# Estimating and Modeling S-Band Lunar Radar Backscatter

## Chandrayaan Mini-RF Team Meeting

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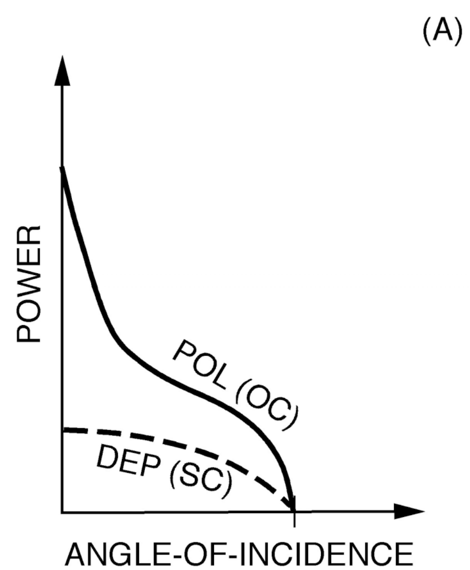
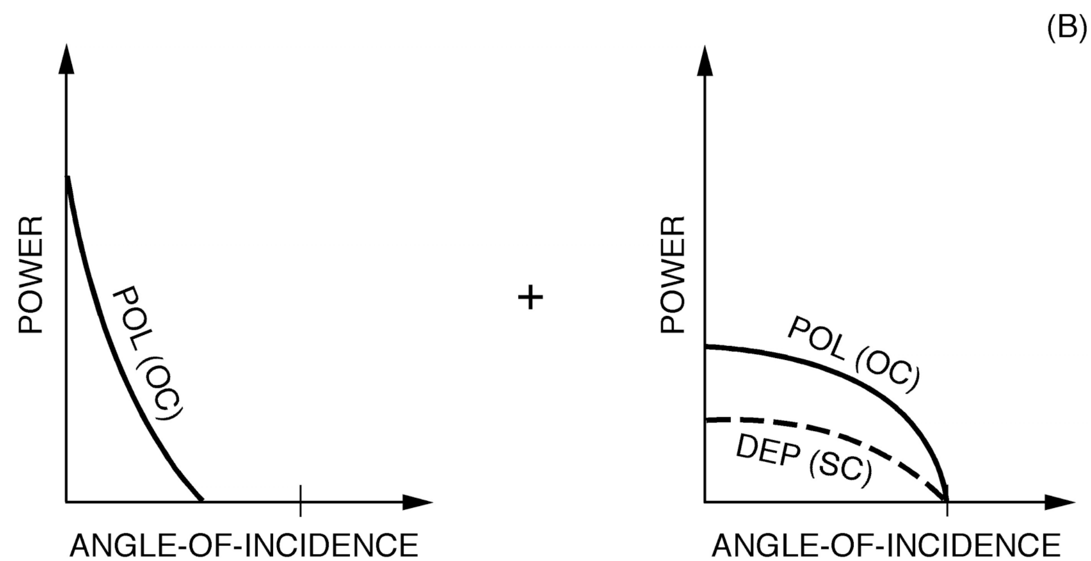
# AVERAGE LUNAR RADAR BEHAVIOR

| OBSERVATIONS | INFERRED SCATTERING MECHANISMS |
|---|---|
| POLARIZED (OC) + DEPOLARIZED (SC) | QUASI-SPECULAR + DIFFUSE |



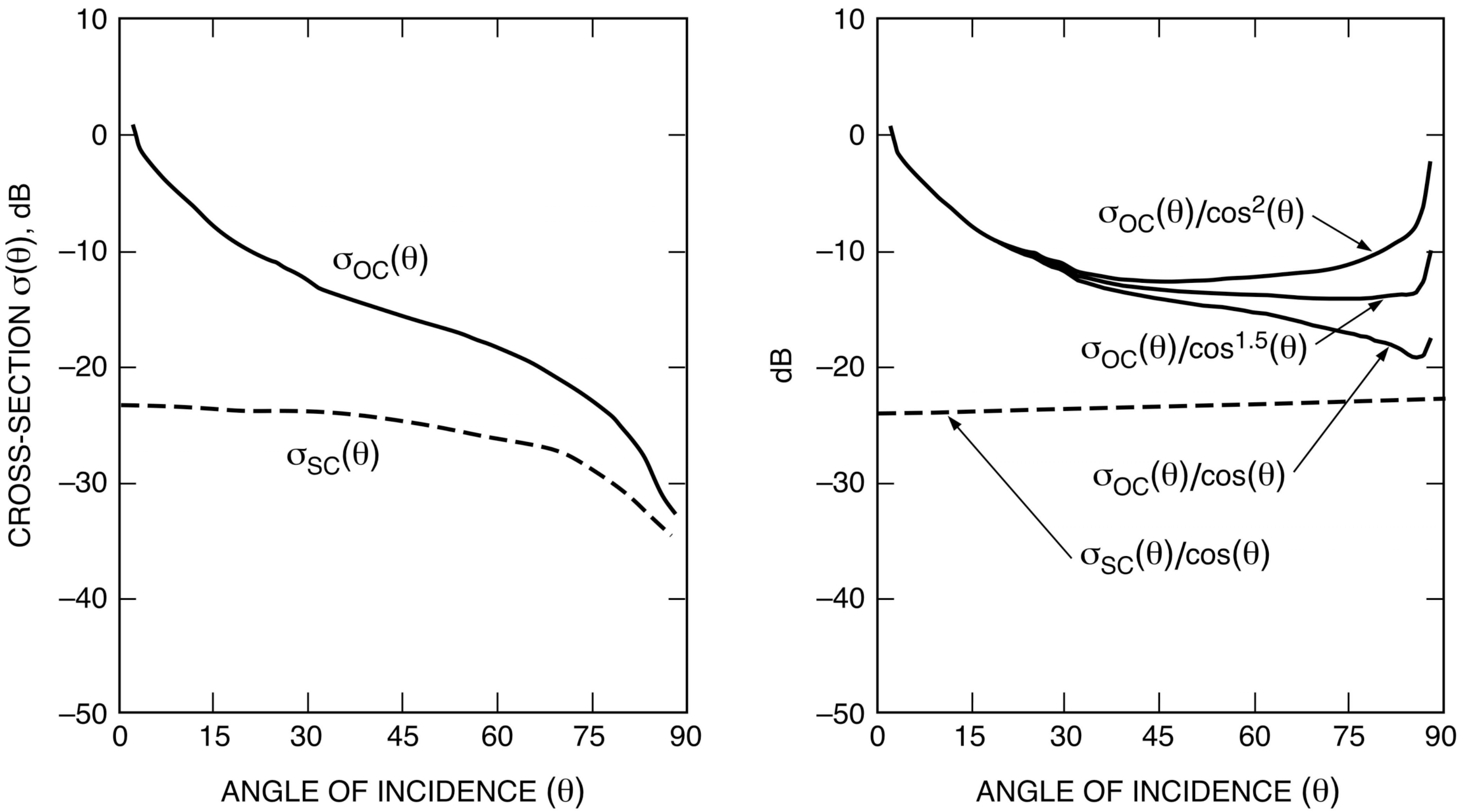




( Evans and Hagfors, 1964)

