

Appendix A: Econometric Analysis of Mortgages

This appendix describes technical details of the econometric models used to estimate the historical and future performance of FHA single-family loans for the FY 2005 Review. We first summarize the model specification and estimation issues arising from the analysis of FHA claim and prepayment rates. Then we describe for the specific explanatory variables used in the analysis. The model estimation output statistics and graphical comparisons of the overall within-sample fit of the models are provided thereafter.

I. Model Specification and Estimation Issues

A. Specification of FHA Mortgage Termination Models

For the FY 2005 Review, the TAC Team has developed and estimated competing risk models for mortgage prepayment and claim terminations. Prepayment and claim rates estimates were based on a multinomial logit model for quarterly conditional probabilities of prepayment and claim terminations. The general approach is based on the multinomial logit models reported by Calhoun and Deng (2002) that were originally developed for application to OFHEO's risk-based capital adequacy test for Fannie Mae and Freddie Mac. The multinomial model recognizes the competing risks nature of prepayment and claim terminations. The use of quarterly data aligns more closely with key economic predictors of mortgage prepayment and claims such as changes in interest rates and housing values.

The loan performance analysis was undertaken at the loan level. Through the use of categorical explanatory variables and discrete indexing of mortgage age, it was possible to achieve considerable efficiency in data storage and reduced estimation times by collapsing the data into a much smaller number of loan strata. In effect, the data were transformed into synthetic loan pools, but without loss of detail on individual loan characteristics beyond that implied by the original categorization of the explanatory variables, which were entirely under control. Sampling weights were used to account for differences in the number of identical loans in each loan strata.

The present analysis differs from the Calhoun-Deng (2002) study in two important ways. First, following the approach suggested by Begg and Gray (1984), we estimated separate binomial logit models for prepayment and claim terminations, and then mathematically recombined the parameter estimates to compute the corresponding multinomial logit probabilities. This approach allowed us to account for differences between the timing of claim terminations and the censoring of potential prepayment outcomes at the onset of default episodes that ultimately lead to claims. This issue is discussed in greater detail below.

A second difference from the Calhoun-Deng (2002) study was the treatment of mortgage age in the models. The traditional models apply quadratic age functions for both mortgage default and prepayment terminations. While the quadratic age function fits reasonably well for estimating conventional mortgage defaults rates, it worked less well for prepayments, as it failed to capture the more rapid increase in conditional prepayment rates early in the life of the loans. FHA conditional claim and prepay rates also show a more rapid increase during the early part of the loan life. We found a quadratic specification to be insufficiently flexible to capture the age patterns of conditional claim and prepayment observed in the FHA data. The approach we adopted was a series of piece-wise linear spline functions. This approach is sufficiently flexible to fit the relatively rapid increase in conditional claim and prepayment rates observed during the first two to three years following mortgage origination, while still providing a good fit over the later ages and limiting the overall number of model parameters that have to be estimated. At the end of this Appendix we present graphical comparisons showing the goodness of fit by age of our final model estimates.

As indicated, the starting point for specification of the loan performance models was a multinomial logit model of quarterly conditional probabilities of prepayment and claim terminations. The corresponding mathematical expressions for the conditional probabilities of claim ($\mathbf{p}_C(t)$), prepayment ($\mathbf{p}_P(t)$), or remaining active ($\mathbf{p}_A(t)$) over the time interval from t to $t+1$ are given by:

$$\mathbf{p}_C(t) = \frac{e^{\mathbf{a}_C + X_C(t)\mathbf{b}_C}}{1 + e^{\mathbf{a}_C + X_C(t)\mathbf{b}_C} + e^{\mathbf{a}_P + X_P(t)\mathbf{b}_P}} \quad (1)$$

$$\mathbf{p}_P(t) = \frac{e^{\mathbf{a}_P + X_P(t)\mathbf{b}_P}}{1 + e^{\mathbf{a}_C + X_C(t)\mathbf{b}_C} + e^{\mathbf{a}_P + X_P(t)\mathbf{b}_P}} \quad (2)$$

$$\mathbf{p}_A(t) = \frac{1}{1 + e^{\mathbf{a}_C + X_C(t)\mathbf{b}_C} + e^{\mathbf{a}_P + X_P(t)\mathbf{b}_P}} \quad (3)$$

where the constant terms \mathbf{a}_C and \mathbf{a}_P and the coefficient vectors \mathbf{b}_C and \mathbf{b}_P are the unknown parameters to be estimated. $X_C(t)$ is the vector of explanatory variables for the conditional probability of a claim termination, and $X_P(t)$ is the vector of explanatory variables for the conditional probability of prepayment. Some elements of $X_C(t)$ and $X_P(t)$ are constant over the life of the loan and are not functions of t .

B. Differences in the Timing of Borrower Default Episodes and Claim Terminations

As mentioned above, timing differences between borrower default episodes and actual FHA claims led us to apply the Begg-Gray method of estimating separate binomial logit models for FHA prepayment and claim terminations and then recombine the parameter estimates to derive

the corresponding multinomial logit model. The issue in this case is the time lag between the time that a borrower decides to cease payment on a loan, *i.e.*, default, and when FHA actually receives the claim from the servicer. Because prepayments are unlikely to occur for defaulting loans on their way to becoming claim terminations, censoring of prepayments actually occurs prior to the observed claim termination date. Failure to account for this particular form of censoring could result in biased estimates of the parameters of the prepayment model.

The claim-rate model is best viewed as a reduced-form of a more complicated model with two components: (1) an option-based model of borrower payment behavior that determines the incidence and timing of default events that ultimately lead to FHA claims; and (2) a model for differences in the waiting time from borrower default until the claim is submitted to FHA. The second component can be properly addressed in conjunction with estimates of loss severity (or loss-given-default), and can vary significantly with differences in state laws on mortgage foreclosure, differences in lender loss-mitigation policies, and with current economic conditions that affect the values and time-to-sale of collateral properties.

For the FY 2005 Review, we apply average loss severity rates observed between the FY 2000 and FY 2004 stratified by six mortgage product types. For consistency with the available data on loss rates, the incidence and timing of mortgage default-related terminations is defined specifically according to FHA claim incidences. The Begg-Gray method of estimating separate binomial logit models is particularly advantageous in dealing with this requirement. In recognition of the potential censoring of prepayment prior to the actual claim termination date, we used information on the timing of the initiation of default episodes leading to claim terminations to create a prepayment-censoring indicator that was applied when estimating the prepayment-rate model.

A separate claim-rate model was estimated that accounted for the censoring of potential claim terminations by observed prepayments. The two sets of parameter estimates were recombined mathematically to produce the final multinomial model for prepayment and claim probabilities. This approach facilitated unbiased estimation of the prepayment function, which would not be possible in a joint multinomial model of claim and prepayment terminations, since one cannot simultaneously censor loans at the onset of default episodes and retain the same observations for estimating subsequent claim termination rates.

The Begg-Gray methodology produces parameter estimates that are theoretically equivalent to those in the multinomial logit model. By estimating the prepayment and claim rate models separately, we can isolate the issues associated with the timing of claims from the estimation of the parameters of the prepayment function. Failure to exclude defaulting loans from the sample of loans assumed to be at risk of prepayment would result in downward bias in the estimates of conditional probabilities of prepayment because loans with zero chance of prepayment would be included in the sample in estimating conditional prepayment rates.

To summarize, estimation of the multinomial logit model for prepayment and claim terminations involved the following steps:

1. Data on the start of a default episode that ultimately leads to an FHA claim was used to define a default censoring indicator for prepayment.
2. A binomial logit model for conditional prepayment probabilities was estimated using the default-censoring indicator to truncate individual loan event samples at the onset of the default episodes and all subsequent quarters.
3. A binomial logit model for conditional claim probabilities was estimated using observed prepayments to truncate individual loan event samples during the quarter of the prepayment event and all subsequent quarters.
4. The separate sets of binomial logit parameter estimates were recombined mathematically to derive the corresponding multinomial logit model for the joint probabilities of prepayment and claim terminations.

B. Computation of Multinomial Logit Parameters from Binomial Logit Parameters

Once the separate binomial claim rate and prepayment rate models have been estimated, the parameter estimates must be combined to compute the multinomial probabilities. The theory underlying the Begg-Gray method is that the values of parameters \mathbf{a}_c , \mathbf{b}_c , \mathbf{a}_p , and \mathbf{b}_p from separate binomial logit (BNL) models are identical to those in the corresponding multinomial logit (MNL) model. Assume that conditional probabilities for claim and prepayment terminations for separate BNL models are given, respectively, by:

$$\mathbf{p}_{BNL}^C = \frac{e^{\mathbf{a}_c + X_c \mathbf{b}_c}}{1 + e^{\mathbf{a}_c + X_c \mathbf{b}_c}}, \quad \mathbf{p}_{BNL}^P = \frac{e^{\mathbf{a}_p + X_p \mathbf{b}_p}}{1 + e^{\mathbf{a}_p + X_p \mathbf{b}_p}}. \quad (4)$$

We have suppressed the time index t to simplify the notation. We can rearrange terms to solve for $e^{\mathbf{a}_c + X_c \mathbf{b}_c}$ and $e^{\mathbf{a}_p + X_p \mathbf{b}_p}$ in terms of binomial probabilities π_{BNL}^C and π_{BNL}^P , respectively,

$$e^{\mathbf{a}_c + X_c \mathbf{b}_c} = \frac{\mathbf{p}_{BNL}^C}{(1 - \mathbf{p}_{BNL}^C)}, \quad e^{\mathbf{a}_p + X_p \mathbf{b}_p} = \frac{\mathbf{p}_{BNL}^P}{(1 - \mathbf{p}_{BNL}^P)}. \quad (5)$$

