Covariances from Light-Element R-Matrix Analyses

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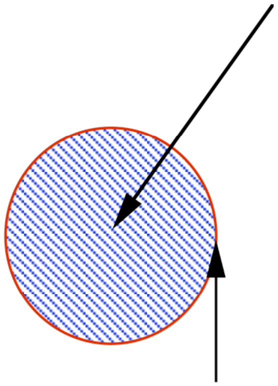
- Summary of R-matrix formalism
- Implementation in EDA code
- Uncertainty propagation in EDA
- Examples: n+p, n+^{6}Li
- Possible extensions
- Conclusions

R-matrix Schematic

INTERIOR (Many-Body) REGION
(Microscopic Calculations)

$$\underbrace{H + \mathcal{L}_B}$$

compact, hermitian operator with real, discrete spectrum; eigenfunctions in Hilbert space



SURFACE

$$|\psi^+) = (H + \mathcal{L}_B - E)^{-1} \mathcal{L}_B |\psi^+)$$

$$\mathcal{L}_B = \sum_c |c)(c| \left(\frac{\partial}{\partial r_c} r_c - B_c \right),$$

$$(\mathbf{r}_c|c) = \frac{\hbar}{\sqrt{2\mu_c a_c}} \frac{\delta(r_c - a_c)}{r_c} [(\phi_{s_1}^{\mu_1} \otimes \phi_{s_2}^{\mu_2})_s^{\mu} \otimes Y_l^m(\hat{\mathbf{r}}_c)]_J^M$$

$$R_{c'c} = (c'|(H + \mathcal{L}_B - E)^{-1}|c) = \sum_{\lambda} \frac{(c'|\lambda)(\lambda|c)}{E_\lambda - E}$$

ASYMPTOTIC REGION
(S-matrix, phase shifts, etc.)

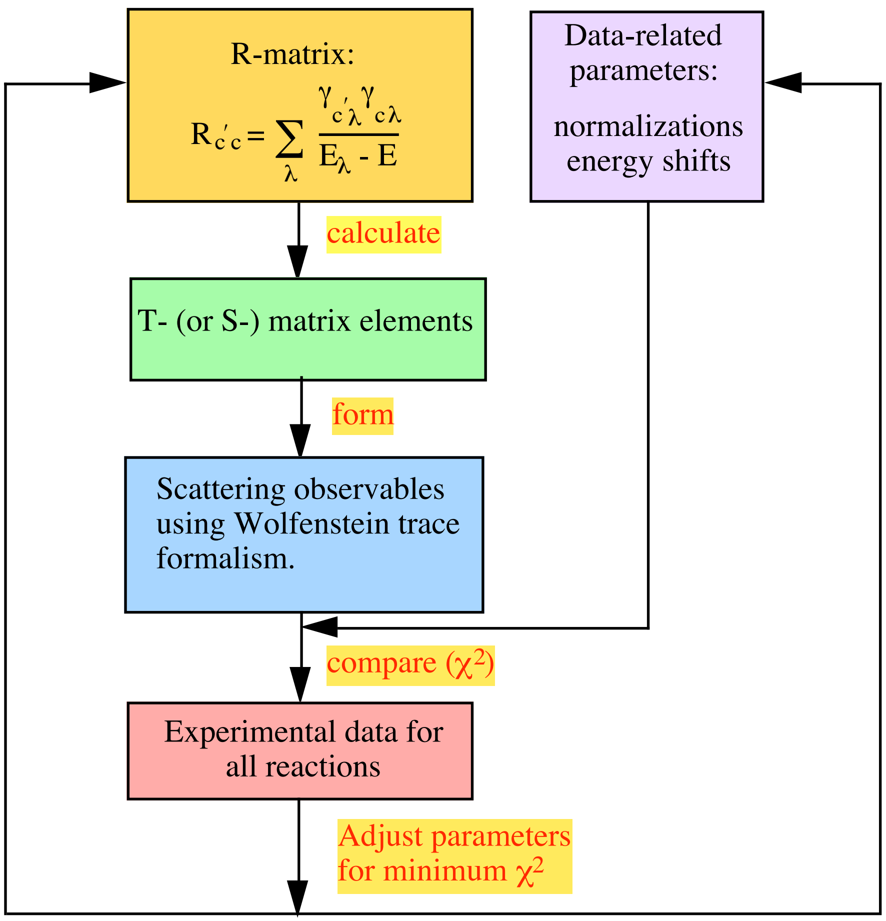
$$(r_{c'} | \psi_c^+ \rangle = -I_{c'}(r_{c'})\delta_{c'c} + O_{c'}(r_{c'})S_{c'c}$$

or equivalently,

$$(r_{c'} | \psi_c^+ \rangle = F_{c'}(r_{c'})\delta_{c'c} + O_{c'}(r_{c'})T_{c'c}$$

Measurements

Energy Dependent Analysis Code



Capabilities and Features

1) Accomodates general (spins, masses, charges) two-body channels

2) Uses relativistic kinematics and R-matrix formulation

3) Calculates general scattering observables for $2 \rightarrow 2$ processes

4) Has rather general data-handling capabilities

5) Uses modified variable-metric search algorithm that gives parameter covariances at a solution.

