Sensitivities+Contours
or equivalently
Statistics & Probabilities is a tricky business!!

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Outline

- What do we measure, or want to measure, in neutrino oscillation experiments.

  (First, that neutrinos are indeed missing…)

- Differentiating between Neutrino Oscillation vs Alternative Hypothesis.

  (Second, that neutrinos are missing in a "particular" way as a function of their energy)

Outline

- Measuring Neutrino Oscillation parameters

  (Third, if indeed they are oscillating we want to know the oscillation parameters->2D Contours and what do they represent)

- Planning for the exciting future: We want to measure ALL neutrino oscillation parameters!! (we have very little or no clue at all what their values are...How do we go about that?-> 2D Sensitivities shown for the third mixing angle, CP Violating Phase and Mass Hierarchy and what do they represent)

1

What do we measure, or want to measure, in neutrino oscillation experiments.

(First, that neutrinos are indeed missing…)

What do we measure, or want to measure, in neutrino oscillation experiments.

- The first thing we want to verify is that neutrinos are indeed "missing" (missing does not mean "oscillating") and how significant the deficit is.

- Therefore we construct a "Null Hypothesis = Neutrinos are not missing" and we test against it.

- Suppose we expect 20 neutrinos and we see 10:

$$\chi^2 = \left(\frac{20-10}{\sqrt{20}}\right)^2 = \left(\frac{10}{4.47}\right)^2 = 5 \Rightarrow$$

$$significance = \sqrt{\chi^2} = \sqrt{5} = 2.23\,\sigma$$

- The above results (oversimplified here) you might see in talks as the following statement "The deficit (of neutrinos) seen is an xxx sigma effect".

What do we measure, or want to measure, in neutrino oscillation experiments con't

- 3 sigma is "good" for strong evidence, 5 sigma is "good" for discovery. All of these are "subjective" and based on the Probabilities.
- 5 sigma effect means that the Probability of this measurement being consistent with the "Null Hypothesis" is < 0.0001%

2 : What is the next thing we want to study

Differentiate between Neutrino Oscillation vs Alternative Hypothesis.

(Second, that neutrinos are missing in a "particular" way as a function of their energy)

Suppose neutrino ARE missing, what next? 3-Flavor Oscillation Formalism

- If neutrinos oscillate, then the interaction eigenstates (what we observe) can be expressed in terms of the mass eigenstates as follows:

$$\nu_{e(\mu)(\tau)} = \sum_{i=1}^{3} U^{*}_{e(\mu)(\tau)i}\,\nu_i$$

Atmospheric — Cross Mixing — Solar

$$U=\begin{bmatrix}1 & 0 & 0\\ 0 & c_{23} & -s_{23}\\ 0 & s_{23} & c_{23}\end{bmatrix}\begin{bmatrix}c_{13} & 0 & -s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ s_{13}e^{-i\delta} & 0 & c_{13}\end{bmatrix}\begin{bmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$c_{ij} = \cos\theta_{ij}$$

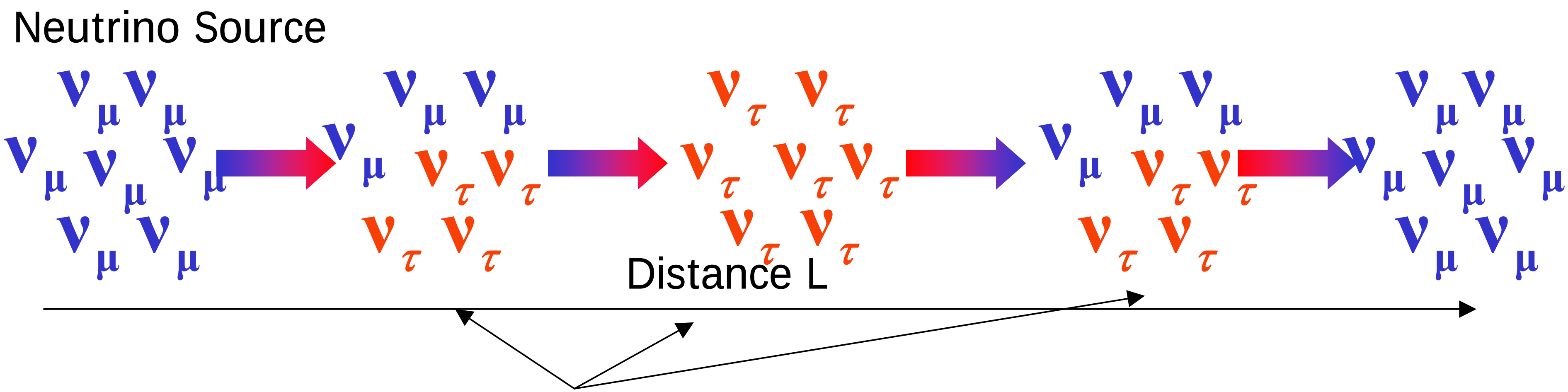
$$s_{ij} = \sin\theta_{ij}$$

Majorana phases

$$\begin{bmatrix}e^{ia_1/2} & 0 & 0\\ 0 & e^{ia_2/2} & 0\\ 0 & 0 & 1\end{bmatrix}$$

$0\nu\beta\beta$ decays

Neutrino oscillations



Count ν_μ's here and you will find them Missing!!