Then we can substitute directly into the MNL probabilities for $e^{\mathbf{a}_c + X_c \mathbf{b}_c}$ and $e^{\mathbf{a}_p + X_p \mathbf{b}_p}$:

$$p_{MNL}^C = \frac{\frac{p_{BNL}^C}{(1-p_{BNL}^C)}}{1 + \frac{p_{BNL}^C}{(1-p_{BNL}^C)} + \frac{p_{BNL}^P}{(1-p_{BNL}^P)}}, \quad p_{MNL}^P = \frac{\frac{p_{BNL}^P}{(1-p_{BNL}^P)}}{1 + \frac{p_{BNL}^C}{(1-p_{BNL}^C)} + \frac{p_{BNL}^P}{(1-p_{BNL}^P)}}. \quad (6)$$

These expressions for the MNL probabilities can be simplified algebraically to:

$$p_{MNL}^C = \frac{p_{BNL}^C \cdot (1-p_{BNL}^P)}{(1-p_{BNL}^C \cdot p_{BNL}^P)}, \quad p_{MNL}^P = \frac{p_{BNL}^P \cdot (1-p_{BNL}^C)}{(1-p_{BNL}^C \cdot p_{BNL}^P)}. \quad (7)$$

Equations (7) were used to derive the corresponding MNL probabilities directly from separately estimated BNL probabilities.

C. Loan Event Data

We used loan-level data to reconstruct quarterly loan event histories by combining mortgage origination information with contemporaneous values of time-dependent factors. In the process of creating quarterly event histories, each loan contributed an additional observed “transition” for every quarter from origination up to and including the period of mortgage termination, or until the last time period of the historical data sample. The term “transition” is used here to refer to any period in which a loan remains active, or in which claim or prepayment terminations are observed.

The FHA single-family data warehouse records each loan for which insurance was endorsed and includes additional data fields updating the timing of changes in the status of the loan. The data set used in this Actuarial Review is based on an extract from FHA’s database as of March 31, 2005. The data set was first filtered for loans with missing or abnormal values of key variables in our econometric model. In addition, lender information was not used in our econometric model, loans with missing lender/servicer information were also excluded from our analysis. Most of those loans were believed to have already been prepaid but the records were not yet updated. Since FY 2004, HUD has been investigating and updating the performance records of these loans.

A dynamic event history sample was constructed from the database of loan originations by creating additional observations for each quarter that the loan was active from the beginning amortization date up to and including the termination date for the loan, or the end of the first quarter of FY 2005 if the loan has not terminated prior to that date.

Additional “future” observations were created for projecting the future performance of loans currently outstanding, and additional future cohorts were created to enable simulation of the performance of future books of business. These aspects of data creation and simulation of future loan performance are discussed in greater detail in Appendix C.

D. Random Sampling

A full 100-percent sample of loan level data from the FHA single-family data warehouse was extracted for the FY 2005 analysis. This produced a starting sample of approximately 20 million single-family loans originated between FY 1975 and the first quarter of FY 2005. At the estimation stage a 10-percent random sample of loans is used to generate loan-level event histories for up to 120 quarters (30 years) of loan life per loan, or until the scheduled maturity date of the loan.

II. Explanatory Variables

Three main categories of explanatory variables were developed:

1. Fixed loan characteristics, such as mortgage product type, amortization term, origination year and quarter, original loan-to-value (LTV) ratio, original loan amount, original mortgage interest rate, and geographic location (MSA, state, Census division);
2. Dynamic variables based entirely on the loan information, such as mortgage age, season of the year, and scheduled amortization of the loan balance; and
3. Dynamic variables derived by combining loan information with external economic data, such as interest rates and house price indexes.

In some cases the two types of dynamic variables are combined, as in the case of adjustable rate mortgage (ARM) loans where external data on changes in Treasury rates are used to update the original coupon rates and payment amounts on ARM loans in accordance with standard FHA loan contract features. This in turn affects the amortization schedule of the loan.

Exhibit A-1 summarizes the specific explanatory variables that are used in the statistical modeling of loan performance. All of the variables listed in Exhibit A-1 were entered as 0-1 dummy variables in the statistical models, with the exception of the mortgage age variables, which were entered directly. The specification of each variable is described in more detail as below.

Mortgage Product Types

Separate statistical models were estimated for the following six FHA mortgage product types:

1. FRM30 Fixed-rate 30-year home purchase mortgages.
2. FRM15 Fixed-rate 15-year home purchase mortgages.
3. ARM Adjustable-rate home purchase mortgages.
4. FRM30_SR Fixed-rate 30-year streamlined refinance mortgages.
5. FRM15_SR Fixed-rate 15-year streamlined refinance mortgages.
6. ARM_SR Adjustable-rate streamlined refinance mortgages.

Specification of Piece-Wise Linear Age Functions

Exhibit A-1 lists the series of piece-wise linear age functions that were used for the six different mortgage product types. For example, we create a piece-wise linear age function for FRM15

loans with knots (the k 's) at 2, 4, 8, and 12 quarters by generating 5 new age variables $age1$ - $age5$ defined as follows:

$$\begin{aligned}
 age1 &= \begin{cases} AGE & \text{if } AGE \leq k_1 \\ k_1 & \text{if } AGE > k_1 \end{cases} \\
 age2 &= \begin{cases} 0 & \text{if } AGE \leq k_1 \\ AGE - k_1 & \text{if } k_1 < AGE \leq k_2 \\ AGE - k_2 & \text{if } AGE > k_2 \end{cases} \\
 age3 &= \begin{cases} 0 & \text{if } AGE \leq k_2 \\ AGE - k_2 & \text{if } k_2 < AGE \leq k_3 \\ AGE - k_3 & \text{if } AGE > k_3 \end{cases} \\
 age4 &= \begin{cases} 0 & \text{if } AGE \leq k_3 \\ AGE - k_3 & \text{if } k_3 < AGE \leq k_4 \\ AGE - k_4 & \text{if } AGE > k_4 \end{cases} \\
 age5 &= \begin{cases} 0 & \text{if } AGE \leq k_4 \\ AGE - k_4 & \text{if } AGE > k_4 \end{cases} \tag{8}
 \end{aligned}$$

Coefficient estimates corresponding to the slopes of the line segments between each knot point and for the last line segment are estimated and reported in Exhibit A-2. The overall AGE function (for this 5-age segment example) is given by:

$$\text{Age Function} = \mathbf{b}_1 \cdot age1 + \mathbf{b}_2 \cdot age2 + \mathbf{b}_3 \cdot age3 + \mathbf{b}_4 \cdot age4 + \mathbf{b}_5 \cdot age5 \tag{9}$$

Age functions with greater or fewer numbers of segments are developed in a similar manner. The number of segments is determined by trial-and-error estimation and review of in sample fit to the observed age profiles of conditional claim and prepayment rates.

Loan Size

Loan size is defined relative to the average sized FHA loan originated in the same state during the same fiscal year. The resulting values were stratified into 5 levels based on direct examination of the data, with the middle category, *category 3*, corresponding to average-sized loans plus or minus 10 percent, *i.e.*, 90 to 110 percent of the size of the average sized loan.

Loan-to-Value Ratio

Loan to value is recorded in the FHA's data warehouse. The LTV ratio variable may exceed 100 percent due to FHA's practice of allowing the financing of some closing costs, so a categorical outcome is included for this possibility. Based on discussions with FHA, any LTV values recorded for streamline refinance products were considered unreliable for use in the analysis. We imputed original LTV values for these loans for the purpose of establishing the starting point for tracking the evolution of the probability of negative equity (see description of this variable below). The imputed values were based on the mean LTV values for FRM30, FRM15, and ARM loans stratified by product, beginning amortization year and quarter, and geographic location (state and county).

Season

The season of an event observation quarter is defined as the season of the year corresponding to the calendar quarter, where 1=Winter (January, February, March), 2=Spring (April, May, June), 3=Summer (July, August, September), and 4=Fall (October, November, December).

Probability of Negative Equity

Following the approach applied by Deng, Quigley, and Van Order (2000), Calhoun and Deng (2002), and others, we computed the equity positions of individual borrowers using *ex ante* probabilities of negative equity. The probability of negative equity is a function of the current loan balance and the probability of individual house price outcomes that fall below this value during the quarter of observation. The distributions of individual housing values relative to the value at mortgage origination were computed using estimates of house price drift and volatility based on OFHEO House Price Indexes (HPIs) published in the first quarter of 2005.

The probability of negative equity is computed as follows:

$$PNEQ = \Phi \left\{ \frac{\ln(UPB(t)) - \ln(P(0) \cdot HPI(t))}{s(t)} \right\} \quad (10)$$

where $\Phi(x)$ is the standard normal cumulative distribution function evaluated at x , $UPB(t)$ is the current unpaid mortgage balance based on scheduled amortization, $P(0)$ is the value of the borrower's property at mortgage origination, $HPI(t)$ is an index factor for the percentage change in housing prices in the local market since origination of the loan, and $s(t)$ is a measure of the diffusion volatility for individual house price appreciation rates over the same period of time. The values of $HPI(t)$ are computed directly from the house price indexes published by OFHEO, while the diffusion volatility is computed from the following equation:

$$s(t) = \sqrt{a \cdot t + b \cdot t^2}. \quad (11)$$

The parameters "a" and "b" in this expression are estimated by OFHEO when applying the three-stage weighted-repeat-sales methodology advanced by Case-Shiller (1987, 1989). Further details on the OFHEO HPI methodology are given in Calhoun (1996).