**OC = Opposite-Sense Circular (Mirror Reflection)**
**SC = Same-Sense Circular**

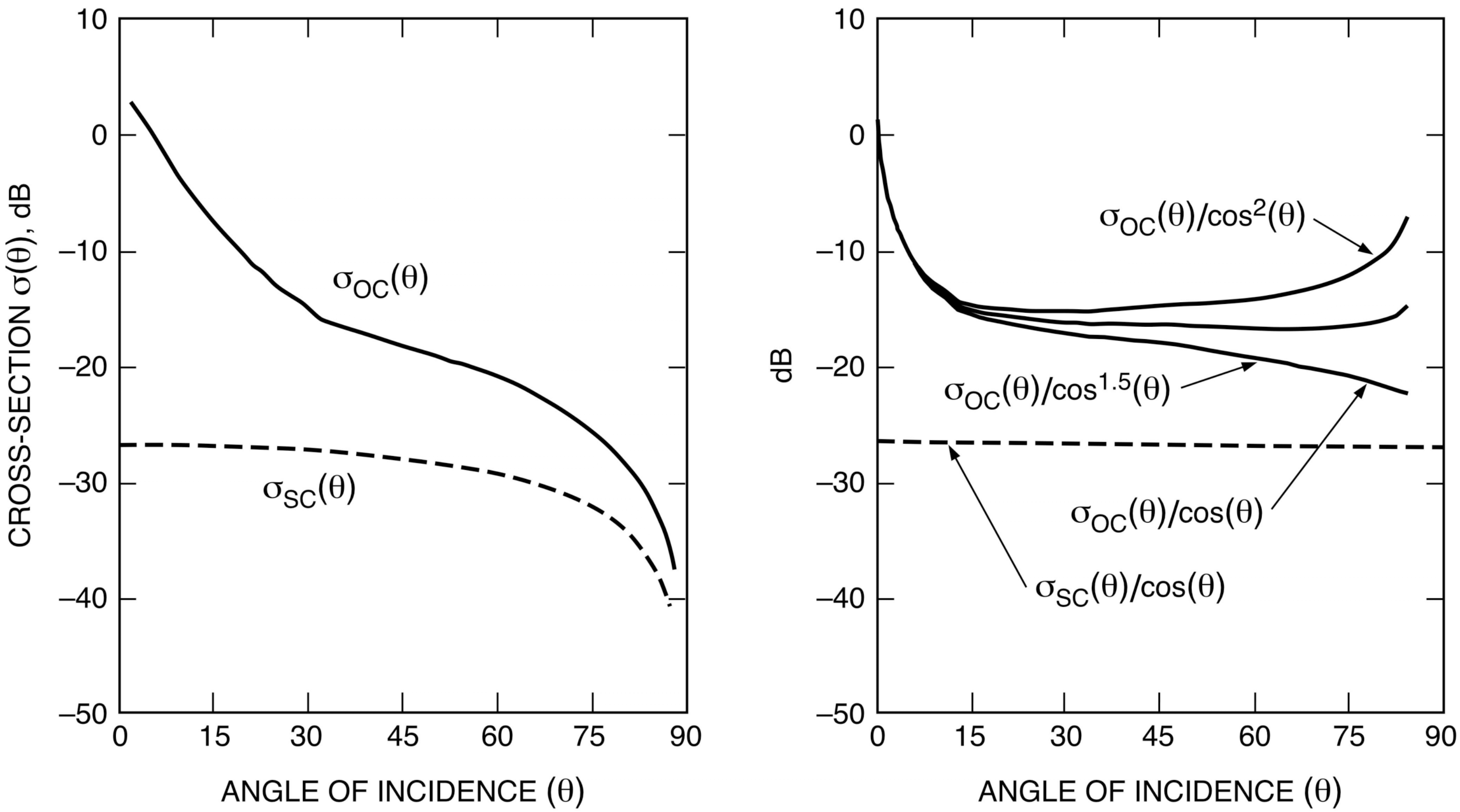
**Quasi-Specular – from flat facets**
**Diffuse – from wavelength-sized roughness**

