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Scaling of percolative current flow to long lengths in biaxially textured conductors

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Abstract. A random mixture of conducting and non-conducting sites or bonds on a two-dimensional lattice is used to model textured polycrystalline superconducting conductors in which high currents are carried only by low-angle grain boundaries. We calculate the frequency with which unfavourable configurations will significantly decrease current flow. For realistic distributions, in very narrow conductors (∼10 grains wide) critical current is reduced, sometimes to zero, over long lengths. For a typical conductor, 1 cm wide with 50 μm grain size, fluctuations in critical current of less than 10% are expected for kilometre lengths.

1. Introduction

Current in polycrystalline superconductors flows through grain boundaries with a distribution of grain boundary misorientations. Various techniques are under development to texture the superconductor so that most of the grain boundaries are low-angle boundaries with high critical current (J_c), including ion beam-assisted deposition (IBAD) [1, 2], inclined-substrate deposition (ISD) [3], and rolling-assisted biaxially-textured substrates (RABiTS^{TM}) [4, 5]. Most often the high-angle boundaries with low J_c are isolated obstacles, around which current can easily flow. However, even if processing conditions are well controlled, there remains a non-zero probability that randomly located low-J_c boundaries will form a barrier which significantly restricts current flow. For this reason, demonstration of high J_c in short lengths of superconductor does not necessarily guarantee that production of long lengths is feasible.

Scale-up to long lengths is a particular concern for the RABiTS^{TM} technique, in which superconducting films are grown epitaxially on metal substrates with grain sizes ∼50 μm. Conducting tapes are thus only ∼100 grains wide, increasing the likelihood of statistical fluctuations. Grain sizes for IBAD and ISD are determined by the deposition technique, and are ∼1 μm, so conducting tapes are $\sim 10^4$ grains wide.

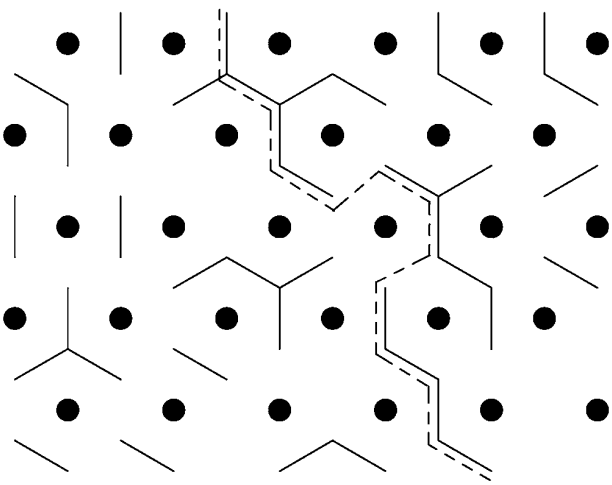

Figure 1. Bond percolation on a hexagonal lattice. Circles represent grains. Full lines represent non-conducting boundaries; conducting boundaries are blank. The dashed line is the non-unique path which limits J_c.

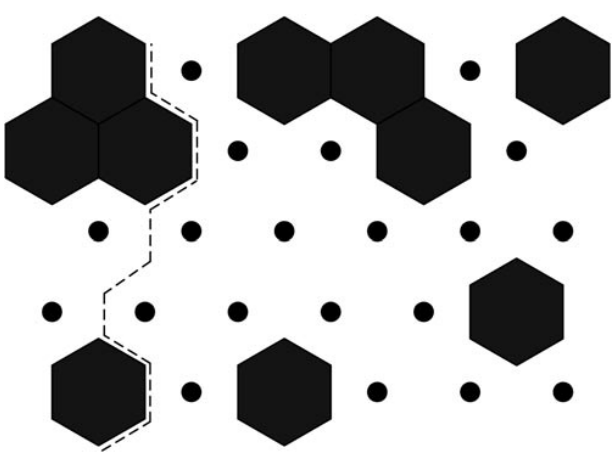

Figure 2. Site percolation on a hexagonal lattice. Circles represent conducting grains. Non-conducting grains are black. The dashed line is the non-unique path which limits J_c.

