Coherence and Phase in Imaging

OR

What REALLY are the limitations on Coherent Diffractive Imaging?

Keith A. Nugent

ARC Centre of Excellence for Coherent X-ray Science & School of Physics

The University of Melbourne

Australia

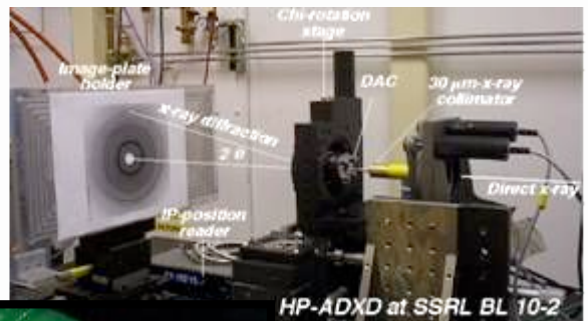
ARC Centre of Excellence for
COHERENT X-RAY SCIENCE

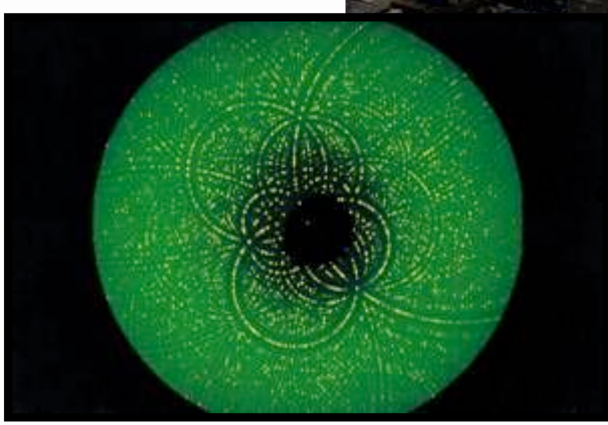
A Lightning Review...

Protein Crystallography

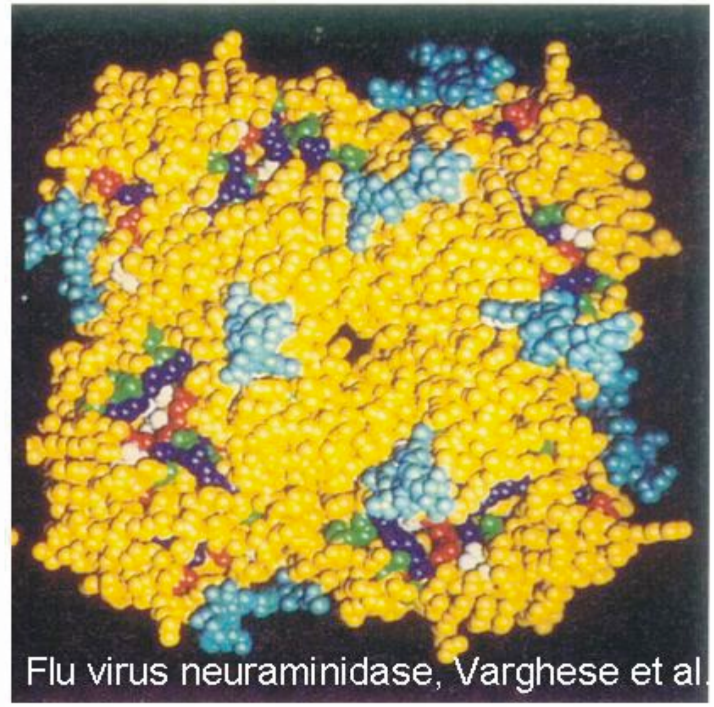
Protein crystal

Diffract x-rays





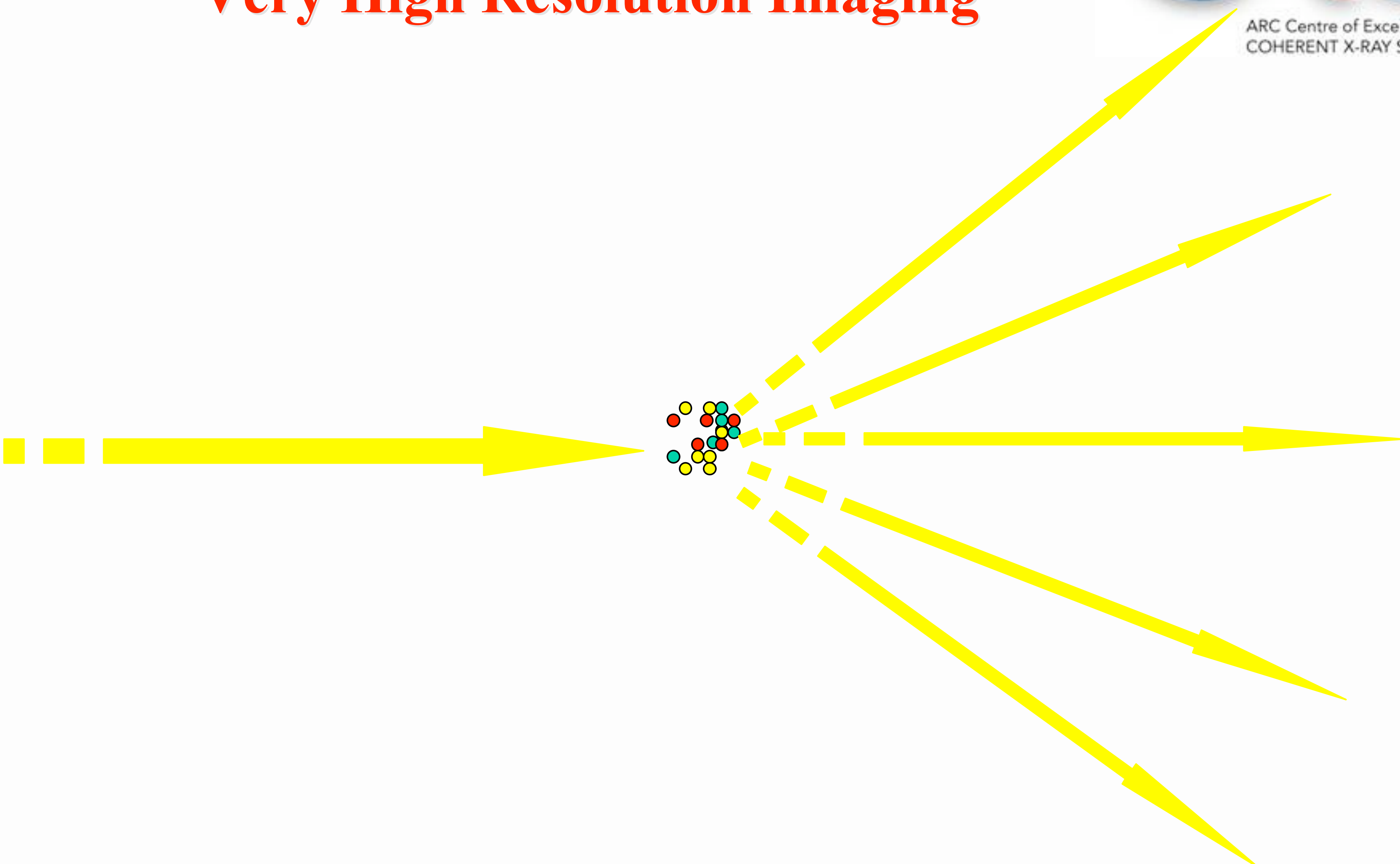
alyse diffraction pattern



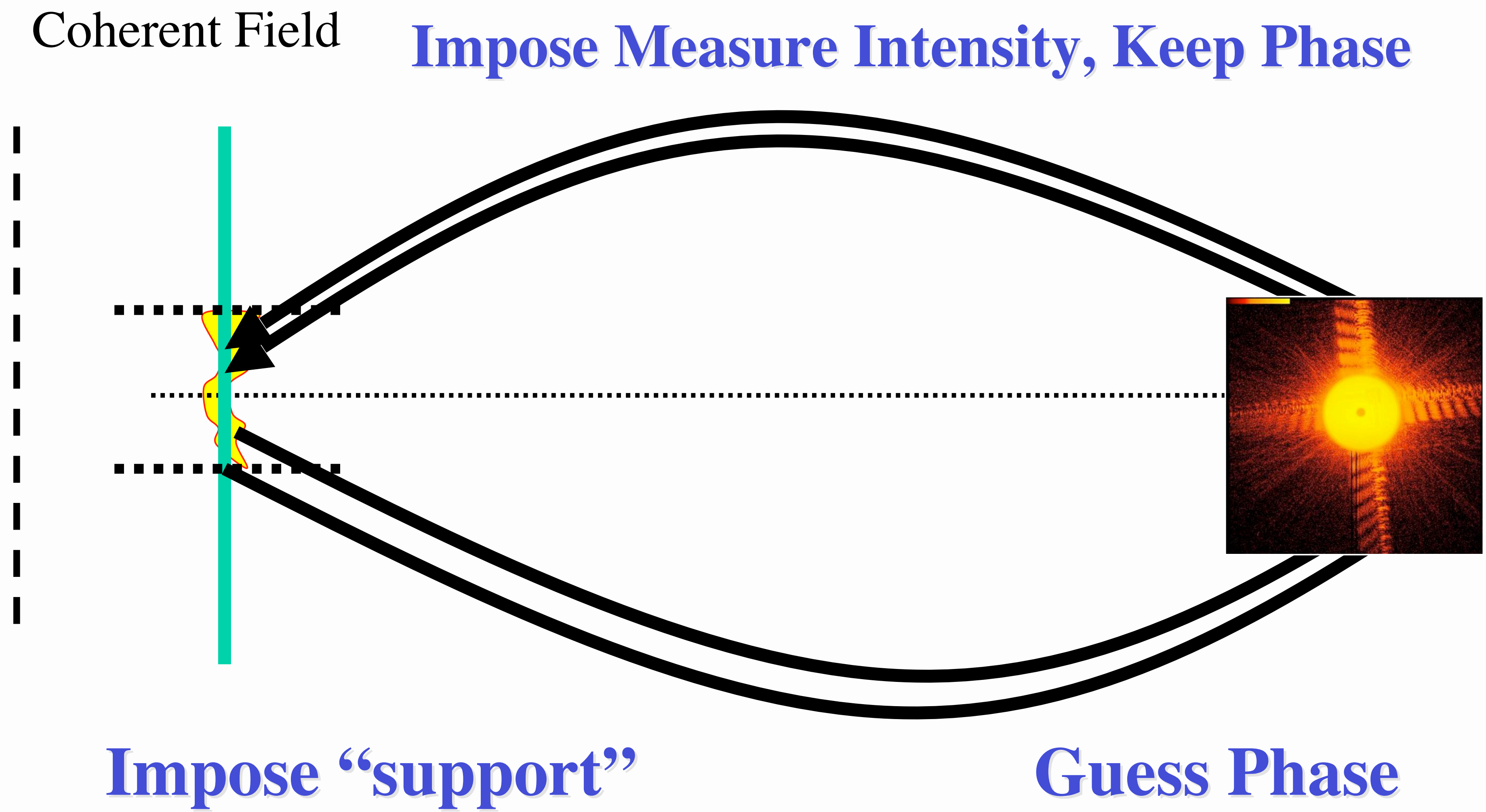
Flu virus neuraminidase, Varghese et al.

Very High Resolution Imaging

ARC Centre of Excellence for
COHERENT X-RAY SCIENCE



Coherent imaging methods are being developed.