# AVERAGE LUNAR RADAR CROSS-SECTION AT 3.8-cm WAVELENGTH (Hagfors, 1970)



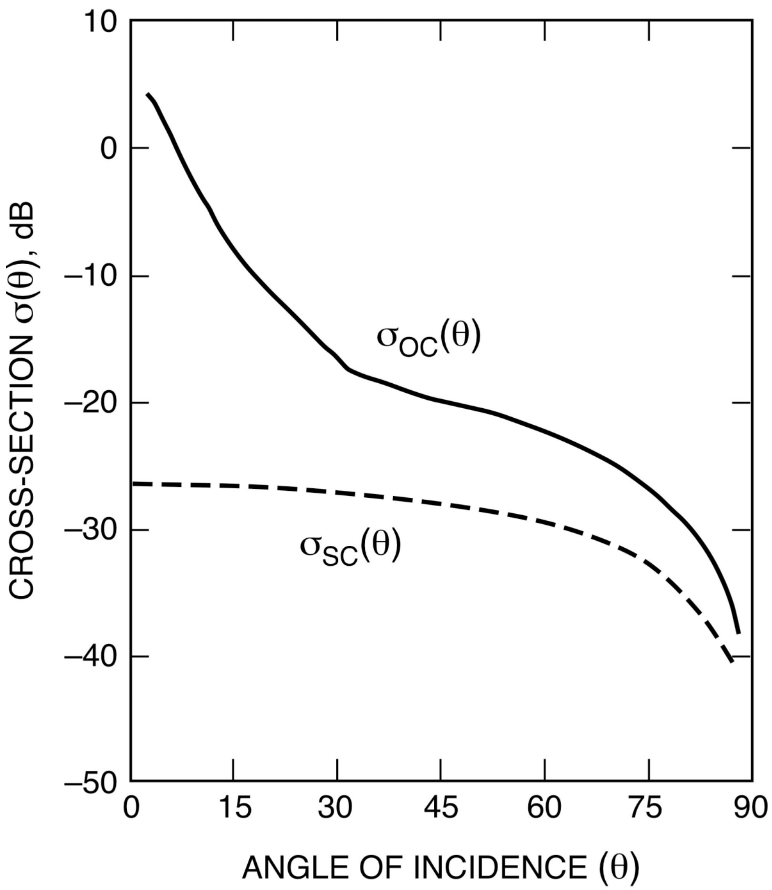

**OC = Opposite-Sense Circular (Mirror Reflection)**
**SC = Same-Sense Circular**

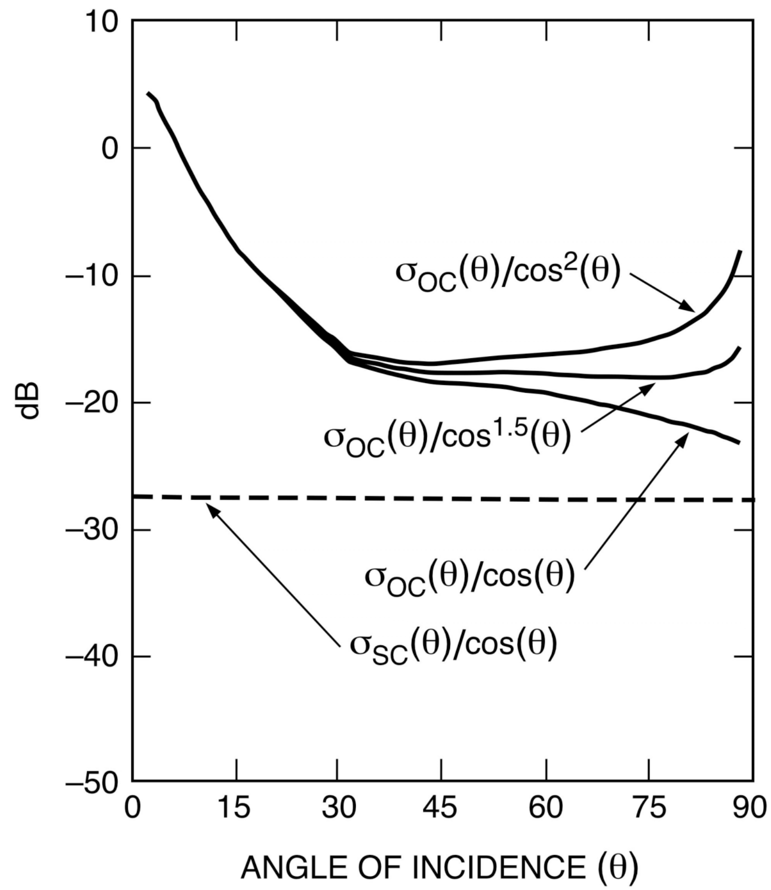
# AVERAGE LUNAR RADAR CROSS-SECTION AT 23-cm WAVELENTGH (Hagfors, 1970)




**OC = Opposite-Sense Circular (Mirror Reflection)**
**SC = Same-Sense Circular**

# AVERAGE LUNAR RADAR CROSS-SECTION AT 68-cm WAVELENTGH (Hagfors, 1970)




**OC = Opposite-Sense Circular (Mirror Reflection)**
**SC = Same-Sense Circular**

# Equations for estimating 13 cm scattering from 3.8 and 23 cm data

**(A) Estimating lunar backscatter at S-Band (13 cm) wavelength from 3.8 and 23 cm**

$$\sigma_{dB}^{(13)} = 0.32\sigma_{dB}^{(3.8)} + 0.68\sigma_{dB}^{(23)}$$

where $0.32 = \ln(23/13)/\ln(23/3.8)$, $0.68 = \ln(13/3.8)/\ln(23/3.8)$.

**(B) Mathematical models for lunar backscatter at S-Band (13 cm) wavelength**

$$\sigma(\Theta) = \sigma_{OCQ}(\Theta) + \sigma_{OCD}(\Theta) + \sigma_{SCD}(\Theta)$$

where:

$\sigma_{OCQ}(\Theta)$ is the quasi-specular component in the OC polarization,

$\sigma_{OCD}(\Theta)$ is the diffuse component in the OC polarization, proportional to $\cos^{1.5}\Theta$

$\sigma_{SCD}(\Theta)$ is the diffuse component in the SC polarization, proportional to $\cos\Theta$

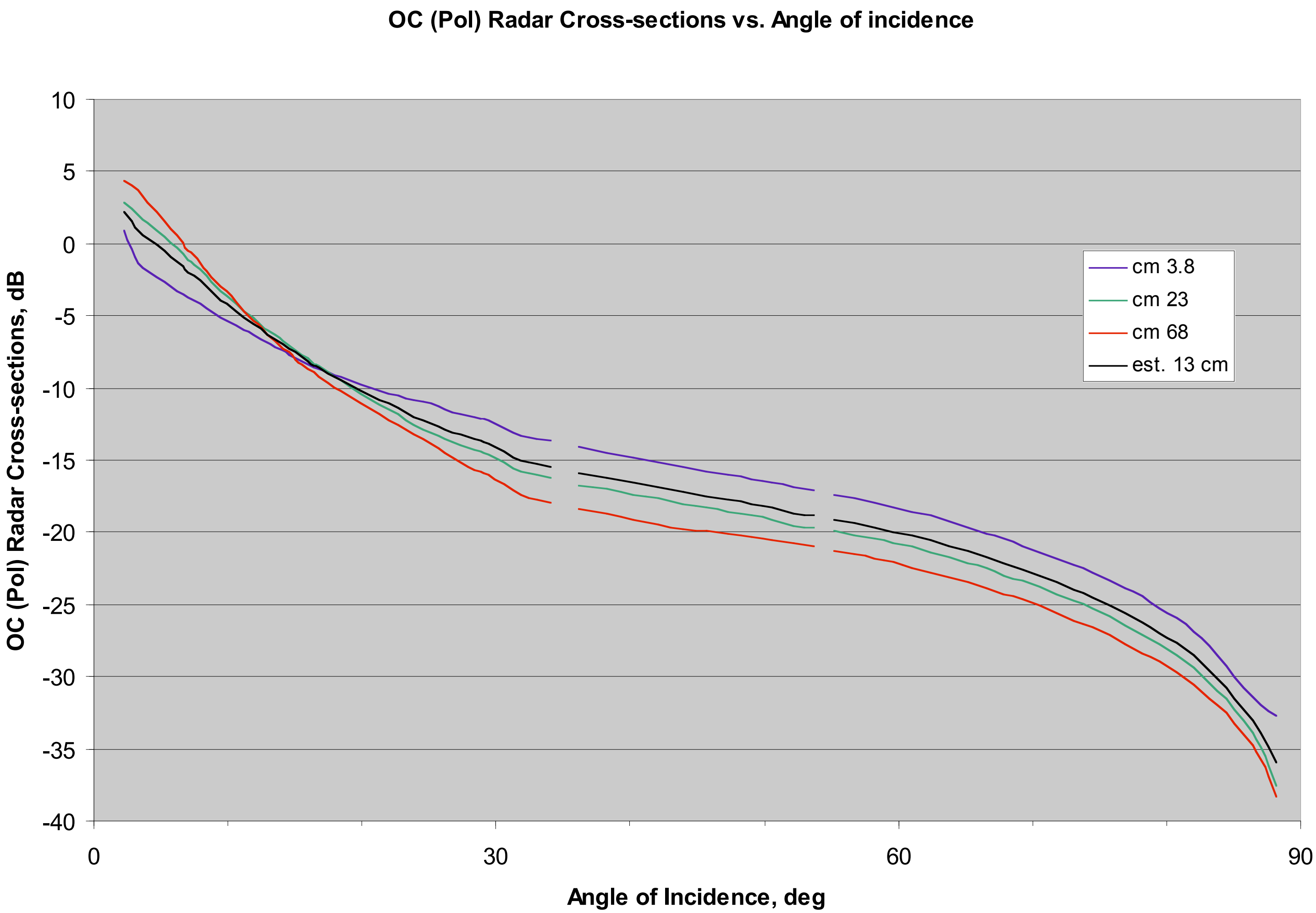
We have found that the $\sigma_{OCQ}(\Theta)$ in dB is modeled by the function $0.25 - 0.07\Theta$.

