Competency 1.2 Radiation protection personnel shall demonstrate a working level knowledge of radioactivity and transformation mechanisms.

## 1. SUPPORTING KNOWLEDGE AND/OR SKILLS

a. Define the following terms:

- Activity
- Radioactive decay constant
- Curie/becquerel
- Radioactive half-life
- Radioactive equilibrium
- Decay products
- Parent nuclide
- Activation
- Specific activity
- Naturally Occurring Radioactive Material (NORM)
- Secular equilibrium
- Transient equilibrium
b. Describe the following processes including any resulting product of decay:
- Alpha decay
- Beta-minus decay
- Beta-plus decay
- Electron capture
- Isomeric transition
- Internal conversion
- X-ray generation
c. Given the Chart of the Nuclides, trace the decay chain for a specified nuclide.
d. Given either the half-life or the radioactive decay constant, solve radioactive decay problems.
e. Using the specific activity or decay constant of an isotope, convert between mass quantities and curies.
f. Convert numerical amounts of radioactivity between curie, becquerel, and dpm.


## 2. SUMMARY

## Definitions

activity: The rate of disintegration (nuclear transformation) or decay of radioactive material. Mathematically, activity (A) can be expressed as the product of the decay constant ( $\lambda$ ) and the number of atoms present in a sample ( N ). Units of activity are the curie and the becquerel.

$$
A=\lambda N
$$

Where:
A $=$ activity in curies, becquerels, or disintegrations per second
$\lambda=$ radioactive decay constant in reciprocal units of time $\left(\mathrm{sec}^{-1}\right)$
$\mathrm{N}=$ the number of atoms present in the sample
activation: Process of producing a radioactive material by bombardment with neutrons, protons, or other nuclear particles.
becquerel: A unit used to describe the intensity of radioactivity (or activity) in a sample of radioactive material in the International System of Units (SI), equal to one disintegration (nuclear transformation) per second. Since the becquerel is a rather small unit, metric prefixes are often applied to aid in designating larger amounts of activity (see chart below).

Becquerel Superunits

| Unit | Abbreviation | disintegrations <br> per second <br> (dps) | disintegrations <br> per minute <br> (dpm) |
| :--- | :---: | :---: | :---: |
| becquerel | Bq | 1 | 60 |
| kilobecquerel | kBq | 1 E 3 | 6 E 4 |
| megabecquerel | MBq | 1 E 6 | 6 E 7 |

byproduct material: any radioactive material made radioactive by exposure to radiation incident to the process of producing or using special nuclear material, or the wastes produced by the extraction of uranium or thorium from ore processed primarily for its source material content, including discrete surface wastes resulting from uranium solution extraction processes.
curie ( $\mathbf{C i}$ ): The basic unit used to describe the intensity of radioactivity (or activity) in a sample of radioactive material. The curie is equal to 37 billion disintegrations per second, which is approximately the rate of decay of 1 gram of radium. A curie is also a quantity of any radionuclide that decays at a rate of 37
billion disintegrations per second. Named for Marie and Pierre Curie, who discovered radium in 1898. Since the curie represents a very large amount of activity, often smaller, and more convenient subunits are used (see chart below).

Curie Subunits

| Unit | Abbreviation | disintegrations <br> per second <br> (dps) | disintegrations <br> per minute <br> (dpm) |
| :--- | :--- | :--- | :--- |
| curie | Ci | 3.7 E 10 | 2.22 E 12 |
| millicurie | mCi | 3.7 E 7 | 2.22 E 9 |
| microcurie | $\mu \mathrm{Ci}$ | 3.7 E 4 | 2.22 E 6 |
| nanocurie | nCi | 3.7 E 1 | 2.22 E 3 |
| picocurie | pCi | $3.7 \mathrm{E}-2$ | 2.22 |

The relationship between the curie and the becquerel is:

$$
1 \mathrm{~Bq}=1 \mathrm{dps}=2.7 \mathrm{E}-11 \mathrm{Ci} \quad 1 \mathrm{Ci}=3.7 \mathrm{E} 10 \mathrm{dps}=3.7 \mathrm{E} 10 \mathrm{~Bq}
$$

Using unit analysis and conversion, activity measurements given in dps, dpm, or curies can be converted to becquerels.
daughter or decay products: Isotopes that are formed by the radioactive decay (disintegration) of some other radionuclide are commonly called a daughter radionuclide. In the case of radium-226, for example, there are 10 successive daughter products, ending in the stable isotope lead-206.
naturally occurring radioactive material (NORM): Any radioactive material that can be considered naturally occurring and is not source, special nuclear, or byproduct material or that is produced in a charged particle accelerator. (DOE Order 5820.2A)
parent nuclide: A radionuclide that upon radioactive decay or disintegration yields a specific nuclide (the daughter).
radioactive equilibrium: A condition where the production or collection of radioactive atoms equals the rate of decay and/or loss of atoms by other mechanisms. The result is the overall activity in a system remains constant over time.
radioactive decay constant: The rate of decay, or fraction of atoms decaying per unit of time. The decay constant is represented by the symbol $\lambda$ and mathematically is the quotient of the natural $\log$ of 2 divided by the half-life of the radionuclide. The formula is:

$$
\lambda=\frac{\ln 2}{T_{1 / 2}}
$$

radioactive half-life: The half-life is the length of time required for the activity of a radionuclide to be reduced to one-half of the beginning, or original, activity. Half-life is specific for each radionuclide and can be used as a method of identifying unknown radionuclides.
secular equilibrium: A condition where the activity of the daughter product equals the activity of the parent. This condition occurs when the half-life of the parent is much, much greater than the half-life of the daughter and the system, where the parent and daughter activity is collected, is a closed system. The daughter activity in secular equilibrium can be calculated by the formula:

$$
A_{b}=A_{a}\left(1-e^{-\lambda_{b} t}\right)
$$

Where:
$A_{a}=$ the activity of the parent
$\mathrm{A}_{\mathrm{b}}=$ the activity of the daughter at time t
$\lambda_{\mathrm{b}}=$ the radioactive decay constant of the daughter
$\mathrm{t}=$ the elapsed time since the build up of the daughter
The activity of the parent and daughter will be equal after about 7 to 10 daughter half-lives.
source material: Uranium or thorium, or any combination, in any physical or chemical form. Ores that contain greater than $0.05 \%$ of the weight of uranium or thorium or the combination.
special nuclear: Plutonium, uranium-233, or uranium enriched in 233 or 235 , or any material enriched in these isotopes.
specific activity: The activity per unit mass of a radionuclide. Generally reported in units of curies per gram $(\mathrm{Ci} / \mathrm{g})$ or becquerels per kilogram $(\mathrm{Bq} / \mathrm{kg})$. Specific activity varies with the half-life and the gram atomic weight of the radionuclide.
transient equilibrium: A condition where the daughter activity, starting at time 0 , builds up, marginally surpasses the parent activity, and then begins to decline at the same rate as the parent. This condition occurs when the half-life of the parent is only slightly greater than the daughter.
transuranic (TRU) elements: actinides with atomic numbers from 93 (neptunium) through 103 (lawrencium)

## Decay Processes

When a radioactive nuclide decays, a transmutation occurs. The decay product, or daughter, has become an atom of a new element with chemical properties entirely unlike the original parent atom. With each transmutation an emission from the nucleus occurs. There are several modes of decay and emissions associated with each mode.

## Alpha Decay

With a few exceptions, only relatively heavy radioactive nuclides decay by alpha emission. An alpha particle is essentially a helium nucleus. It consists of two protons and two neutrons, giving it a mass of 4 atomic mass unit (amu). Because of the two protons it has an electric charge of +2 . The symbol $\alpha$ is used to designate alpha particles.

A nucleus emitting an alpha particle decays to a daughter element, reduced in atomic number $(Z)$ by 2 and reduced in mass number $(A)$ by 4 . The standard notation for alpha decay is:

$$
{ }_{\mathrm{Z}}^{\mathrm{A}} \mathrm{X} \rightarrow{ }_{\mathrm{Z}-2}^{\mathrm{A}-4} \mathrm{Y}+{ }_{2}^{4} \alpha
$$

For example, radium-226 decays by alpha emission to produce radon-222 as follows:

$$
{ }_{88}^{226} \mathrm{Ra} \rightarrow{ }_{86}^{222} \mathrm{Rn}+{ }_{2}^{4} \alpha
$$

Alpha particles are the least penetrating of the three most common types of radiations. They can be absorbed, or stopped, by a few centimeters of air or a sheet of paper.

## Beta-minus Decay

A nuclide that has an excess number of neutrons (i.e., the neutron to proton, $\mathrm{n}: \mathrm{p}$, ratio is high) will usually decay by beta-minus emission. The intranuclear effect would be the changing of a neutron into a proton, thereby decreasing the $n$ :p ratio, resulting in the emission of a beta-minus particle. Beta-minus particles are negatively charged particles. They have the same mass as an electron ( $1 / 1836$ of proton or $5.49 \mathrm{E}-4 \mathrm{amu}$ ) as well as the same charge ( -1 ) and can be considered high speed electrons. Beta particles originate in the nucleus, in contrast with ordinary electrons, which exist in orbits around the nucleus. The symbol $\beta^{-}$is used to designate beta-minus particles.

In beta-minus emitters, the nucleus of the parent gives off a negatively charged particle due to a neutron changing into a proton. The atomic number increases by one, but the mass number is unchanged. There is also the emission of an antineutrino, symbolized by the Greek letter nu with a bar above it ( $\bar{v}$ ).

The standard notation for beta-minus decay is:

$$
{ }_{\mathrm{Z}}^{\mathrm{A}} \mathrm{X} \rightarrow{ }_{\mathrm{Z}+1}^{\mathrm{A}} \mathrm{Y}+\beta^{-}+\overline{\mathrm{V}}
$$

For example, lead-214 (Pb-214) decays by beta-minus emission to produce bismuth-214 (Bi-214) as follows:

$$
{ }_{82}^{210} \mathrm{~Pb} \rightarrow{ }_{83}^{210} \mathrm{Bi}+\beta^{-}+\bar{v}
$$

Beta-minus particles are emitted with kinetic energies ranging up to the maximum value of the decay energy, $\mathrm{E}_{\text {max }}$. The average energy of beta-minus particles is about $1 / 3 \mathrm{E}_{\max }$. They travel several hundred times the distance of alpha particles in air and require a few millimeters of aluminum to stop them.