Chi-square Expressions and Covariances

$$\chi^2 = \sum_{i,j} (X_i(\mathbf{p}) - M_i)(\mathbf{V}_M^{-1})_{ij}(X_j(\mathbf{p}) - M_j),$$

with $M_i = R_i S$, and $\mathbf{V}_{ij}^M = \underbrace{S^2 (\Delta R_i)^2 \delta_{ij}}_{\text{diagonal piece}} + \underbrace{R_i R_j (\Delta S)^2}_{\text{rank-1 piece}}$ if R_i, S uncorrelated.

$$\chi^2_{\text{EDA}} = \sum_i \left[\frac{n X_i(\mathbf{p}) - R_i}{\Delta R_i} \right]^2 + \left[\frac{nS - 1}{\Delta S / S} \right]^2$$

$$\rightarrow \chi_0^2 + (\mathbf{p} - \mathbf{p}_0)^{\mathrm{T}} \mathbf{g}_0 + \tfrac{1}{2} (\mathbf{p} - \mathbf{p}_0)^{\mathrm{T}} \mathbf{G}_0 (\mathbf{p} - \mathbf{p}_0)$$

The parameter covariance matrix is $\mathbf{C}_0 = 2\mathbf{G}_0^{-1}$, and so first-order error propagation gives

$$\operatorname{cov}[\sigma_{\mathrm{i}}(E)\sigma_j(E')] = \left[\nabla_{\mathbf{p}} \sigma_{\mathrm{i}}(E)\right]^{\mathrm{T}} \mathbf{C}_0 \left[\nabla_{\mathbf{p}} \sigma_{\mathrm{j}}(E')\right]\Big|_{\mathbf{p}=\mathbf{p}_0}$$

$$= \Delta\sigma_{\mathrm{i}}(E) \Delta\sigma_{\mathrm{j}}(E') \rho_{ij}(E, E')$$

if $\mathbf{C}_0 \rightarrow \varepsilon \mathbf{1}$,

$$\tilde{\rho}_{ij}(E, E') = \frac{\left[\nabla_{\mathbf{p}} \sigma_{\mathrm{i}}(E)\right] \cdot \left[\nabla_{\mathbf{p}} \sigma_{\mathrm{j}}(E')\right]}{\sqrt{\left[\nabla_{\mathbf{p}} \sigma_{\mathrm{i}}(E)\right]^2 \left[\nabla_{\mathbf{p}} \sigma_{\mathrm{j}}(E')\right]^2}}\Bigg|_{\mathbf{p}=\mathbf{p}_0}$$

Convergence of EDA Solutions

Last 16 iterations of *N*-*N* (*n*-*p*) solution:

							$\sqrt{\mathbf{g}^T\mathbf{g}}$				χ^2/d.o.f.
it	1	hden	-1.62361E-06	fms	1.0000E+00	rmsq	2.799010E-01	time	160.444	wv	8.30020593E-01
it	2	hden	-2.70673E-07	fms	1.0000E+00	rmsq	5.057829E-02	time	172.095	wv	8.30020593E-01
it	3	hden	2.33652E-09	fms	1.0000E+00	rmsq	1.566784E-02	time	183.743	wv	8.30020593E-01
it	4	hden	-7.06336E-09	fms	1.0000E+00	rmsq	1.794353E-02	time	195.403	wv	8.30020593E-01
it	5	hden	1.46237E-11	fms	1.0000E+00	rmsq	1.492338E-02	time	207.047	wv	8.30020593E-01
it	6	hden	-4.36149E-08	fms	1.0000E+00	rmsq	3.261380E-02	time	218.701	wv	8.30020593E-01
it	7	hden	7.59279E-10	fms	1.0000E+00	rmsq	1.986385E-02	time	230.359	wv	8.30020593E-01
it	8	hden	-1.06823E-08	fms	1.0000E+00	rmsq	2.645074E-02	time	242.037	wv	8.30020593E-01
it	9	hden	1.19068E-10	fms	1.0000E+00	rmsq	2.243988E-03	time	253.693	wv	8.30020593E-01
it	10	hden	-1.85667E-10	fms	1.0000E+00	rmsq	3.034362E-03	time	265.350	wv	8.30020593E-01
it	11	hden	-1.07883E-12	fms	1.0000E+00	rmsq	2.767857E-03	time	277.018	wv	8.30020593E-01
it	12	hden	1.04047E-09	fms	1.0000E+00	rmsq	2.397856E-03	time	288.659	wv	8.30020593E-01
it	13	hden	2.24879E-12	fms	1.0000E+00	rmsq	2.951332E-03	time	300.303	wv	8.30020593E-01
it	14	hden	-2.08016E-09	fms	2.5000E-01	rmsq	4.356623E-03	time	317.050	wv	8.30020593E-01
it	15	hden	5.43813E-11	fms	6.5051E-01	rmsq	3.488838E-03	time	328.691	wv	8.30020593E-01
it	16	hden	-4.04937E-10	fms	1.0000E+00	rmsq	6.601823E-04	time	340.345	wv	8.30020593E-01