The levels for $\sigma_{OCD}(\Theta)$ and $\sigma_{SCD}(\Theta)$ come from 3.8 and 23 cm observations

Thus

$$\sigma_{avg}(\Theta) = (0.25 - 0.07\Theta) - 14\log_{10}(\cos^{1.5}\Theta) + 24.5\log_{10}(\cos\Theta)$$

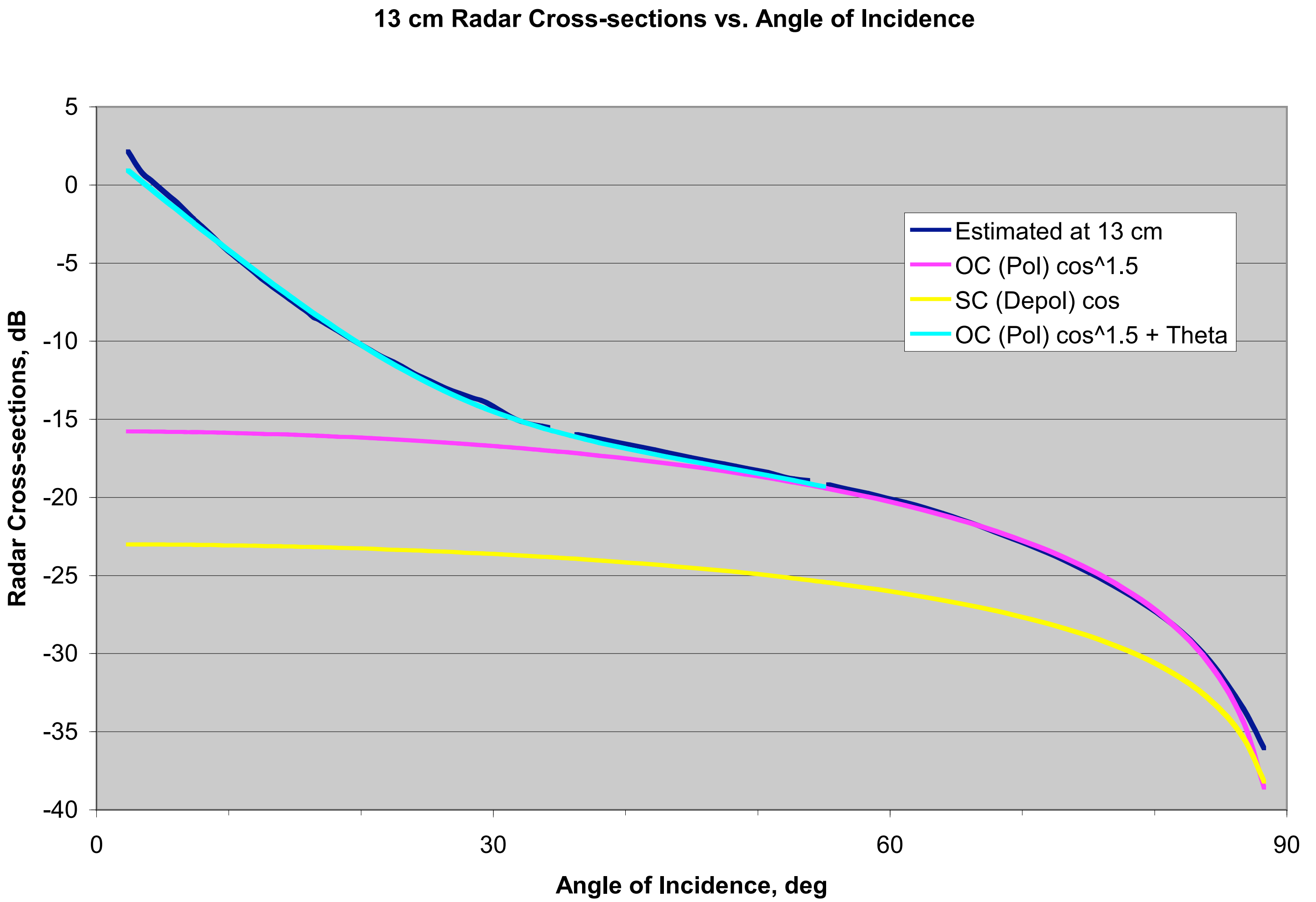
# Inferred 13-cm Average Scattering -- Interpolation from 1960's Observations (MIT)




( Hagfors, 1970)

