## Ninth DOE ACTS Collection Workshop <br> Building Robust, Scalable and Portable Software

## ScaLAPACK

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## ScaLAPACK: Functionalities

| $A x=b$ | Simple Driver | Expert Driver | Factor | Solve | Inversion | Conditioning Estimator | Iterative Refinement |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Triangular |  |  |  | X | X | X | x |
| SPD <br> SPD Banded <br> SPD Tridiagonal | $\begin{aligned} & x \\ & x \\ & x \end{aligned}$ | x | $\begin{aligned} & x \\ & x \\ & x \end{aligned}$ | $\begin{aligned} & x \\ & x \\ & x \end{aligned}$ | x | x | x |
| General <br> General Banded <br> General Tridiagonal | x | x | $\begin{aligned} & x \\ & x \\ & x \end{aligned}$ | $\begin{aligned} & x \\ & x \\ & x \end{aligned}$ | x | x | x |
| Least Squares <br> GQR <br> GRQ | X |  | $\begin{aligned} & x \\ & x \\ & x \end{aligned}$ | X |  |  |  |
| $A x=\lambda x$ or $A x=\lambda B x$ | Simple <br> Driver | Expert <br> Driver | Reduction | Solution |  |  |  |
| Symmetric <br> General <br> Generalized BSPD <br> SVD | $\begin{aligned} & x \\ & x \\ & x \end{aligned}$ | $\begin{aligned} & x \\ & x \end{aligned}$ | $\begin{aligned} & \mathrm{x} \\ & \mathrm{x} \\ & \mathrm{x} \\ & \mathrm{x} \end{aligned}$ | $\begin{aligned} & \mathrm{x} \\ & \mathrm{x} \\ & \mathrm{x} \\ & \mathrm{x} \end{aligned}$ |  |  |  |
| symmetric positive definite <br> generalized QR <br> generalized RQ <br> banded SPD |  | tinct | standard <br> putation | es of p task | oblems |  |  |

SPD: symmetric positive definite
GQR: generalized QR
GRQ: generalized RQ
BSPD: banded SPD
SVD: singular value decomposition

## Outline

- Motivation
- ScaLAPACK: software structure
- Basic Linear Algebra Subprograms (BLAS)
- Linear Algebra PACKage (LAPACK)
- Basic Linear Algebra Communication Subprograms (BLACS)
- Parallel BLAS (PBLAS)
- ScaLAPACK: details
- Data layout
- Array descriptors
- Error handling
- Performance
- Hands-on


## ScaLAPACK: Applications



Induced current (white arrows) and charge density (colored plane and gray surface) in crystallized glycine due to an external field: courtesy of Louie, Yoon, Pfrommer and Canning (UCB and $\angle B N L$ ).


Two Scal APACK routines, PZGETRF and PZGETRS, are used for solution of linear systems in the spectral algorithms based AORSA code (Batchelor et al.), which is intended for the study of electromagnetic waveplasma interactions. The code reaches $68 \%$ of peak performance on 1936 processors of an IBM SP.
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## Application: Cosmic Microwave Background (CMB) Analysis

- The statistics of the tiny variations in the CMB (the faint echo of the Big Bang) allows the determination of the fundamental parameters of cosmology to the percent level or better.
- MADCAP (Microwave Anisotropy Dataset Computational Analysis Package)
- Makes maps from observations of the CMB and then calculates their angular power spectra. (See http://crd.lbl.gov/~borrill).
- Calculations are dominated by the solution of linear systems of the form $M=A^{-1} B$ for dense $n \times n$ matrices $A$ and $B$ scaling as $\mathrm{O}\left(n^{3}\right)$ in flops. MADCAP uses ScaLAPACK for those calculations.
- On a Cray T3E (original code):
- Cholesky factorization and triangular solve.
- Typically reached $70-80 \%$ peak performance.
- Solution of systems with $n \sim 10^{4}$ using tens of processors.
- The results demonstrated that the Universe is spatially flat, comprising $70 \%$ dark energy, $25 \%$ dark matter, and only $5 \%$ ordinary matter.
- On an IBM SP:
- Porting was trivial but tests showed only $20-30 \%$ peak performance.
- Code rewritten to use triangular matrix inversion and triangular matrix multiplication $\rightarrow$ one-day work !
- Performance increased to $50-60 \%$ peak.
- Solution of previously intractable systems with $n \sim 10^{5}$ using hundreds of processors.