Simple Math:

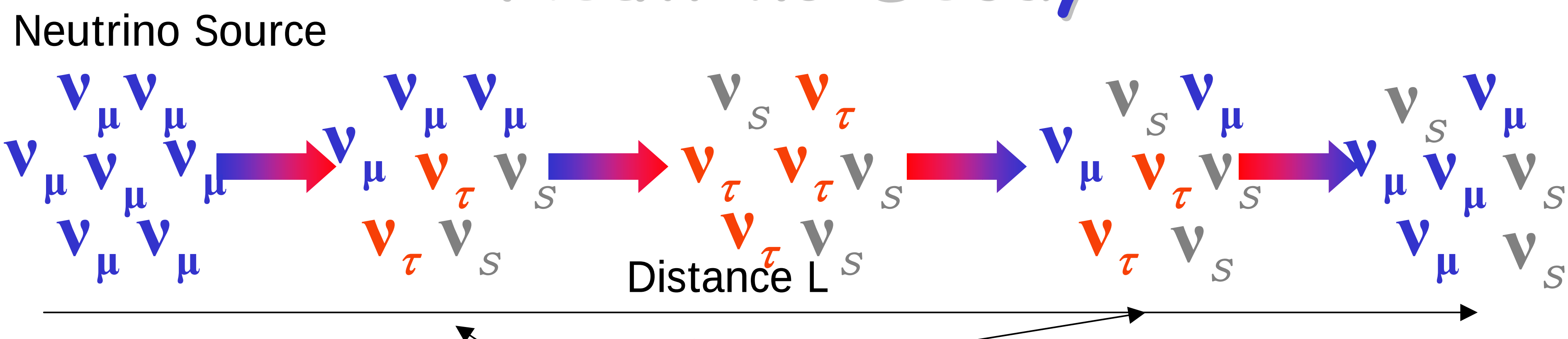
$$P = \alpha \cdot \sin^2\left(\frac{1.267 \cdot \Delta m^2 \cdot L}{E}\right)$$

2 parameters we don't know and we want to measure

L: distance from source

E: energy of neutrinos

Neutrino Decay



Count ν_μ's here and you will find them Missing!!

Count all ν's and you will find them Missing too…

Simple Math:

$$P = \left(\sin^2(\theta) + \cos^2(\theta)\cdot e^{-\frac{mL}{2\tau E}}\right)^2$$

2 parameters we don't know and we want to measure

L: distance from source

E: energy of neutrinos

Neutrino Decoherence

NO PICTORIAL VIEW! (NOT EASY)

But it is a theory (theorist are extremely bright with wild/vivid imagination)

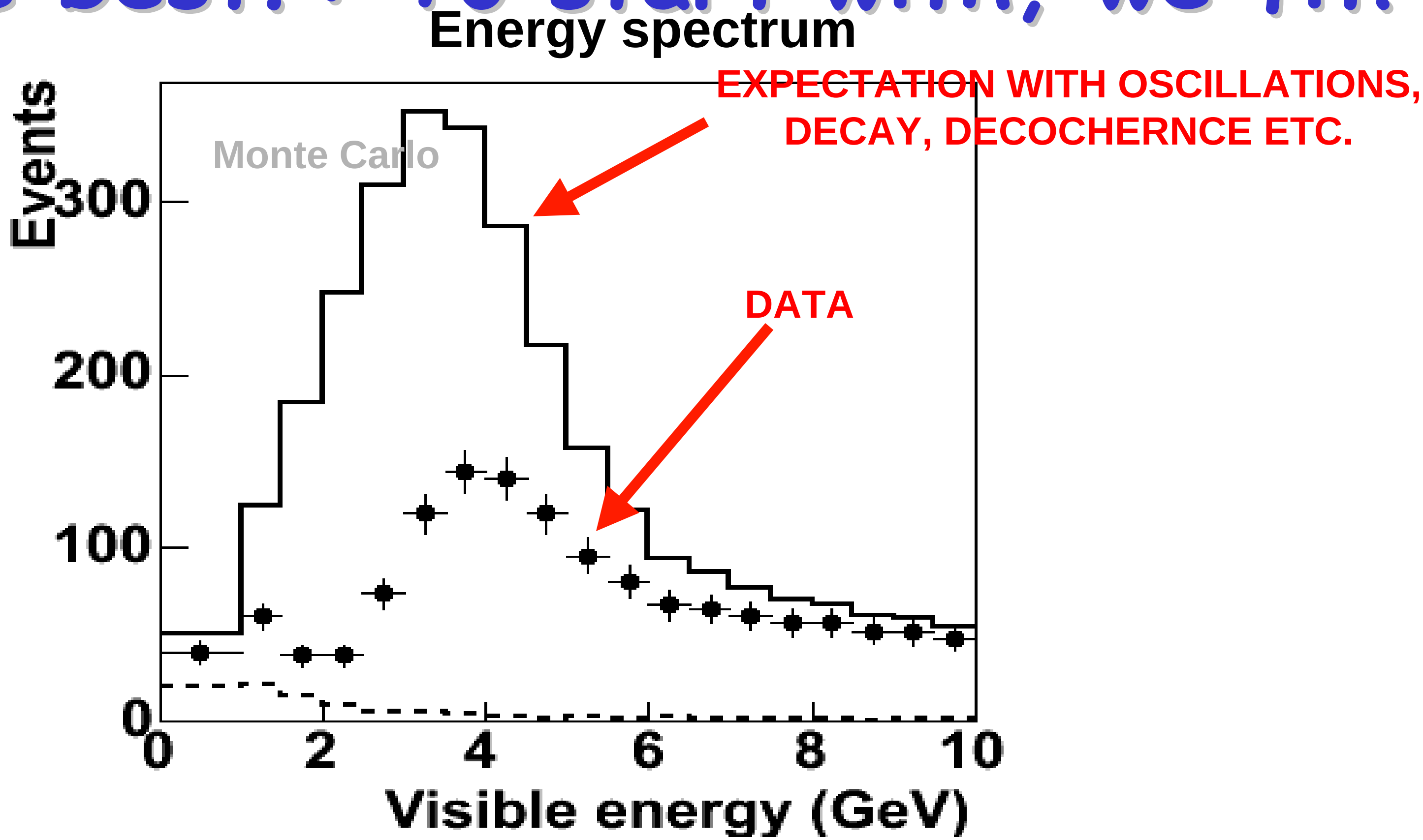
$$P = 1 - \frac{\sin^2 2\theta}{2}\left(1 - e^{-\frac{\mu^2 L}{2E}}\right)$$