Coherent Diffractive Imaging

- **The diffraction pattern (equivalently, the autocorrelation function) of an object <u>with finite support</u> "almost" uniquely defines the object**
- **If we can find an object that is consistent with the measured diffraction pattern and the (assumed known) support, then we have almost certainly found the object.**
- **Complete <u>coherence</u> is implicitly assumed**

R.H.T.Bates, Optik, **61**, 247 (1982)

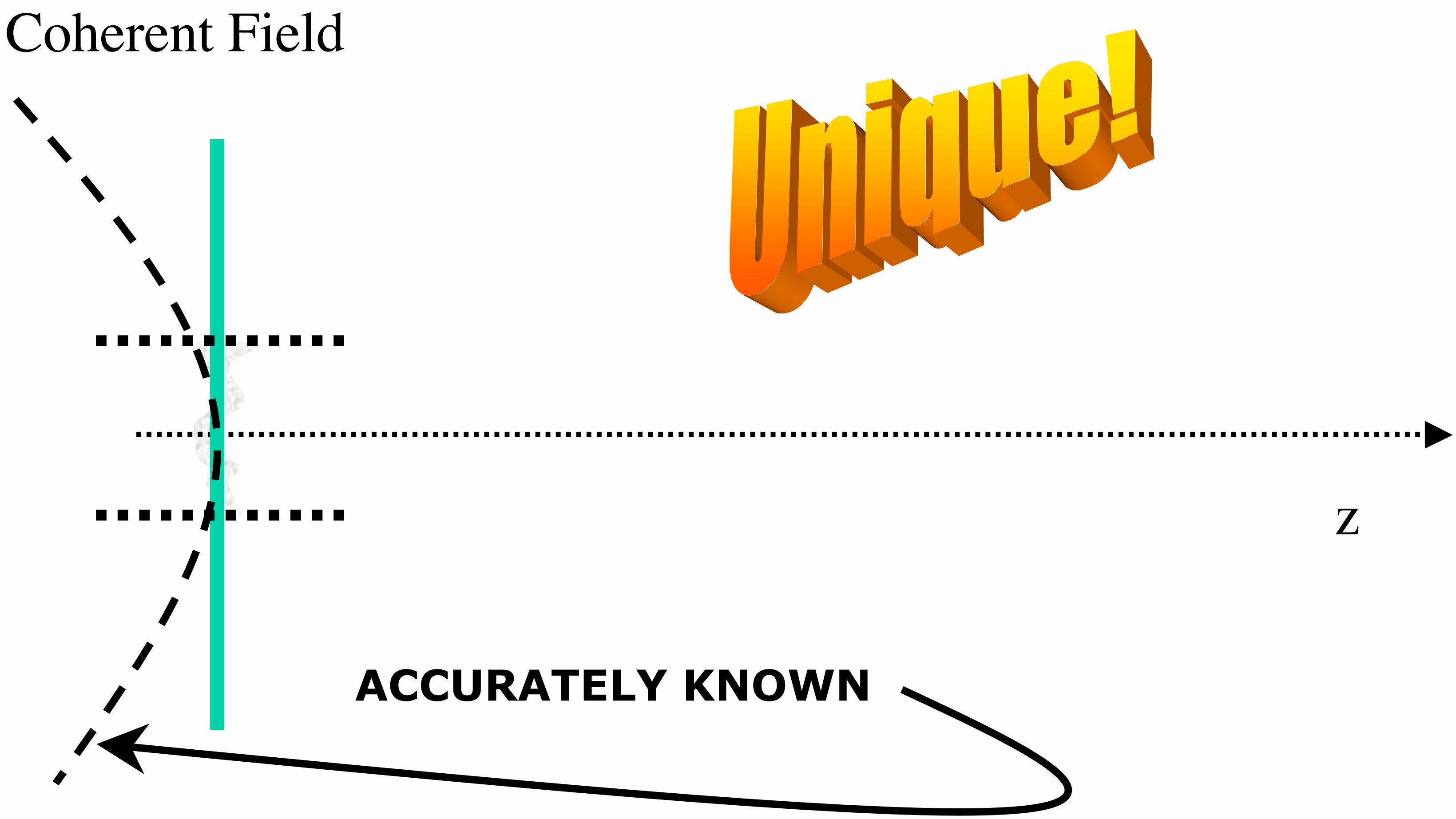
Coherent Diffractive Imaging makes a number of implicit assumptions

ARC Centre of Excellence for COHERENT X-RAY SCIENCE

- **Planar incident wave**
- **Complete coherence**
- **Finite support**

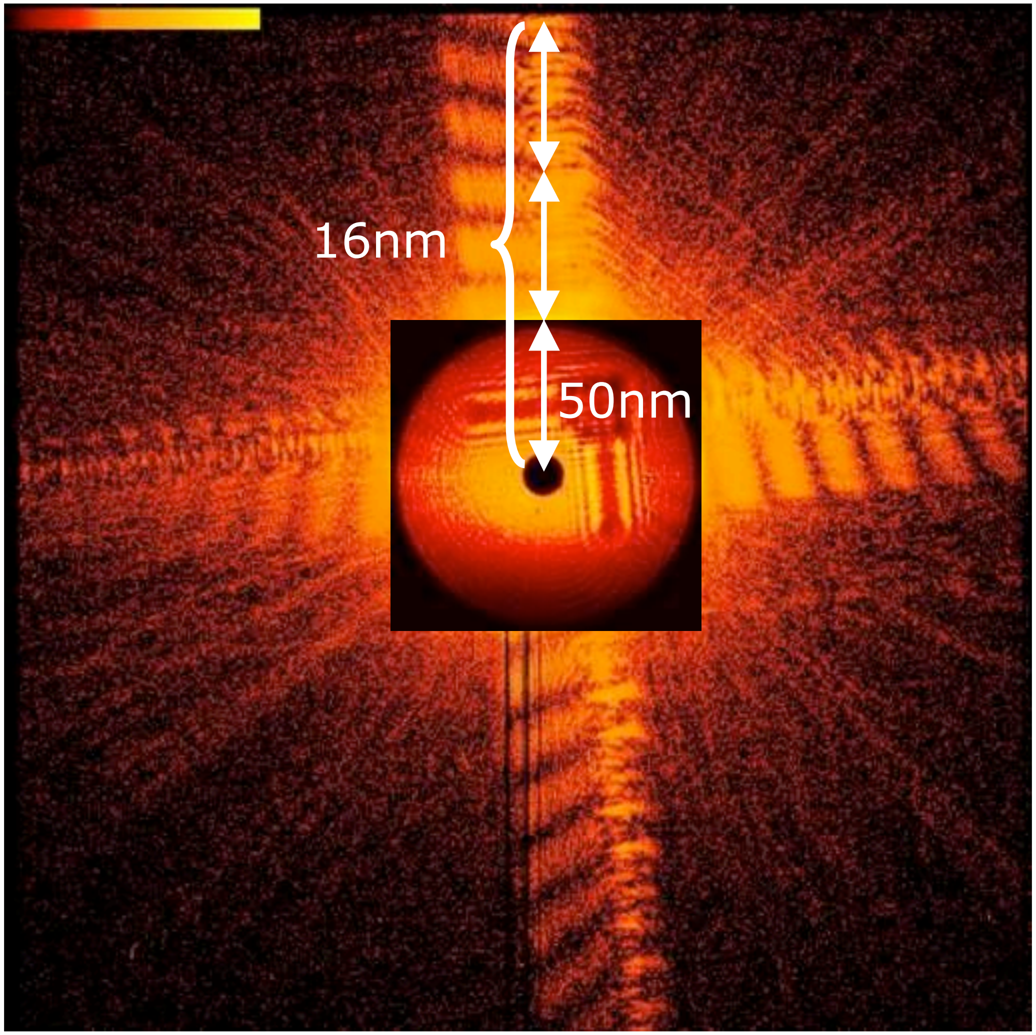
In this presentation I will question all three …

Coherent Diffractive Imaging

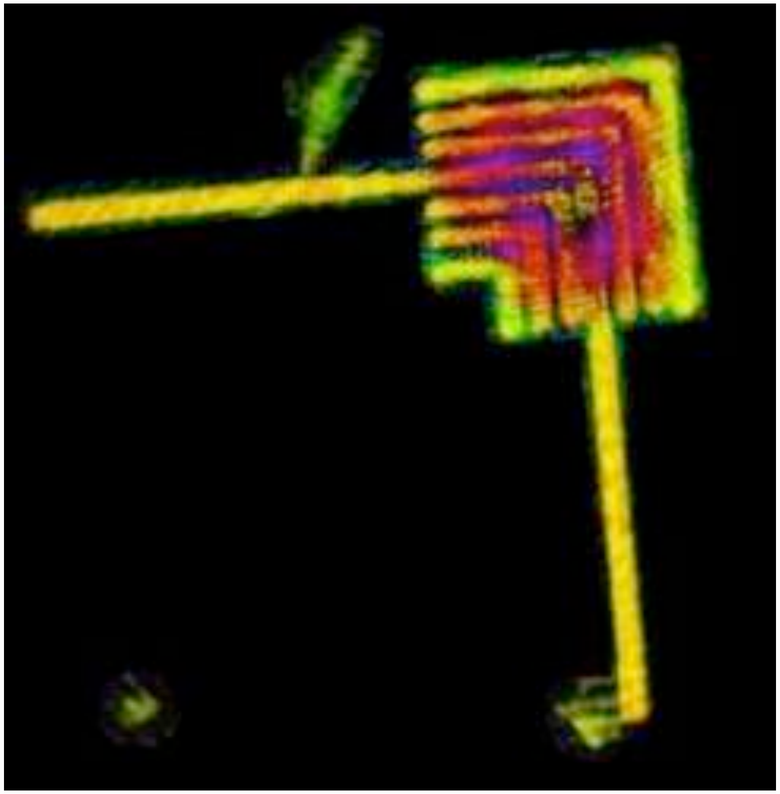


HM Quiney et al, "Iterative image reconstruction algorithms using wave-front intensity and phase variation", Optics Letters **30**, 1638-1640 (2005)

Fresnel diffraction imaging



Fresnel diffraction imaging



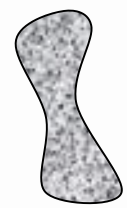
GJ Williams, et al, “Fresnel Coherent Diffractive Imaging”, Physical Review Letters, **97,** 025506 (2006)

What are the effects of partial coherence?

Note: X-ray science is almost always concerned with beams and so the paraxial and quasi-monochromatic assumptions are adopted