Neutrinos ( $v$ ) and anti-neutrinos ( $\bar{v}$ ) are neutral (uncharged) particles with negligible rest mass that travel at the speed of light and are very non-interacting. They account for the energy distribution among positrons and beta particles from given radionuclides in the positron- and beta-decay processes respectively. Since they are so non-interacting, no energy is deposited in tissue; therefore, no dose results to personnel exposed to neutrinos.

## Positron Decay

A nuclide that has a low n:p ratio (too many protons) will tend to decay by positron emission. A positron is often mistakenly thought of as a positive electron. If positive electrons existed, when they encountered an ordinary negative electron the Coulomb force would cause the two particles to accelerate toward each other. They would collide and the two equal but opposite charges would mutually cancel, which would leave two neutral electrons. Actually, a positron is the anti-particle of an electron. This means that it has the opposite charge $(+1)$ of an electron (or beta particle). Thus, the positron is a positively charged, high-speed particle which originates in the nucleus. Because of its positive charge and rest mass equal to that of a beta particle, a positron is sometimes referred to as "beta-plus." The symbol $\boldsymbol{\beta}^{+}$is used to designate positrons.

With positron emitters, the parent nucleus changes a proton into a neutron and gives off a positively charged particle. This results in a daughter less positive by one unit of charge. Because a proton has been replaced by a neutron, the atomic number decreases by one and the mass number remains unchanged. The emission of a neutrino also occurs in conjunction with the positron emission.

Positron decay is illustrated by the following notation:

$$
{ }_{Z}^{A} \mathrm{X} \rightarrow{ }_{\mathrm{Z}-1}^{\mathrm{A}} \mathrm{Y}+\mathrm{B}^{+}+v
$$

For example, nickel-57 (Ni-57) decays by positron emission to cobalt-57 (Co-57):

$$
{ }_{28}^{57} \mathrm{Ni} \rightarrow{ }_{27}^{57} \mathrm{Co}+\beta^{+}+v
$$

The possibility of "pair production" is present when an incident photon of energy greater than 1.022 MeV disappears in the vicinity of a nucleus and in its place appears a pair of electrons: one negatively charged and one positively charged (positron). The pair production electrons travel through matter causing ionizations and excitations, until it loses all of its kinetic energy and is joined with an atom. The positron also produces ionizations and excitations until it comes to rest. While at rest, the positron attracts a free electron which then results in annihilation of the pair, converting both into electromagnetic energy. Two photons of 511 keV each arise at the site of the annihilation (accounting for the rest mass of the particles). Then the fate of the annihilation photons is either photoelectric absorption or Compton scattering followed by photoelectric absorption.

## Electron Capture

For radionuclides having a low n:p ratio, another mode of decay known as orbital electron capture (EC) can occur. In this radioactive decay process the nucleus captures an electron from an orbital shell of the atom, usually the K-shell since the electrons in that shell are closest to the nucleus. The nucleus might conceivably capture an L-shell electron, but K-electron capture is much more probable. This mode of decay is frequently referred to as K-capture. The transmutation resembles that of positron emission, as follows:

$$
{ }_{\mathbf{Z}}^{\mathbf{A}} \mathbf{X}+{ }_{-1}^{0} \mathbf{e} \rightarrow{ }_{\mathbf{Z}-1}^{\mathrm{A}} \mathbf{Y}+v
$$

The electron combines with a proton to form a neutron, followed by the emission of a neutrino. Electrons from higher energy levels immediately move in to fill the vacancies left in the inner, lower-energy shells. The excess energy emitted in these moves results in a cascade of characteristic x-ray photons (see the discussion on the next page under X-ray Generation).

Either positron emission or electron capture can be expected in nuclides with a low n:p ratio. The intranuclear effect of either mode of decay would be to change a proton into a neutron, thus increasing the n :p ratio.

## Gamma Emission

Nuclear decay reactions resulting in a transmutation generally leave the resultant nucleus in an excited state. Nuclei, thus excited, may reach an unexcited, or ground state, by emission of a gamma ray.

Gamma waves ( $\gamma$ ), x-rays and visible light are all types of electromagnetic radiation differing in frequency. They behave as small bundles, or packets, of energy, called photons, and travel at the speed of light. Gamma waves are usually of higher energy, whereas x-rays are usually of lower energy. The basic difference between gamma waves and x-rays is their origin; gamma waves are emitted from the nucleus of unstable atoms, while x-rays originate in the electron shells. No nuclide decays solely by gamma emission. Gamma rays are produced only to relieve excitation energy. They are emitted from nuclei of excited atoms following a radioactive transformation, and occur only after decay has occurred by $\alpha$ emission, $\beta$ emission or electron capture.

All of the transmutation examples given could be accompanied by gamma emission. Although most nuclear decay reactions do have gamma emissions associated with them, there are some radionuclide species which decay by particulate emission with no gamma emission.