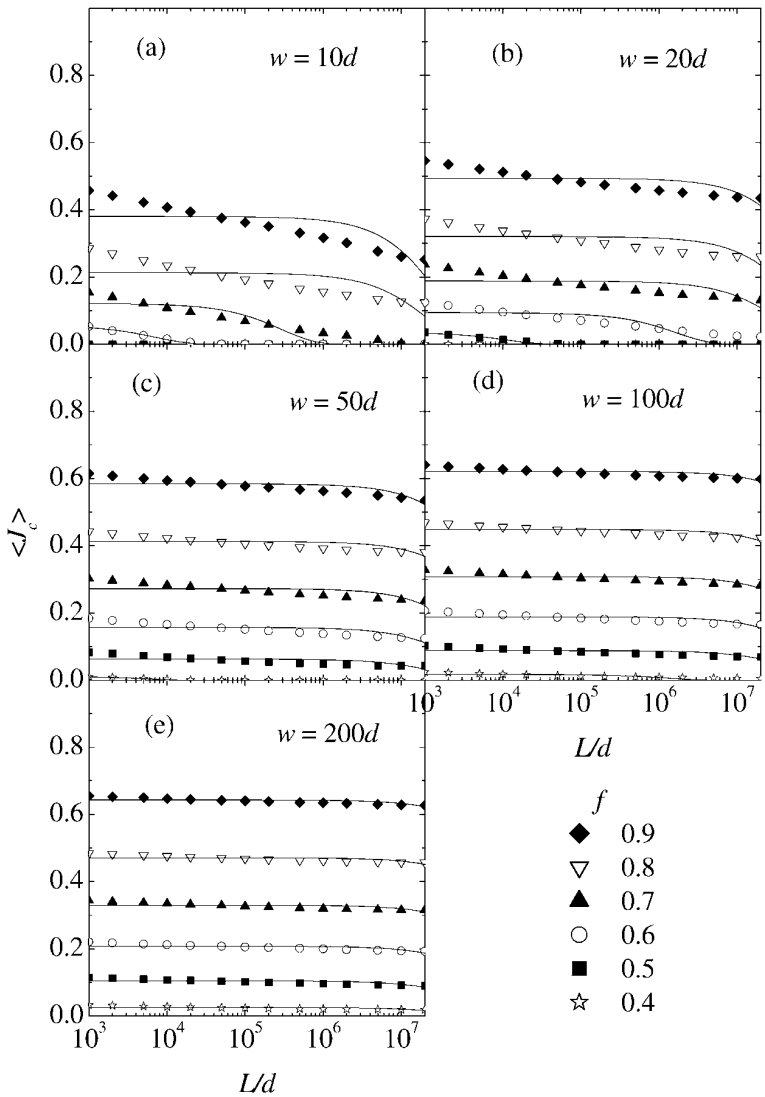

Figure 3. $\langle J_c \rangle$ for various fractions f of conducting grain boundaries on a hexagonal lattice. Curves are least-squares fits to equation (3).

Deutscher [6] has considered the problem of current transport through a large number of grains with random conductivity, for films with thickness less than the 3D percolation length. By considering the probability that conductivity is completely blocked in a system close to the percolation threshold, Deutscher finds that average conductivity of a strip with grain size d, length L and width w can be approximated as

$$\langle J_c \rangle \sim \exp\left\{\frac{L}{\xi}\ln[1-(1-d/\xi)^{w/d}]\right\}. \qquad (1)$$