A Partially Coherent Description

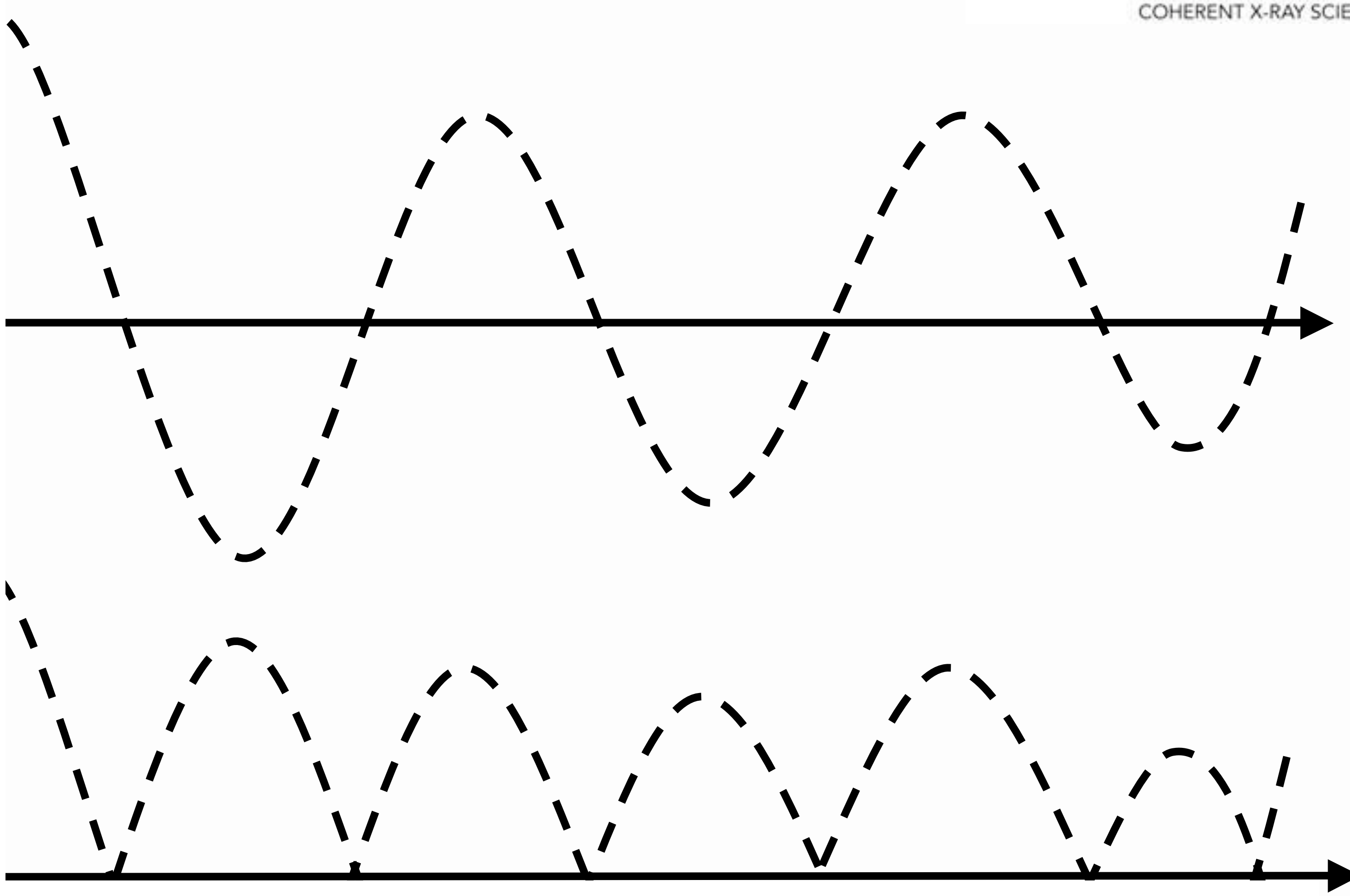
$$g\left(\boldsymbol{r}_1 - \boldsymbol{r}_2\right) = exp\left[-\frac{\left|\boldsymbol{r}_1 - \boldsymbol{r}_2\right|^2}{2\ell_c^2}\right]$$



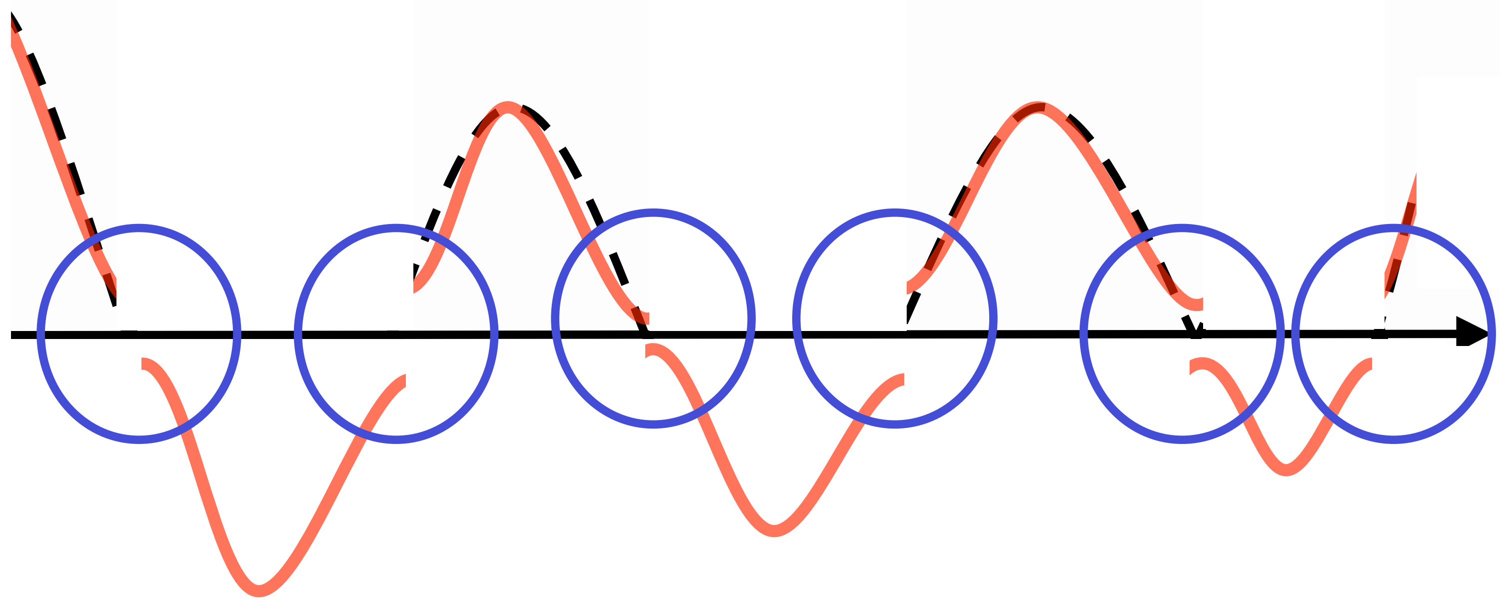
$$I_{\infty}^{sc}\left(s\right) = A_0^2 \int \sigma\left(r\right)\sigma\left(r + x\right)dr \exp\left[-\frac{\left|x\right|^2}{\ell_c^2}\right] \exp\left[-ik_0 s \bullet x\right]dx$$

GJ Williams et al, “Coherent Diffractive Imaging and Partial Coherence”, Phys Rev B, **75**, 104102 (2007)

A Partially Coherent Description



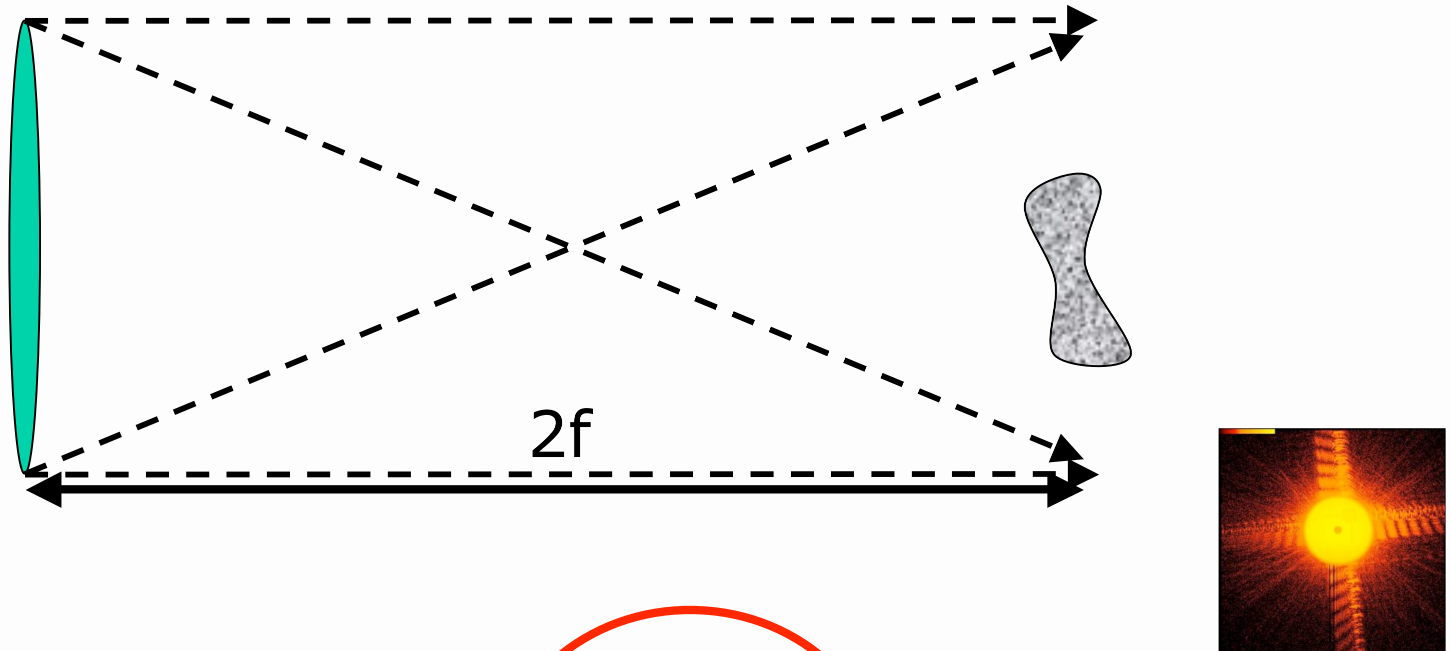
A Partially Coherent Description



There is no object that is consistent with both the data and the prior knowledge if perfect coherence is assumed

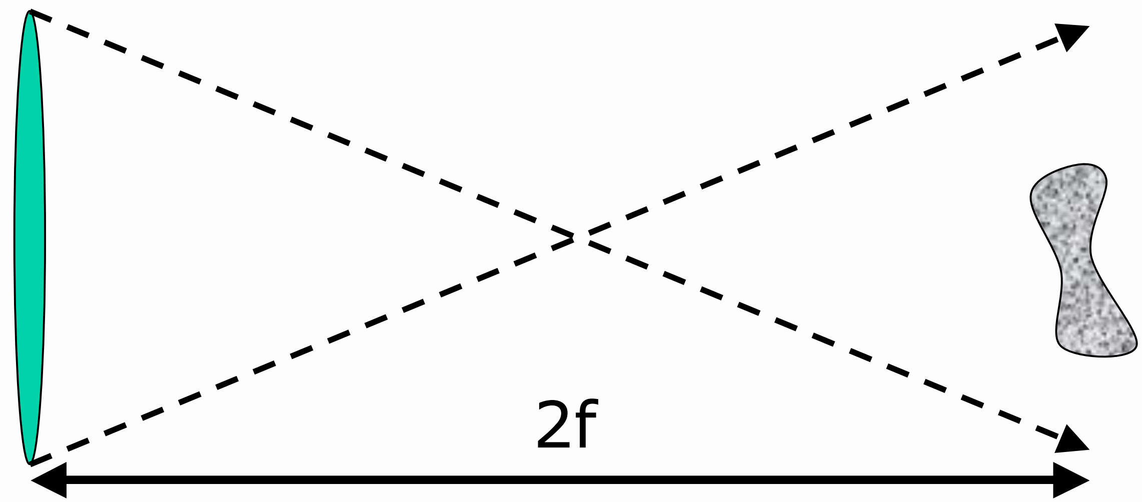
<u>Partial coherence can prevent the iteration from ever converging</u>

Spherical Incident Wave



$$I(s) \sim \int \sigma(r)\sigma(r+x)\exp\left[-ik_0 \frac{r \bullet x}{f}\right] dr \exp\left[-ik_0 s \bullet x\right] dx$$

A Partially Coherent Description

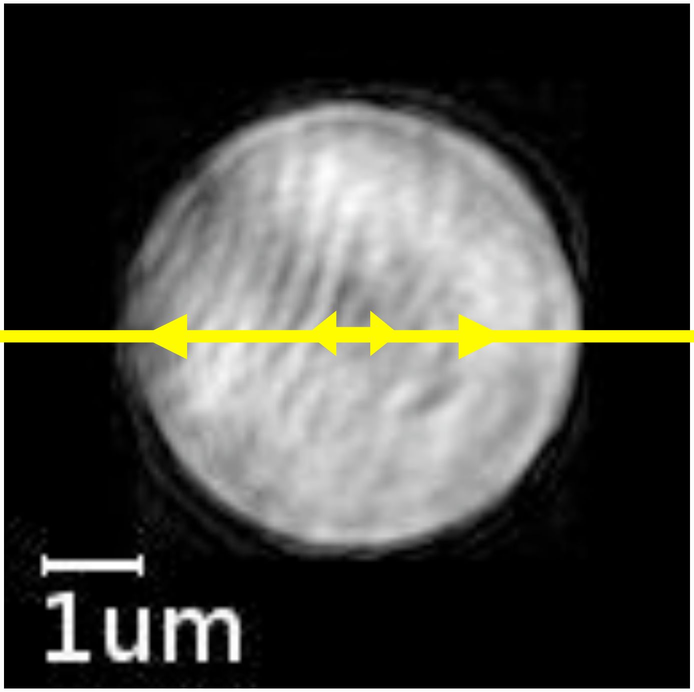


$$g(\mathbf{r}_1 - \mathbf{r}_2) = exp\left[-\frac{|\mathbf{r}_1 - \mathbf{r}_2|^2}{2\ell_c^2}\right] \qquad \mu \equiv \frac{1}{2\ell_c^2} + \frac{ik_0}{f}$$

$$I_\infty^{sc}(\mathbf{s}) = \mathcal{A}_0^2 \int \sigma(\mathbf{r})\sigma^*(\mathbf{r}+\mathbf{x}) exp\left[-ik_0 \frac{\mathbf{r}\bullet\mathbf{x}}{f}\right] d\mathbf{r}\, exp\left[-\mu|\mathbf{x}|^2\right] exp\left[-ik_0 \mathbf{s}\bullet\mathbf{x}\right] d\mathbf{x}$$