## Isomeric Transition

Isomeric transition commonly occurs immediately after particle emission; however, the nucleus may remain in an excited state for a measurable period of time before dropping to the ground state at its own characteristic rate. A nucleus that remains in such an excited state is known as an isomer because it is in a metastable state; that is, it differs in energy and behavior from other nuclei with the same atomic number and mass number. Generally, the isomer achieves ground state by emitting delayed (usually greater than $10^{-9}$ seconds) gamma radiation.

The metastable, or excited state, is usually represented by a small m following the mass number, A , in the standard nuclide notation. For example, technetium-99m (Tc-99m) and technetium-99 (Tc-99) are isomers. ${ }_{43}^{99} \mathrm{Tc}$ will decay to ${ }_{43}^{99} \mathrm{Tc}$ with the emission of a 140.5 keV gamma. Further radioactive decay can still occur from the ground state. In this case, Tc-99 decays to ruthenium-99 (Ru-99), which is stable.

## Internal Conversion

This phenomena occurs when a gamma photon does not escape the electron cloud surrounding the nucleus, but transfers enough energy to one of the orbital electrons to eject it from the atom. The photon is said to have undergone internal conversion. The conversion electron is ejected from the atom with kinetic energy equal to the gamma energy minus the binding energy of the orbital electron. This process usually takes place in the K-shell. There will then follow emission of characteristic x-rays as with electron capture. In principle, it is similar to the photoelectric effect.

Radiation Protection Competency 1.2

X-ray Generation

X-rays are generated by both man-made and natural methods. In general, x-rays are generated by two processes, either the transition of orbital electrons from higher energy levels to lower energy levels, or the change in energy or path of a high speed charged particle (i.e., bremsstrahlung).

Transitions of orbital electrons occur by adding energy to the atom. Energy is most commonly added to the atom by interactions with ionizing radiations. Examples would be gamma radiation causing a K-shell photoelectric effect and leaving a K-shell vacancy. The vacant spot will be filled by orbital electrons cascading down higher energy orbits. In order to conserve energy, characteristic x-rays are emitted from the electronic shell of the atom. The energy of the x-ray will be the difference between the electron shell energy levels. Charged particles interacting by ionizations and excitations can also result in x-ray emission. An electron can be raised to a higher energy level by a passing charged particle. After the charged particle passes, the electron will transition back to the ground state by emitting an x-ray.

Medical departments generate x-rays for diagnostic purposes by bombarding a high Z material, such as tungsten metal, with high speed electrons. The electrons come via thermionic emission by passing a current through a filament. The electrons are accelerated in a vacuum by a high voltage electric field into the tungsten target. When the electron passes near the proximity of the positively charged nucleus, it is defected from its path and decelerated. Since the electron has undergone a loss of energy, an x-ray is given off equal to the energy lost by the electron. This type of x-ray generation is known as bremsstrahlung radiation, which is German for "braking radiation".

## Chart of the Nuclides

## General Arrangement

In arranging the nuclides in chart form, the number of neutrons $(N)$ is plotted horizontally on the $x$-axis against the number of protons (atomic number, $Z$ ) on the $y$-axis. Such a plot at once reveals the continuity in composition in progressing from the lighter to the heavier elements. The full-size Chart of the Nuclides (poster) is much easier to follow than the Nuclides and Isotopes volume which contains all of the material from the chart in book form. A guide for using the chart is found in the lower right-hand corner of the chart or on pages 18 and 19 of the book.

## Specific Nuclide Representation

Each specific nuclide is represented in the Chart of the Nuclides by a block. The coloring and labeling of each block specifies certain information concerning the properties of the nuclide. Z values are given along the left side of the grid, and N values are found along the bottom.

A grey block denotes a stable nuclide. A typical example is stable sodium $\left({ }_{11}^{23} \mathrm{Na}\right)$. A key to the listed data
within the block is shown below.

| $\mathbf{N a 2 3}$ |
| :---: |
| 100 |
| $\boldsymbol{\sigma}_{\gamma}(.40+.13), 32$ |
| 22.989767 |


| Nuclide: | sodium-23 |
| :--- | ---: |
| Grey color: | stable |
| Percent Abundance: | $100 \%$ |
|  |  |
| Neutron activation cross section in barns-- |  |
| (n, $\gamma$ ) interaction |  |
| Atomic mass: | 22.989767 amu |

Unlike sodium, most elements have more than one stable isotope. For example, magnesium $(\mathrm{Mg})$ has three stable isotopes as shown below.

|  | $\underline{\mathbf{M g}-\mathbf{2 4}}$ | $\underline{\mathbf{M g - 2 5}}$ | $\underline{\mathbf{M g - 2 6}}$ |  |
| :--- | :--- | :--- | :--- | :--- |
| Percent <br> abundance | 78.99 | 10.00 | 11.01 |  |
| amu | 23.985042 |  | 24.985837 | 25.982594 |