## ScaLAPACK: software structure

## http://acts.nersc.gov/scalapack



## BLAS

## ( $\underline{B}$ asic Linear Algebra Subroutines)

- Clarity: code is shorter and easier to read.
- Modularity: gives programmer larger building blocks.
- Performance: manufacturers (usually) provide tuned machine-specific BLAS.
- Portability: machine dependencies are confined to the BLAS.
- Key to high performance: effective use of memory hierarchy (true on all architectures).


## BLAS: 3 levels

- Level 1 BLAS: vector-vector

$$
\prod \Longleftarrow \rrbracket+\square \star \rrbracket
$$

- Clarity
- Portability
- Performance: development of blocked algorithms is important for performance!
- Level 2 BLAS: matrix-vector

- Level 3 BLAS: matrix-matrix

2.2 GHz AMD Opteron



## LAPACK: main features

## (http://www.netlib.org/lapack)

- Linear Algebra library written in Fortran 77 (Fortran 90)
- Combine algorithms from LINPACK and EISPACK into a single package.
- Efficient on a wide range of computers (RISC, Vector, SMPs).
- Built atop level 1, 2, and 3 BLAS Basic problems:
- Linear systems: $A x=b$
- Least squares: $\min \|A x-b\|_{2}$
- Singular value decomposition: $A=U \Sigma V^{T}$
- Eigenvalues and eigenvectors: $A z=\lambda z, A z=\lambda B z$
- LAPACK does not provide routines for structured problems or general sparse matrices (i.e. sparse storage formats such as compressed-row, column, -diagonal, skyline ...).


## BLACS

## ( $\underline{B}$ asic Linear $\underline{\text { Algebra Communication } \underline{S} u b r o u t i n e s) ~}$

- A design tool, they are a conceptual aid in design and coding.
- Associate widely recognized mnemonic names with communication operations. This improves:
- program readability
- self-documenting quality of the code.
- Promote efficiency by identifying frequently occurring operations of linear algebra which can be optimized on various computers.


## BLACS: basics

- Promote efficiency by identifying common operations of linear algebra that can be optimized on various computers.
- Processes are embedded in a two-dimensional grid.

- An operation which involves more than one sender and one receiver is called a scoped operation.

| Scope | Meaning |
| :--- | :--- |
| Row | All processes in a process row participate. |
| Column | All processes in a process column participate. |
| All | All processes in the process grid participate. |

## BLACS: communication routines

## Send/Receive:

```
_XXSD2D(ICTXT, [UPLO, DIAG] ,M, N, A, LDA, RDEST, CDEST) _XXRV2D(ICTXT, [UPLO, DIAG] , M, N, A, LDA , RSRC , CSRC )
```

| _ (Data type) | xx (Matrix type) |
| :--- | :--- |
| I: Integer, | GE: General rectangular matrix |
| S: Real, | TR: Trapezoidal matrix |
| D: Double Precision, |  |
| C: Complex, |  |
| Z: Double Complex. |  |

## Broadcast:

_XXBS2D(ICTXT, SCOPE, TOP, [UPLO, DIAG], M, N, A, LDA)
_XXBR2D(ICTXT, SCOPE, TOP, [UPLO, DIAG], M, N, A, LDA, RSRC, CSRC)

| SCOPE | TOP |
| :--- | :--- |
| 'Row' | ' '(default) |
| 'Column' | 'Increasing Ring' |
| 'All' | '1-tree' ... |