ξ is a 2D correlation length,

$$\xi \sim d(f-f_c)^{-\nu} \qquad (2)$$

where f is the fraction of conducting material, f_c is the percolation threshold, and ν is a critical exponent. Percolation crosses over from 2D to 1D at a critical length $L_c = \xi\exp(w/\xi)$. For $L \ll L_c$ the conductor behaves as a 2D sample, and J_c does not depend strongly on the sample dimensions. For $L \gg L_c$, the conductor obeys 1D percolation, and conductivity decreases rapidly with length:

$$J_c \sim \exp(-L/L_c). \qquad (3)$$

Small changes in ξ have a large effect on the critical length L_c. Consider, for example, a sample with width $w = 200d$ (1 cm for 50 μm grains): when ξ/d changes from 7.5 to 15, L_c/d changes from 3×10^{12} to 9×10^{6}. The scaling relationship used to find ξ (equation (2)) is valid only close to f_c, where J_c will be low. A useful conductor must be far from threshold. The scaling results of equations (1)–(3) may provide a qualitative description of the crossover from 1D to 2D percolation, but they cannot be used for a reasonable estimate of the length scale at which 1D percolation effects occur.

In order to obtain an order of magnitude estimate of percolation effects in long lengths, we present numerical results in which conductivity is modelled using a simple coupling scheme. A variable fraction of the grain boundaries are non-conducting and the conducting boundaries all have

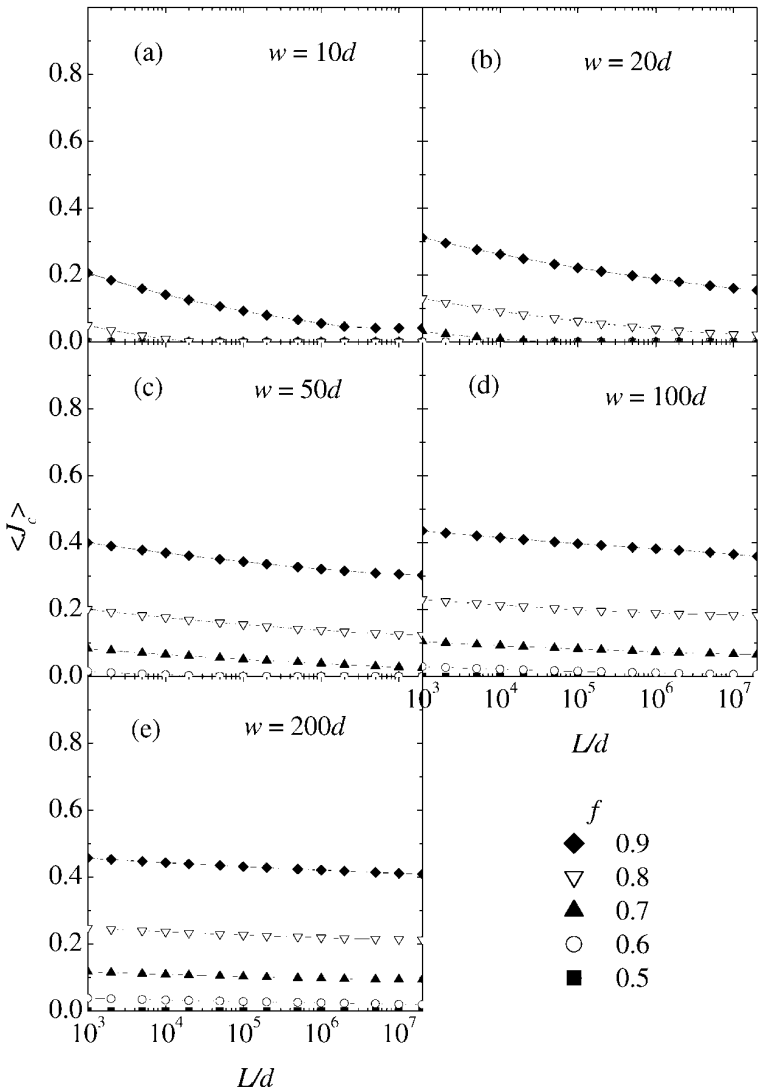

Figure 4. $\langle J_c \rangle$ for various fractions f of conducting grains on a hexagonal lattice. Curves are guides to the eye.