A white block denotes an artificially produced radioactive nuclide. A typical example is $\left({ }_{26}^{59} \mathrm{Fe}\right)$. A key to data listed within the block is shown below:

| $\mathrm{Fe}-59$ | - nuclide |
| :--- | :--- |
| 45.51 d | - half-life (yellow color: 10 to 100 days) |
| $\beta .466, .271$ | - beta energies in MeV |
| $\gamma 1099.2,1291.6$, | - gamma energies in keV |
| E 1.56 | - disintegration energy in MeV |

A white block with a black triangle in the lower right hand corner denotes an artificially produced radionuclide resulting from slow neutron fission (fission product). An example follows:

| Sr-90 | - nuclide |
| :--- | :--- |
| 29.1 a | - half-life |
| $\beta . .546$ | - beta energy in MeV |
| no $\gamma$ | - no associate gamma emission |
| E .546 | - beta decay energy in MeV |

A grey block with a black bar across the top denotes a long-lived, naturally-occurring radioactive isotope. Uranium-238 $\left({ }_{92}^{238} \mathrm{U}\right)$ is a good example:

| U-238 | - nuclide |
| :--- | :--- |
| 99.2745 | - percent abundance |
| 4.47 E 9 a | - half-life |
| $\alpha 4.197,4.147$ | - mode of decay, radiation and energy |
| $\gamma 49.6$ | - gamma energy in keV |

## Depicting Nuclear Processes

As a result of decay, radionuclides shift from block to block within the Chart of the Nuclides. The diagram below (taken from the guide for using the Chart of the Nuclides) shows the relative locations of the products of various nuclear processes.


As can be seen, the relative locations (displacements) of the primary modes of decay are:

| Alpha $(\alpha)$ | down 2, left $2(\downarrow \downarrow, \leftarrow \leftarrow)$ |
| :--- | :--- |
| Beta $\left(\beta^{-}\right)$ | up 1, left 1 ( $\uparrow \hookleftarrow)$ |
| Positron $\left(\beta^{+}\right) /$EC | down 1, right $1(\downarrow \rightarrow)$ |

Displacements can also occur as a result of nuclear reactions brought about through bombarding given nuclides with various nuclear particles or gamma photons. These changes are depicted in the guide for using the Chart of the Nuclides.

## Chart of the Nuclides Summary

The Chart of the Nuclides provides considerable information about the behavior of nuclides. There is continuity in composition of the nuclides. For example, a line drawn through the stable nuclides forms a rather smooth curve extending from the lower left to the upper right corner of the Chart of the Nuclides.

Nuclides below this line are characterized by having an excess of neutrons and will, in general, be beta-minus emitters.

Nuclides above this line are characterized by having an excess of protons and will, in general, decay by positron emission or electron capture.

Nuclides lying beyond the line of stability will, in general, demonstrate a tendency to seesaw between alpha decay and beta decay. All nuclides, if followed through their various decay schemes, will eventually end in a grey box (stable isotope).

The Chart presents, in compact style, valuable information concerning the properties of the nuclides. This includes data for:

1. Stable nuclides
a. Relative abundance
b. Cross section for activation
2. Radioactive nuclides
a. Types of emissions
b. Energy of emissions
c. Half-life.

## Decay Equation

The activity of a sample containing radioactive material decreases, or decays, at a fixed rate, which is a characteristic of that particular radionuclide. No known physical or chemical agents (such as temperature, pressure, dissolution, or combination) can influence this rate. The rate may be characterized by observing the fraction of activity that remains after a period of time.

For convenience, we choose a fraction that is easy to work with; one-half ( $1 / 2$ ). In using this fraction we can observe the decay of a radionuclide with the passing of time. We can observe how long it takes for the activity to be reduced to one-half of the original activity. The period of time required to reduce the original activity by one-half is call the half-life. If successive half-lives are observed, we can see a reduction each time by a fraction of one-half, and the effect will be cumulative. In other words, one half-life reduces to $(1 / 2)^{1}$; two
half-lives reduces to $1 / 2 \times 1 / 2=(1 / 2)^{2}$ or $1 / 4$; three half-lives will reduce to $1 / 2 \times 1 / 2 \times 1 / 2=(1 / 2)^{3}$ or $1 / 8$, etc. In the general case the fraction of activity remaining after any number of half lives will be $(1 / 2)^{n}$, where n is the number of half-lives that have elapsed.


FIGURE 1 - Radioactive Decay (Linear Scale)

In Figure 1, it can be seen that as time passes radioactive decay occurs at an exponential rate. In using the half-life for our time value, we express this exponential function as $(1 / 2)^{n}$. To put it still another way, the reduction in activity occurs at an exponential rate, which can be expressed as a natural logarithm function.

The decay scale, shown in figure 1, represents an exponential decrease in activity over time. This reduction in activity fits a natural logarithm base function, generally denoted $e$, and represents the number 2.718. Since
the half-life of any specific radionuclide is always the same, both future or past activity of a source can be accurately calculated using the following formula.

$$
A_{t}=A_{0} e^{-\lambda t}
$$

and

$$
\lambda=\frac{\ln 2}{T^{1 / 2}}
$$

Where:
$\mathrm{A}_{\mathrm{o}}=$ the original activity in curies, becquerels, or dps
$\mathrm{A}_{\mathrm{t}}=$ the activity at time t
$\mathrm{e}=$ natural log
$\lambda=$ radioactive decay constant in reciprocal units of time (i.e., $\sec ^{-1}$ )
$\mathrm{t}=$ elapsed time
$\mathrm{T} 1 / 2=$ half-life

When solving problems, the units of time must be consistent between $t$ and the radioactive decay constant.