**OC = Opposite-Sense Circular (Mirror reflection)**

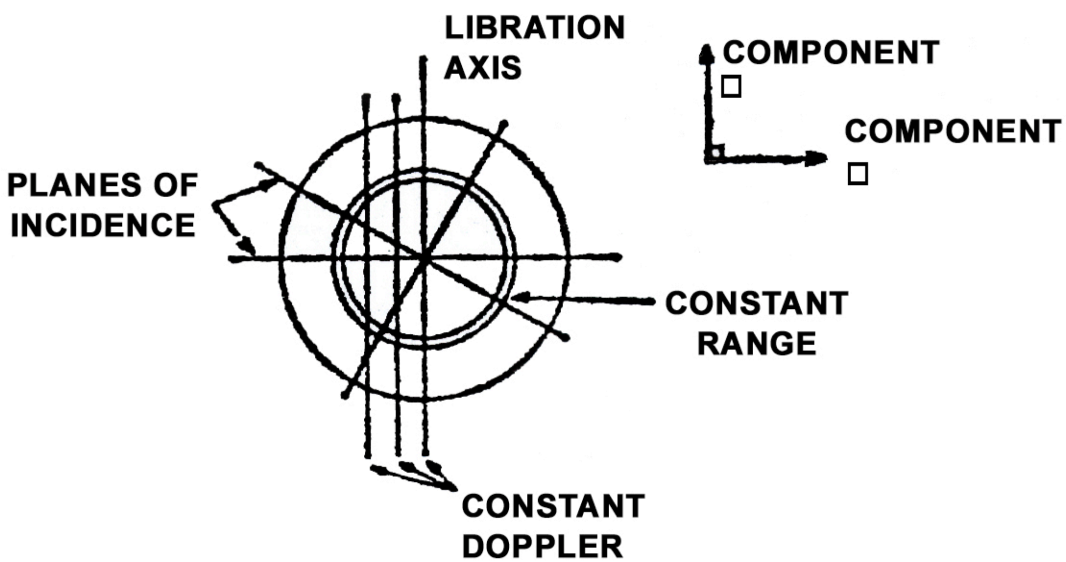
# Modeled 13-cm Average Scattering




**OC = Opposite-Sense Circular (Mirror Reflection)**
**SC = Same-Sense Circular**

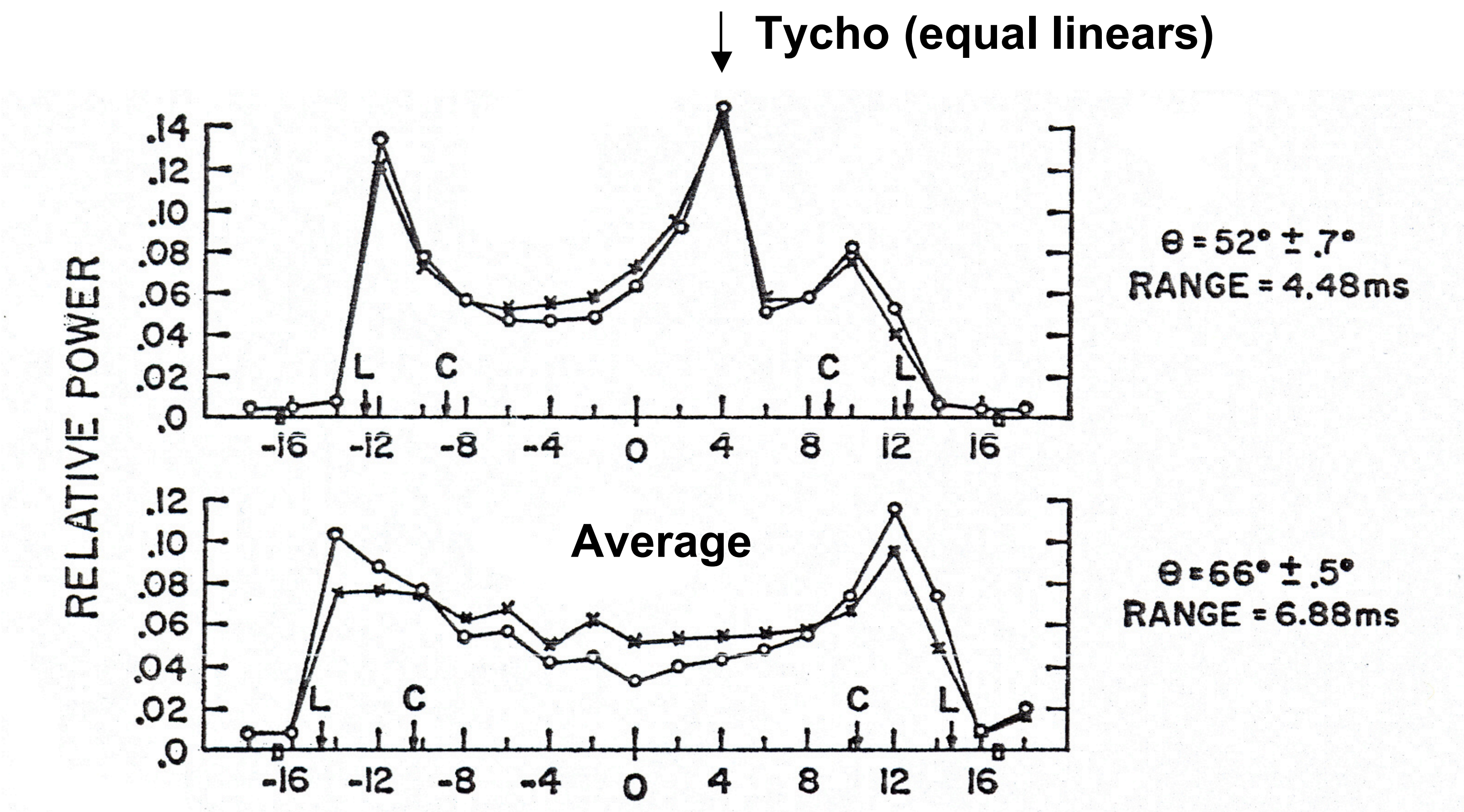
# Differences in Horizontal and Vertical Linears -- (Hagfors, et al., 1965)

**Transmit Circular, Receive cross-fed linear polarizations at 23 cm wavelength**



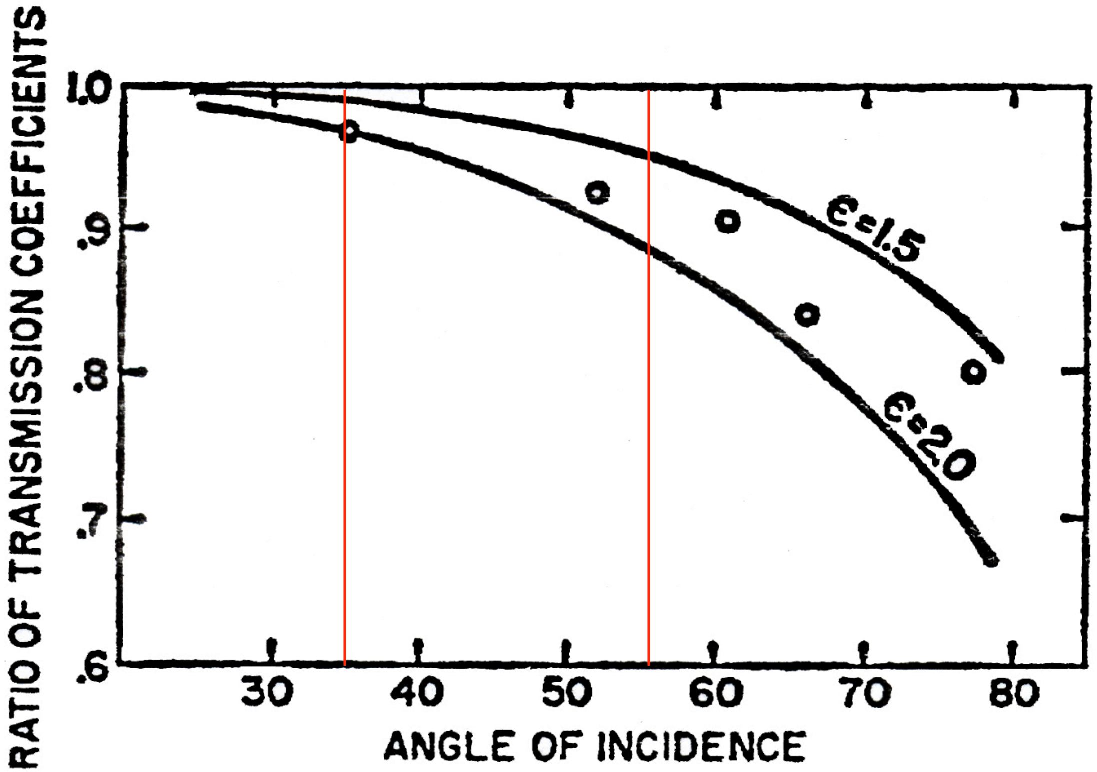

**Schematic view of the lunar disk with range-doppler coordinates and with the two received linearly polarized components**

# Differences in Horizontal and Vertical Linears -- (Hagfors, et al., 1965)




**Normalized frequency spectra of the Moon (C = cross-over orientation / L = limb)**
**Curves: -- x--, *E*-field aligned with libration axis; --o--, E-field normal to libration axis**