the same critical current. The non-conducting boundaries are distributed in two ways. The first models the ideal case in which all the grains are close in orientation to the average texture. Random deviations in grain orientation lead to random deviations in misorientation at grain boundaries, so a random fraction f of the boundaries are assigned as conducting (figure 1). This is a bond percolation model, in which current passes through the low-angle boundaries. If the orientations are primarily uniaxial, with a Gaussian distribution with FWHM F, the grain boundary misorientations will have a Gaussian distribution with FWHM $\sqrt{2}F$, in the absence of any local correlations. Measurements of texture on a microscopic scale show evidence that the grain boundary misorientations in most cases are less, reflecting a bias towards low-angle boundaries [7]. Measurements of grain boundary critical current show that grain boundaries carry a high current when misorientation is below $\phi_c \sim 5^\circ$ [8], so we make the simple assumption that all boundaries with misorientation below ϕ_c have the same critical current and those with higher misorientation are non-conducting. Assuming a Gaussian distribution as above, the probability that each boundary is conducting is $\mathrm{erf}(1.18\phi_c/F)$, where

$$\mathrm{erf}(x) = 2\int_0^x \exp(-y^2)\,\mathrm{d}y/\sqrt{\pi}. \tag{4}$$

For example, if the FWHM of the grain distribution is $F = 6^\circ$ and boundaries are conducting below $\phi_c = 5^\circ$, 83% of the boundaries will be conducting. Measurements of texture on a microscopic scale [8] show evidence that the grain boundary misorientations may be smaller than predicted by a random distribution of grain orientations; due to these local correlations, this probability represents a lower bound.

The second distribution models less ideal textures, in which the orientation of a fraction of the grains deviates markedly from the average orientation. For example, thermomechanical processing can lead to a microstructure in which most of the material is $\{100\}\langle 100\rangle$ textured (cube-textured) with a small fraction of $\{221\}\langle 221\rangle$ texture (twin-textured). None of the boundaries of the $\{221\}\langle 221\rangle$ grains will carry high currents. This is a site percolation model,