The resulting values of PNEQ were stratified into seven levels ranging from less than 5-percent to more than 30-percent probability of negative equity as listed in Exhibit A-1.

Mortgage Premium (Spread)

The financial incentive of a borrower to refinance is measured using a variable for the relative spread between the current mortgage contract interest rate and the current market mortgage rate:

$$MP(t) = \left\{ \frac{C(t) - R(t)}{C(t)} \right\}. \quad (12)$$

Where $C(t)$ is the current note rate on the mortgage and $R(t)$ is the current market average fixed-rate mortgage rate. This variable is as an approximation to the call option value of the mortgage given by the difference between the present value of the "anticipated" future stream of mortgage payments discounted at the current market rate of interest, $R(t)$, and the present value of the mortgage evaluated at the current note rate, $C(t)$. Additional details are given in Deng, Quigley, and Van Order (2000) and Calhoun and Deng (2002).

The relative mortgage premium values for ARMs and FRMs are derived in exactly the same manner, except that the current coupon is always equal to the coupon at origination for FRMs. ARM coupon rates are updated over the life of the mortgage as described below.

ARM Coupon Rate Dynamics

To estimate the current financial value of the prepayment option for ARM loans, we required the path of the coupon rate over the active life of individual ARM loans. The coupon rate resets

periodically to a new level that depends on the underlying index, plus a fixed margin, subject to periodic and lifetime caps and floors that specify the maximum and minimum amounts by which the coupon can change on any one adjustment and over the life of the loan. Accordingly, the ARM coupon rate at time t , $C(t)$, was computed as follows:

$$C(t) = \max\{ \min[\text{Index}(t-S) + \text{Margin}, \\ C(t-1) + A(t) \cdot \text{Period_UpCap}, C(0) + \text{Life_UpCap}], \\ C(t-1) - A(t) \cdot \text{Period_DownCap}(t), \max(C(0) - \text{Life_DownCap}, \text{Life_Min}) \} \quad (13)$$

where $\text{Index}(t)$ is the underlying rate index value at time t , S is the “lookback” period, and Margin is the amount added to $\text{Index}(t-S)$ to obtain the “fully-indexed” coupon rate. The periodic adjustment caps are given by Period_UpCap and Period_DownCap , and are multiplied by dummy variable $A(t)$ which equals zero except during scheduled adjustment periods. Maximum lifetime adjustments are determined by Life_UpCap and Life_Down_Cap , and Life_Min is the overall minimum lifetime rate level.

Yield Curve Slope

Expectations about future interest rates and differences in short-term and long-term borrowing rates associated with the slope of the Treasury yield curve influence the choice between ARM and FRM loans and the timing of refinancing. We use the ratio of the ten-year Constant Maturity Treasury yield to the one-year Constant Maturity Treasury yield to measure the slope of the Treasury yield curve.

Burnout Factor

A burnout factor is included to identify borrowers who have foregone recent opportunities to refinance. The burnout factor is included to account for individual differences in propensity to prepay, often characterized as unobserved heterogeneity. In addition, unmeasured differences in borrower equity at the loan level may give rise to unobserved heterogeneity that can impact both prepayment and claim rates. Borrowers with negative equity are less likely to prepay due to the difficulty of qualifying and more likely to exercise the default option.

Changes were introduced to the burnout factor for the FY 2005 Review. The previous burnout factor, which was identical to that used in the OFHEO risk-based capital stress test model, took the value one if the mortgage note rate exceeds the market mortgage rate by 200 basis points or more in any two of the preceding eight quarters. Empirical evidence now suggests that borrowers who refinance tend to do so at much lower thresholds. The new burnout factor

measures the average of the number of basis points the borrower was in the money, for all quarters during which the borrower was in the money, anytime during the preceding 8 quarters. The resulting measure was categorized into smaller 50 basis point categories that provide a more refined measure of burnout.

Pre-1986 Origination

An indicator for loans originated prior to FY 1986 Q3 is included to account for improvements in FHA underwriting requirements.

Post-1995 Origination

An indicator for loans originated after FY 1995 is included to account for a loosening of FHA underwriting requirements.

Exposure Year/Quarter FRM Rate

A variable measuring the market average FRM mortgage rate is included to distinguish high-rate and low-rate market environments.

Changes in Metropolitan Area Unemployment Rates

For the FY 2005 Review we undertook to develop a measure of changes in metropolitan area unemployment rates. Data on metropolitan area unemployment rates were obtained from the Bureau of Labor Statistics and converted into times series from which we computed a dynamic measure for the percentage change in the unemployment rate over the preceding year.

The unemployment rate variables did not perform well in any of the preliminary models that were estimated, and are not included in the final model specifications. No consistent pattern was observed between mortgage claims and increases in local area unemployment rates, in contrast to the strong relationship between loan performance and borrower equity. This outcome is consistent with prior experience using this variable in loan-level models in which borrower behavior is more strongly linked to changes in the borrower's equity position or changes in the value of the mortgage instrument due to changes in interest rates. Changes in these variables have a direct impact on property and mortgage values, whereas the local area unemployment measure has a much weaker connection to individual borrowers.

ARM Payment Burden

Another new variable considered for the FY 2005 Review was the ARM payment burden. This variable measured the percentage change in the monthly payment since origination on ARM loans. The percentage change was categorized into 5 levels ranging from no increase to more than 30-percent increase.

The ARM payment burden variables did not perform well in the preliminary models that were estimated and were generally not statistically significant. This variable is highly collinear with the mortgage premium (spread) and burnout variables (for loans that do not prepay), particularly over the early years before there is substantial amortization of the loan balance. As a result, this variable contributes little to the explanation of loan performance once the other variables are included.

Source of Down Payment Assistance

The FHA single-family program has experienced a significant increase in the use of down payment assistance from relatives, non-profit organizations, and government programs in the past two years, and loans to borrowers utilizing down payment assistance have been observed to generate higher claim rates.

For the FY 2005 Review we included a series of indicators for the use of different types of down payment assistance.

Exhibit A-1

Logit Model Explanatory Variables							
Variable Name			Values				Description
Mortgage Age Function							
	FRM30	FRM15	ARM	FRM30_SR	FRM15_SR	ARM_SR	<p>Piece-wise linear age functions for ages up to specified knot points.</p> <p>Estimated parameters give the slope of the age function for each segment.</p> <p>Functions differ by mortgage product type as indicated.</p>
age1	2	2	2	2	2	2	
age2	4	4	4	4	4	4	
age3	8	8	8	8	8	8	
age4	12	12	12	12	12	12	
age5	16	16	16	> 12	16	16	
age6	20	> 16	20		20	20	
age7	24		24		24	24	
age8	28		28		> 24	> 24	
age9	32		32				
age10	40		40				
age11	60		> 40				
age12	80						
age13	> 80						
Loan Size							
loancat_cat_1			0 < X = 60				Relative loan size measured as percent difference from average size loan originated in same state in the same year.
loancat_cat_2			60 < X = 90				
loancat_cat_3			90 < X = 110				
loancat_cat_4			110 < X = 140				
loancat_cat_5			X > 140				
Loan-to-Value							
ltvcat_cat_1			0 < X = 80				Loan-to-value at origination. Missing or zero values replaced with update file provided by FHA. Additional missing values imputed as mean LTV by state, origination FY, and product type.
ltvcat_cat_2			80 < X = 90				
ltvcat_cat_3			90 < X = 95				
ltvcat_cat_4			95 < X = 97				
ltvcat_cat_5			97 < X = 98				
ltvcat_cat_6			98 < X = 100				
ltvcat_cat_7			X > 100				

(continued on following page)

Exhibit A-1

Logit Model Explanatory Variables		
Variable Name	Values	Description
Season		
season_cat_1	$X = 1$	Calendar quarter of mortgage origination.
season_cat_2	$X = 2$	
season_cat_3	$X = 3$	
season_cat_4	$X = 4$	
Probability of Negative Equity		
pneqcat_cat_1	$0.00 < X = 0.05$	Probability of negative equity. Based on OFHEO house price drift and volatility estimates. MSA-level estimates used for selected MSAs, otherwise, Census Division level estimates are used.
pneqcat_cat_2	$0.05 < X = 0.10$	
pneqcat_cat_3	$0.10 < X = 0.15$	
pneqcat_cat_4	$0.15 < X = 0.20$	
pneqcat_cat_5	$0.20 < X = 0.25$	
pneqcat_cat_6	$0.25 < X = 0.30$	
pneqcat_cat_7	$X > 0.30$	
Mortgage Premium (Spread)		
spreadcat_cat_1	$X = -30$	Mortgage premium value measured as difference between current coupon rate and average FRM market rate relative to current coupon rate.
spreadcat_cat_2	$-30 < X = -20$	
spreadcat_cat_3	$-20 < X = -10$	
spreadcat_cat_4	$-10 < X = 0$	
spreadcat_cat_5	$0 < X = 10$	
spreadcat_cat_6	$10 < X = 20$	
spreadcat_cat_7	$20 < X = 30$	
spreadcat_cat_8	$X > 30$	
Yield Curve Slope		
yslopecat_cat_1	$0.0 = X = 1.0$	Yield curve slope measured as ratio of 10-year CMT to 1-year CMT.
yslopecat_cat_2	$1.0 < X = 1.2$	
yslopecat_cat_3	$1.2 < X = 1.5$	
yslopecat_cat_4	$X > 1.5$	