## Calculating Activity and/or Mass

Activity of a sample is related to the rate at which a radionuclide decays and the number of atoms present in the sample. Using this relationship allows the calculation of the mass of a radionuclide, if the activity is known or the activity if the mass of the radionuclide is known. Avagarodo discovered that one mole of any element contains the same number of atoms, exactly 6.023 E23 atoms. One mole of any given element equals the atomic weight of that element. For example, one mole of uranium-238 (U-238) requires 238 grams of U-238. The number of atoms in a sample changes in direct proportion to the number of grams of material.

The activity is calculated by the formula:

$$
A=\lambda N
$$

Where:
$A=$ activity in disintegrations per second

$$
\begin{aligned}
& \lambda=\text { decay constant of the radionuclide }\left(\mathrm{in} \mathrm{sec}^{-1}\right) \\
& \mathrm{N}=\text { number of atoms present in the sample }
\end{aligned}
$$

The number of atoms present in the sample can be calculated by the formula:

$$
N=\text { weightofsample }(g)\left(\frac{1 \text { mole }}{\text { atomic weight }(g)}\right)\left(\frac{6.023 \times 10^{23} \text { atoms }}{1 \text { mole }}\right)
$$

## Calculating Mass from Activity

If the activity is given for each specific radioactive isotope of an element, it is by definition the specific activity of a unit mass of the element. The specific activity will be given in $\mathrm{Ci} / \mathrm{g}$ and is useful for calculating between mass and activity. For example, if the specific activity of plutonium-239 (Pu-239) is $6.13 \times 10^{-2} \mathrm{Ci} / \mathrm{g}$, then the mass of 1 Curie of $\mathrm{Pu}-239$ is 16.3 g .

In another example, calculate the mass of $1 \mu \mathrm{Ci}$ of $\mathrm{U}-238$.
Given:
U-238 half-life $=4.5 \times 10^{9}$ years
$1 \mu \mathrm{Ci}=3.7 \times 10^{4} \mathrm{dps}$

Solution:

$$
A=\lambda N
$$

Where:

$$
\frac{\text { disintegrations }}{\sec }=\left(\frac{0.693}{T 1 / 2(\mathrm{sec})}\right)\left[\text { weight of sample }(g)\left(\frac{1 \text { mole }}{\text { atomic weight }(g)}\right)\left(\frac{6.023 \times 10^{23} \text { atoms }}{1 \text { mole }}\right)\right]
$$

Convert the half-life into seconds and solve for the sample weight. Assume one alpha disintegration per atom.

Answer:

$$
\text { weight of sample }(g)=\left(\frac{3.7 \times 10^{4} \text { atoms }}{\sec }\right)\left(\frac{4.5 \times 10^{9} y r}{0.693}\right)\left(\frac{365 d}{1 y r}\right)\left(\frac{86,400 \mathrm{sec}}{1 d}\right)\left(\frac{238 g}{1 \text { mole }}\right)\left(\frac{1 \text { mole }}{6.023 \times 10^{23} \text { atoms }}\right)
$$

gram weight of sample $=2.99$ grams

## Converting Units of Activity

The units by which physical quantities are measured are established in accordance with an agreed standard. Measurements made are thereby based on the original standard which the unit represents. The various units that are established, then, form a system by which all measurements can be made.

Since the exchange of scientific information is world-wide today, international committees have been set up to standardize the names and symbols for physical quantities. In 1960, the International System of Units (abbreviated SI from the French Name Le Système Internationale d'Unites) was adopted by the 11th General Conference of Weights and Measures. The metric system, which is the basis for the SI System, is based on the decimal (base 10) numbering system. First devised in France around the time of the French Revolution, the metric system has been refined and currently consists of a set of specifically defined units that serve as an internationally accepted system of measurement. Nearly all countries in the world use metric or SI units for business and commerce as well as for scientific applications.

The metric system is completely decimalized and uses prefixes for the base units of meter (m) and gram (g) as well as for derived units, such as the liter (l).

Metric prefixes are used with units for various magnitudes associated with the measurement being made. Units with a prefix whose value is a negative power of ten are called subunits. Units with a prefix whose value is a positive power of ten are called superunits.

For example, as a point of reference, the meter is a little longer than a yard. Try using a yard stick to measure the size of a frame on film for a camera. Instead you would use inches, because it is a more suitable unit. With the metric system, in order to measure tiny lengths, such as film size, the prefix milli- can be attached to the meter to make a millimeter, or $1 / 1000$ of a meter. A millimeter is much smaller and is ideal in this situation. On the other hand, we would use a prefix like kilo- for measuring distances traveled in a car. A kilometer would be more suited for these large distances than the meter.