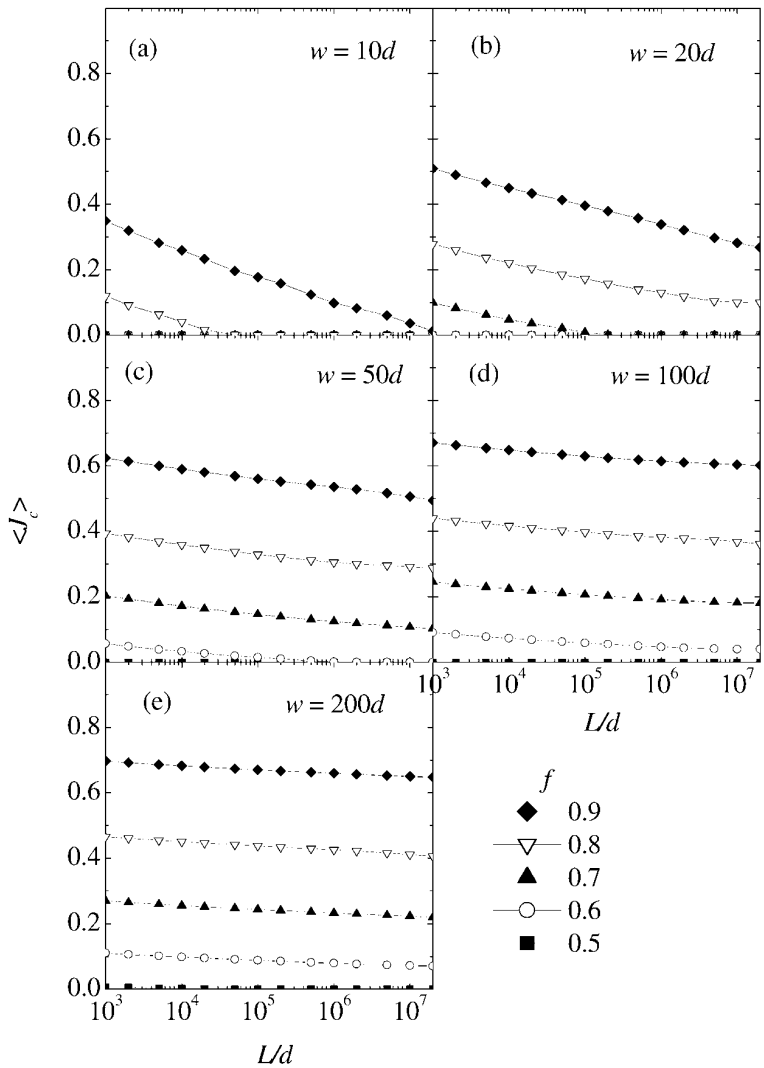

Figure 5. $\langle J_c \rangle$ for various fractions f of conducting grain boundaries on a square lattice. Curves are guides to the eye.

Table 1. Percolation threshold: fraction f_c of conducting sites (i.e. grains) or bonds (i.e. grain boundaries) at which long-range percolation first occurs.

Lattice	Percolation	f_c
Hexagonal	Bond	0.35
Hexagonal	Site	0.50
Square	Bond	0.50
Square	Site	0.59

in which current passes through the cube-textured grains (figure 2).

Calculations are shown for site and bond percolation on both a hexagonal lattice, which more accurately models the microstructure, and on a square lattice as in other work [9]. The percolation thresholds f_c for all four cases are shown in table 1.

2. Method

Following Rhyner and Blatter [9], we use the limiting-path method to determine J_c for the model systems. For the bond percolation cases, a fraction f of bonds are randomly assigned as conducting (figure 1). The site percolation cases are converted to bond percolation problems by first randomly assigning a fraction f of grains as conducting, then assigning boundaries as conducting when both neighbouring grains are conducting (figure 2). J_c is normalized so that $J_c = 1$ when all boundaries are conducting. Sample length and width are measured in units of the near-neighbour grain separation. In order to eliminate end effects, periodic boundary conditions are applied in the direction of current flow. As shown in figures 1 and 2, we include returning paths [10] which are inclined at 30° with respect to current flow, but not those inclined at 90°.

In order to find J_c for a range of sample lengths, we calculate J_c for 20 000 samples, each 1000 grains long, and tabulate the distribution. Thus the total length sampled is 2×10^7 grains (1 km for 50 μm grains). We model sample widths of 10, 20, 50, 100, and 200 grains. We compute the frequency distribution of J_c and its average $\langle J_c \rangle$ for longer conductors by noting that if a 1000-grain segment has $J_c \geqslant j$ with frequency p, a conductor with length L will have $J_c \geqslant j$ with probability $p^{L/(1000d)}$.