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Exhibit A-1

Logit Model Explanatory Variables		
Variable Name	Values	Description
Burnout Factor		
		Burnout factor equal to the average number of basis points the prepayment option was in the money during those quarters the option was in the money during the preceding 8 quarters.
in_moneycat_cat_1	$X = 0$	
in_moneycat_cat_2	$0 < X = 50$	
in_moneycat_cat_3	$50 < X = 100$	
in_moneycat_cat_4	$100 < X = 150$	
in_moneycat_cat_5	$150 < X = 200$	
in_moneycat_cat_6	$X > 200$	
Pre-1986 Origination		
pre_fy86_cat_1	$X = 1986$	Post- or pre-FY1986 Q3 origination. Included to account for changes in FHA underwriting standards.
pre_fy86_cat_2	$X < 1986$	
Post-1995 Origination		
post_fy95_cat_1	$X = 1995$	Pre-or post-FY1995 origination. Included to account for changes in FHA underwriting standards.
post_fy95_cat_2	$X > 1995$	
Exposure Year/Quarter FRM Rate		
ey_ratecat_cat_1	$X = 6$	FRM average mortgage rate during exposure year and quarter. Included to distinguish high-rate and low-rate environments.
ey_ratecat_cat_2	$6 < X = 7$	
ey_ratecat_cat_3	$7 < X = 8$	
ey_ratecat_cat_4	$8 < X = 9$	
ey_ratecat_cat_5	$9 < X = 10$	
ey_ratecat_cat_6	$X > 10$	

(continued on following page)

Exhibit A-1

Logit Model Explanatory Variables		
Variable Name	Values	Description
Metropolitan Unemployment Rates		
uechngcat_1	$X = -30$	Percent change over the preceding year in the metro-area unemployment rate.
uechngcat_2	$-30 < X = -20$	
uechngcat_3	$-20 < X = -10$	
uechngcat_4	$-10 < X = 0$	
uechngcat_5	$0 < X = 10$	
uechngcat_6	$10 < X = 20$	
uechngcat_7	$20 < X = 30$	
uechngcat_8	$30 < X = 50$	
uechngcat_9	$50 < X = 100$	
uechngcat_10	$100 < X = 150$	
uechngcat_11	$X > 150$	
ARM Payment Burden		
arm_paymentcat_1	$X = 0$	Percent increase in monthly payment since origination.
arm_paymentcat_2	$0 < X = 10$	
arm_paymentcat_3	$10 < X = 20$	
arm_paymentcat_4	$0 < X = 30$	
arm_paymentcat_5	$X > 30$	
Source of Down Payment Assistance		
gift_ltr_src_cat_1	None Recorded	Source of down payment assistance.
gift_ltr_src_cat_2	Relatives	
gift_ltr_src_cat_3	Non-Profit	
gift_ltr_src_cat_4	Government	
gift_ltr_src_cat_5	Other	

III. Model Estimation Results

Exhibit A-2 and A-3 present the coefficient estimates for the binomial logit models for conditional claim and prepayment probabilities.

Exhibit A-2							
Results for Conditional Claim Rate Model Estimation							
Variable	FRM 30	FRM 15	ARM	SR FRM 30	SR FRM 15	SR ARM	
loancat_cat_2	-0.016732	-0.160244	-0.072824	0.273969	-0.008372	0.084128	*
loancat_cat_3	-0.128082	-0.307136	-0.270335	0.441777	-0.320789	0.249221	
loancat_cat_4	-0.197559	-0.752877	-0.340092	0.470568	-0.546842	0.260696	
loancat_cat_5	-0.249766	-0.490569	-0.423801	0.365607	-0.781124	0.066382	*
ltvcat_cat_2	0.469594	0.747209	0.559814				
ltvcat_cat_3	0.334264	0.902079	0.591527				
ltvcat_cat_4	0.371866	1.188054	0.591202				
ltvcat_cat_5	0.244715	0.947665	0.453571				
ltvcat_cat_6	0.272767	0.700217	0.866739				
ltvcat_cat_7	0.382445	0.581725 *	0.647210				
season_cat_2	-0.045690	-0.135457	-0.086338	-0.021745	0.201557	0.030289	*
season_cat_3	-0.018125	-0.187107	-0.053147	-0.012551 *	0.218215	-0.207372	
season_cat_4	0.000286 *	-0.172602	-0.091357	-0.005937 *	-0.251634	0.076085	
pneqcat_cat_2	0.477840	0.639057	0.355824	0.641124	0.970934	0.558920	
pneqcat_cat_3	0.590987	0.883998	0.443204	1.021999	0.928073	0.990068	
pneqcat_cat_4	0.764703	1.076217	0.622707	1.248694	0.431279	1.488170	
pneqcat_cat_5	0.893551	0.732395	0.826702	1.444939	1.376952	1.575700	
pneqcat_cat_6	1.062103	1.552369	1.010598	1.586848	1.592882	2.044242	
pneqcat_cat_7	1.400141	1.844753	1.609487	2.399955	2.191905	2.762291	
yslopecat_cat_2	-0.081311	-0.126868	-0.147397	-0.426936	0.018613 *	-0.020076 *	*
yslopecat_cat_3	-0.011497	-0.034818 *	-0.189458	-0.273201	0.225656	0.075550 *	*
yslopecat_cat_4	-0.066557	-0.142677	-0.187546	-0.148120	0.144324 *	-0.013022 *	*
spreadcat_cat_2	0.479155	0.340711	0.197414	-0.399250		0.097840 *	*
spreadcat_cat_3	0.657314	-0.062450 *	0.320994	-0.255871		0.169858 *	*
spreadcat_cat_4	0.887306	0.215390	0.216201	0.169508		-0.059099 *	*
spreadcat_cat_5	0.967931	0.266151	0.228617	0.406086		-0.079644 *	*
spreadcat_cat_6	1.088967	0.391725	0.317799	0.708497		-0.079644 *	*
spreadcat_cat_7	1.280974	0.379263	0.317799	0.891223		-0.079644 *	*
spreadcat_cat_8	1.442082	0.582245		1.021441			
inmoneycat_cat_2	-0.046575	-0.128041	0.495487	-0.213929	0.473322	0.544196	
inmoneycat_cat_3	0.142659	-0.013059 *	0.630699	0.019480 *	0.659423	0.480549	
inmoneycat_cat_4	0.393570	0.202241	1.188455	0.253725	1.422897	0.480549	
inmoneycat_cat_5	0.618083	0.504391	1.630496	0.420967	1.340174	0.480549	
inmoneycat_cat_6	0.837551	0.575533	1.630496	0.594618	1.855513	0.480549	

(continued on following page)

Exhibit A-2						
Results for Conditional Claim Rate Model Estimation						
Variable	FRM 30	FRM 15	ARM	SR FRM 30	SR FRM 15	SR ARM
gift_ltr_src_cat_2	0.343228	0.400345	0.342958			
gift_ltr_src_cat_3	1.007345	1.366716	0.906482			
gift_ltr_src_cat_4	0.662357	2.460486	0.921824			
gift_ltr_src_cat_5	0.726536					
pre_fy86_cat_2	0.699994	0.861123				
post_fy95_cat_2	0.467352	0.316874	0.679368	0.562291	-0.046727 *	1.119083
ey_ratecat_cat_2			0.075879 *			-0.134383
ey_ratecat_cat_3			-0.151703			-0.615481
ey_ratecat_cat_4			-0.365531			-0.777643
ey_ratecat_cat_5			-0.194312			-1.314773
ey_ratecat_cat_6			0.014893 *			-0.675084
age1	1.611829	0.620501	2.492865	1.182654	0.555414	1.762337
age2	0.707664	1.016088	1.041914	0.838917	0.487529	0.854395
age3	0.196007	0.217370	0.257009	0.159520	0.358573	0.257120
age4	0.026338	0.048903	0.105508	0.070478	0.056284	0.104720
age5	0.005000	0.035729	0.008198	-0.025621	0.079832	-0.017089
age6	-0.014888	-0.055026	-0.029256		-0.042060	
age7	-0.043340		0.024139		-0.094686	
age8	-0.031681		-0.057743		-0.105049	
age9	-0.035792		-0.037102			
age10	-0.013896		-0.058138			
age11	-0.045270		-0.025721			
age12	-0.063514					
age13	-0.057111					
_cons	-12.781940	-11.622790	-14.716460	-12.000470	-11.846360	-13.101790
Statistics	FRM 30	FRM 15	ARM	SR FRM 30	SR FRM 15	SR ARM
Log likelihood	-6487906.6	-74540.766	-506651.19	-486983.98	-31438.213	-46818.861
Number of obs	297610690	8403630	19682180	34420250	9547900	2381300
LR χ^2	1053509.12	12291.26	81949.02	78761.55	3909.78	10043.75
Prob > χ^2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