2 parameters we don't know and we want to measure

L: distance from source

E: energy of neutrinos

How do we know which "Hypothesis" is the best?: To start with, we fit!



Fit the Data to the hypothesis , looping over all possible values of hypothesis parameters (2 in the cases we considered) and obtain the best χ^2 :

$$\chi^2(\Delta m^2, \sin^2 2\theta, s_1, \ldots, s_{nsys}) = \sum_{i=1}^{nbins} 2(e_i - o_i) + 2o_i \ln(o_i / e_i) + \sum_{j=1}^{nsys} \Delta s_j^2 / \sigma_{s_j}^2$$

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2. Calculate $\Delta\chi^2$ of the best fits for the various hypothesis

   Suppose $\chi^2(A) = 45$ & $\chi^2(B) = 65$. $\Delta\chi^2 = 65-45=20$. This $\Delta\chi^2$ corresponds to 1 degree of freedom.

   Parenthesis : Degrees of Freedom = (Number of observations - number of constraints)

$$Significance = \sqrt{\Delta\chi^2} = \sqrt{20} = 4.47\,\sigma$$

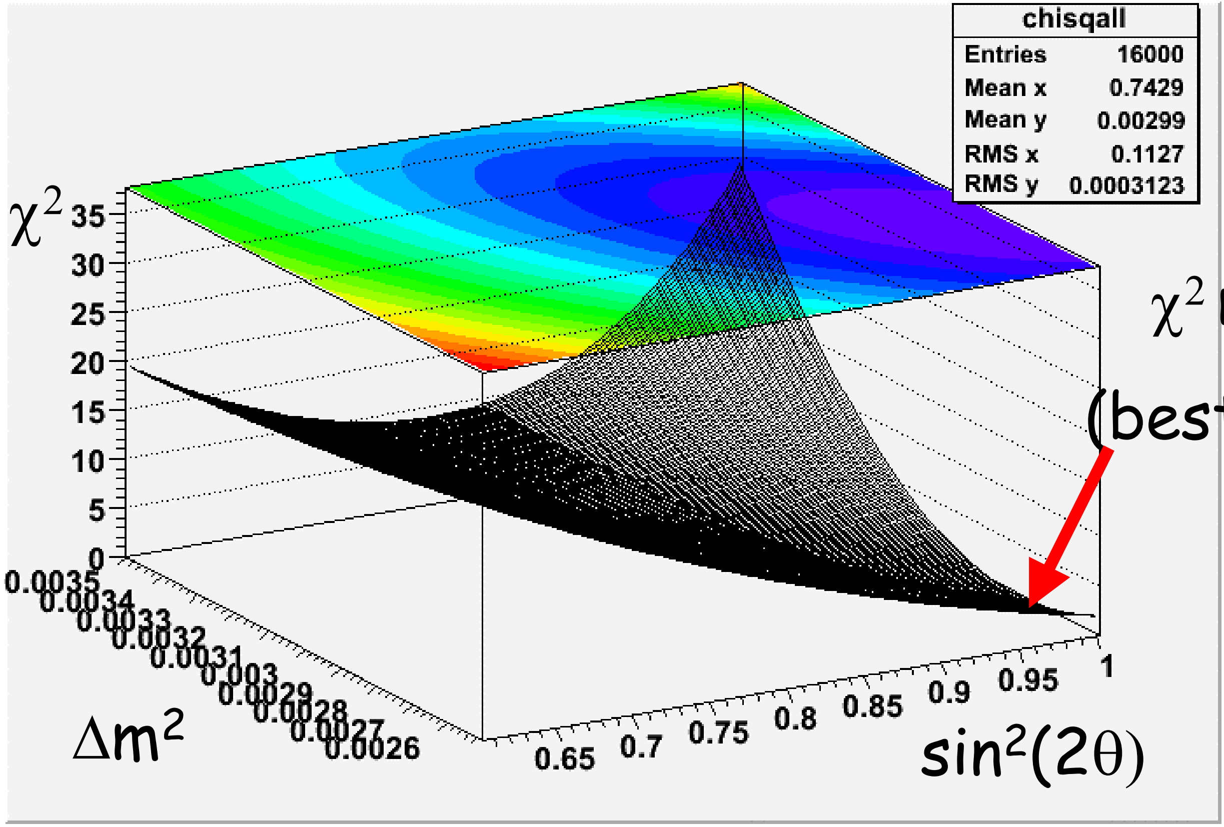
3: What is the next thing we want to study

Measure Neutrino Oscillation parameters

(Third, if indeed they are oscillating we want to know the oscillation parameters->2D Contours and what do they represent)

How do we estimate oscillation parameters : 2D Contours

1. Loop over all possible values of the hypothesis parameters (2 in the cases we considered) and for each $(\Delta m^2, \sin^2(2\theta))$ point calculate and save the corresponding χ^2.: That way we create a 2D χ^2 surface:



2D Contours : What do they mean?