Relativistic, Charge-Independent Analysis of *N*-*N* Scattering up to 30 MeV

Channel	a_c (fm)	l_{max}
$p+p$	3.26	3
$n+p$	3.26	3
$\gamma+d$	40	1

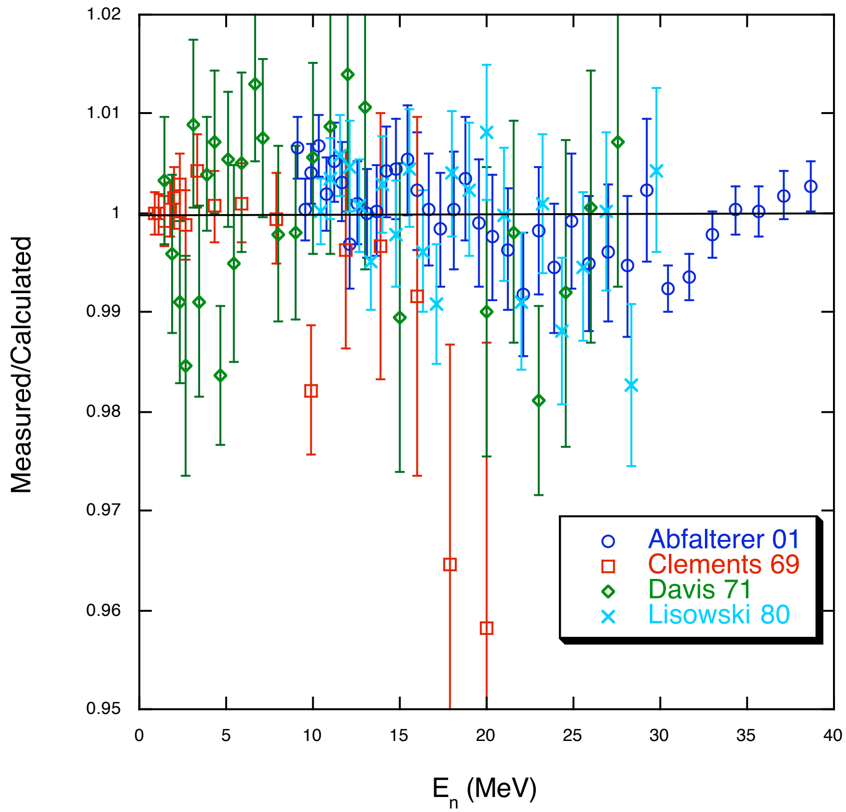
Reaction	# Pts.	χ^2	Observable Types
$p(p,p)p$	692	815	$\sigma(\theta)$, $A_y(p)$, $C_{x,x}$, $C_{y,y}$, $K_x^{x'}$, $K_y^{y'}$, $K_z^{x'}$
$p(n,n)p$	4378	3232	σ_T, $\sigma(\theta)$, $A_y(n)$, $C_{y,y}$, $K_y^{y'}$
$p(n,\gamma)d$	80	133	σ_{int}, $\sigma(\theta)$, $A_y(n)$
$d(\gamma,n)p$	59	35	σ_{int}, $\sigma(\theta)$, $\Sigma(\gamma)$, $P_y(n)$
Norms.	129	72	
Total	5338	4287	19

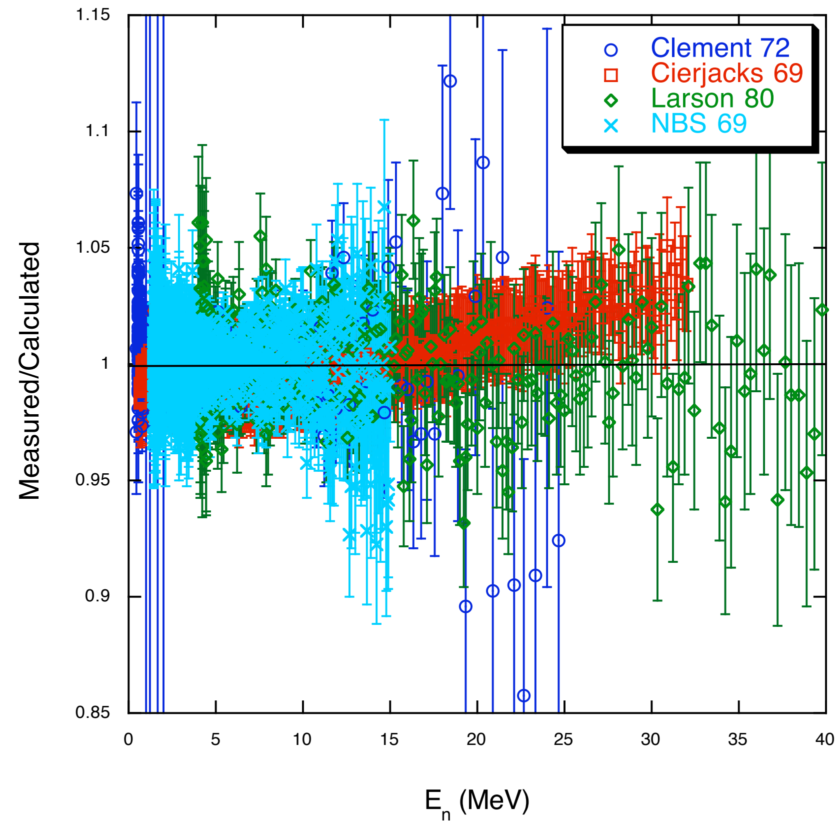
free parameters = 44+129 $\Rightarrow$ χ^2/degree of freedom = 0.830