* Not significant for 0.05-level asymptotic normal test

Exhibit A-3								
Results for Conditional Prepay Rate Model Estimation								
Variable	FRM 30	FRM 15	ARM	SR FRM 30	SR FRM 15	SR ARM		
loancat_cat_2	0.370862	0.256887	0.277454	0.321970	0.132616	0.317468		
loancat_cat_3	0.644319	0.399674	0.477953	0.543630	0.225987	0.544409		
loancat_cat_4	0.823667	0.513876	0.624842	0.671676	0.261625	0.620688		
loancat_cat_5	0.965692	0.599617	0.686423	0.774034	0.389178	0.816958		
ltvcat_cat_2	-0.167545	-0.020742	-0.093570					
ltvcat_cat_3	-0.116537	-0.030026	-0.021562					
ltvcat_cat_4	-0.019245	0.034244	0.081163					
ltvcat_cat_5	0.044646	0.106757	0.082819					
ltvcat_cat_6	-0.130374	-0.020729	*					
ltvcat_cat_7	0.060673	0.094637	*					
season_cat_2	-0.041567	-0.066184	0.011142	0.022039	0.022357	0.106471		
season_cat_3	0.011018	-0.045468	0.056560	0.056933	-0.081255	-0.017400		
season_cat_4	-0.149179	-0.182362	-0.085513	-0.138809	-0.218921	-0.092460		
pneqcat_cat_2	-0.187764	-0.244514	-0.271937	-0.248319	-0.215832	-0.188917		
pneqcat_cat_3	-0.237237	-0.445660	-0.409251	-0.332121	-0.391674	-0.352293		
pneqcat_cat_4	-0.307093	-0.606166	-0.537343	-0.438045	-0.574717	-0.420086		
pneqcat_cat_5	-0.459252	-0.524166	-0.631915	-0.650952	-0.408219	-0.795630		
pneqcat_cat_6	-0.594571	-0.691834	-0.820448	-0.801052	-0.783135	-0.795086		
pneqcat_cat_7	-0.638595	-0.834639	-1.081210	-1.107349	-0.789614	-1.302474		
ycslopecat_cat_2	-0.039655	-0.060205	-0.478498	-0.087066	0.224313	-0.212156		
ycslopecat_cat_3	-0.077359	-0.102052	-0.376750	-0.297373	0.022689	*	-0.286571	
ycslopecat_cat_4	0.491302	0.331956	-0.526625	0.543495	0.722667		-0.290775	
spreadcat_cat_2	0.686341	0.057182	*	0.293635	-0.657686		0.315479	
spreadcat_cat_3	0.641292	0.442555		0.518244	-0.662849		0.529468	
spreadcat_cat_4	0.777440	0.723854		0.786113	-0.386373		0.770454	
spreadcat_cat_5	1.260226	1.064783		1.115632	0.049462		1.027561	
spreadcat_cat_6	1.871985	1.393165		1.231015	0.539379		1.243956	
spreadcat_cat_7	2.098209	1.523365		1.231015	0.755196		1.243956	
spreadcat_cat_8	2.094521	1.514091		0.821478				
inmoneycat_cat_2	0.140066	0.116783	0.012252	0.326640	0.433841		-0.169788	
inmoneycat_cat_3	0.233292	0.118487	-0.193520	0.380531	0.514983		-0.299316	
inmoneycat_cat_4	0.287868	0.142322	-0.163184	*	0.334196	0.600402	0.020028	*
inmoneycat_cat_5	0.176030	-0.007592	*	0.456368	0.241120	0.543969	0.020028	*
inmoneycat_cat_6	-0.008856	-0.193809		0.456368	0.112217	0.466013	0.020028	*
gift_ltr_src_cat_2	0.026838	-0.027810	*	-0.062109				
gift_ltr_src_cat_3	0.032066	0.698979		-0.045678				
gift_ltr_src_cat_4	-0.225389	-0.075738	*	-0.059354				
gift_ltr_src_cat_5	0.094901	*		0.452919	*			
pre_fy86_cat_2	0.145497	0.027091						

(continued on following page)

Exhibit A-3						
Results for Conditional Prepay Rate Model Estimation						
Variable	FRM 30	FRM 15	ARM	SR FRM 30	SR FRM 15	SR ARM
post_fy95_cat_2	0.357813	0.322949	0.340555	0.479892	0.271979	0.6122433
ey_ratecat_cat_2			-0.329457			-0.2454614
ey_ratecat_cat_3			-0.602031			-0.6117676
ey_ratecat_cat_4			-1.067716			-0.9149861
ey_ratecat_cat_5			-1.622924			-1.045389
ey_ratecat_cat_6			-2.452726			-3.024939
age1	0.559038	0.523853	0.847535	0.249767	0.439417	0.5152128
age2	0.233554	0.331733	0.285669	0.069065	0.135992	0.0963906
age3	0.036981	0.060857	0.034597	-0.034810	0.040098	-0.0369361
age4	0.019908	0.043802	-0.018869	0.010536	-0.011580	-0.0299088
age5	-0.008118	-0.048640	-0.037177	-0.009297	0.063282	-0.0004549 *
age6	-0.028365	0.005899	-0.036166		0.034037	
age7	0.003968		0.000685 *		-0.057383	
age8	-0.000668 *		0.007282		0.010947	
age9	-0.001789		0.012696			
age10	-0.001749		-0.001236 *			
age11	-0.024703		-0.026716			
age12	0.002702					
age13	0.000637					
_cons	-7.235682	-6.812974	-5.259501	-4.657524	-5.676237	-4.277679
Statistics	FRM 30	FRM 15	ARM	SR FRM 30	SR FRM 15	SR ARM
Log likelihood	-38554779	-1105290.8	-3549272.2	-6323305.4	-1273136.1	-509327.44
Number of obs	301615260	8607600	20139030	35759420	9821870	2481800
LR χ^2	7546626.41	120436.49	518527.82	1016237.49	95378.97	56856.57
Prob > χ^2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

* Not significant for 0.05-level asymptotic normal test

IV. Graphical Comparisons of Goodness-of-Fit by Age of Loan

Exhibits A-4 to A-15 present within-sample comparisons of the overall goodness-of-fit of the binomial logit models for claim and prepayment probabilities. Separate comparisons are given for each of the six mortgage product types. The graphs compare the average value by mortgage age of the observed and predicted conditional claim and prepayment probabilities. The large fluctuations in the observed probabilities at higher values of mortgage age are the result of sampling variation due to the small numbers of surviving loans.

