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1. Introduction

The purpose of this report is to document the details of the governing equations and physical parameterizations of the sea ice model component in the first version of the NCAR Climate System Model (CSM). It should be noted that this model is very similar to the sea ice component used by Washington and Meehl (1996a, 1996b) in a fully coupled model of the atmosphere, ocean, and sea ice. Many of the thermodynamic ice growth and melt processes are taken from previous work, namely Semtner (1976), Parkinson and Washington (1979), Harvey (1988), and Pollard and Thompson (1994). Ice dynamics are based upon the cavitating fluid solution described by Flato and Hibler (1990, 1992), and used by Pollard and Thompson (1994), where the shear and tensile strength of the ice are neglected and the compressive strength is used to iterate to a representative ice velocity each timestep.

The sea ice component of the CSM is driven by the heat, momentum, and freshwater fluxes provided at the upper and lower ice boundaries by the atmospheric and oceanic model components, respectively. In return, the sea ice model provides the appropriate boundary fluxes required by the atmosphere and ocean in the presence of ice. The CSM Flux Coupler (Bryan *et al.*, 1996) facilitates and manages the exchange of fluxes between the CSM component models and exercises the required care to assure conservation of heat, momentum, and freshwater within the model climate system.

2. Interaction with the Flux Coupler

a. Spatial and Temporal Coordination

While the CSM is designed to accommodate component models that have varying spatial domains and horizontal grid structures, the sea ice model and ocean model lie on the same horizontal grid within the present configuration. The Arakawa-B grid scheme is used in spherical coordinates, with a resolution of 2.4° in longitude, and a "stretched" latitudinal grid that is 1.2° in latitude poleward of 60° in both hemispheres (NCAR Oceanography Section, 1996).

Because the atmospheric component of the CSM resolves a diurnal cycle, it is necessary for the ice model to update the temperature over the ice/snow surface frequently each day. To minimize complications in computing the energy balance between the ice and atmosphere, the synchronous period for the exchange of information between the ice and atmosphere, through the Flux Coupler, matches the timestep of the atmospheric model. Under normal circumstances this timestep is much shorter than is otherwise required for the sea ice model based upon computational stability criteria (approximately two hours for the above stated grid resolution). Because the component models of the CSM integrate asynchronously, and since this ice model is considerably less computationally intensive than either the atmosphere or the ocean model, the extra time spent in the ice model to resolve the diurnal cycle is not significant.

b. Fluxes Exchanged

The Flux Coupler is the computational interface that gathers the required information from each of the component models within the CSM and calculates the information necessary to provide input to each component for the next time period of integration. The following information is provided to the sea ice model at the beginning of each synchronous timestep:

$\vec{\tau}_c$	the vector sum of the wind stress at the top of the ice and the portion of the ocean stress that is independent of ice velocity
S_1	the net solar radiation penetrating the ice/snow surface
H_1	the sum of the latent and sensible heat fluxes and the downward longwave radiation exchanged with the atmosphere
dH_1	the derivative of H_1 with respect to surface temperature; used in the calculation of the new surface temperature (see Section 3.e)
$L\uparrow_1$	the upward longwave radiation emitted to the atmosphere from the surface
Q_H	the net heat exchange (and implied water flux) between the ice and ocean due to ocean freezing or the potential heat available in the upper ocean to melt sea ice (NCAR Oceanography Section, 1996, for details)

F_1 the freshwater flux (presently all convective and large-scale precipitation) onto the ice/snow surface (includes evaporation of snow and ice)

H_4' the total surface heat flux through the atmosphere-hydrosphere interface; used by some sea ice models to control the rate of lateral ice growth into leads (**not** used by this version of the model)

$\vec{\nabla}\eta$ sea surface slope

At the end of the synchronous timestep, the ice model provides the Flux Coupler with the following output information:

$\vec{U}_c = \vec{U}_i$ the ice velocity

$T_c = T_s$ the surface temperature over the ice, or snow-covered ice, surface

$f_c = A_i$ the ice concentration (fraction of grid cell covered by sea ice)

$\vec{\alpha}_c$ the sea ice albedos for direct and diffuse solar radiation for the visible and near-infrared wavelengths

$d_c = h_s$ the snow depth on top of the ice

S_4 the solar radiation passing through the ice and into the ocean below

H_4 the heat flux into the ocean due to ice/snow melt in response to upper ocean temperatures above freezing as given by the Q_H input flux; unless all the ice and snow melt, $H_4 = Q_H$

F_4 the freshwater flux into the ocean due to ice/snow melt, or due to ice growth due to vertical energy balance within the ice; does not include flux due to ice growth dictated by Q_H

The CSM sign convention for all boundary fluxes is consistent with the convention that incident solar radiation is positive. That is, all fluxes are defined as positive downward. Therefore, by definition, a positive flux of any quantity across an interface acts to **increase** the value of that quantity in the layer below that interface. For example, the Q_H flux input into the sea ice model represents the heat available from the ocean to either grow or melt ice. If the flux is such that ice grows, the change of state has occurred at the expense of holding the ocean temperature at its freezing point, and the flux is positive.

