## 6. Econometric Specification

For our multivariate analyses, we estimate hazard models of exits from and re-entry into the Food Stamp Program and binary choice models of employment. The transitions out of and into the Food Stamp Program are specified to depend on employment. We apply Lillard's (1993) simultaneous hazards procedure to address problems of omitted variables in the models of all three processes and to account for the endogeneity of employment in the food stamp hazard models. The econometric specification is discussed in more detail below.

To examine the determinants of the timing of exits from food stamps, we estimate a continuous-time log hazard model

Food stamp exit model:  $\ln h_{FS}(t) = A_{FS}'T_{FS}(t) + \delta_{FS}E(t) + B_{FS}'X_{FS}(t) + \eta.$ (1)

The hazard,  $h_{FS}(t)$  is the probability of exiting the Food Stamp Program at time *t* conditional on having remained in the program until at least *t*. In equation (1),  $T_{FS}(t)$  represents a vector of duration variables; these are functions of the length of time that an ongoing spell of program participation has lasted and include controls for typical recertification deadlines. Among the other terms in equation (1), E(t) is an indicator for employment;  $X_{FS}(t)$  is a vector of other observed and possibly time-varying explanatory variables;  $\eta$  is an unobserved, time-invariant variable, and  $A_{FS}$ ,  $\delta_{FS}$  and  $B_{FS}$  are coefficients.

The presence of unobserved heterogeneity (equivalently, the problem of omitted variables) in the hazard function is a substantial complication. Unobserved heterogeneity arises because we are not able to measure all of the characteristics that are relevant to people's food stamp participation decisions, such as their precise food needs or their attitudes regarding assistance. Failure to account for such heterogeneity can lead to biased estimates of the coefficients and especially to spurious indications of negative duration dependence. Following Lillard (1993), we assume that the variable representing these characteristics,  $\eta$ , is normally distributed with mean 0 and variance  $\sigma_{\eta}^2$ . We then use a maximum likelihood procedure that accounts for the distribution of food stamp participation spell lengths under this assumption. The procedure is similar to the one developed by Butler and Moffitt (1982) for random-effect panel probit models in that it specifies the hazard function conditional on  $\eta$  and then integrates over the distribution and possible values of  $\eta$ .

A second complication is that our explanatory measures include employment, which is a behaviorally-determined, or endogenous, measure. We address this problem by estimating models of food stamp participation and employment jointly and by allowing the unobserved determinants of these outcomes to be correlated. The key assumption underlying this approach is that the source of bias is a time-invariant unobserved variable. This is similar to the

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assumption that is invoked when fixed effects or difference-in-difference estimators are used to address endogeneity. The correlated random effects approach is even more restrictive than a fixed effects estimator, however, because it requires the omitted variables to be conditionally independent of the observed variables in  $X_{FS}(t)$ .

Along with the model for exits from food stamps, we also estimate a model of the timing of re-entry into food stamps (equivalently, exits from non-participation and spells of non-participation). The log hazard for this outcome is specified as

Food stamp re-entry model: 
$$\ln h_{NF}(t) = A_{NF}'T_{NF}(t) + \delta_{NF}E(t) + B_{NF}'X_{NF}(t) + \mu$$
(2)

where  $T_{NF}(t)$  is a vector of duration variables, E(t) is defined as before,  $X_{NF}(t)$  is a vector of other observed variables,  $\mu$  is an unobserved, time-invariant variable, and  $A_{NF}$ ,  $B_{NF}$  and  $\delta_{NF}$  are coefficients. The unobserved variable  $\mu$  is assumed to be normally distributed with mean 0 and variance  $\sigma_{\mu}^2$ . The analysis allows for multiple, alternating spells of food stamp participation and non-participation.

A discrete-time, binary-choice specification is used to model employment. In the model, the net benefits of employment for the primary informant of the household at time t are specified to be a linear function such that

Employment model:

$$E^{*}(t) = \mathbf{B}_{E}' \mathbf{X}_{E}(t) + \mathbf{v} + \varepsilon(t)$$
(3)

where  $X_E(t)$  is a vector of observed variables, v is a normally distributed, time-invariant, unobserved variable with mean 0 and variance  $\sigma_v^2$ , and  $\varepsilon(t)$  is a normally distributed, transitory, unobserved variable with mean 0 and variance 1. We assume that the primary informant works to earn more than \$250 if the net benefits are positive (E(t) = 1 if  $E^*(t) > 0$ ) and does not work this much otherwise (E(t) = 0 if  $E^*(t) \le 0$ ). The unobserved transitory variable  $\varepsilon(t)$  is assumed to be serially uncorrelated and independent of the other unobserved variable v. With this assumption, employment is modeled as a random-effects probit.

The transitory error is also assumed to be independent of the other two time-invariant, unobserved variables,  $\eta$  and  $\mu$ . However,  $\eta$ ,  $\mu$ , and  $\nu$  are allowed to be freely correlated (the correlation coefficients are  $\rho_{\eta\mu\nu}$ ,  $\rho_{\eta\nu\nu}$ , and  $\rho_{\mu\nu\nu}$ ). The two log hazard models and the random effects probit model are estimated jointly as a single system using the aML software package (Lillard and Panis 2003). The aML package employs Gaussian quadrature—a numerical approximation procedure—to evaluate the integrals over the three sources of time-invariant, unobserved heterogeneity. We report estimates from models that used ten quadrature points in each dimension, or 1,000 points total.<sup>6</sup>

<sup>&</sup>lt;sup>6</sup> For more information on the Gaussian quadrature technique, please see Butler and Moffitt (1982) and Lillard and Panis (2003).