Exhibit A-4

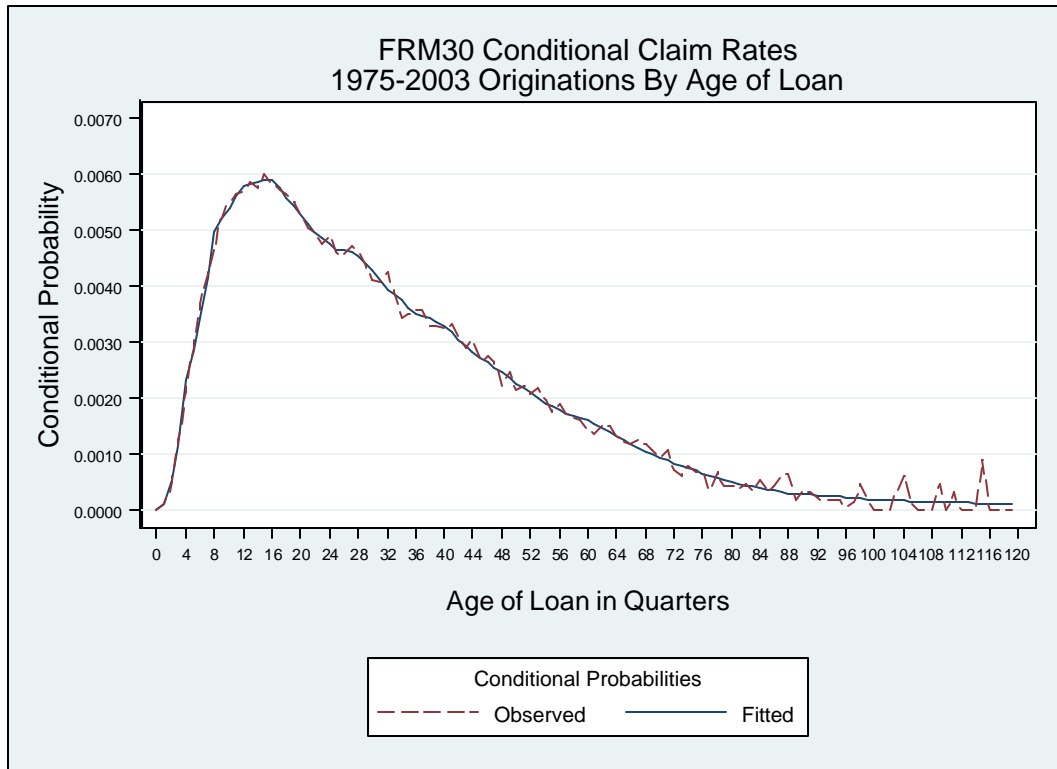


Exhibit A-5

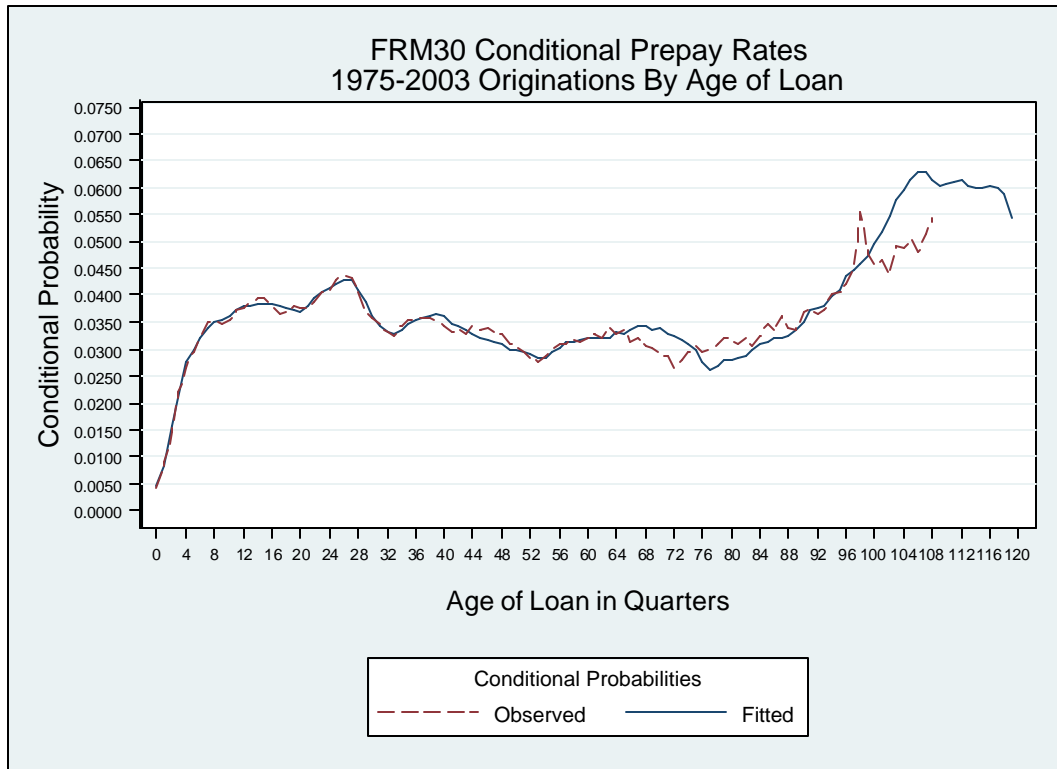


Exhibit A-6

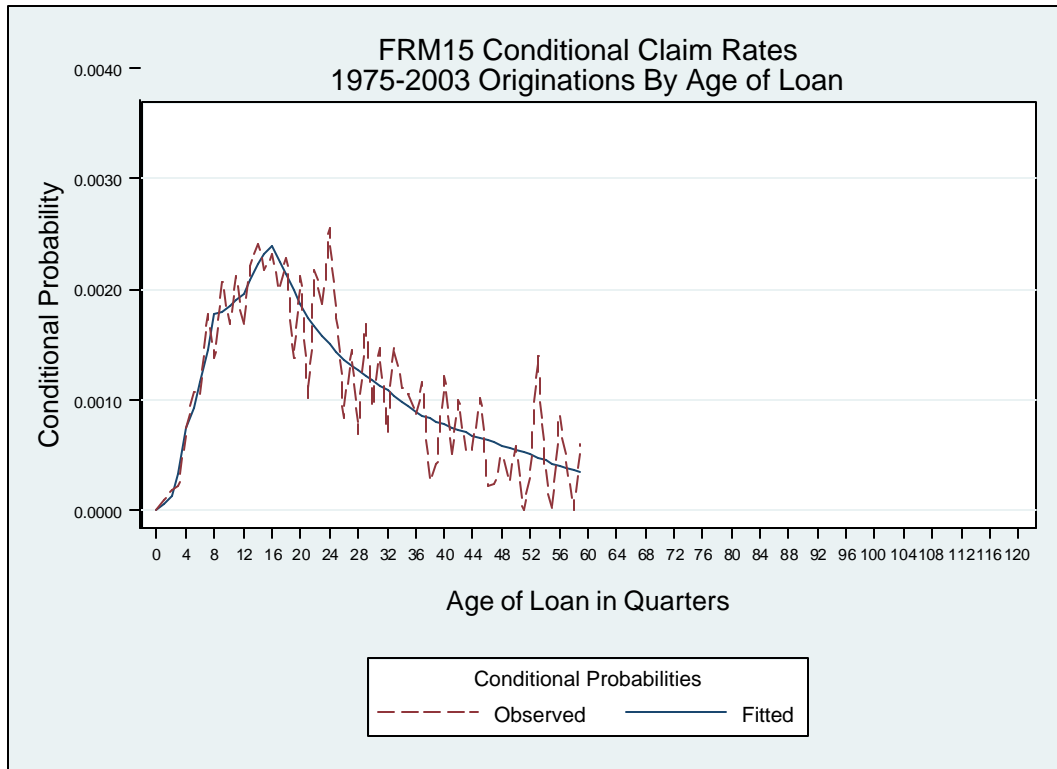


Exhibit A-7

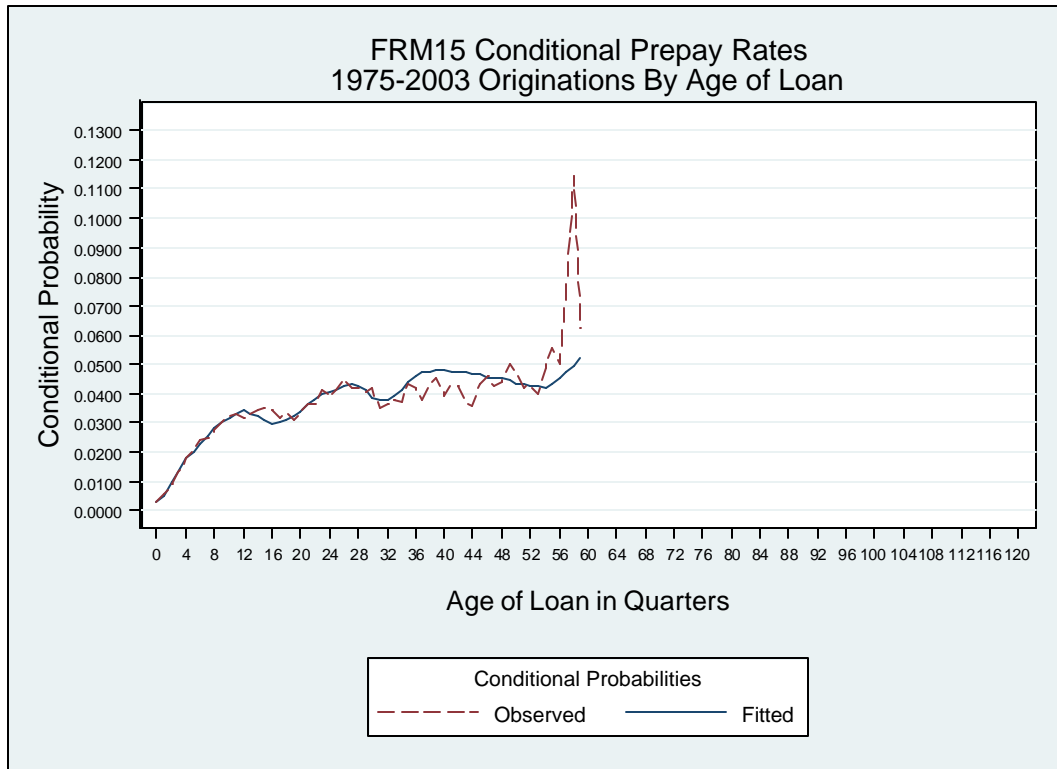


Exhibit A-8