n-p Scattering Lengths

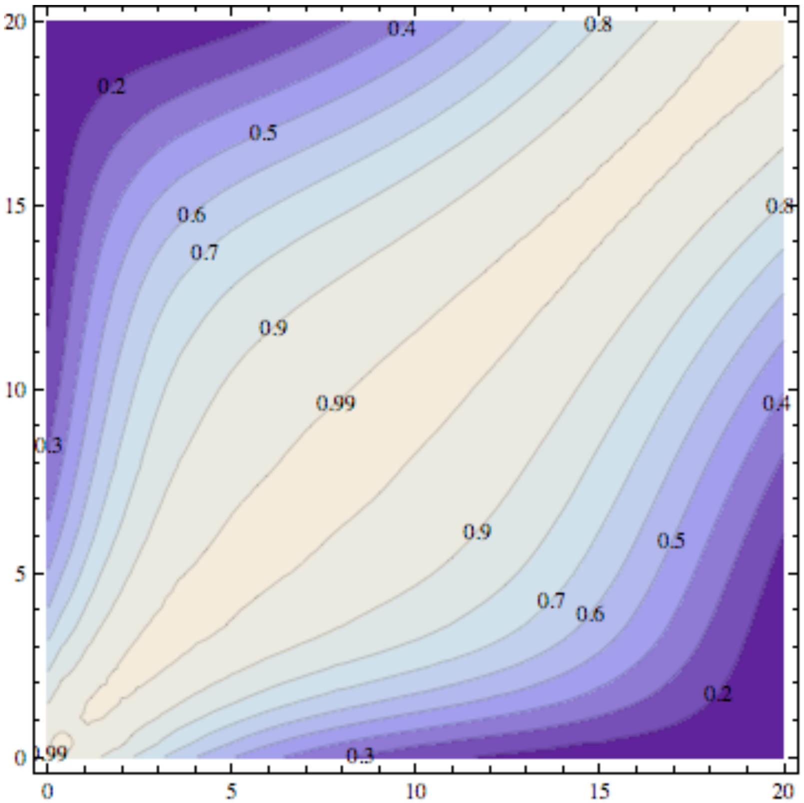
From the analysis,
a_0 = -23.719(5) fm, a_1 = 5.414(1) fm,
giving
$a_c = (3a_1 + a_0)/4 = -1.8693$ fm,
$\sigma_{pol} = (a_1^2 - a_0^2)/4 = -1.3332$ b,
$\sigma_{sc} = \pi(3a_1^2 + a_0^2) = 20.437$ b.
The first two agree exactly with experimental values, while the last one agrees with the measurement of Houk, (20.436 ± 0.023) b, but not with that of Dilg, (20.491 ± 0.014) b.
The spin-dependent scattering lengths from AV18 are
a_0 = -23.732 fm, a_1 = 5.419 fm,
in good agreement with those from the analysis.