# Differences in Horizontal and Vertical Linears -- (Hagfors, et al., 1965)

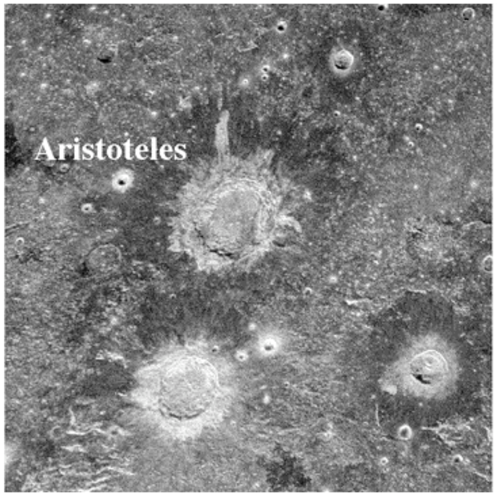



**Ratio of transmission coefficients derived from data as compared with theoretical predictions – Therefore we expect differences of less than 10% for Chandrayaan Mini-RF (S-band, 35 to 55 degrees)**

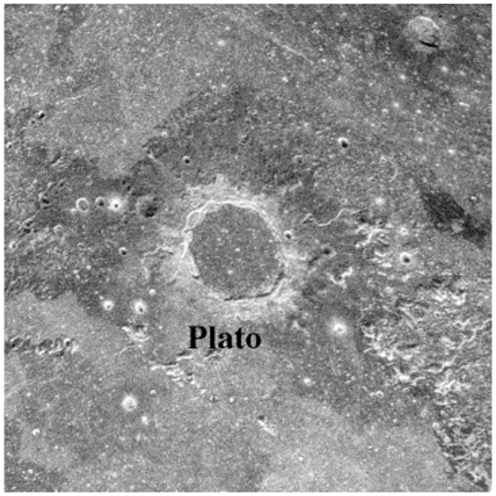
# Lunar Radar Image at 70 cm
# Examples of Slopes and Roughness Effects

| Aristoteles<br>Diameter - 88 km<br>Eratosthenian<br>(Young) | Plato<br>Diameter - 109 km<br>Upper Imbrian<br>(Middle Age ) | Maurolycus<br>Diameter - 114 km<br>Nectarian<br>(Old) |
|---|---|---|

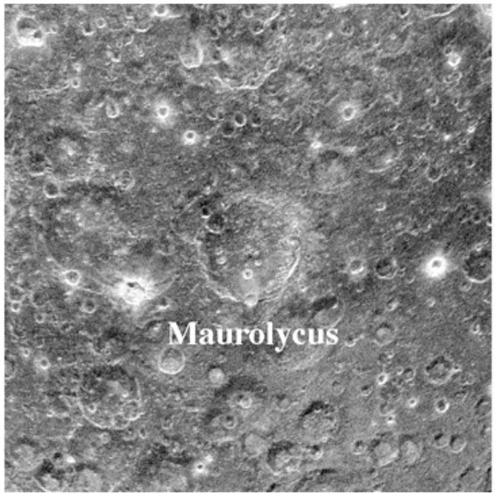
Radar illumination




Radar illumination




Radar illumination



# Estimating differences in scattering from slopes and roughness

**(A) Deviations from average lunar backscatter at S-Band (13 cm) wavelength for slopes**

$$\Delta\sigma = \sigma(\Theta + \Delta\Theta)/\sigma(\Theta)$$

**(B) Predicting deviations from average lunar backscatter at S-Band (13 cm) wavelength for roughness (increases in the diffuse components; Thompson, unpublished)**

$$\sigma_{OC,c} = \alpha\sigma_{OC,avg,d} + (1 \pm 1)\sigma_{OC,avg,q}$$
$$\sigma_{SC,c} = \alpha\sigma_{SC,avg}$$

where

$\sigma_{OC,c}$ = crater opposite-sense (OC) cross-section (enhanced with respect to average)

$\sigma_{SC,c}$ = crater same-sense (SC) cross-section (enhanced with respect to average)

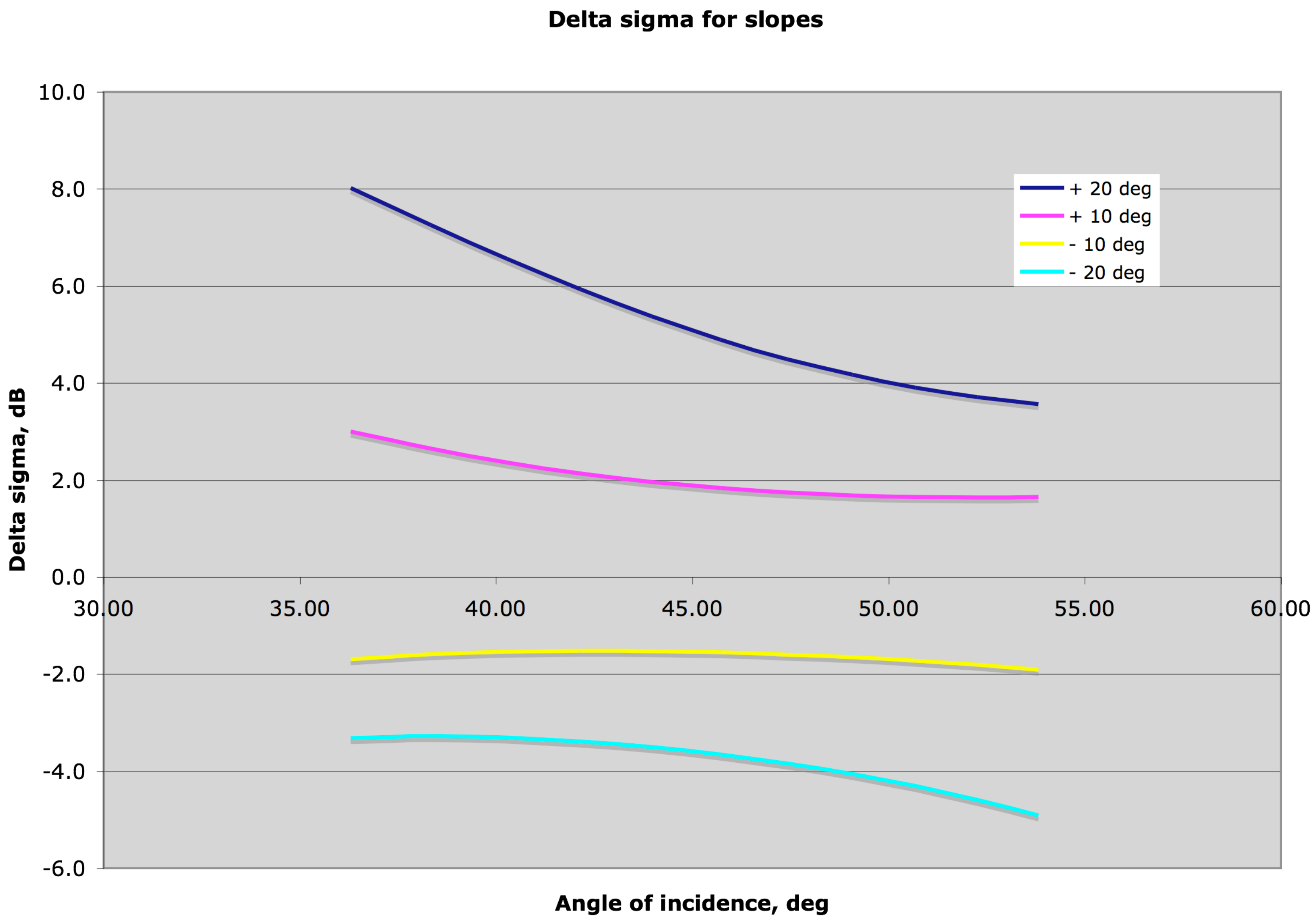
$\alpha$ = crater diffuse scattering enhancement = $\sigma_{SC,c} / \sigma_{SC,avg}$