Metric Prefixes

| PREFIX | POWER | ABBR. |
| :--- | :---: | :---: |
| exa- | $10^{18}$ | E |
| peta- | $10^{15}$ | P |
| tera- | $10^{12}$ | T |
| giga- | $10^{9}$ | G |
| mega- | $10^{6}$ | M |
| kilo- | $10^{3}$ | k |
| hecto- | $10^{2}$ | h |
| PREFIX | POWER | ABBR. |
| deka- | $10^{1}$ | da |
| deci- | $10^{-1}$ | d |
| centi- | $10^{-2}$ | c |
| milli- | $10^{-3}$ | m |
| micro- | $10^{-6}$ | $\mu$ |
| nano- | $10^{-9}$ | n |
| pico- | $10^{-12}$ | p |
| femto- | $10^{-15}$ | f |
| atto- | $10^{-18}$ | a |

Using these prefixes, the metric system has two systems used for the quantities length, mass, and time: the MKS and the CGS systems. The units for these systems are given below.

Metric Systems

| Physical Quantity | CGS | MKS |
| :--- | :---: | :---: |
| Length: | centimeter | meter |
| Mass: | gram | kilogram |
| Time: | second | second |

## Units and the Rules of Algebra

Remember that a measurement consists of a number and a unit. When working problems with measurements, it should be noted that the units follow the same rules as the values. Some examples are provided below.

$$
\begin{aligned}
& (\mathrm{cm}) \times(\mathrm{cm})=\mathrm{cm}^{2} \\
& \frac{f t^{3}}{f t}=f t^{2} \\
& \frac{1}{y r}=y r^{-1}
\end{aligned}
$$

As mentioned above, measurements are subject to algebraic laws of operation and can therefore be multiplied, divided, etc., in order to convert to a different system of units. Obviously, in order to do this, the units must be the same.

## Steps for Unit Analysis and Conversion

1) Determine given units and desired units.
2) Build (or obtain) conversion factor(s)

A conversion factor is a ratio of two equivalent physical quantities expressed in different units. When expressed as a fraction, the value of all conversion factors is 1 . Because it equals 1 , it does not matter which value is placed in the numerator or denominator of the fraction.

Examples of conversion factors are:

$$
\frac{365 \text { days }}{1 \text { year }}=\frac{12 \text { inches }}{1 \text { foot }}=\frac{1 \text { foot }^{3}}{2.832 E 4 \mathrm{~cm}^{3}}
$$

Building conversion factors involving the metric prefixes for the same unit can be tricky. This involves the conversion of a base unit to, or from, a subunit or superunit.

To do this, use the following steps:
a) Place the base unit in the numerator and the subunit/ superunit in the denominator (or vice versa).
b) Place a 1 in front of the subunit/superunit.

Example: 1 gram to milligrams
$\underline{g}$
$m g$ $\frac{g}{1 m g}$
$\mathrm{m}($ milli- $)=10^{-3}$ or $1 \mathrm{E}-3$
$\frac{1 E-3 g}{1 m g}$

Also remember that algebraic manipulation can be used when working with metric prefixes and bases. For example, 1 centimeter $=10^{-2}$ meters. This means that 1 meter $=1 / 10^{-2}$ centimeters, or 100 cm . Therefore, the two conversion factors below are equal:

$$
\frac{1 E-2 m}{1 \mathrm{~cm}} \quad \frac{1 \mathrm{~m}}{100 \mathrm{~cm}}
$$

3) Set up an equation by multiplying the given units by the conversion factor(s) to obtain desired units.

When a measurement is multiplied by a conversion factor, the units (and probably the magnitude) will change; however, the actual measurement itself does not change. For example, 1 foot and 12 inches are still the same length; only different units are used to express the measurement.

By using a "ladder" or "train tracks," a series of conversions can be accomplished in order to get to the desired units. By properly arranging the numerator and denominator of the conversion factors, given and intermediate units will cancel out by multiplication or division, leaving the desired units. Some examples of the unit analysis and conversion process follow.

## Example 1

Convert 3 millicuries to becquerels.
$\underline{\text { Step } 1 \text { - Determine given and desired units }}$
Given units: millicuries
Desired units: Becquerels
Step 2 - Build/obtain conversion factor(s)
We can use multiple conversion factors to accomplish this problem:

$$
\begin{array}{ll}
1 \text { curie } & =3.7 \mathrm{E} 10 \mathrm{dps} \\
1 \text { millicurie } & =1 \mathrm{E}-3 \text { curies } \\
1 \text { becquerel } & =1 \mathrm{dps}
\end{array}
$$

Step 3 - Analyze and cancel given and intermediate units. Perform multiplication and division of numbers.

$$
3 m / C i\left(\frac{1 E-3 C i}{1 m C i}\right)\left(\frac{3.7 E 10 d p s s}{1 C h}\right)\left(\frac{1 B q}{1 d p s}\right)=1.11 E 8 B q .
$$

## Example 2

$$
\text { What is the activity of a solution in } \frac{\mu C i}{m l} \text { if it has } 2,000 \frac{d p m}{\text { gallon }} ?
$$

Step 1 - Determine given and desired units.