## BLACS: example (send/receive)

```
: (out) uniquely identifies each prockss
* Get system information (out) number of processes available
CALL BLACS_PINFO( IAM, NPROCS ) (in) integer handle indicating the context
    !
* Get default system context
    CALL BLACS_GET( 0, 0, ICTXT )
    :
* Define \(1 \times(N P R O C S / 2+1)\) process grid
    NPROW = 1
    NPCOL = NPROCS / 2 + 1
    CALL BLACS_GRIDINIT( ICTXT, 'Row', NPROW, NPCOL
    CALL BLACS_GRIDINFO( ICTXT, NPROW, NPCOL, MYROW, MYCOL )
* If I'm not in the grid, go to end of program
IF ( MYROW.NE.-1 ) THEN
    IF ( MYROW.EQ. 0 .AND. MYCOL.EQ. 0 ) THEN
        CALL DGESD2D ( ICTXT, 5, 1, X, 5, 1, 0 )
        ELSE IF( MYROW.EQ. 1 .AND. MYCOL.EQ. 0 ) THEN
            CALL DGERV2D ( ICTXT, 5, 1, Y, 5, 0, 0 )
        END IF
    CALL BLACS_GRIDEXIT ( ICTXT ) \(\longleftarrow\) leave context
END IF
\(\vdots\)
CALL BLACS_EXIT ( 0 ) を exit from the BLACS
END
```

- The BLACS context is the BLACS mechanism for partitioning communication space.
- A message in a context cannot be sent or received in another context.
- The context allows the user to
- create arbitrary groups of processes
- create multiple overlapping and/or disjoint grids
- isolate each process grid so that grids do not interfere with each other
- BLACS context $\Leftrightarrow$ MPI communicator


## See http://www.netlib.org/blacs for more information.

## PBLAS

## (로arallel Basic Linear Algebra Subroutines)

- Similar to the BLAS in portability, functionality and naming.
- Built atop the BLAS and BLACS
- Provide global view of matrix

CALL DGEXXX( M, N, A( IA, JA ), LDA, ... )


CALL PDGEXXX( M, N, A, IA, JA, DESCA, ... )
PBLAS


Array descriptor (see next slides); stores information about the distribution of the matrix $A$.

## ScaLAPACK: structure of the software




## ScaLAPACK: goals

## - Efficiency

- Optimized computation and communication engines
- Block-partitioned algorithms (Level 3 BLAS) for good node performance
- Reliability
- Whenever possible, use LAPACK algorithms and error bounds.
- Scalability
- As the problem size and number of processors grow
- Replace LAPACK algorithm that did not scale (new ones into LAPACK)
- Portability
- Isolate machine dependencies to BLAS and the BLACS
- Flexibility
- Modularity: build rich set of linear algebra tools (BLAS, BLACS, PBLAS)
- Ease-of-Use
- Calling interface similar to LAPACK


## ScaLAPACK: data layouts

- 1D block and cyclic column distributions


| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 3 | 2 | 3 | 2 | 3 | 2 | 3 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 2 | 3 | 2 | 3 | 2 | 3 | 2 | 3 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 2 | 3 | 2 | 3 | 2 | 3 | 2 | 3 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 2 | 3 | 2 | 3 | 2 | 3 | 2 | 3 |

- 1D block-cycle column and 2D block-cyclic distribution
- 2D block-cyclic used in ScaLAPACK for dense matrices


## ScaLAPACK: 2D Block-Cyclic Distribution

$5 \times 5$ matrix partitioned in $2 \times 2$ blocks

| and | at2 | $a_{43}$ | $a_{14}$ | 215 |
| :---: | :---: | :---: | :---: | :---: |
| $a_{31}$ | $a_{22}$ | $a_{23}$ | $\mathrm{a}_{24}$ | 325 |
| $\mathrm{a}_{31}$ | a32 | $a_{33}$ | $\mathrm{a}_{34}$ | $\mathrm{a}_{35}$ |
| anı | an2 | ans | and | $\mathrm{a}_{45}$ |
| $\mathrm{a}_{51}$ | $\mathrm{a}_{52}$ | $\mathrm{a}_{53}$ | $\mathrm{a}_{54}$ | ${ }^{\text {ass }}$ |