The polarized (OC) enhancement, $\gamma$, for a depolarized (SC) enhancement of $\alpha$ is given by

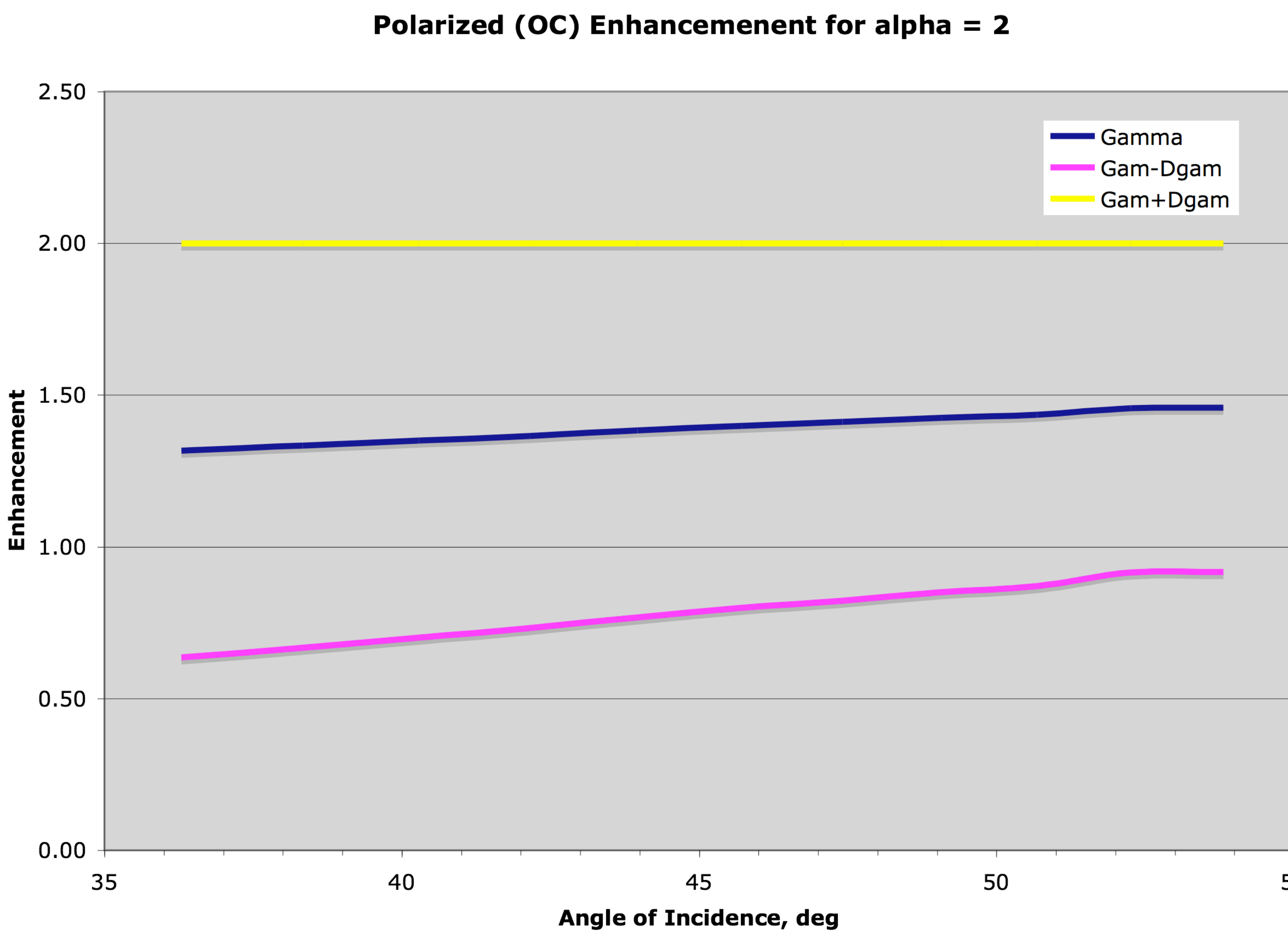
$$\gamma = \frac{\sigma_{OC,c}}{\sigma_{OC,avg}} = \frac{\alpha + (1 \pm 1)\beta}{1 + \beta}, \quad \text{where} \quad \beta = \frac{\sigma_{OC,avg,q}}{\sigma_{OC,avg,d}}$$