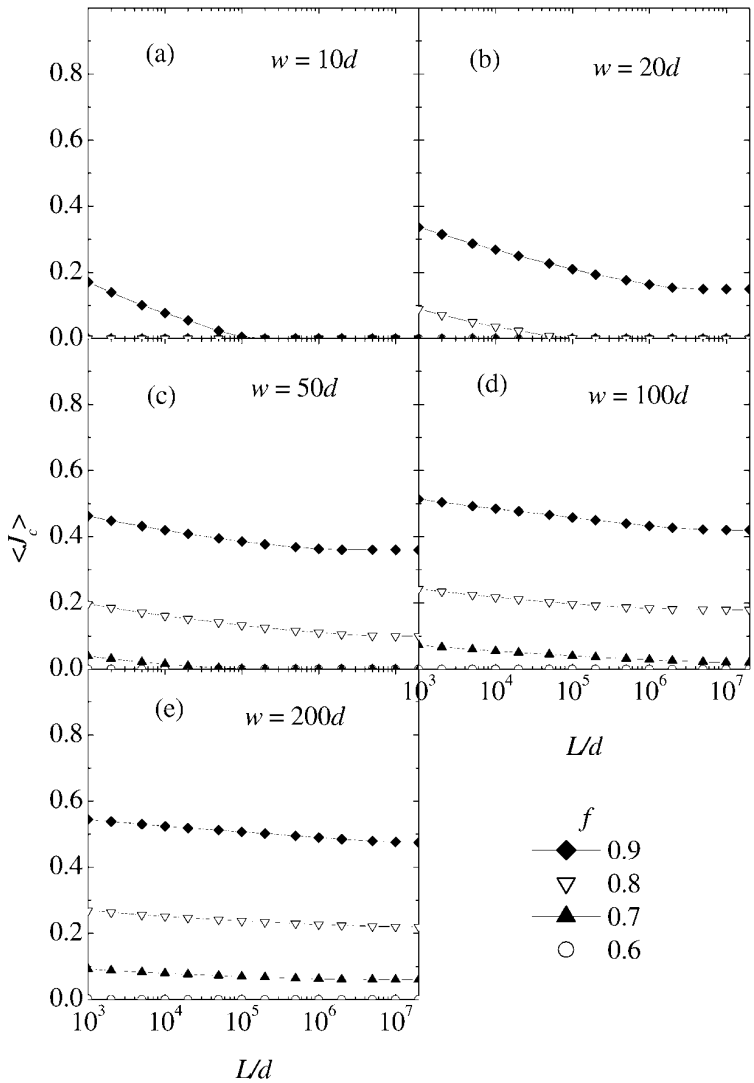

Figure 6. $\langle J_c \rangle$ for various fractions f of conducting grains on a square lattice. Curves are guides to the eye.

3. Results

Figures 3–6 show $\langle J_c \rangle$. The lines in figure 3 are least-squares fits to equation (3); agreement is poor. Equation (3) is derived by assuming that $\langle J_c \rangle$ is determined by clusters which completely block conduction; the frequency of these clusters corresponds to L_c. Our results show that $\langle J_c \rangle$ is in fact affected by conduction bottlenecks with a range of J_c, so there is no single critical length L_c. Note that $\langle J_c \rangle$ is increased far more by an increase in w than $\langle J_c \rangle$ is decreased by an increase in L by a similar factor. For this reason, decreasing the grain size will increase $\langle J_c \rangle$.

The results are summarized in figures 7 and 8. Figure 7 shows the minimum J_c for each set of 20 000 simulations, i.e. the $\langle J_c \rangle$ for a long-length sample. Figure 8 compares J_c^{max} and J_c^{min}, respectively the J_c for the best and worst 1000-grain segment for each set of simulations. Plotted is $(J_c^{max} - J_c^{min})/J_c^{max}$, the maximum relative decrease expected when comparing short and long samples. The data are linear on a log–log scale, with a slope corresponding to $(J_c^{max} - J_c^{min})/J_c^{max} \sim w^{-1/2}$. The results scale approximately as $f - f_c$; non-conducting grains have a larger effect than a similar fraction of non-conducting grain boundaries, since f_c is larger for site percolation than bond percolation.

For the best polycrystalline superconductors, J_c is ∼0.5 that observed for epitaxial thin films [10], consistent with $f = 0.83$ as predicted from the texture (see above). For superconductors grown epitaxially on thermomechanically processed metal substrates, the texture is determined by the substrate, with a grain size of ∼50 μm, and conductors are typically ∼100 grains wide. For the most realistic model, bond percolation on a hexagonal lattice, this implies that fluctuations in J_c will be ∼10%. This may be too small to observe, since variations in deposition conditions can cause larger fluctuations in J_c. Other texturing techniques yield much smaller grain sizes, making sample widths larger (in units of grains) and statistical fluctuations in J_c insignificant.