GJ Williams et al, “Coherent Diffractive Imaging and Partial Coherence”, Phys Rev B, **75**, 104102 (2007)

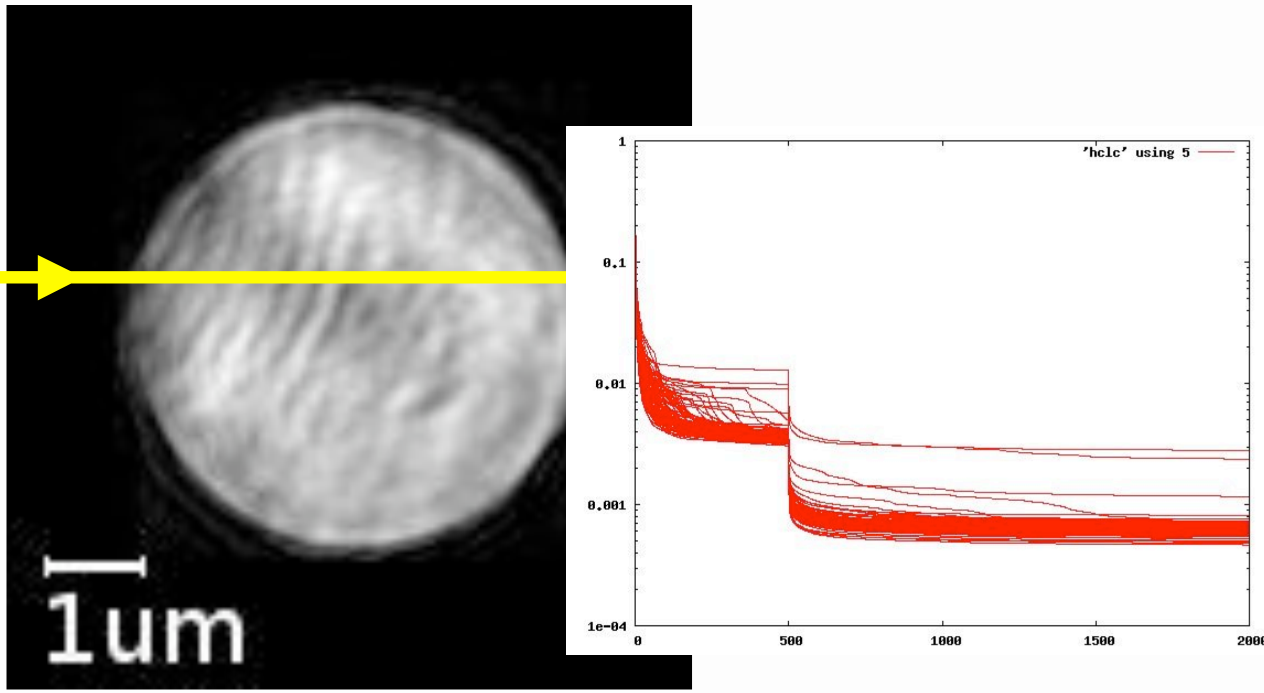
The Effects of Coherence



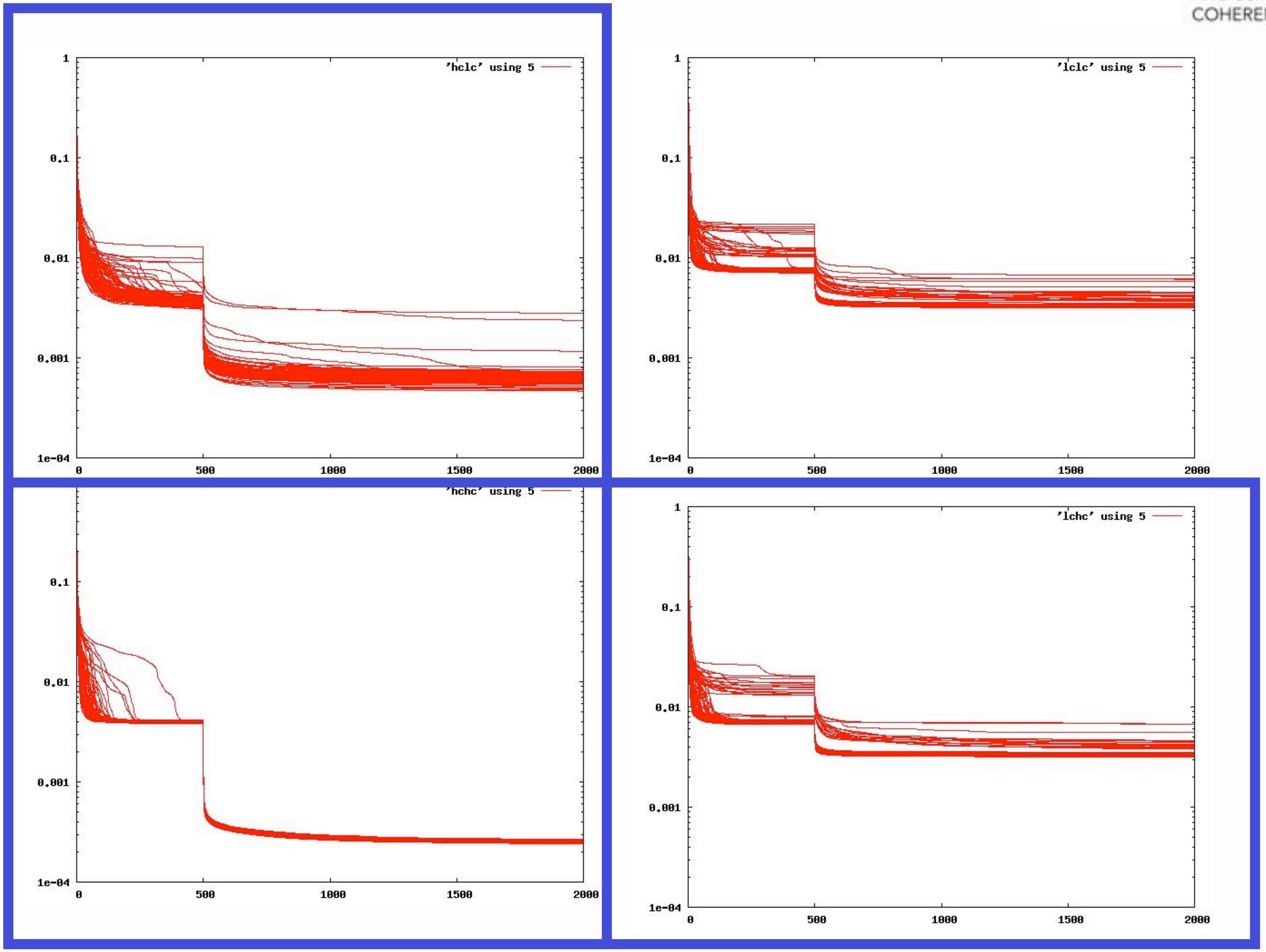
Fresnel# = 0.846

The Effects of Coherence

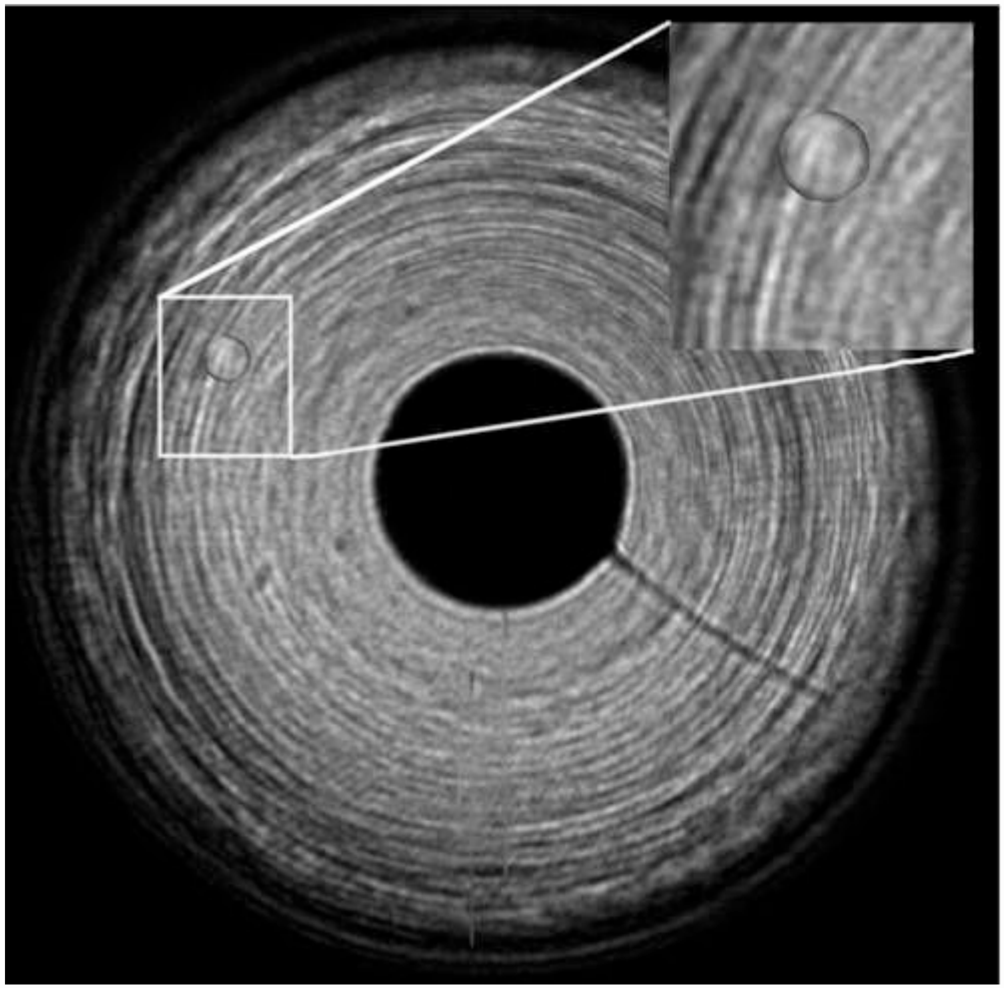
FFesnel##==03846



The Effects of Coherence



Is the Image Correct?