3. Thermodynamics

The sea ice thermodynamic processes involve ice growth in leads (lateral ice growth), lateral ice melt, and changes in the ice state due to energy balance considerations at the top and bottom of the ice. In addition, the sea ice model accounts for snow on top of the ice, accumulating snow when it falls, and deleting snow when the surface energy balance warrants melt or when the ice under the snow disappears. Finally, through surface energy balance considerations, the sea ice model computes and makes available to the Flux Coupler the temperature of the snow/ice surface. Each process is described in detail below. Unless otherwise stated, the values used for physical constants in the following equations can be found in Appendix A.

a. Lateral Ice Growth in Leads

The growth of new ice in leads follows closely the method described by Parkinson and Washington (1979). The net energy flux Q_H into a grid cell is balanced by the time rate of change of ice formation. That is:

$$Q_H = Q_i \, \frac{d\,(A_i\, h_i)}{dt} \; , \tag{3.1}$$

where Q_i is the heat of fusion of sea ice, A_i is the ice concentration, h_i is the ice thickness, and t is time. Since the energy balance determines only the ice volume, as discussed in Washington and Parkinson (1986), Eq. (3.1) can be rearranged such that:

$$\Delta\, A_i = Q_H \, \frac{\Delta t}{Q_i\, h_i^*} \; . \tag{3.2}$$

Here, $\Delta\, A_i$ is the ice concentration increment, and h_i^* is the ice thickness increment. Thus, the ice concentration increase within the lead of a grid cell can be computed after first assigning an appropriate value of ice thickness. Consistent with the references given above, h_i^* is 20 cm.

Once the new ice volume increment within a grid cell is determined, it is "rehomogenized" with any ice that may have previously existed using conservation of ice mass. That is, the new ice thickness in the grid cell is determined from:

$$A_i' \, h_i' = A_i \, h_i + \Delta \, A_i \, \Delta h_i \; . \tag{3.3}$$

Here, A_i' and h_i' are the new ice concentration and thickness, respectively, and $A_i' = A_i + \Delta A_i$.

A maximum value of ice concentration is imposed since small amounts of leads can exist even within thick multi-year ice (Pollard and Thompson, 1994; Harvey, 1988). In a manner similar to Pollard and Thompson (1994), we use maximum values of 0.99 in the Northern Hemisphere and 0.96 in the Southern Hemisphere. Furthermore, following Harvey (1988), when the new total ice concentration in a grid cell exceeds the maximum value A_{max}, the ice concentration is recomputed by:

$$A_i' = \begin{cases} A_{max} & \text{h}' \leq 1 \text{ meter} \\ 1 - (1 - A_{max}) \; \exp\left[-(h_i' - 1)/\tau_1\right] & \text{h}' > 1 \text{ meter} \end{cases} \; . \tag{3.4}$$

As in Harvey (1988), $\tau_1 = 3$ m. After this procedure is applied, the total ice thickness in the grid cell is adjusted as described above so as to conserve ice mass. The net effect of this procedure is that the ice concentration within a grid cell can exceed the maximum value when the ice is greater than 1 meter thick, but it does so at a rate that slowly approaches unity as a function of ice thickness.

b. Lateral Ice Melt

The existing concentration of sea ice is reduced when heat is available to melt ice. Within the CSM, this occurs when Q_H, the heat exchange between the ocean and the sea ice, is negative. The formulation of the lateral sea ice melt follows Parkinson and Washington (1979), where the total amount of heat required to melt all of the ice in a grid cell is given by:

$$H_T = A_i \, \rho_i \, h_i \left[c_{pi} \; (T_{melt} - T_i) + L_f\right] \; , \tag{3.5}$$

where ρ_i is the density of ice, T_{melt} is the melting point of ice, T_i is the temperature of the ice, c_{pi} is the specific heat of ice, and L_f is the latent heat of fusion. H_T is compared to the heat available to melt ice:

$$H_A = Q_H \, dt \ . \tag{3.6}$$

The change in the total concentration in the grid cell is then given by the proportion H_A/H_T, and the ice thickness in the grid cell remains unchanged. When more heat is available to melt ice than is contained in the grid cell's ice volume (H_A/H_T greater than unity), all of the ice disappears and the residual heat is returned to the ocean.

c. Vertical Energy Balance

The method for computing the vertical balance of heat within the ice/snow, and the variables describing the ice/snow state, closely follows the thermodynamic model, with minor modifications, described by Semtner (1976), which should be consulted for complete details. Briefly, the energy balance, and computation of the ice/snow thermodynamic state variables, is based upon the diffusion of heat through the external and internal boundaries of the three-layer system illustrated in Fig. 3.1.