- Every experimental measurement has experimental errors: Statistical and Systematic. Quoting the "best fit" point only, does not convey much information.
- One should report on what the allowed range of the parameters of interest is taking into account statistical and systematic errors.
- This allowed range should correctly encapsulate the following:

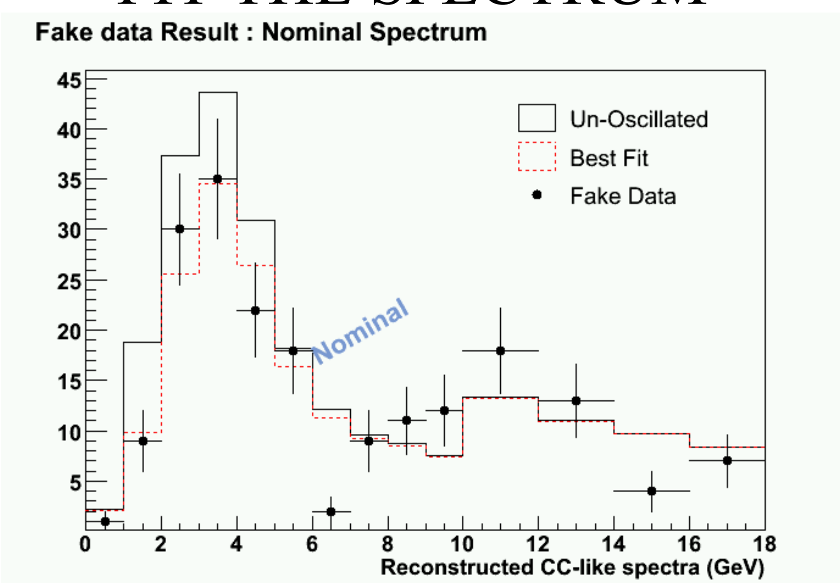
The Probability that the "TRUE" parameters of interest can lie to within this range given this experimental results is Y. (Typical values of Y are 68%, 90%, 95%).

2D Contours : What do they mean?

- *Given a minimum χ^2_{min}, and given the known number of degrees of freedom one can estimate for a desired Y (confidence level) which points should be included from the 2D χ^2 surface*

(points with $\chi^2 = \chi^2_{min} + UP$)

FIT THE SPECTRUM ⟶ GET THE CONTOUR



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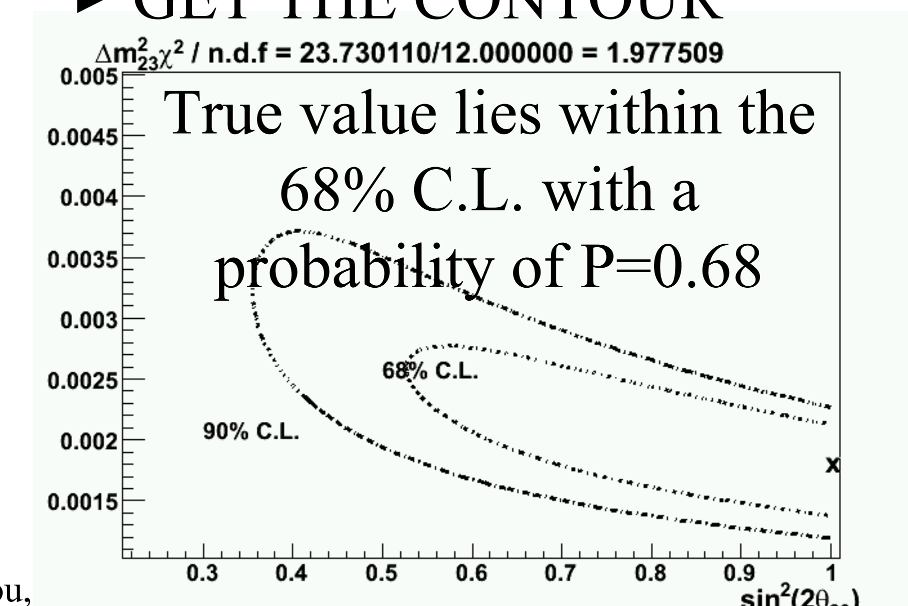
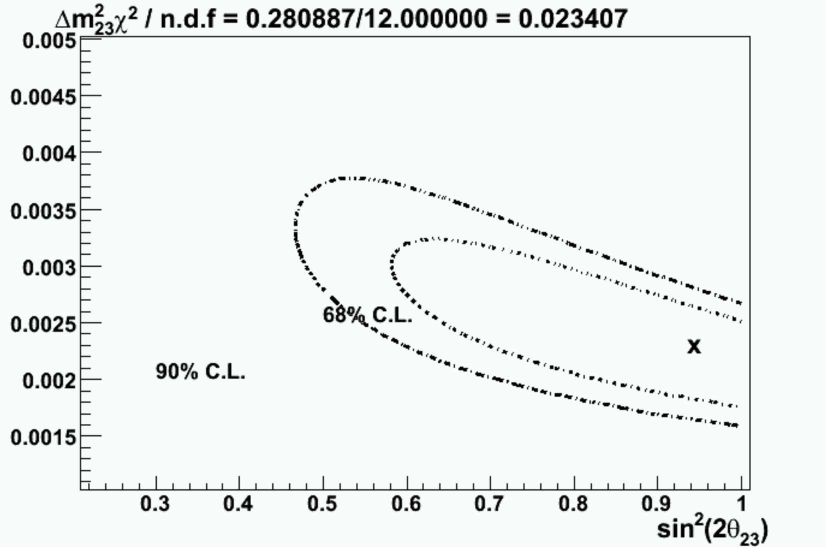
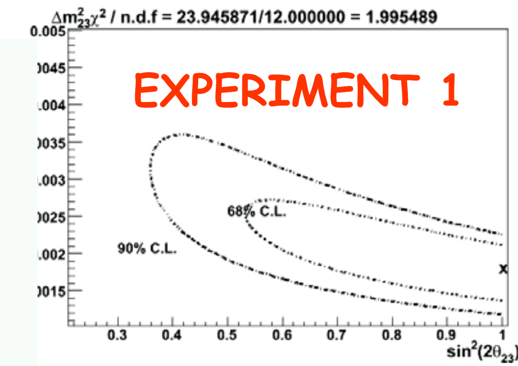
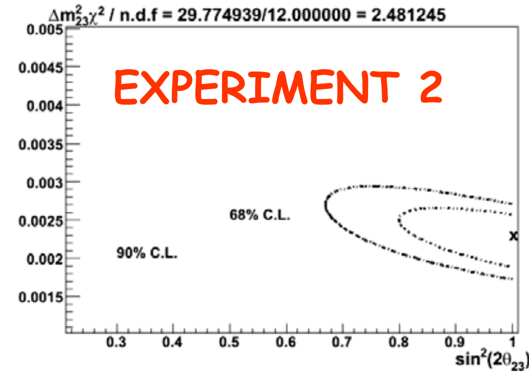