Coherent Diffractive Imaging makes a number of implicit assumptions

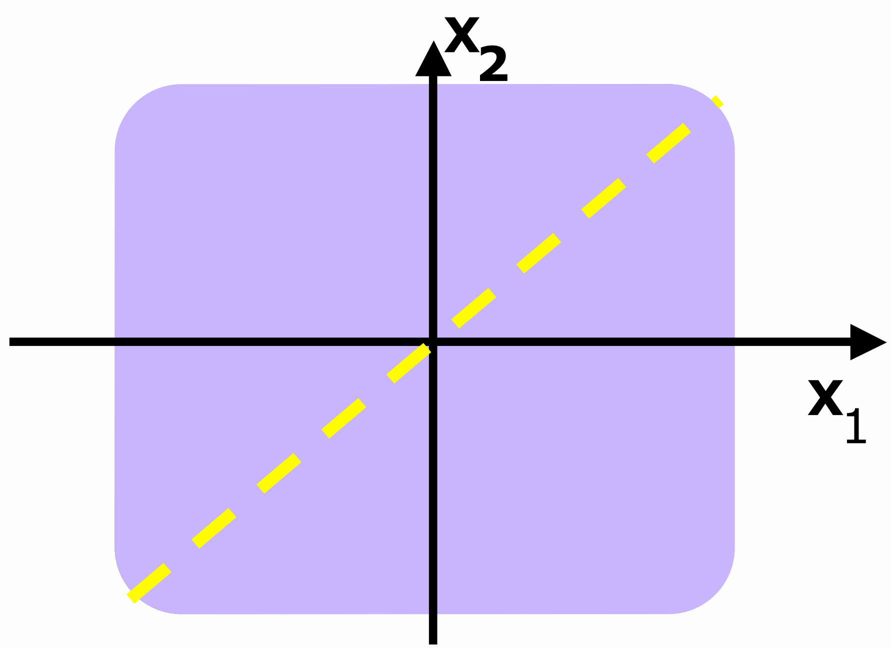
- **Planar incident wave**
- **Complete coherence**
- **Finite support**

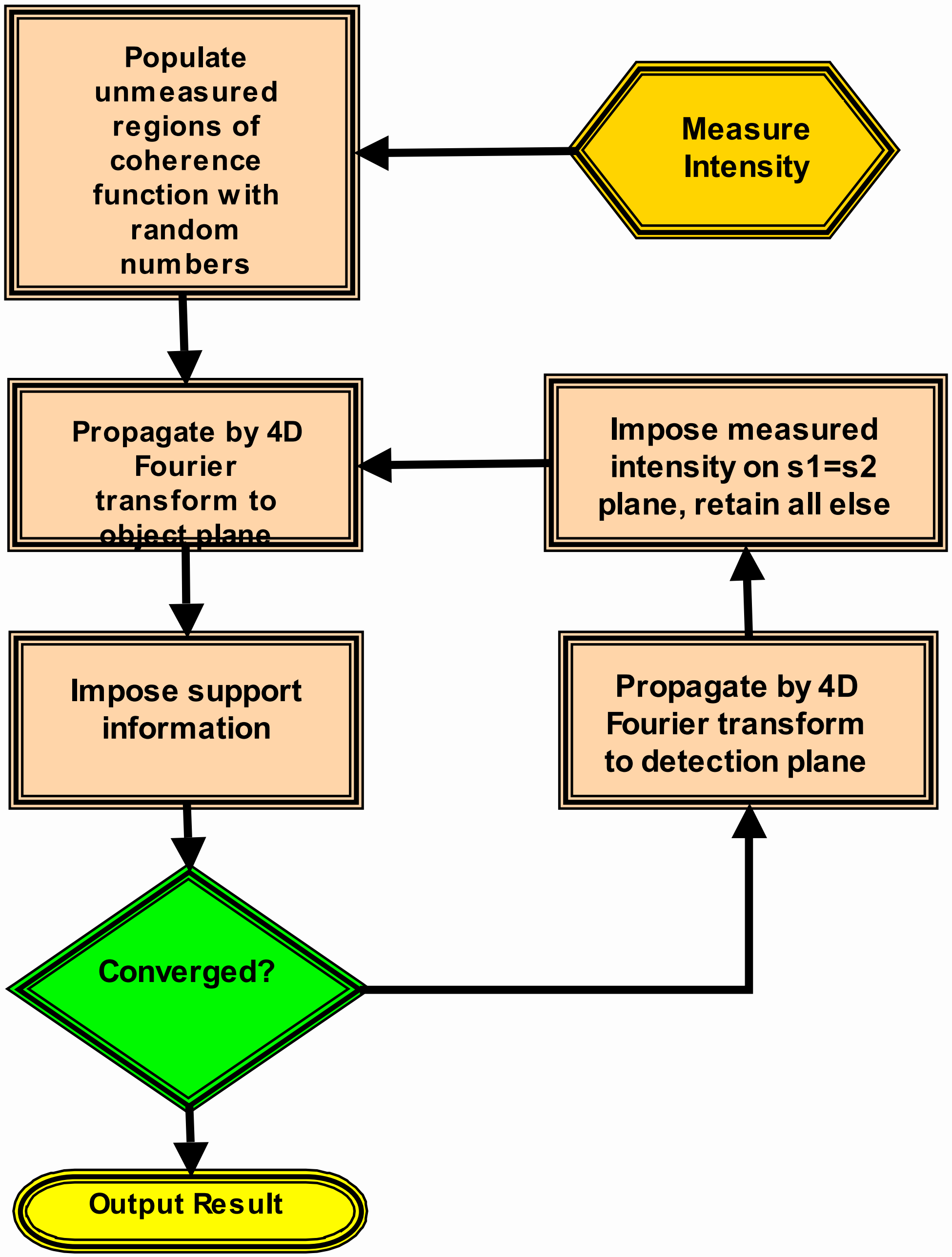
ARC Centre of Excellence for
COHERENT X-RAY SCIENCE

Can we build partial coherence into the reconstruction?

Partially coherent diffraction?

$$J_{out}(r_1, r_2) = T(r_1)T^*(r_2)J_{inc}(r_1, r_2)$$





ARC Centre of Excellence for COHERENT X-RAY SCIENCE

Potential algorithm for recovering partially coherent images

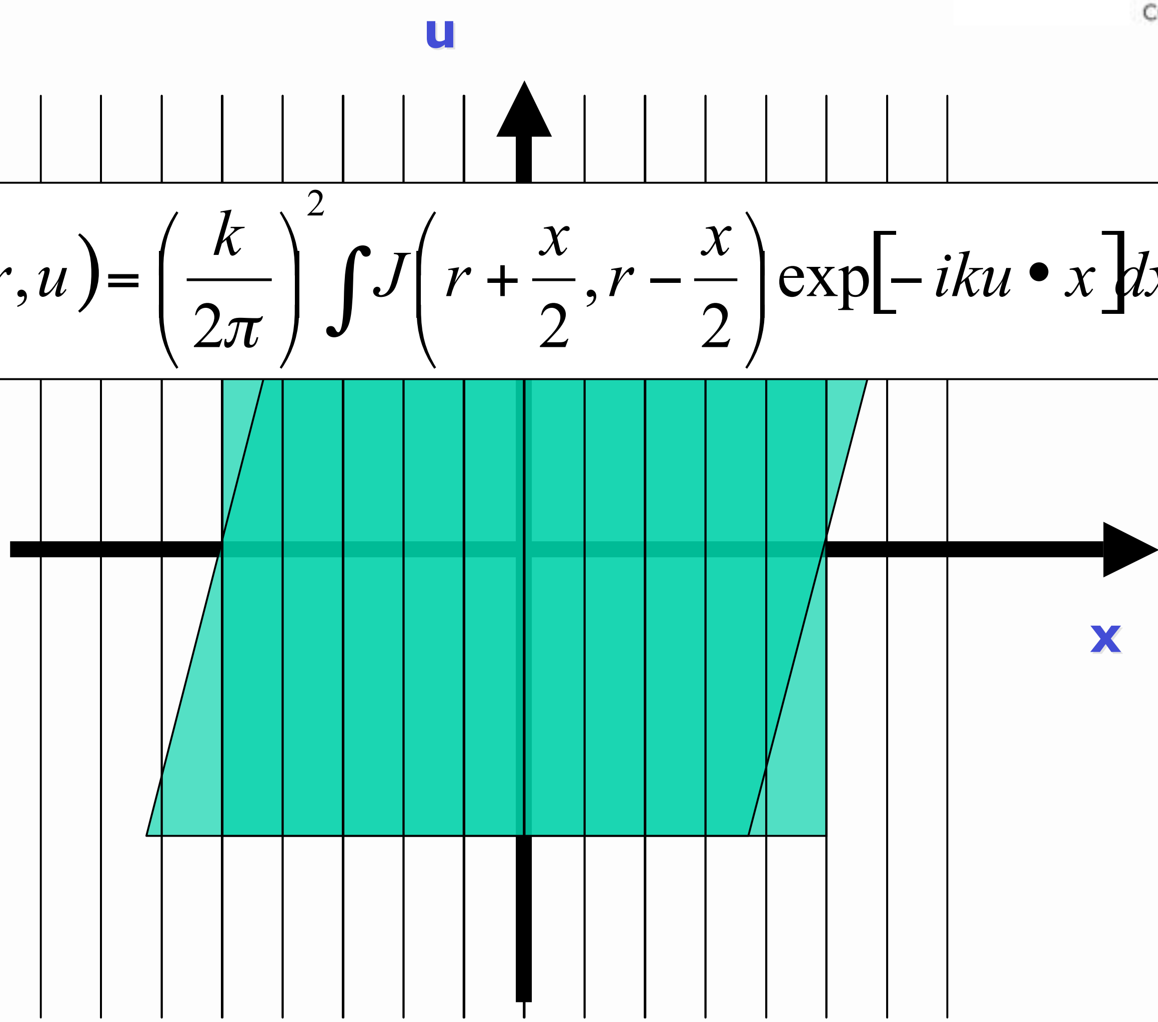
ARC Centre of Excellence for
COHERENT X-RAY SCIENCE

This needs a measurement of the coherence function

Phase Space Tomography

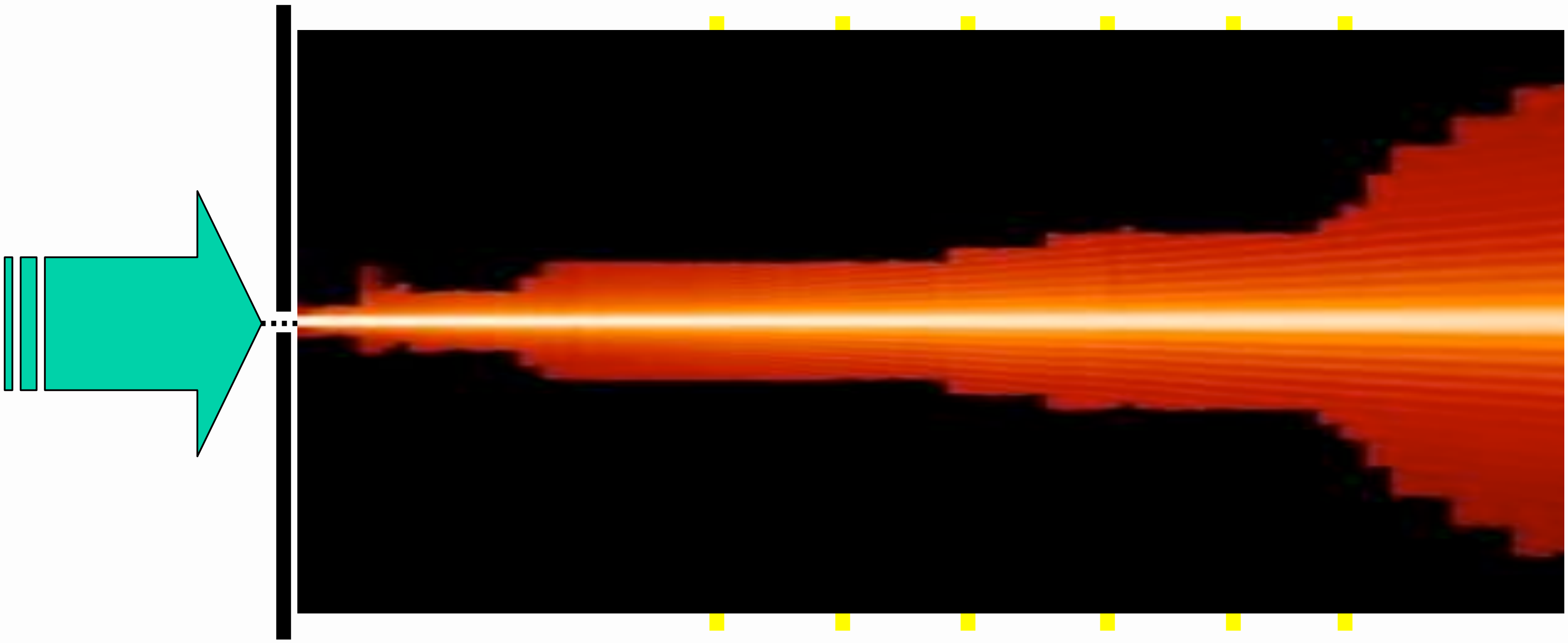
ARC Centre of Excellence for
COHERENT X-RAY SCIENCE

$$B(r,u)=\left(\frac{k}{2\pi}\right)^2\int J\left(r+\frac{x}{2},r-\frac{x}{2}\right)\exp\left[-iku\bullet x\right]dx$$