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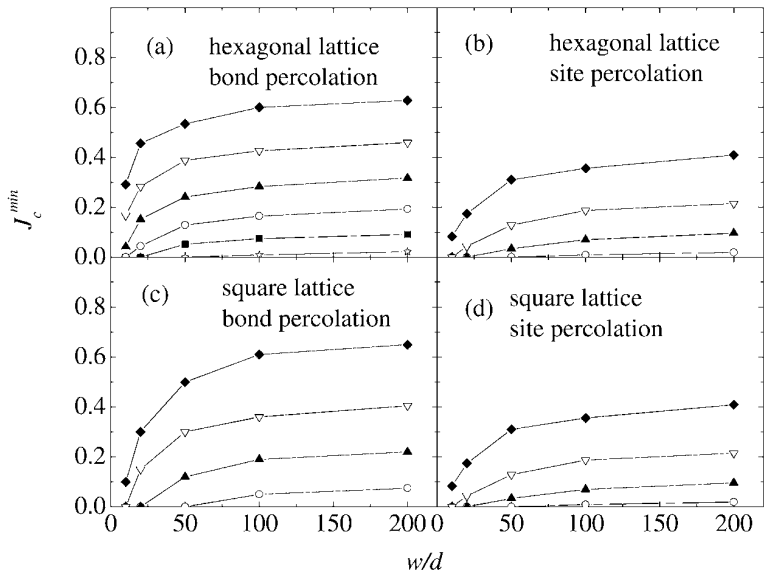

Figure 7. Minimum J_c observed in 2×10^7 grains of simulated length for fraction f of conducting grains or boundaries: 0.9 (♦), 0.8 (▽), 0.7 (▲), 0.6 (○), 0.5 (■), and 0.4 (⋆).

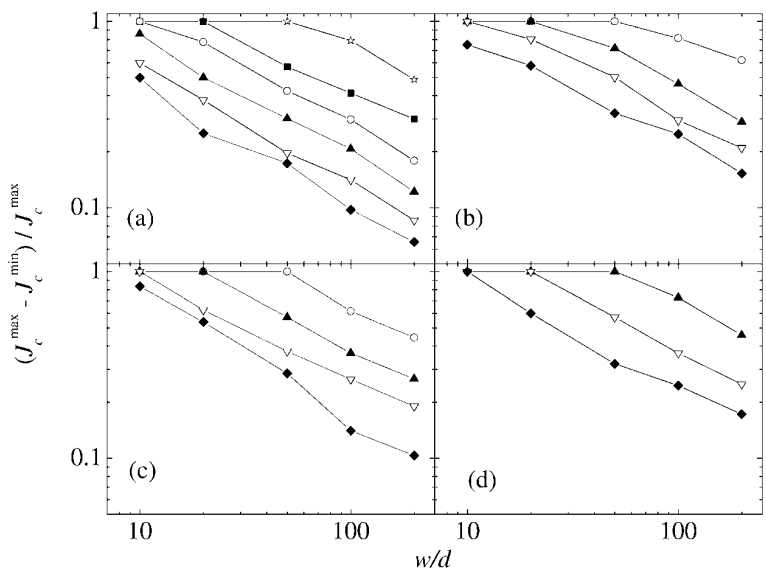

Figure 8. Relative variation in J_c: (*a*) hexagonal lattice, fraction f of conducting grain boundaries, (*b*) hexagonal lattice, fraction f of conducting grains, (*c*) square lattice, fraction f of conducting grain boundaries, and (*d*) square lattice, for fraction f of conducting grains or boundaries: 0.9 (♦), 0.8 (▽), 0.7 (▲), 0.6 (○), 0.5 (■), and 0.4 (⋆).

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