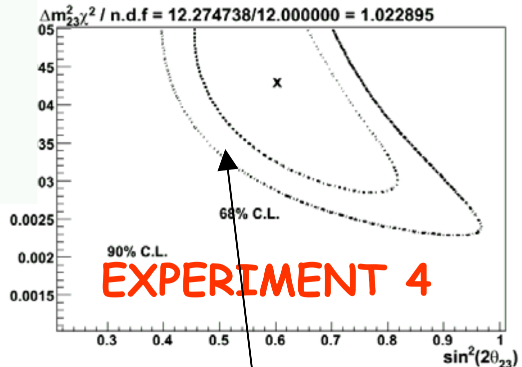
Statistics can be "tricky"

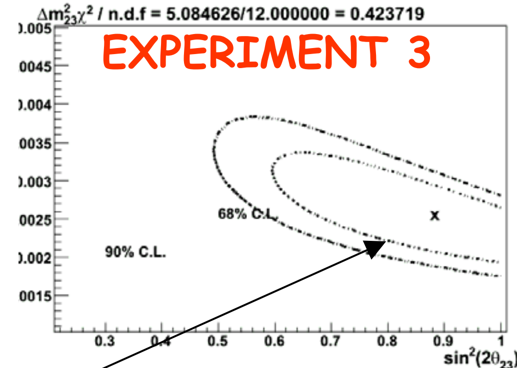
HIGH STATISTICS SAMPLE (SCALED)