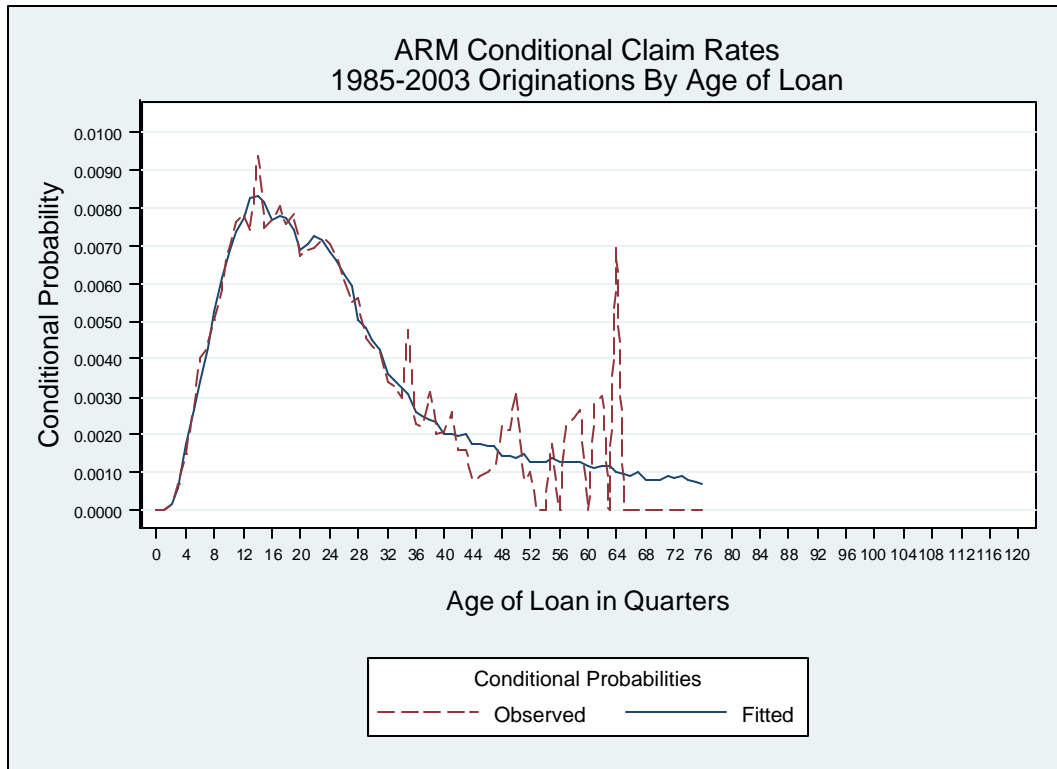


Exhibit A-9

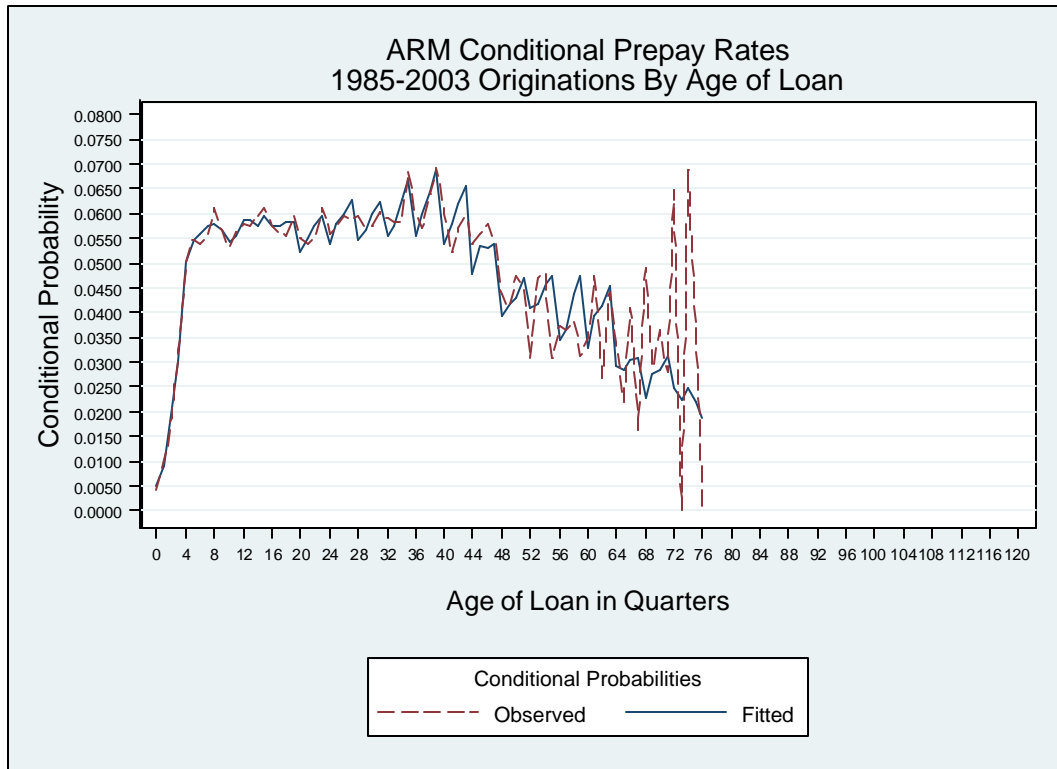


Exhibit A-10

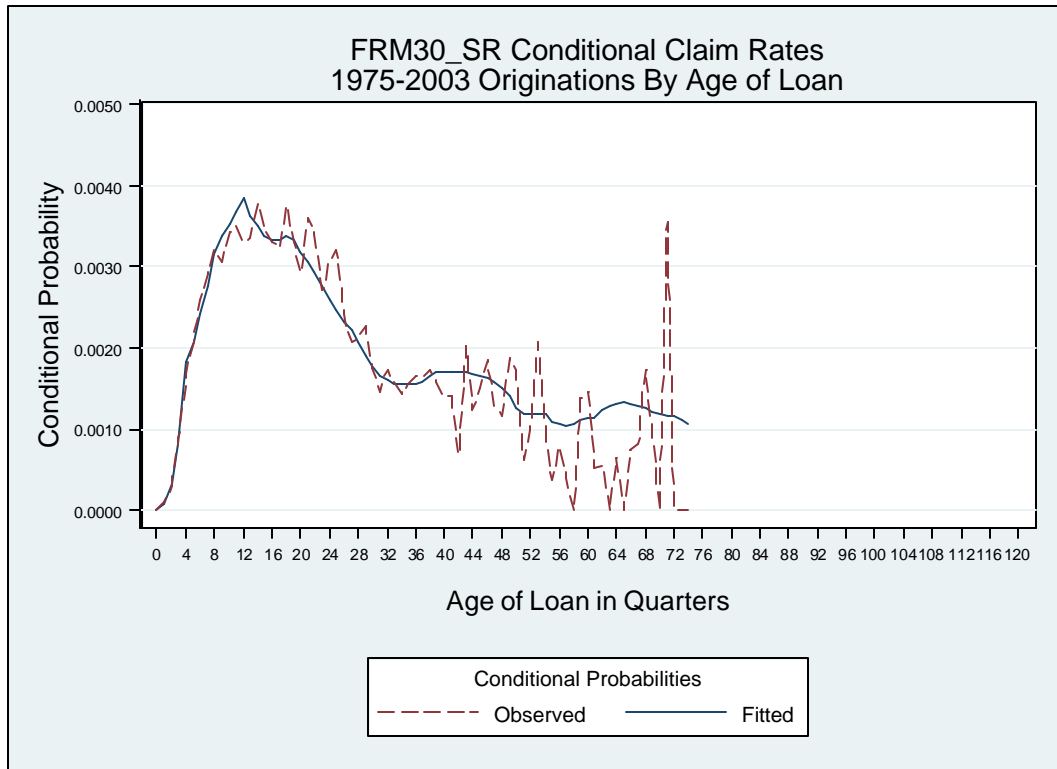


Exhibit A-11

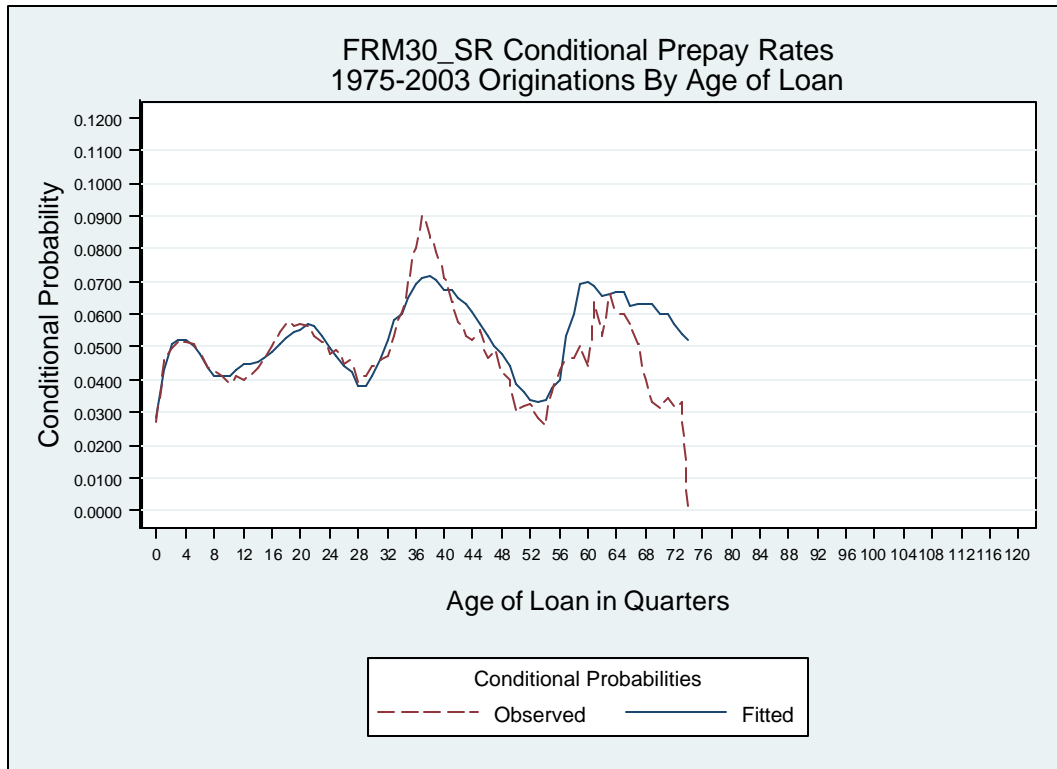


Exhibit A-12

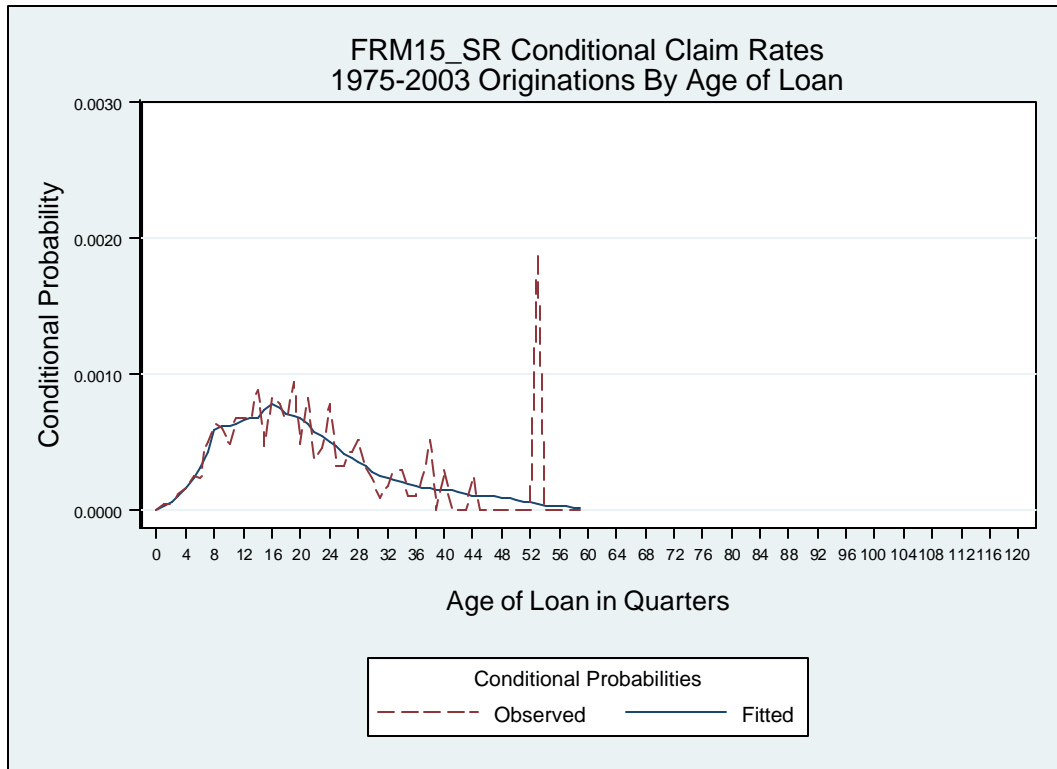