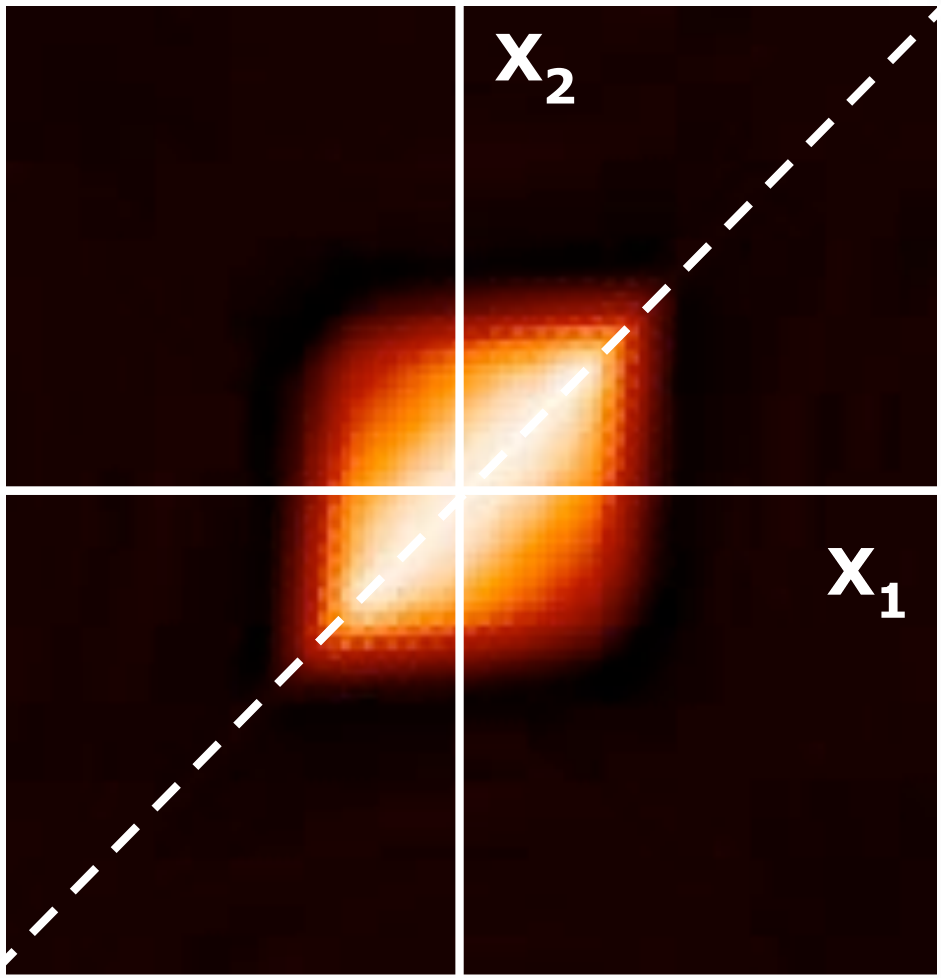


Phase Space Tomography

ARC Centre of Excellence for
COHERENT X-RAY SCIENCE



Recovered Coherence Function



C.Q. Tran, et al, Synchrotron Beam Coherence Measured using Phase-Space Tomography”, Optics Letters, **30**, 204-206 (2005).

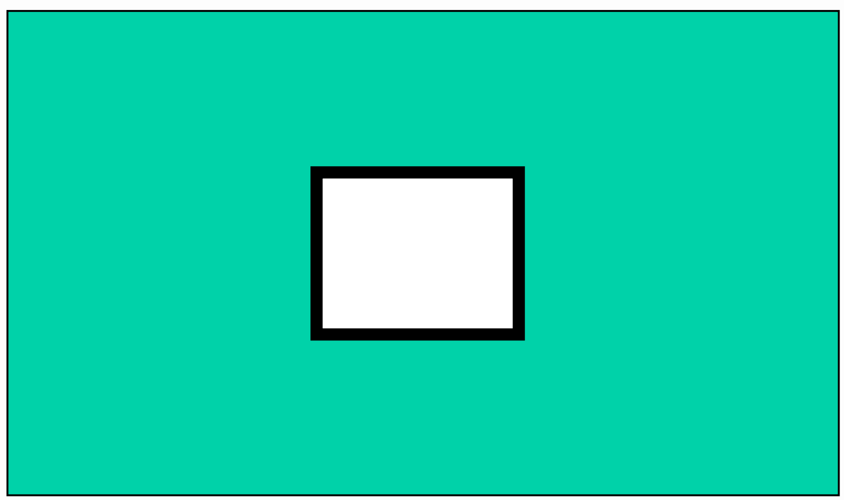
1D is OK, but how do we do a two-dimensional field?

A separable field – one that can be written as a product of a function of x and a function of y – can be reduced to two one-dimensional problems.

A Gaussian field passing through a rectangular aperture will be separable.

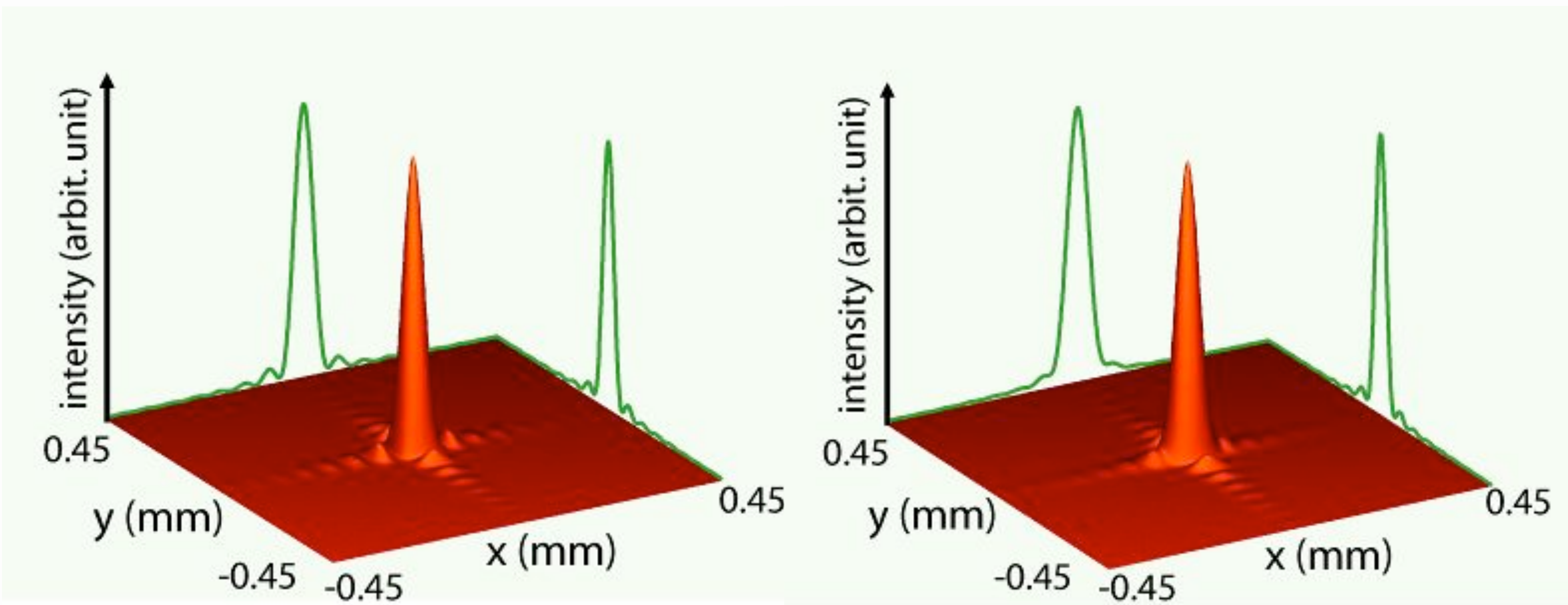
$$J(x_1, x_2, y_1, y_2) = J_x(x_1, x_2)J_y(y_1, y_2)$$

$$I(x, y) = X(x)Y(y)$$



Two-dimensional diffraction patterns

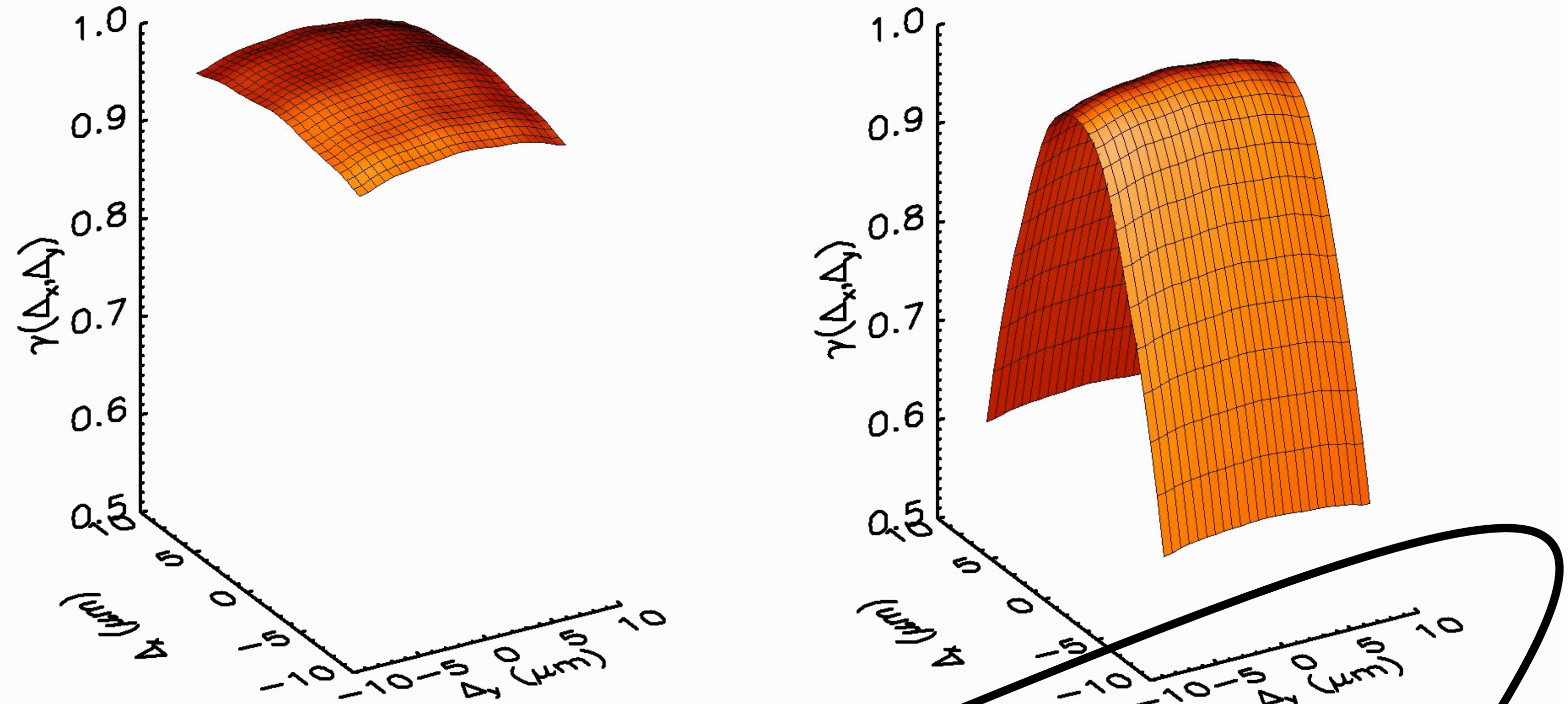
Data is consistent a separable field



$$I(x, y) = X(x)Y(y)$$

The Complex Degree of Coherence

CQ Tran et al, "Experimental Measurement of the Four-Dimensional Mutual Optical Intensity for an Undulator X-ray Source", *Physical Review Letters*, accepted