$2 \times 2$ process grid point of view



## 2D Block-Cyclic Distribution

|  | 0 |  |  | 1 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{a}_{11}$ | $\mathrm{a}_{12}$ |  | $\mathrm{a}_{13}$ | $\mathrm{a}_{14}$ |
| 0 | $\mathrm{a}_{21}$ | a |  | $\mathrm{a}_{23} 1$ | $\mathrm{a}_{24}$ |
|  | $\mathrm{a}_{51}$ | $\mathrm{a}_{52}$ | $\mathrm{a}_{55}$ | $\mathrm{a}_{53}$ | $\mathrm{a}_{54}$ |
| 1 | $\mathrm{a}_{31}$ | $\mathrm{a}_{32}$ 2 | $\mathrm{a}_{35}$ | $a_{23}$ | $\mathrm{a}_{34}$ |
|  | $a_{41}$ | $\mathrm{a}_{42}$ | $\mathrm{a}_{45}$ | $\mathrm{a}_{43}$ | $a_{44}$ |

CALL BLACS_GRIDINFO( ICTXT, NPROW, NPCOL, MYROW, MYCOL )
LDA is the leading dimension of the local array (see slides 22-24); helps in mapping a 2D

IF ( MYROW.EQ. 0 .AND. MYCOL.EQ. 0 ) THEN
$A(1)=$ a11; $A(2)=a 21 ; A(3)=a 51 ;$
$A(1+L D A)=a 12 ; A(2+L D A)=a 22 ; A(3+L D A)=a 52 ;$
$A(1+2 * L D A)=a 15 ; A(2+3 * L D A)=a 25 ; A(3+4 * L D A)=a 55 ;$
ELSE IF ( MYROW.EQ.0 .AND. MYCOL.EQ.1 ) THEN
$A(1)=$ a13; $A(2)=a 23 ; A(3)=a 53 ;$
$A(1+L D A)=a 14 ; A(2+L D A)=a 24 ; A(3+L D A)=a 54 ;$
ELSE IF ( MYROW.EQ.1 .AND. MYCOL.EQ. 0 ) THEN
$A(1)=a 31 ; A(2)=a 41 ;$
$A(1+L D A)=a 32 ; A(2+L D A)=a 42 ;$
$A\left(1+2^{*} L D A\right)=a 35 ; A\left(2+3^{*} L D A\right)=a 45 ;$
ELSE IF ( MYROW.EQ.1 .AND. MYCOL.EQ.1 ) THEN
$A(1)=a 33 ; A(2)=a 43 ;$
$A(1+L D A)=a 34 ; A(2+L D A)=a 44 ;$
END IF
Array descriptor for A (see slides 22-24)


CALL PDGESVD( JOBU, JOBVT, M, N, A, IA, JA, DESCA, S, U, IU, JU, DESCU, VT, IVT, JVT, DESCVT, WORK, LWORK, INFO )


## 2D Block-Cyclic Distribution

- Ensures good load balance $\rightarrow$ performance and scalability (analysis of many algorithms to justify this layout).
- Encompasses a large number of data distribution schemes (but not all).
- Needs redistribution routines to go from one distribution to the other.
- See http://acts.nersc.gov/scalapack/hands-on/datadist.html



## ScaLAPACK: array descriptors

- Each global data object is assigned an array descriptor.
- The array descriptor:
- Contains information required to establish mapping between a global array entry and its corresponding process and memory location (uses concept of BLACS context).
- Is differentiated by the DTYPE_ (first entry) in the descriptor.
- Provides a flexible framework to easily specify additional data distributions or matrix types.
- User must distribute all global arrays prior to the invocation of a ScaLAPACK routine, for example:
- Each process generates its own submatrix.
- One processor reads the matrix from a file and send pieces to other processors (may require message-passing for this).