n-p Total Cross Sections

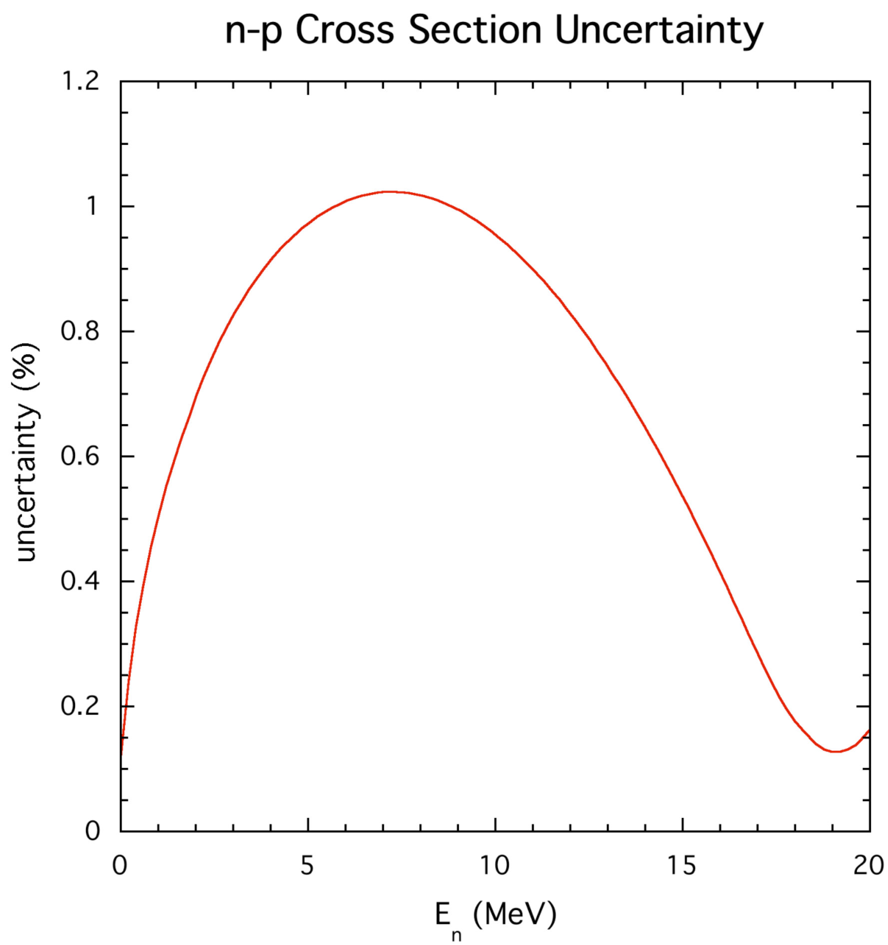




Covariances for *n-p* Scattering Cross Sections



$\rho_{nn}(E,E')$

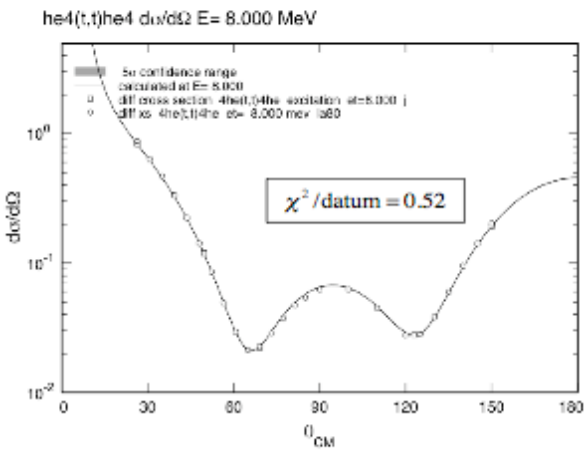


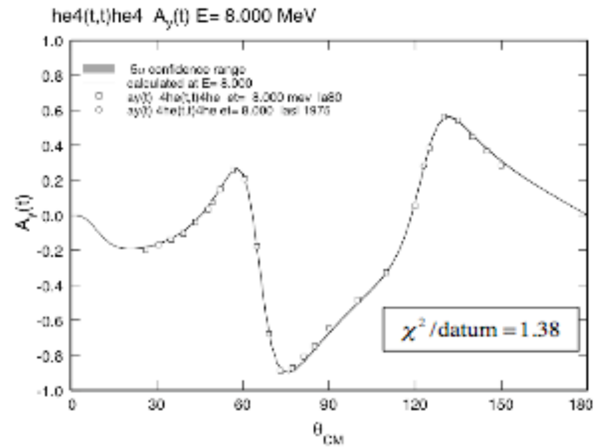
Summary of ^{7}Li Analysis

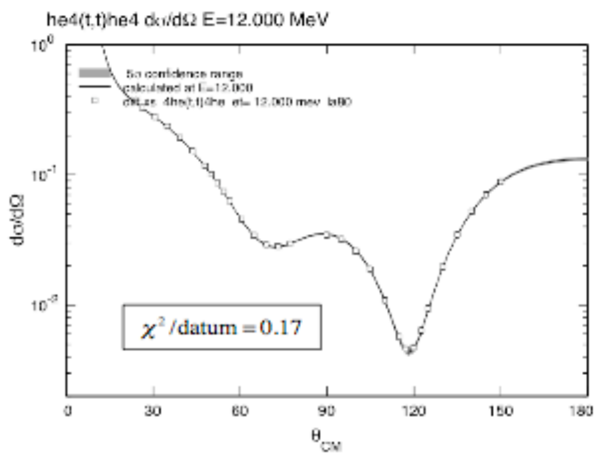
Channel	a_c (fm)	l_{max}
t+^{4}He	4.02	5
n+^{6}Li	5.0	3
n+^{6}Li*	4.5	1

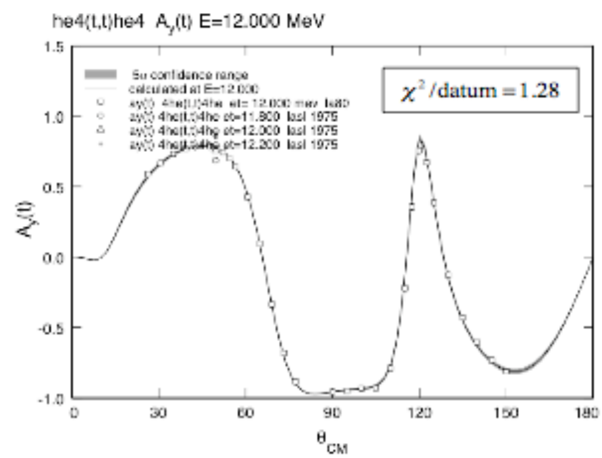
Reaction	Energy Range	# Pts.	χ^2/Pt.
^{4}He(t,t)^{4}He	E_t = 0-14 MeV	1622	0.930
^{4}He(t,n)^{6}Li	E_t = 8.75-14.4 MeV	35	1.658
^{4}He(t,n)^{6}Li*	E_t = 12.9 MeV	3	2.964
^{6}Li(n,t)^{4}He	E_n = 0-4 MeV	860	1.039
^{6}Li(n,n)^{6}Li	E_n = 0-4 MeV	798	1.391
Total	χ^2/d.o.f.	3318	1.164