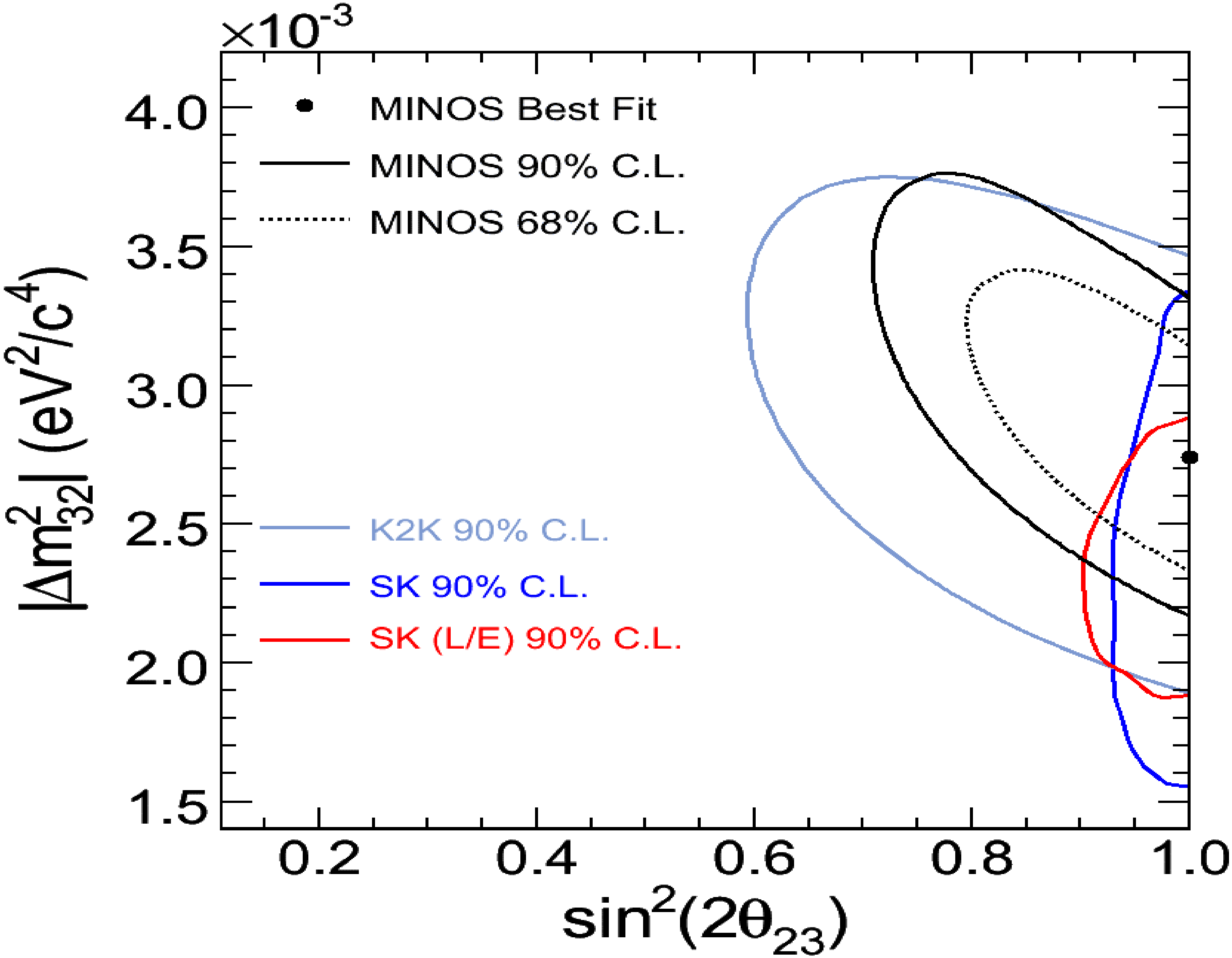






Due to statistical fluctuations each experiment can yield a different Δm^2 & $\sin^2(2\theta)$ along with quite "different" 2D contours. All these fake "experiments" however correspond to the same TRUE Oscillation Parameters….One can get "lucky" or "unlucky"

REAL LIFE EXAMPLE



4: What is the next thing we want to study

-Plan for the exciting future: We want to measure ALL neutrino oscillation parameters!!
-We have very little or no clue at all what their values are…

-How do we go about that?-> 2D Sensitivities shown for the third mixing angle, CP Violating Phase and Mass Hierarchy and what do they represent.

Questions of interest for the oscillations of muon to electron neutrinos

a) Is the third mixing angle, θ_{13}, zero or not?

For a given "experiment" how can we "predict" for what ranges of θ_{13}, δ_{cp} we will be able to give a 3,5, etc sigma answer that the angle is indeed non zero?? (for both normal and inverted hierarchies).

b) Is CP violated or not in the neutrino sector? (which would mean δ_{cp} !=0 and δ_{cp} !=π)

For a given "experiment" how can we "predict" for what ranges of θ_{13}, δ_{cp} we will be able to give a 3,5, etc sigma answer that CP is indeed violated ?? (for both normal and inverted hierarchies).

Questions of interest for the oscillations of muon to electron neutrinos

c) What is the mass hierarchy in neutrinos: normal or inverted?

For a given "experiment" how can we "predict" for what ranges of θ_{13}, δ_{cp} I will be able to give a 3,5, etc sigma answer whether hierarchy is normal or inverted ??

θ_{13} Sensitivity :

Null hypothesis : θ_{13} = 0

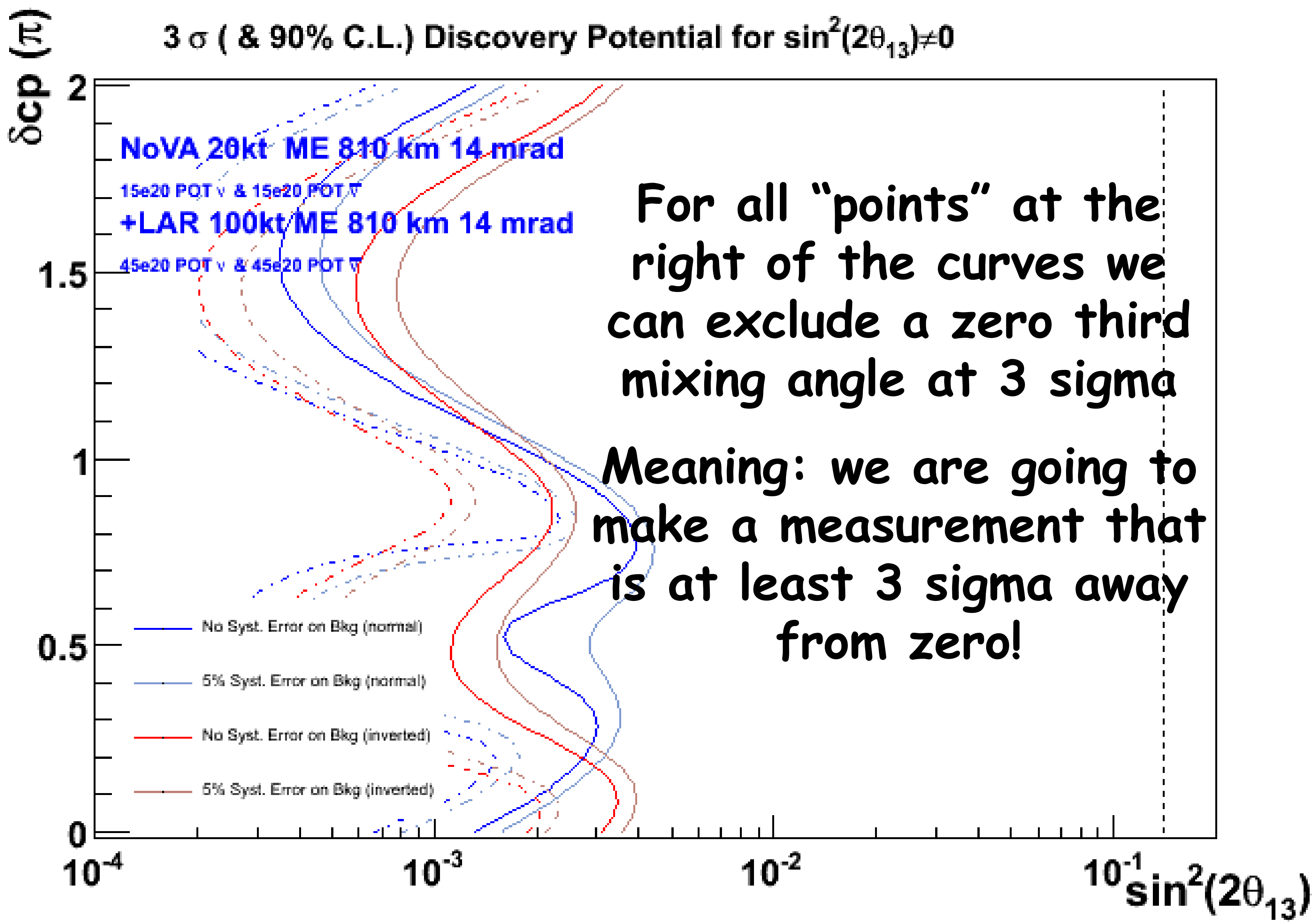
Both δcp and sign of Δm^2_{31} allowed to float in the fit