Given units: $\frac{d p m}{\text { gallon }}$
Desired units: $\frac{\mu C i}{m l}$

Step 2 - Build conversion factor(s).

$$
\begin{aligned}
& 1 \text { liter }=0.26418 \text { gallons } \\
& 1 \mathrm{dpm}=4.5 \mathrm{E}-07 \mu \mathrm{Ci} \\
& 1 \text { liter }=1,000 \mathrm{ml}
\end{aligned}
$$

Step 3 - Analyze and cancel given and intermediate units. Perform multiplication `and division of numbers.

$$
\left(\frac{2,000 \mathrm{dpm} m}{\mathrm{~g} \not \mathrm{l} l}\right)\left(\frac{4.5 E-7 \mu C i}{1 d p m}\right)\left(\frac{0.26418 \mathrm{~g} \not \mathrm{ll}}{1 \not 2}\right)\left(\frac{1 \not \ell}{1,000 \mathrm{ml}}\right)=2.38 E-7 \frac{\mu C i}{m l}
$$

## 3. SELF STUDY SCENARIOS/ACTIVITIES AND SOLUTIONS

## Activity 1:

Given 10 mCi of phosphorus-32 (P-32), which has a half-life of 14.2 days, find the quantity remaining after 200 hours.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Activity 2: Unit Conversion Practical Exercises

Convert the following measurements:

1. $5,000 \mathrm{dpm}$ to mCi
2. 2.22 TBq to Ci
3. 3.7 E 10 Bq to dpm
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Activity Solutions:

Solution, Activity 1

$$
A_{t}=A_{0} e^{-\lambda t}
$$

Where:
$A_{o}=$ the original activity $(10 \mathrm{mCi})$
$\mathrm{A}_{\mathrm{t}}=$ the activity at the end of time t (elapsed time, 200 hours)
$\mathrm{e}=$ natural $\log (\ln 2=0.693)$
$\lambda=$ radioactive decay constant reciprocal units of time (0.693/14.2 d)

$$
\lambda=\frac{\ln 2}{14.2 \text { days }}=\frac{0.693}{14.2 \text { days }}=0.0488 \text { days }^{-1}
$$

Substitute the values and solve for $\mathrm{A}_{\mathrm{t}}$. No algebraic manipulation is necessary for this problem.

$$
A_{t}=(10 m C i)\left(e^{\frac{-0.0488}{d a y} \frac{200 \mathrm{hrs}}{1} \frac{1 \text { day }}{24 \mathrm{hrs}}}\right)
$$

$$
\begin{aligned}
& A_{t}=(10 \mathrm{mCi}) e^{-0.4} \\
& \mathrm{~A}_{\mathrm{t}}=6.65 \mathrm{mCi} \text { after } 200 \text { hours }
\end{aligned}
$$

Solution, Activity 2: Answers to Unit Conversions_

1. $5,000 \mathrm{dpm}$ to mCi

$$
\left(\frac{5 E 3 \mathrm{dis}}{\min }\right)\left(\frac{1 C i}{2.22 E 12 \frac{\mathrm{dis}}{\mathrm{~min}}}\right)\left(\frac{1 E 3 \mathrm{mCi}}{1 C i}\right)=2.25 E-6 \mathrm{mCi}
$$

2. 2.22 TBq to Ci

$$
\text { 2.22 } \mathrm{TBq}\left(\frac{1 E 12 \mathrm{~Bq}}{1 \mathrm{TBq}}\right)\left(\frac{1 d p s}{1 B q}\right)\left(\frac{1 \mathrm{Ci}}{3.7 E 10 d p s}\right)=60 \mathrm{Ci}
$$

3. 3.7 E 10 Bq to dpm

$$
\text { 3.7E } 10 \mathrm{~Bq}\left(\frac{1 d p s}{1 \mathrm{~Bq}}\right)\left(\frac{60 \mathrm{dpm}}{1 d p s}\right)=2.22 E 12 \mathrm{dpm}
$$

## Radiation Protection Competency 1.2

## 4. SUGGESTED ADDITIONAL READINGS AND/OR COURSES

## Readings

- Argonne National Laboratory. (1988). Department of Energy Operational Health Physics Training (ANL-88-26). Argonne, IL.
- Cember, Herman. (1996). Introduction to Health Physics (3rd ed.). McGraw-Hill: New York.
- Gollnick, Daniel A. (1991). Basic Radiation Protection Technology (2nd ed.). Pacific Radiation Corporation: Altadena, CA.


## Courses

- Nuclear Physics/Radiation Monitoring -- DOE.
- DOE/EH-0450 (Revision 0), Radiological Assessors Training (for Auditors and Inspectors) Fundamental Radiological Control, sponsored by the Office of Defense Programs, DOE
- Applied Health Physics -- Oak Ridge Institute for Science and Education.
- Health Physics for the Industrial Hygienist -- Oak Ridge Institute for Science and Education.
- Radiological Worker Training -- DOE-EH.
- Radiological Control Technician -- DOE-EH.
- Safe Use of Radionuclides -- Oak Ridge Institute for Science and Education.
- Radiation Protection General Technical Base Qualification Standard Training -- GTS Duratek.