t-^{4}He Scattering

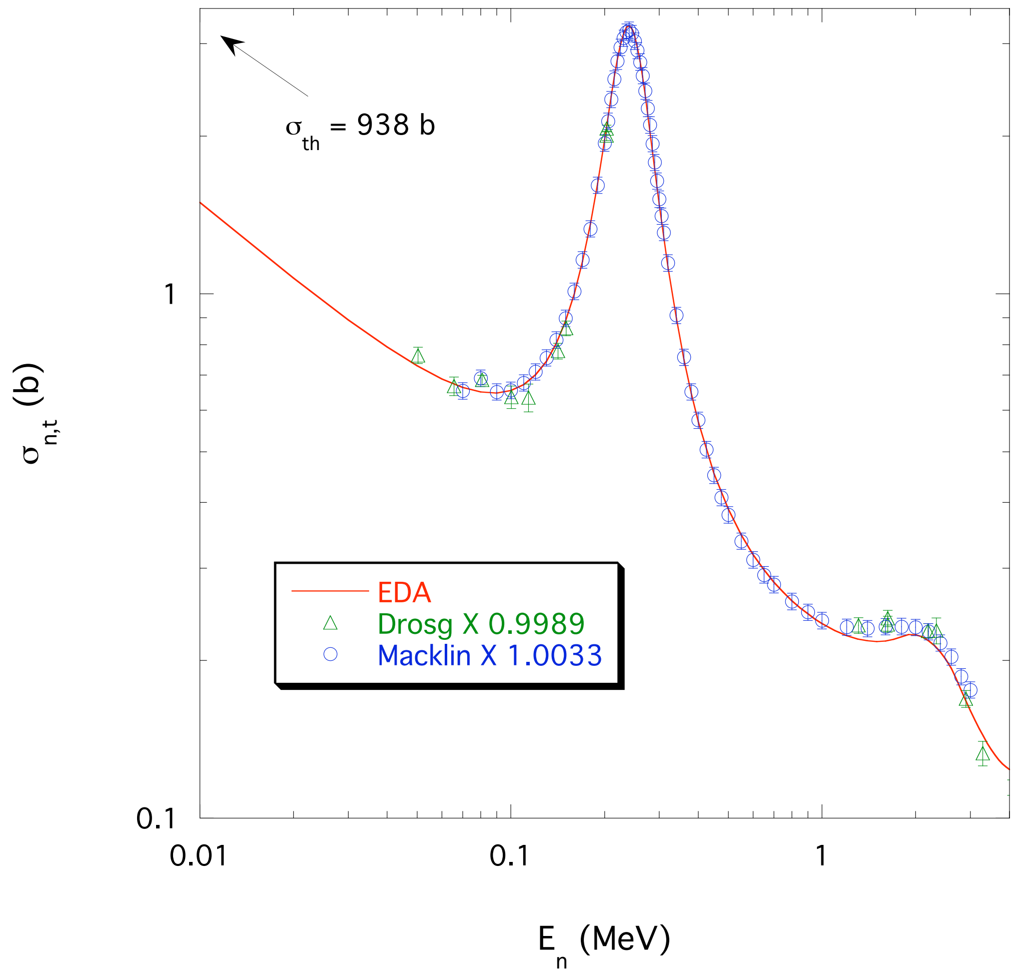




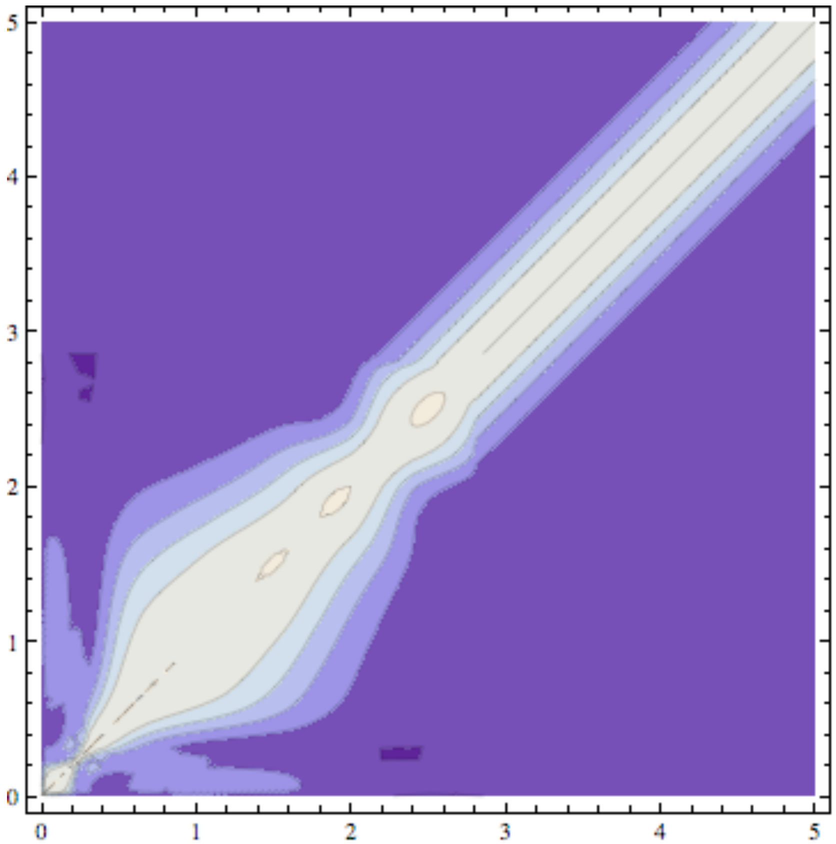




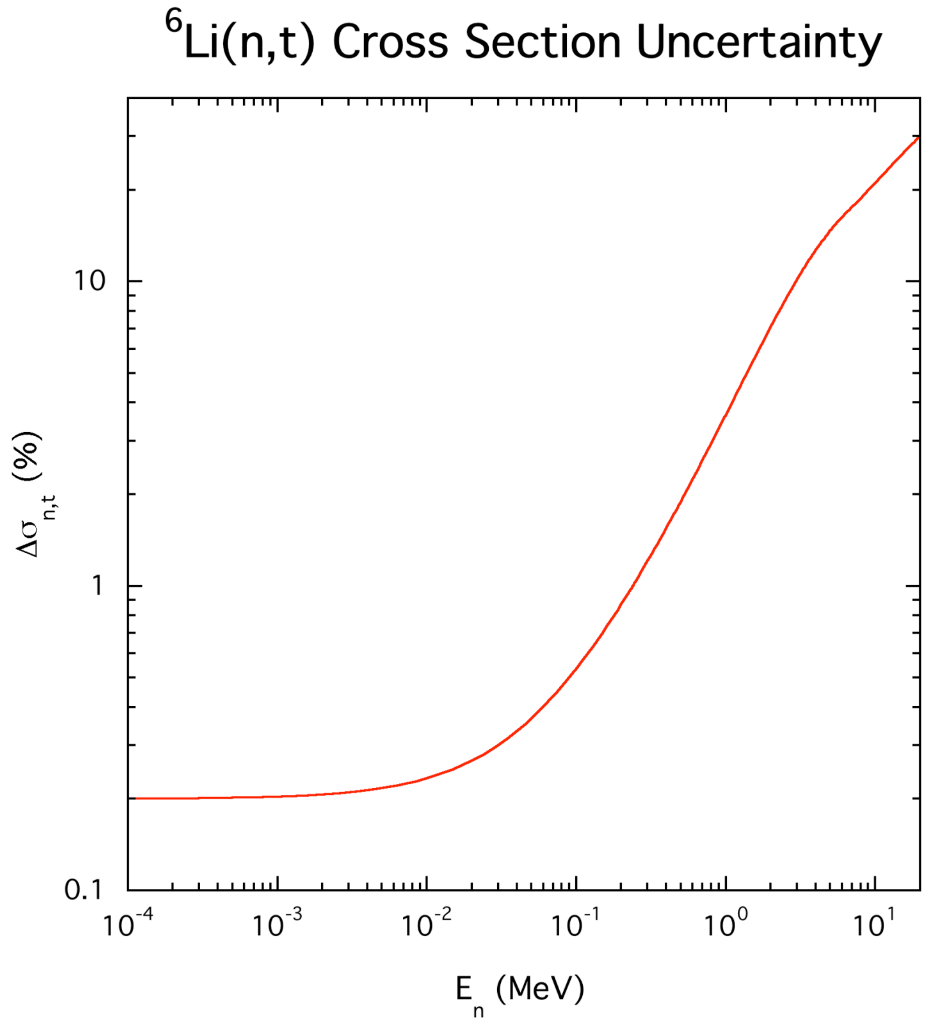
$^{6}Li(n,t)$ Cross Section



Covariances for ^{6}Li(*n,t*) Cross Sections

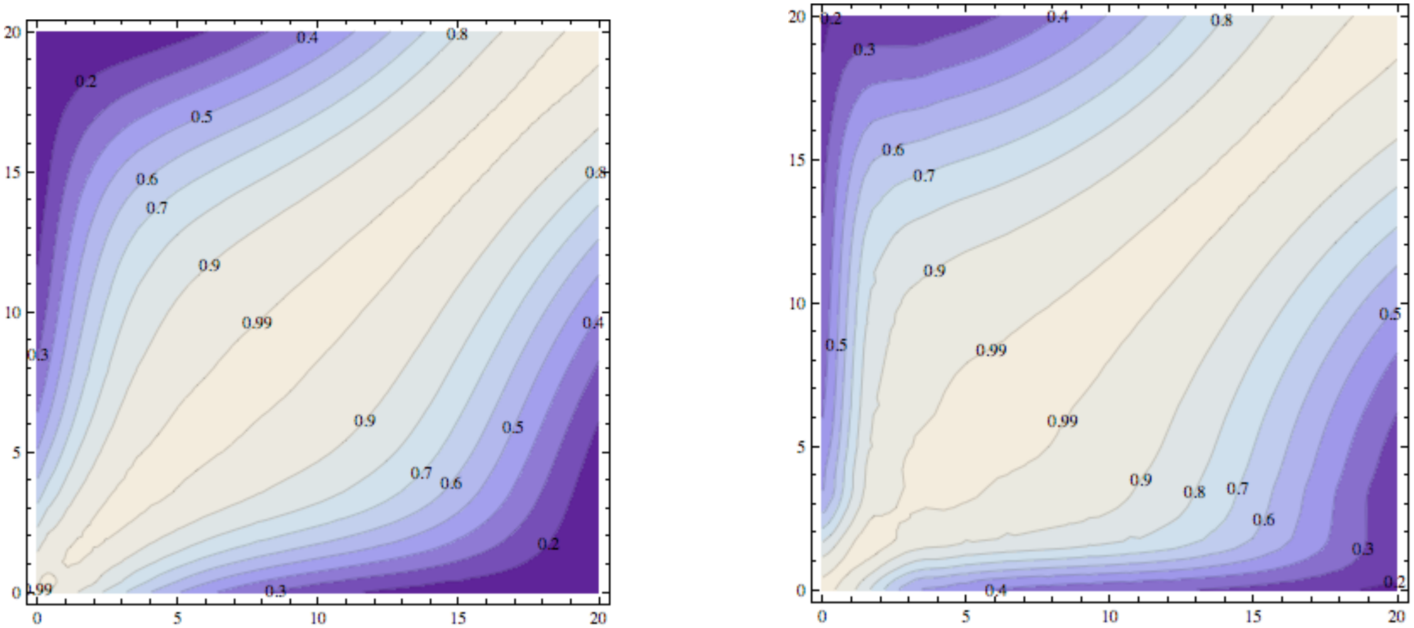


$\rho_{nt}(E,E')$