## Array Descriptor for Dense Matrices

| DESC 0 | Symbolic Name | Scope | Definition |
| :---: | :---: | :---: | :---: |
| 1 | DTYPE_A | (global) | Descriptor type DTYPE $A=1$ for dense matrices. |
| 2 | CTXT_A | (global) | BLACS context handle. |
| 3 | M A | (global) | Number of rows in global array A. |
| 4 | N A | (global) | Number of columns in global array A. |
| 5 | MB_A | (global) | Blocking factor used to distribute the rows of array A. |
| 6 | NBA | (global) | Blocking factor used to distribute the columns of array A. |
| 7 | RSRC A | (global) | Process row over which the first row of the array $A$ is distributed. |
| 8 | CSRC A | (global) | Process column over which the first column of the array A is distributed. |
| 9 | UDA | (local) | Leading dimension of the local array. |

## Array Descriptor for Narrow Band Matrices

| DESC () | Symbolic Name | Scope | Definition |
| :---: | :---: | :---: | :---: |
| 1 | DTYPE_A | (global) | Descriptor type DTYPE A=501 for $1 \times$ Pc process grid for band and tridiagonal matrices block-column distributed. |
| 2 | CTXT_A | (global) | BLACS context handle. |
| 3 | NA | (global) | Number of columns in global array A. |
| 4 | NB A | (global) | Blocking factor used to distribute the columns of array A. |
| 5 | CSRC A | (global) | Process column over which the first column of the array A is distributed. |
| 6 | LDA | (local) | Leading dimension of the local array. For the tridiagonal subroutines, this entry is ignored. |
| 7 | - | - | Unused, reserved. |

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## Array Descriptor for Right Hand Sides for Narrow Band Linear Solvers

| DESC () | Symbolic Name | Scope | Definition |
| :---: | :---: | :---: | :---: |
| 1 | DTYPE B | (global) | Descriptor type DTYPE $B=502$ for $\operatorname{Pr} \times 1$ process grid for block-row distributed matrices |
| 2 | CTXT_B | (global) | BLACS context handle |
| 3 | M B | (global) | Number of rows in global array B |
| 4 | MB B | (global) | Blocking factor used to distribute the rows of array B |
| 5 | RSRC B | (global) | Process row over which the first row of the array $B$ is distributed |
| 6 | பDB | (local) | Leading dimension of the local array. For the tridiagonal subroutines, this entry is ignored |
| 7 | - | - | Unused, reserved |

## ScaLAPACK: error handling

- Driver and computational routines perform global and local input error-checking.
- Global checking $\rightarrow$ synchronization
- Local checking $\rightarrow$ validity
- No input error-checking is performed on the auxiliary routines.
- If an error is detected in a PBLAS or BLACS routine program execution stops.


## ScaLAPACK: debugging hints

- Look at ScaLAPACK example programs.
- Always check the value of INFO on exit from a ScaLAPACK routine.
- Query for size of workspace, $\mathrm{LWORK}=-1$.
- Link to the Debug Level 1 BLACS (specified by BLACSDBGLVL=1 in Bmake.inc).
- Consult errata files on netlib: http://www.netlib.org/scalapack/errata.scalapack http://www.netlib.org/blacs/errata.blacs


## ScaLAPACK: Performance

- The algorithms implemented in ScaLAPACK are scalable in the sense that the parallel efficiency is an increasing function of $\mathrm{N}^{2} / \mathrm{P}$ (problem size per node).
- Maintaining memory use per node constant allows efficiency to be maintained (in practice, a slight degradation is acceptable).
- Use efficient machine-specific BLAS (not the Fortran 77 source code available in http://www.netlib.gov) and BLACS (nondebug installation).
- On a distributed-memory computer:
- Use the right number of processors
- Rule of thumb: $\mathrm{P}=\mathrm{MxN} / 10^{6}$ for an MxN matrix, which provides a local matrix of size approximately 1000 -by- 1000 .
- Do not try to solve a small problem on too many processors.
- Do not exceed the physical memory.
- Use an efficient data distribution
- Block size (i.e., MB,NB) $\geq 64$.
- Square processor grid: Prow = Pcolumn.