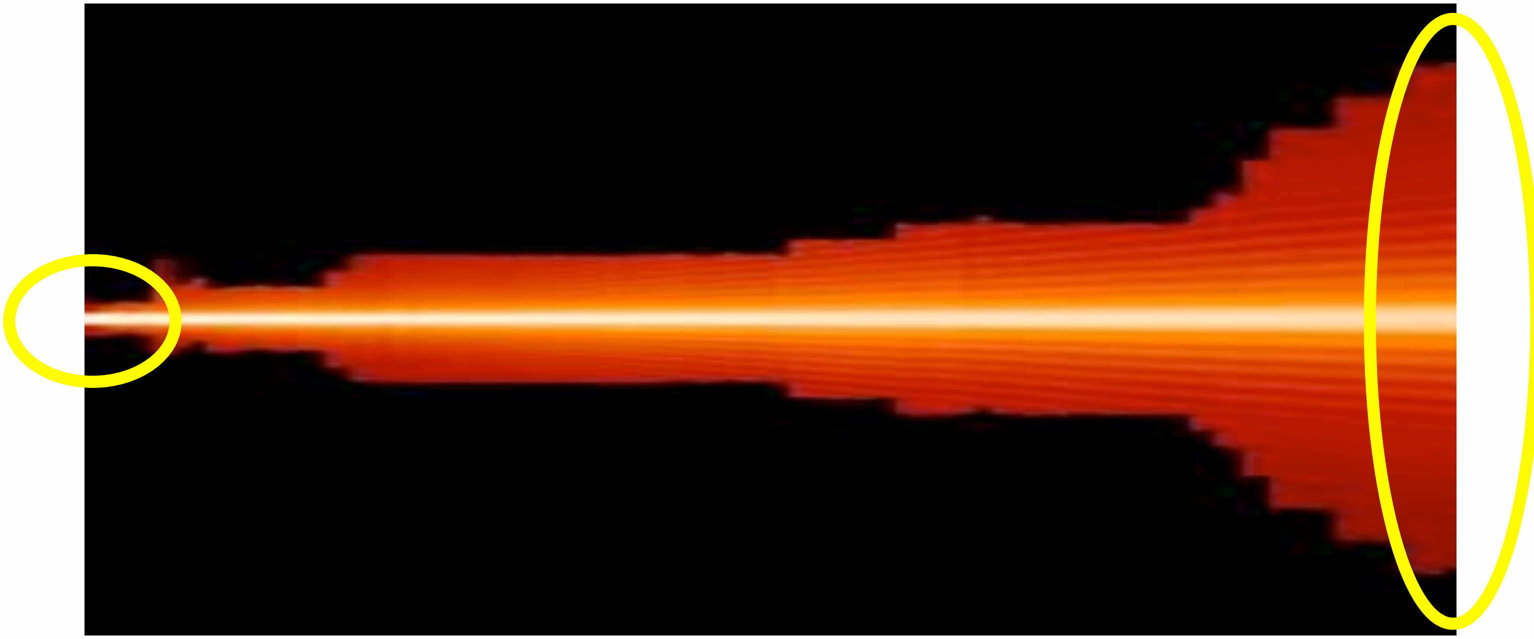


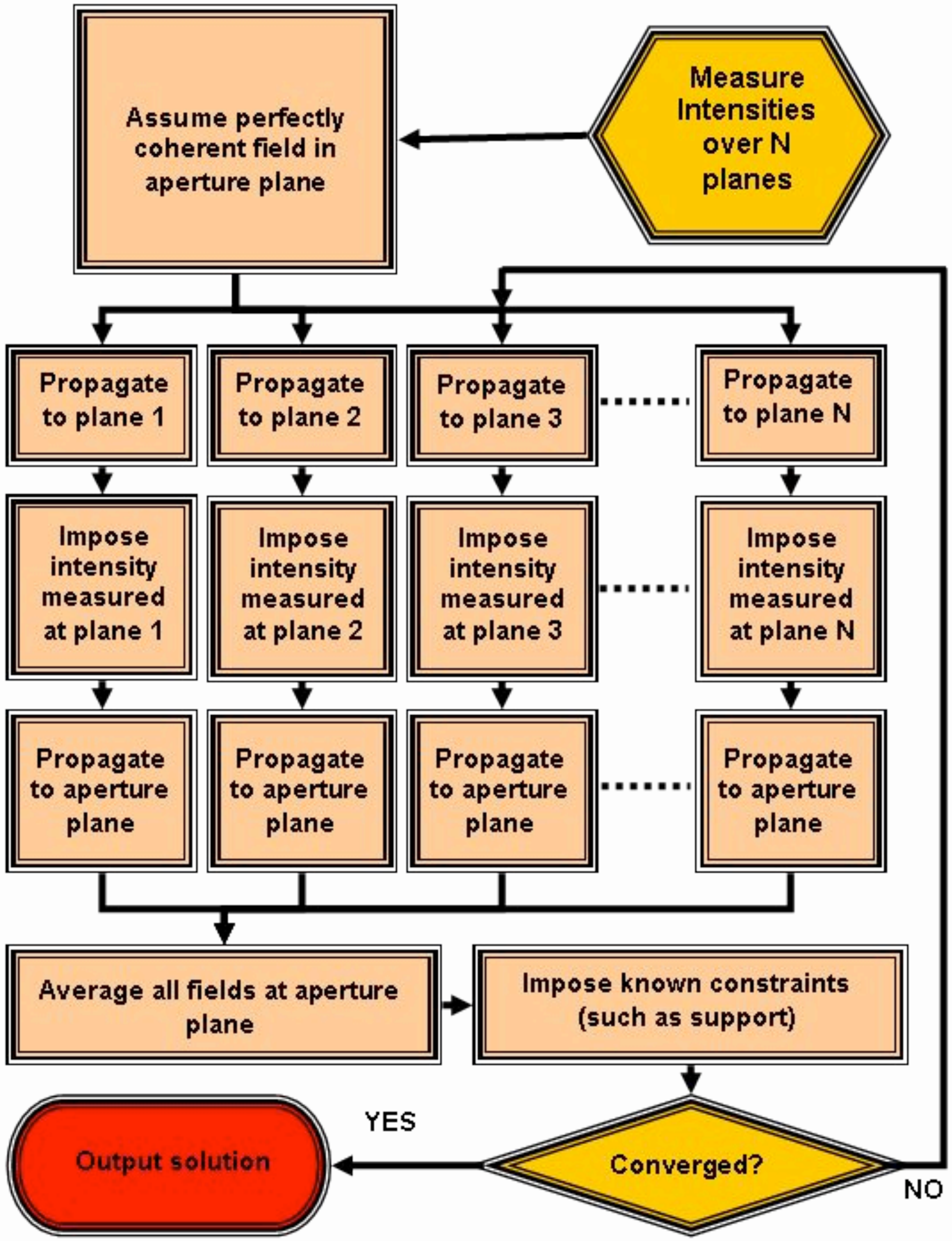
Experimental observation

$$\gamma(x_1,x_2,y_1,y_2)=\frac{J(x_1,x_2,y_1,y_2)}{\sqrt{I(x_1,y_1)I(x_2,y_2)}}\cong\gamma_x(x_1-x_2)\gamma_y(y_1-y_2)$$

Can we improve the spatial resolution?

ARC Centre of Excellence for COHERENT X-RAY SCIENCE

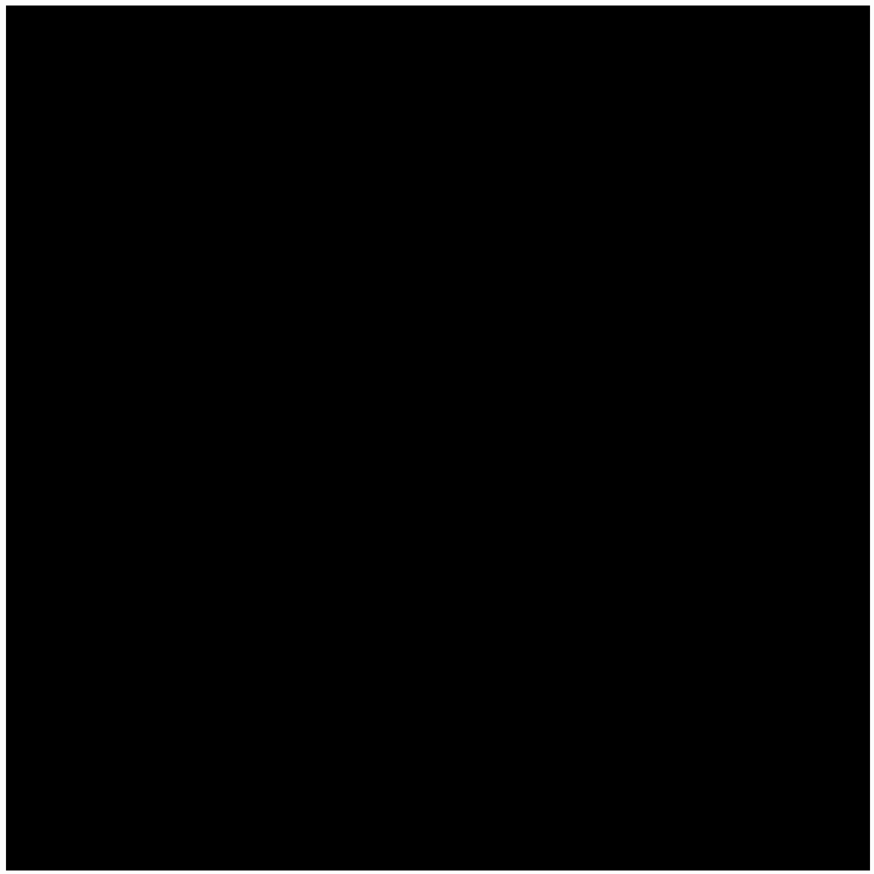




ARC Centre of Excellence for COHERENT X-RAY SCIENCE

An algorithm for recovering the MOI

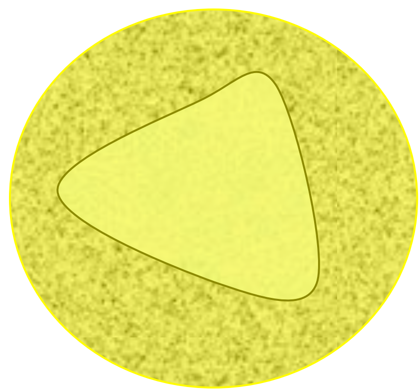
Iterative Recovery of the Mutual Optical Intensity



