

However the expectation value of the product of two uncorrelated samples, e.g. N_1 and N_2 is zero

$$< S_1 \ \cdot S_2 >= 0$$

Thus

$$< \ (\sigma')^2 \ > = \ \sigma^2 \ - \ \frac{2a \ \sigma^2 \ N_1}{N_o \ N_4} \ + \ \frac{a^2(\sigma^2 \ N_1 \ + \ \sigma^2 \ N_2)}{N_o^2}$$

We now note that $N_4 \approx N_o$, and define ϵ by $N_1 = \epsilon N_o$. In the ideal situation of no transit time fluctuations $\epsilon = 1$. As the fluctuations become significant so ϵ is reduced. At the CERN ISR $\epsilon \approx\ 0.5$.

So using this definition of ϵ we obtain.

$$< \ (\sigma')^2 \ > = \ \sigma^2 \ \left(1 \ - \ \frac{2a\epsilon \ + \ a^2}{N_o}\right)$$

or if $< \ (\sigma')^2 \ > = \ \sigma^2(1-f)$

$$f \ = \ \frac{2a\epsilon \ - \ a^2}{N}$$

13.2.6 Optical Stochastic Cooling

13.3 Friction Cooling

13.4 Muonium Cooling