## LU on 2.2 GHz AMD Opteron (4.4 GFlop/s peak performance)



LU+solve on 2.2 GHz AMD Opteron (4.4 GFlop/s peak performance)


## ScaLAPACK: Grid Size Effects

Execution time of PDGESV for various grid shape


60 processors, Dual AMD Opteron 1.4GHz Cluster with Myrinet Interconnect, 2GB Memory


## ScaLAPACK: Commercial Use

ScaLAPACK has been incorporated in the following commercial packages:

- Fujitsu
- Hewlett-Packard
- Hitachi
- IBM Parallel ESSL
- NAG Numerical Library
- Cray LIBSCI
- NEC Scientific Software Library
- Sun Scientific Software Library
- Visual Numerics (IMSL)


## ScaLAPACK: Summary

- Library of high performance dense linear algebra routines for distributed-memory computing
- Reliable, scalable and portable
- Calling interface similar to LAPACK
- New developments on the way
- LAPACK/ScaLAPACK Forum: http://icl.cs.utk.edu/lapack-forum
- ScaLAPACK Users' Guide: http://www.netlib.org/scalapack/slug


## Hands-on: http://acts.nersc.gov/scalapack/hands-on

| Hands-On Exercises for ScaLAPACK |  |  |
| :---: | :---: | :---: |
| $\begin{array}{\|l\|l\|} \text { The ScaLAPACK Team } \\ \text { June } 2007 \end{array}$ |  |  |
| Introduction |  |  |
| These exercises provide basic and more advanced programming instruction for writing parallel programs calling the BLACS, PBLAS, and ScaLAPACK. A basic knowledge of Fortran, parallel programming with message-passing, nd MPI is assumed. Some of the exercises also require an understanding of two-dimensional block cyclic data distribution. |  |  |
| Exercises 1 and 2 give an introduction to parallel programming with the Basic Linear Algebra Communication Subprograms (BLACS). Exercises 3, 4, and 5 provide a range of simplistic to more complex parallel programs calling PBLAS and ScaLAPACK. |  |  |
| The instructions for the exercises assume that the underlying system is an IBM SP; using up to six processes that do message-passing. These example programs use MPI as the underlying message-passing layer. These hands-on YarKhan, C. Hastings, S. Blackford, C. Whaley, A. Petitet and O. Marques. |  |  |
| Exercise 1: BLACS - Hello World Example |  |  |
| Exercise 2: BLACS - Pi Example |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
| Addition Resources: |  |  |
| - Block Cyclic Data Distribution |  |  |
|  |  |  |
| - Detailed information on the BLACS |  |  |
| - Detailed information on the PBLAS |  |  |
|  |  |  |
|  |  |  |
| ScaLAPACK | Tools | Home |

## Hands-on: instructions

- Do a "module load acml".
- Do a "cp-r /usr/common/acts/SCALAPACK/hands-on hands-on".
- There are six subdirectories under hands-on:
- Example 1: BLACS, "hello world" example
- Example 2: BLACS, "pi" example
- Example 3: PBLAS example
- Example 4: ScaLAPACK example 1 (PSGESV)
- Example 5: ScaLAPACK example 2 (PSGESV)
- additional exercises
- Examples 1-5 are written in Fortran. For a successful compilation and execution of Example 5, you will have to correct some lines in the code, in particular the lines starting with ${ }^{* * *}$ (commented lines).
- Examples 1-5 can be compiled with "make", which will generate an executable file with ". $x$ ".
- Try also http://acts.nersc.gov/scalapack/hands-on/datadist.html with a bigger matrix and different block/grid sizes.