STRATEGY TO ESTIMATE FOR WHICH VALUES OF θ_{13}, δ_{cp} WE CAN EXLUCDE THE NULL HYPOTHESIS TO , 3, 5 etc sigma:

Create "pseudo" experiments for each (θ_{13}, δ_{cp}) point.

1. *Fit each "pseudo" experiment under the null hypothesis assumption and estimate minimum $x^{null}{}_{min}{}^2$*
2. *Fit each "pseudo" experiment and get its best $x_{min}{}^2$*
3. *Calculate $\Delta x^2 = x^{null}{}_{min}{}^2 - x_{min}{}^2$. Significance is $\sqrt{\Delta\chi^2}$*
4. *Plot only these points (pseudo experiments) for which Significance is >= that your desired value*

Sensitivity for a particular experiment



For all "points" at the right of the curves we can exclude a zero third mixing angle at 3 sigma

Meaning: we are going to make a measurement that is at least 3 sigma away from zero!

δ_{cp} Sensitivity :

Null hypothesis : δ_{cp} = 0 or $\delta_{cp} = \pi$ (take worst χ^2)

Both θ_{13} and sign of Δm^2_{31} allowed to float in the fit

STRATEGY TO ESTIMATE FOR WHICH VALUES OF θ_{13}, δ_{cp} WE CAN EXLUCDE THE NULL HYPOTHESIS TO , 3, 5 etc sigma:

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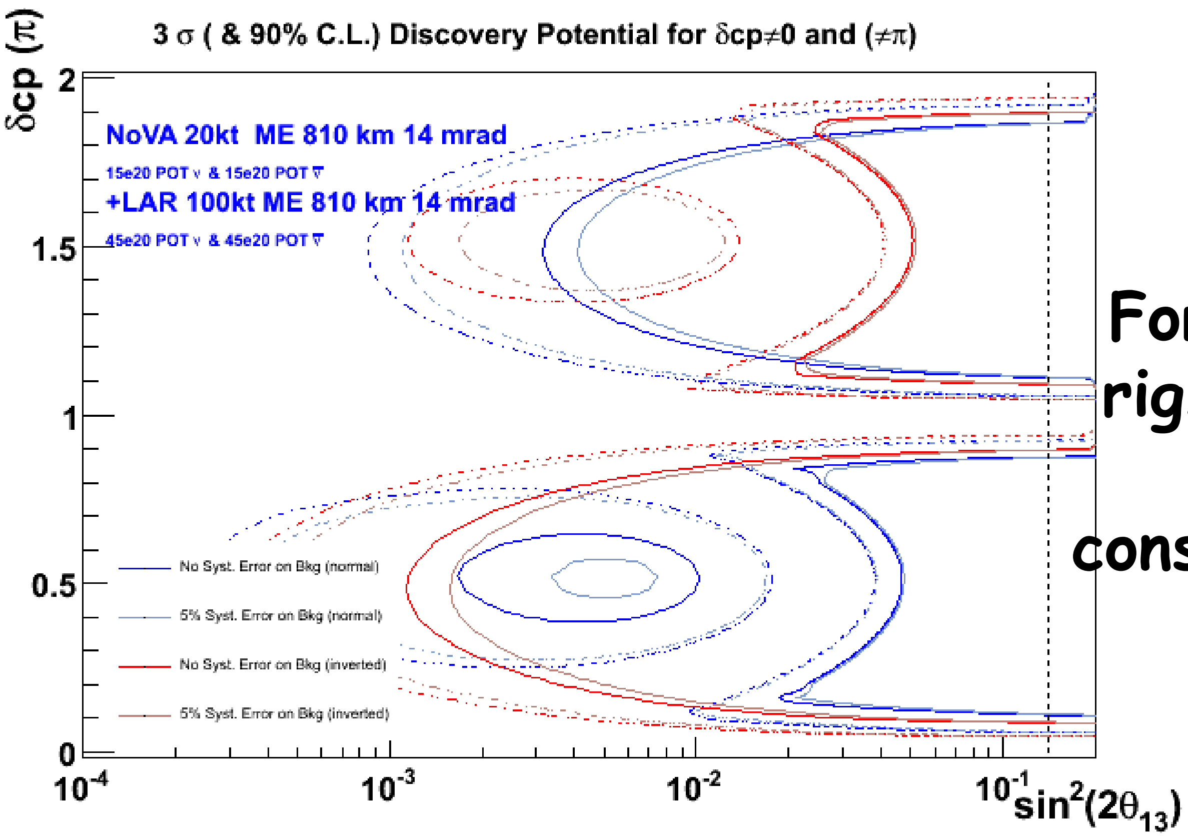