Coherent Diffractive Imaging makes a number of implicit assumptions

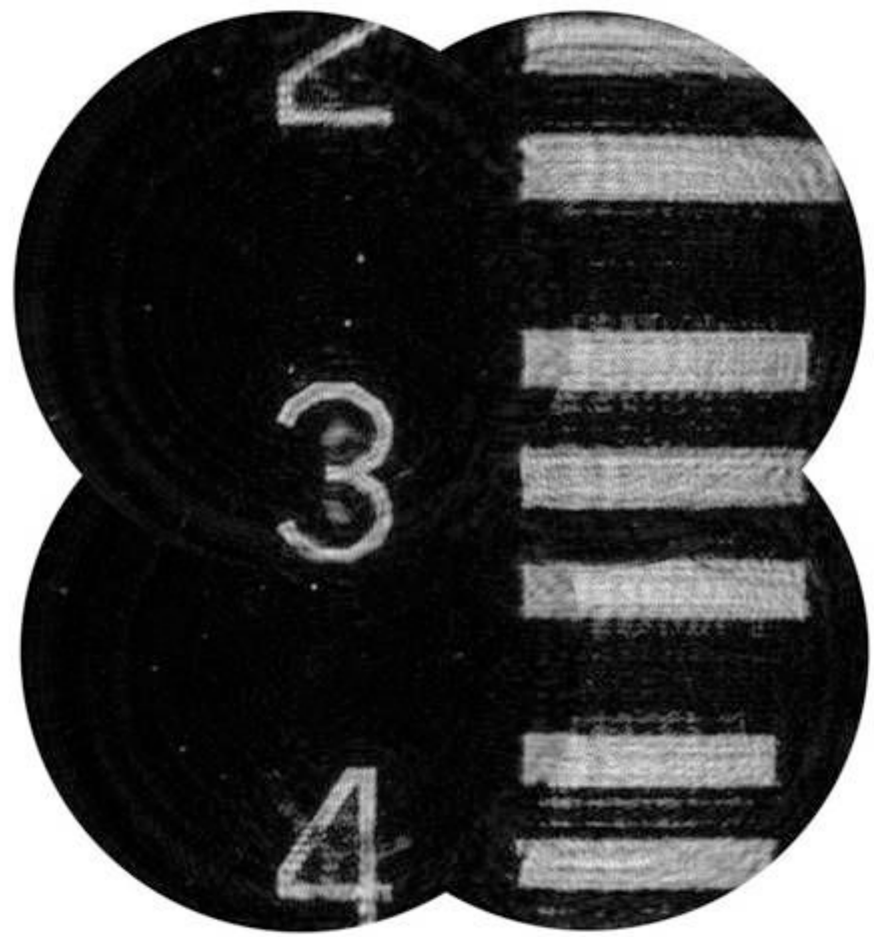
- Planar incident wave
- Complete coherence
- Finite support

What do we mean by “support”?

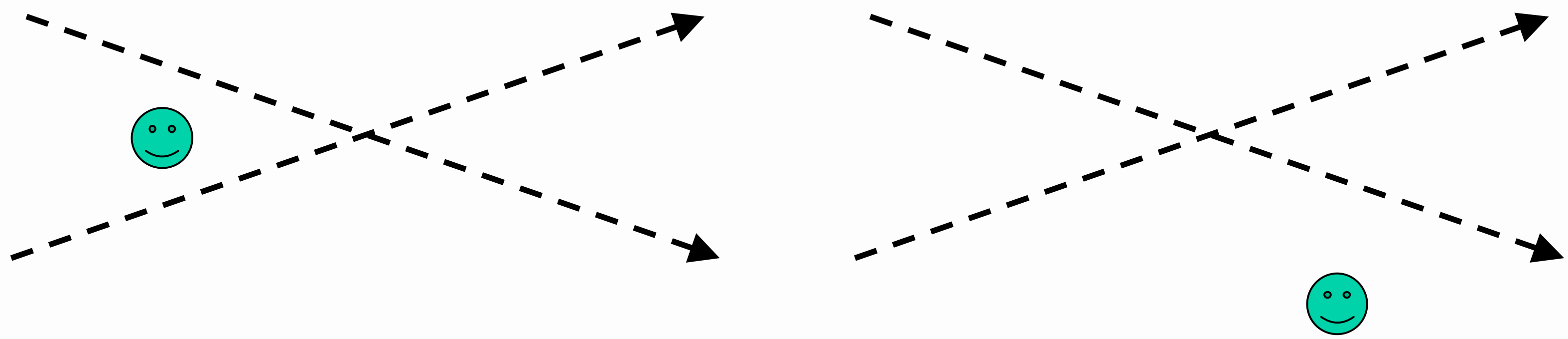


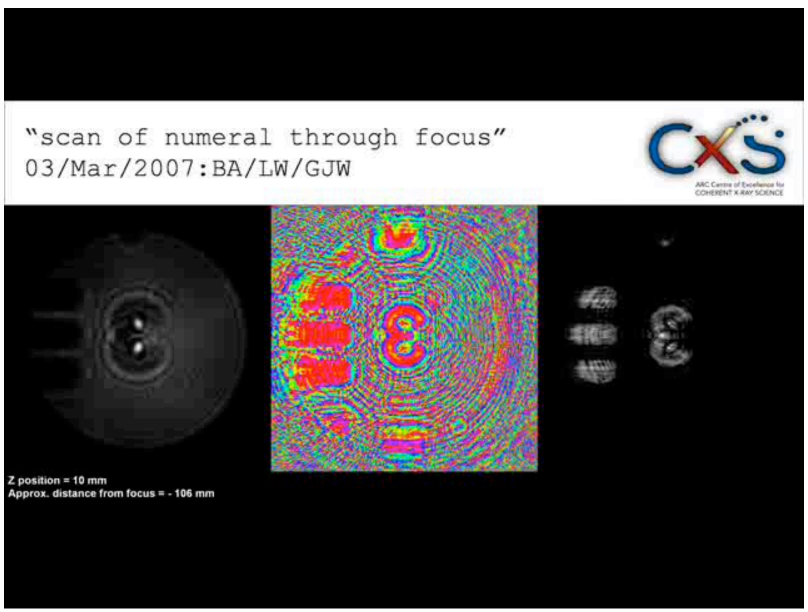
Independent Fresnel CDI images of a large object

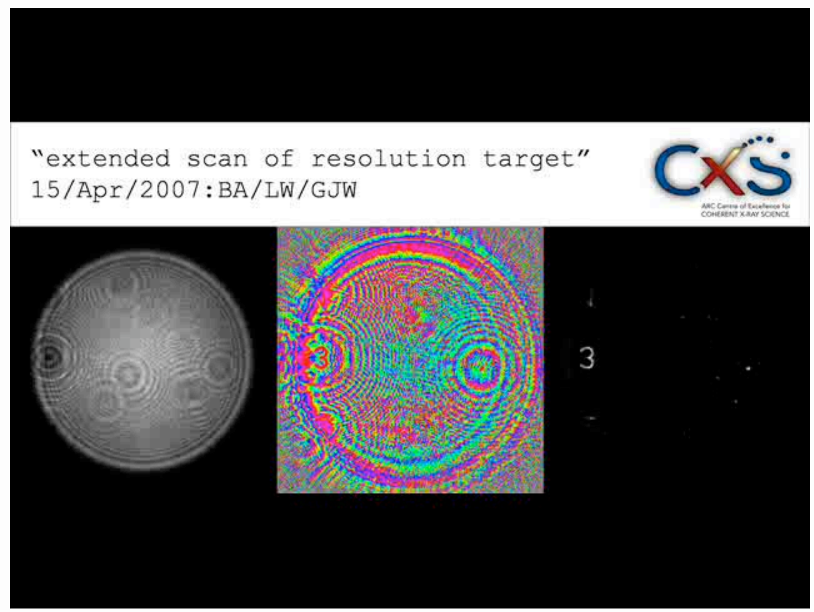
Experimental data obtained using coherent visible light



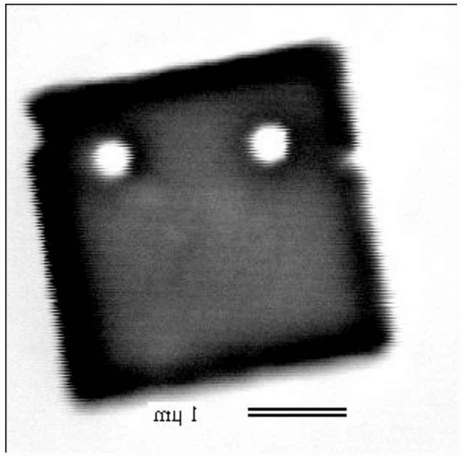
ARC Centre of Excellence for
COHERENT X-RAY SCIENCE

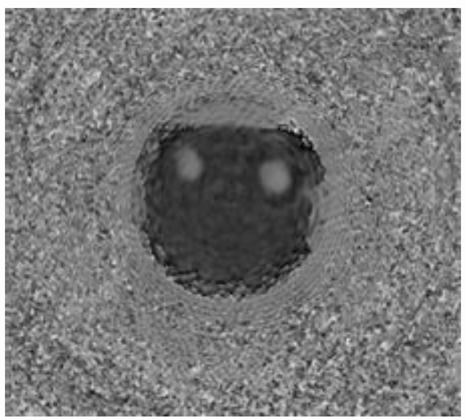


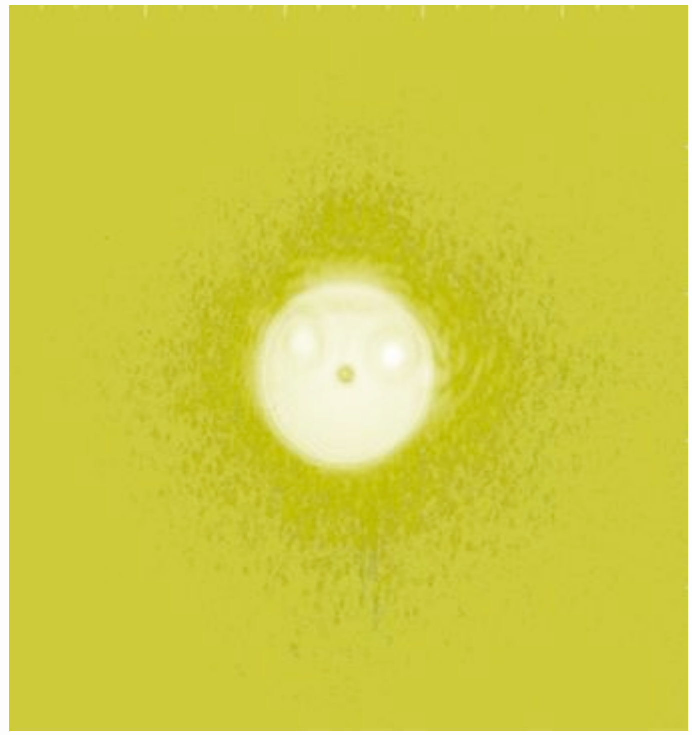




ARC Centre of Excellence for
COHERENT X-RAY SCIENCE



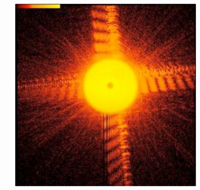




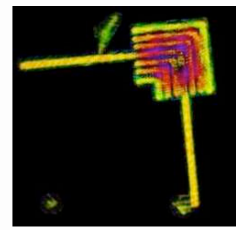
ARC Centre of Excellence for
COHERENT X-RAY SCIENCE

Summary

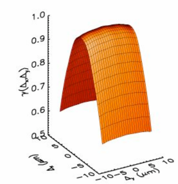
- **Coherent x-ray techniques are emerging rapidly**



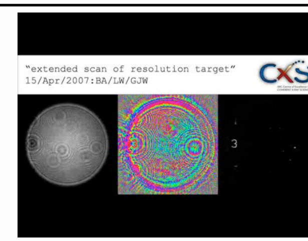
- **Curved-beam diffraction can have significant advantages**



- **May be able to include a coherence measurement into the recovery method**



- **Assumptions about “support” may be unnecessarily restrictive**



Collaborators

ARC Centre of Excellence for
COHERENT X-RAY SCIENCE

- Harry Quiney (UM)
- Andrew Peele (La Trobe)
- Garth Williams (UM)
- Lachlan Whitehead (UM)
- Brian Abbey (UM)
- Sam Flewett (UM)
- Chanh Tran (now at La Trobe)
- Jesse Clark (La Trobe)
- Bipin Dhal (UM)
- David Paterson (Australian Synchrotron)
- Martin de Jonge (APS)
- Ian McNulty (APS)

ARC Centre of Excellence for
COHERENT X-RAY SCIENCE