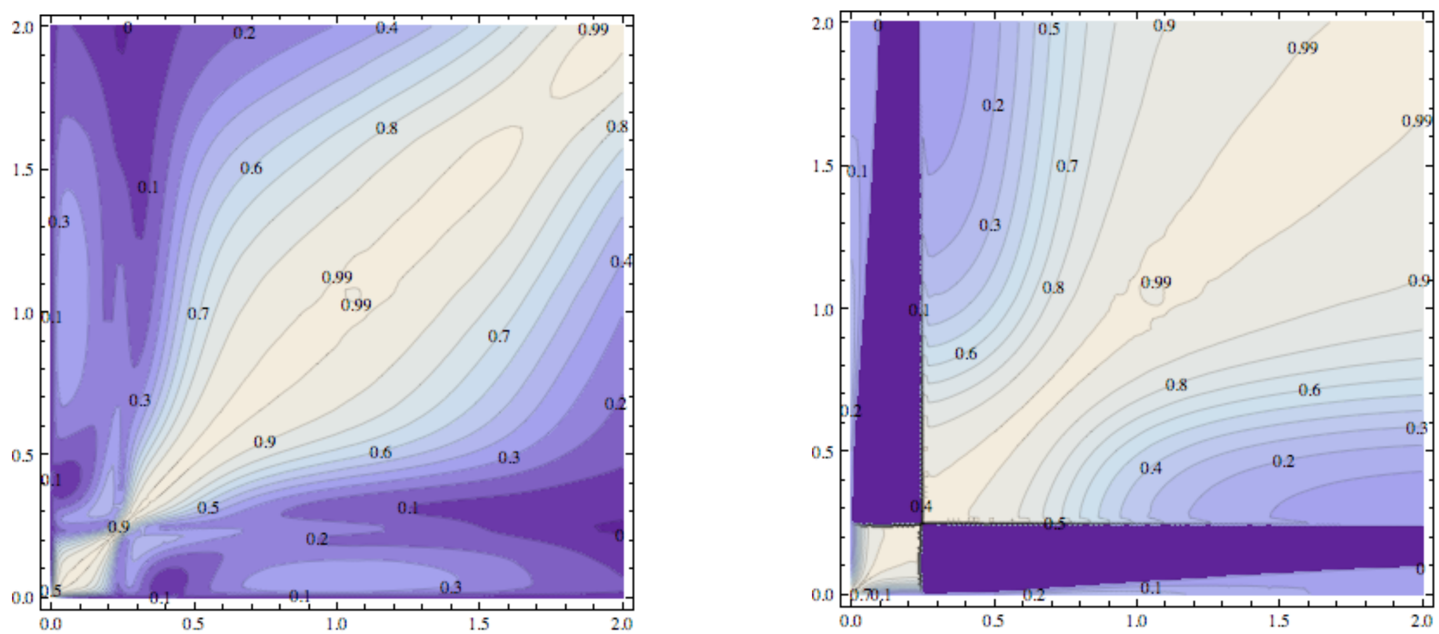


“Pure Theoretical” Correlations?

$^{1}H(n,n)$



$^{6}Li(n,t)$



Summary/Conclusions

- R-matrix approach is ideal for obtaining detailed covariances for light-element reactions, but not always over the full energy range desired.
- Implementation in EDA gives accurate covariance information (assuming that relative and absolute values of measured data are determined independently) from first-order error propagation for all reactions.
- “Pure theoretical” correlations from microscopic calculations may be useful for extending R-matrix covariances to higher energies.