For all "points" at the right of the curves we can exclude *CP* conservation at 3 sigma

Mass Hierarchy Sensitivity :

Null hypothesis : **Mass hierarchy is Normal (or inverted)**

Both θ_{13} and δcp allowed to float in the fit

STRATEGY TO ESTIMATE FOR WHICH VALUES OF θ_{13}, δ_{cp} WE CAN EXLUCDE THE NULL HYPOTHESIS TO , 3, 5 etc sigma:

Create "pseudo" experiments for each (θ_{13}, δ_{cp}) point.

1. ***Fit each "pseudo" experiment under the null hypothesis assumption and estimate minimum $\chi^{null}{}_{min}{}^{2}$***
2. ***Fit each "pseudo" experiment and get its best $\chi_{min}{}^{2}$***
3. ***Calculate $\Delta\chi^2 = \chi^{null}{}_{min}{}^{2} - \chi_{min}{}^{2}$. Significance is $\sqrt{\Delta\chi^2}$***
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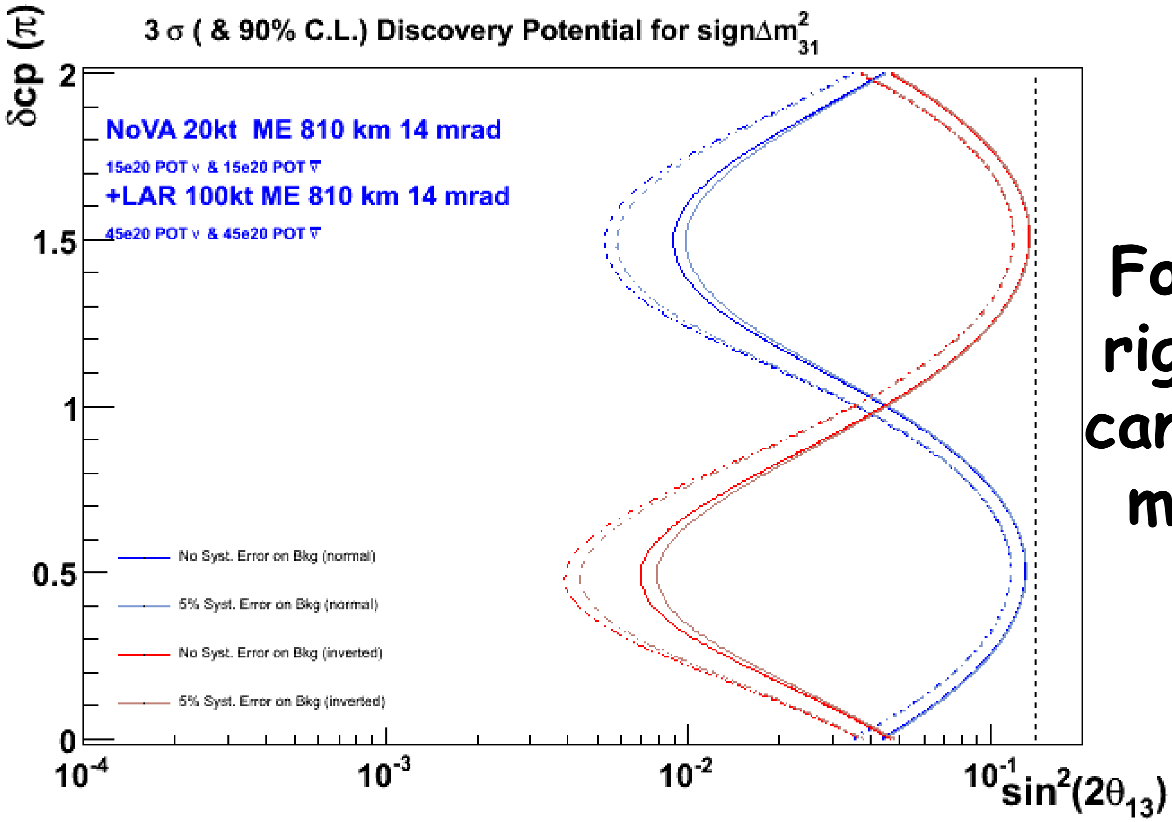
Sensitivity for a particular experiment



For all "points" at the right of the curves we can determine neutrino mass hierarchy at 3 sigma

QUESTIONS ☺