Exhibit A-13

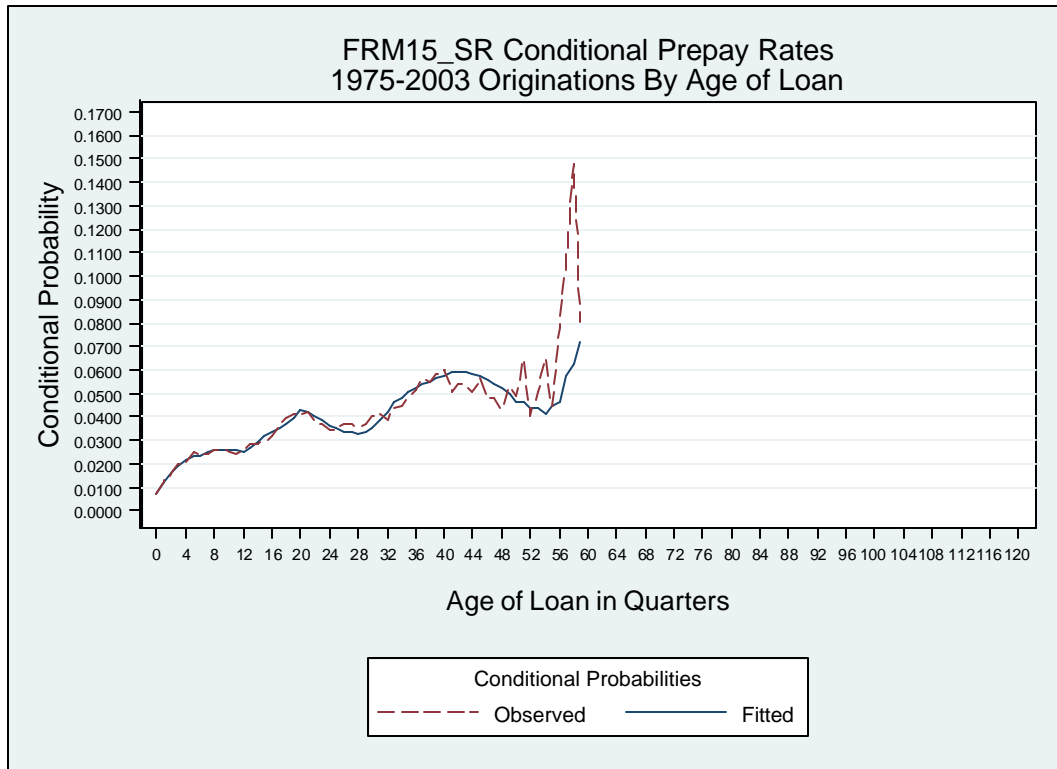


Exhibit A-14

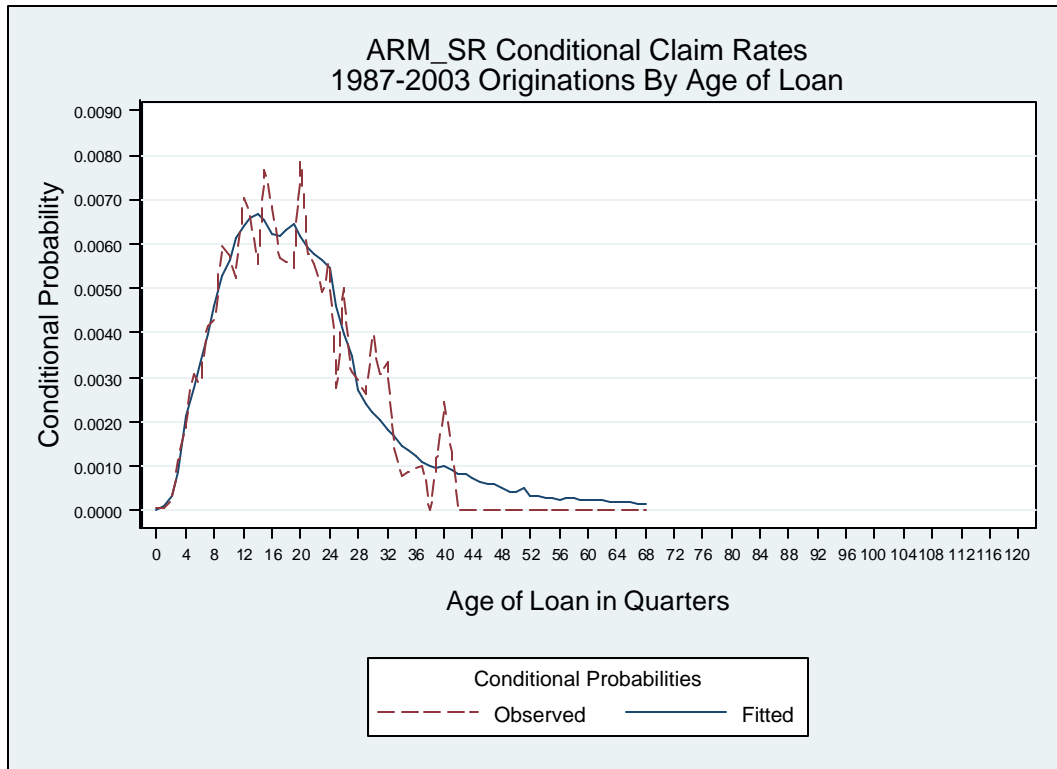
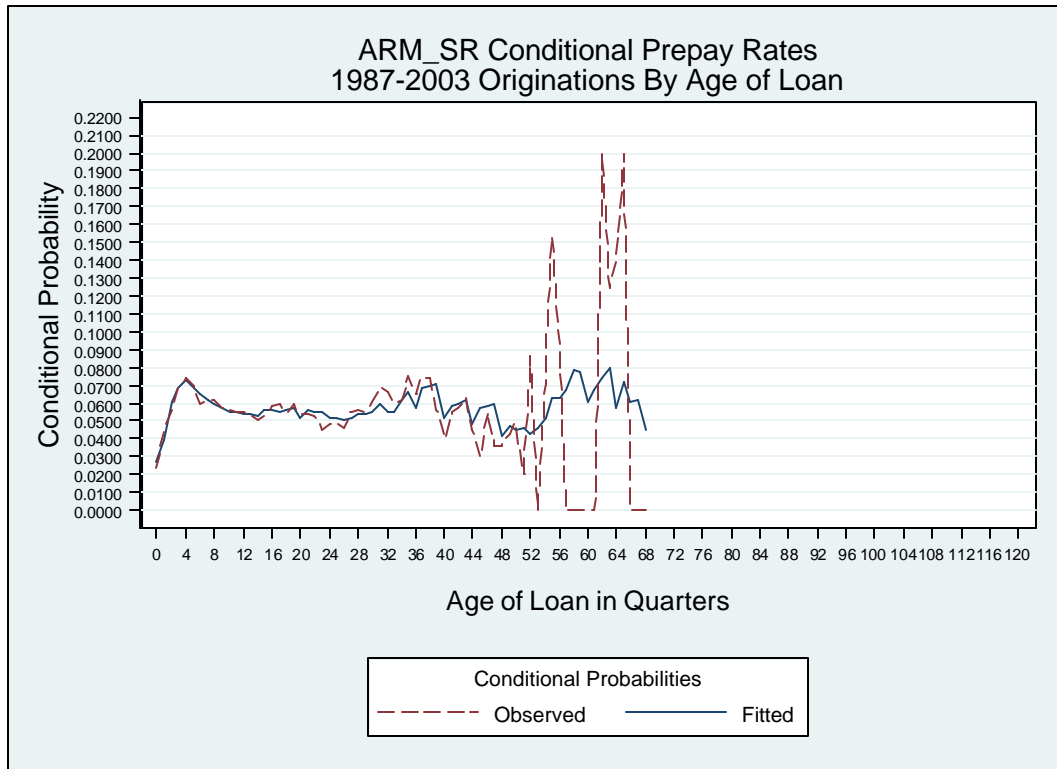


Exhibit A-15



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