## Hands-on: additional examples (directory etc)

- example1.f, example2.f, and example3.f: show how to generate a ScaLAPACK matrix (instructions for compiling and running are given in the header of those files).
- pddttrdrv.c (pddttrdrv.f): shows the use of the ScaLAPACK routines PDDTTRF and PDDTTRS to factor and solve a (diagonally dominant) tridiagonal system of linear equations $T x=b$.
- pdpttr_2.c (pdpttr_2.f): shows the use of the ScaLAPACK routines PDPTTRF and PPPTTRS to factor and solve a symmetric positive definite tridiagonal system of linear equations $T x=b$, in two distinct contexts (see next slide).
- pdgesvddrv.f: reads a (full) matrix $A$ from a file, distributes $A$ among the available processors and then call the ScaLAPACK subroutine PDGESVD to compute the SVD of $A, A=U^{*} S^{*} V^{\wedge} T$. It requires the file pdgesvddrv.dat, which should contain: line 1, the name of the file where $A$ will be read from; line 2 , the number of rows of $A$; line 3 : the number of columns of $A$.
- if $m=n=10$ the results are given in the file A.SVD
- if $m=10, n=7: \operatorname{diag}(S)=\left[\begin{array}{lllllll}4.4926 & 1.4499 & 0.8547 & 0.8454 & 0.6938 & 0.4332 & 0.2304\end{array}\right]$
- if $m=7, \mathrm{n}=10: \operatorname{diag}(S)=\left[\begin{array}{llllllll}4.5096 & 1.1333 & 1.0569 & 0.8394 & 0.8108 & 0.5405 & 0.2470\end{array}\right]$
- pzdtt_col_major.c (pzdtt_row_major.c): shows the use of the ScaLAPACK routines PZDTTRF and PZDTTRS to factor and solve a tridiagonal system of linear equations in nprow distinct contexts, column-major (row-major ) ordering.


## Block Cyclic Distribution

Consider the 12-by-10 matrix:

$$
A=\left[\begin{array}{cccccccccc}
a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} & a_{1,6} & a_{1,7} & a_{1,8} & a_{1,9} & a_{1,10} \\
a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} & a_{2,6} & a_{2,7} & a_{2,8} & a_{2,9} & a_{2,10} \\
a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & a_{3,5} & a_{3,6} & a_{3,7} & a_{3,8} & a_{3,9} & a_{3,10} \\
a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} & a_{4,5} & a_{4,6} & a_{4,7} & a_{4,8} & a_{4,9} & a_{4,10} \\
a_{5,1} & a_{5,2} & a_{5,3} & a_{5,4} & a_{5,5} & a_{5,6} & a_{5,7} & a_{5,8} & a_{5,9} & a_{5,10} \\
a_{6,1} & a_{6,2} & a_{6,3} & a_{6,4} & a_{6,5} & a_{6,6} & a_{6,7} & a_{6,8} & a_{6,9} & a_{6,10} \\
a_{7,1} & a_{7,2} & a_{7,3} & a_{7,4} & a_{7,5} & a_{7,6} & a_{7,7} & a_{7,8} & a_{7,9} & a_{7,10} \\
a_{8,1} & a_{8,2} & a_{8,3} & a_{8,4} & a_{8,5} & a_{8,6} & a_{8,7} & a_{8,8} & a_{8,9} & a_{8,10} \\
a_{9,1} & a_{9,2} & a_{9,3} & a_{9,4} & a_{9,5} & a_{9,6} & a_{9,7} & a_{9,8} & a_{9,9} & a_{9,10} \\
a_{10,1} & a_{10,2} & a_{10,3} & a_{10,4} & a_{10,5} & a_{10,6} & a_{10,7} & a_{10,8} & a_{10,9} & a_{10,10} \\
a_{11,1} & a_{11,2} & a_{11,3} & a_{11,4} & a_{11,5} & a_{11,6} & a_{11,7} & a_{11,8} & a_{11,9} & a_{11,10} \\
a_{12,1} & a_{12,2} & a_{12,3} & a_{12,4} & a_{12,5} & a_{12,6} & a_{12,7} & a_{12,8} & a_{12,9} & a_{12,10}
\end{array}\right]
$$

Do the following block cyclic distributions:

- 3-by-3 blocking on a 3-by-2 process grid
- 4-by-4 blocking on a 2-by-3 process grid

Use http://acts.nersc.gov/scalapack/hands-on/datadist.html to compare

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$\vdots$