and factor $(1 \pm 1)$ = values of 0, 1, 2

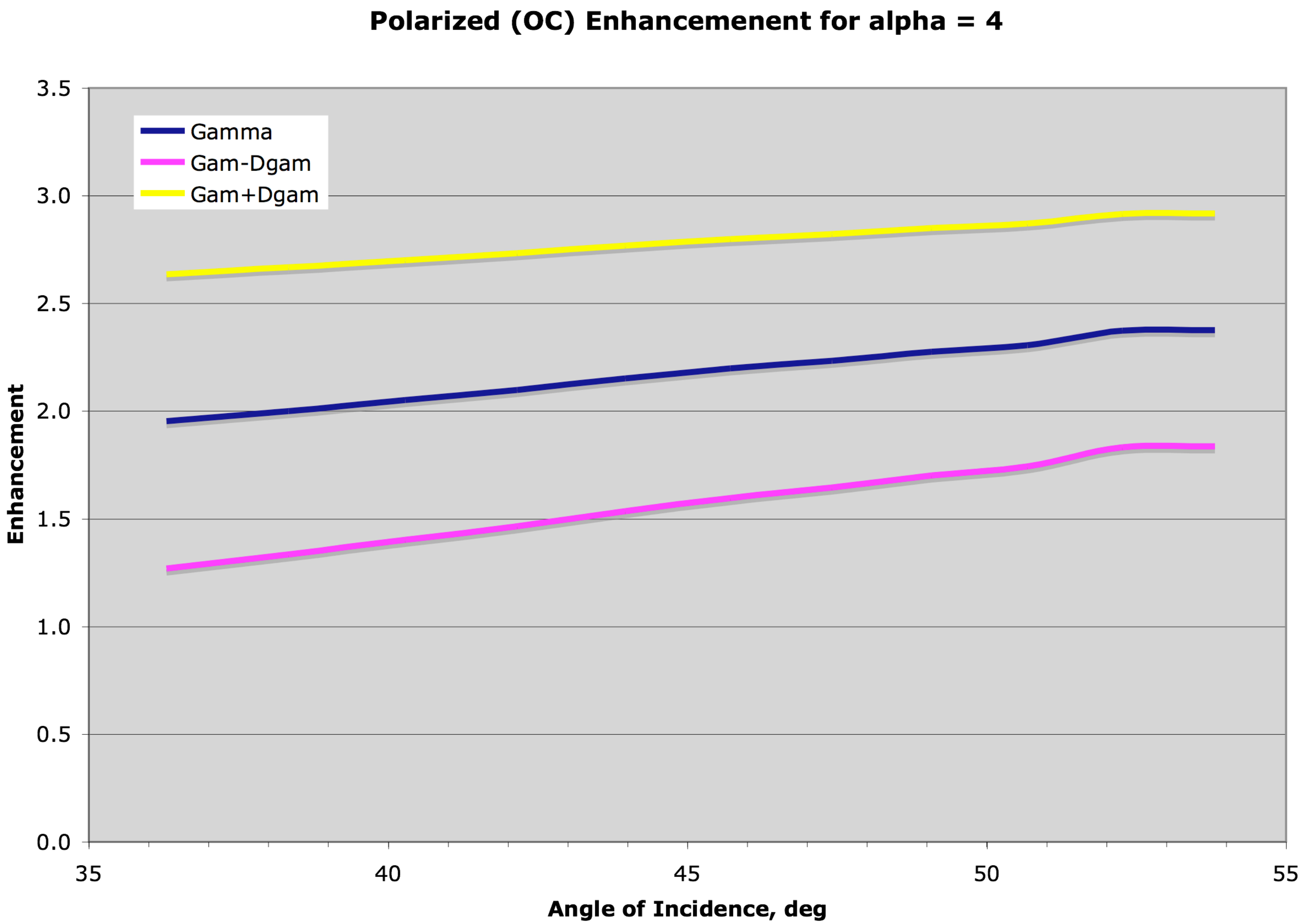
# Differences induced by Slopes



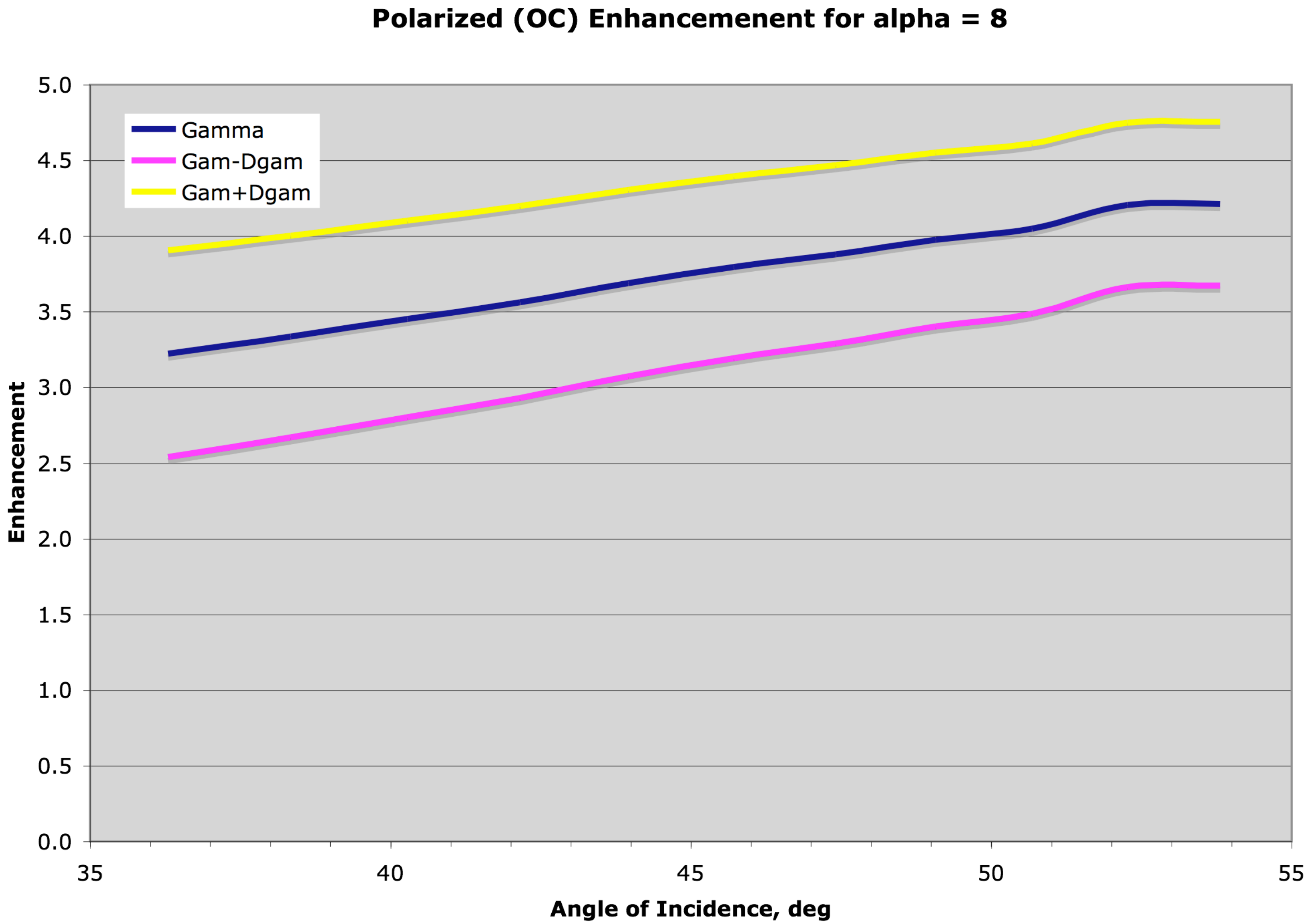
# Differences from Differences in Roughness (x2 diffuse)



# Differences from Differences in Roughness
# (x4 diffuse)



# Differences from Differences in Roughness (x8 diffuse)



# Summary

- Average lunar radar backscatter cross-sections at 3.8, 23, and 68-cm wavelength provides a basis for estimating and modeling the average lunar radar backscatter at 13-cm wavelength
- Hagfors, et al., 1965 (23-cm wavelength) observations suggest that we'll see linear polarization differences of only a few percent
- Scattering differences associated with slopes can be estimated simply from our 13-cm scattering model
- Scattering differences from increased roughness can be estimated assuming a 2-component (diffuse and quasi-specular) model for the average scattering of the Moon

## Back-up References


- J.V. Evans and G.H. Pettengill (1963), The scattering behavior of the Moon at wavelengths of 3.6, 68, and 784 centimeters, JGR, v.68, No.2, pp. 423 – 447.
- J.V. Evans and T. Hagfors (1964), On the interpretation of radar reflections from the Moon, Icarus, v.3, pp. 151 – 160.
- T. Hagfors, Brockelman R.A., Danforth H.H., Hanson I., and Hyde G.M. (1965), Tenuous surface layer on the Moon: Evidence derived from radar observations, Science, v.150, pp. 1153 – 1156.
- T. Hagfors (1970), Remote probing of the Moon by infrared and microwave emissions and by radar, Radio Science, v.5, No.2, pp. 189 – 227.
- T.W. Thompson (unpublished manuscript), Radar Scattering Models for Lunar Craters.