As in Semtner (1976), two layers of ice are maintained as long as the total ice thickness is greater than 50 cm. When the thickness falls below 50 cm, but is above 25 cm, one layer of ice is maintained. Below an ice thickness of 25 cm, the zero-layer model suggested by Semtner (1976) is employed. The third layer in this model represents the snow, if present, on top of the ice.

The Semtner model computes the vertical growth of existing ice at the ice-ocean interface by the heat conduction through the ice. This is distinct from the growth of new ice in the ocean.

It should be noted that the model applies all of Q_H, the net heat from the ocean to grow or melt ice, to the lateral ice growth or melt processes. Therefore, for the purpose of computing the vertical balance of heat within an ice column, the heat flux at the bottom of the ice is set to zero. This assumes that the heat fluxes through the open water lead are much greater than the fluxes underneath thick ice. For a completely ice-covered grid cell, any additional lateral ice growth is rehomogenized into the existing ice according to Eqs. (3.3) and (3.4).

When the ice grows to compensate for the loss of heat due to a change in the surface energy balance, the thickness of the ice is applied uniformly to the ice slab with no change in ice concentration within the grid cell. However, when the ice thickness decreases (melts) via this process, the ice concentration with the grid cell is decremented slightly using a method suggested by Harvey (1988). Assuming the ice thickness to be uniformly distributed between zero and twice the ice thickness within the grid cell (i.e., a "wedge" of ice), and using simple geometry, the change in ice concentration is:

$$\Delta A_i = \frac{\Delta h_i}{2h_i} \cdot A_i \ . \tag{3.7}$$

Given the change in ice thickness and in ice concentration, the ice is once again rehomogenized using Eq. (3.3).

d. Snow Cover

Snowfall onto the CSM sea ice is derived from the freshwater flux onto the ice/snow surface. It consists of the precipitation (usually frozen, i.e., snowfall), as well as the evaporation of snow and ice, from the atmospheric component of CSM. As such, this flux can be positive (more precipitation than evaporation) or negative (more evaporation than precipitation).

To simplify the complexities of the snow distribution on top of the sea ice, it is assumed that any snow cover on the ice evenly covers all of the ice in the grid cell. When the ice grows laterally within the cell, the snow cover is maintained evenly through conservation of snow mass. When the freshwater flux at the top of the ice/snow is positive, snow accumulates evenly over all of the ice. On the other hand, when the ice fraction within a grid cell shrinks, the column of snow that laid on top of the ice that disappeared is added to the freshwater flux going into the ocean. Likewise, any snow that melts during the vertical energy balance calculation of ice/snow is added to the freshwater flux into the ocean. Finally, if precipitation is falling onto the ice/snow surface, and the surface temperature is at the melting point of fresh, frozen water, the precipitation does not accumulate as snowfall but falls directly into the ocean as a freshwater flux. In all cases, any snow cover or precipitation that becomes eligible for freshwater flux into the ocean is assumed to be totally fresh (no salinity content).

While it is assumed that existing snow evenly covers all of the ice within a grid cell, it should be noted that for the purpose of computing the albedo of the ice/snow surface, a combined ice/snow albedo is parameterized (see Section 3.h). For melting snow and ice, melt ponds on top of the ice are not modeled explicitly but are parameterized by reductions in albedo.

e. Surface Temperature

The surface temperature calculation over ice/snow follows that of the Semtner (1976) three-level thermodynamic ice model with minor modifications for its implementation into CSM. It is designed to take the energy fluxes that are passed from the Flux Coupler and compute a surface temperature based on a surface energy balance.

For a snow-covered surface, the energy balance is

$$S_1 + H_1 + L\uparrow_1 + k_s \frac{T_o - T_s}{h_s/2} = 0\ , \tag{3.8}$$

where S_1 is the net absorbed solar radiation, H_1 is the sum of incoming longwave radiation, latent and sensible heat flux, $L\uparrow_1$ is outgoing longwave radiation ($=\sigma T_s^4$), k_s is conductivity of snow, T_s is the surface temperature, T_o is the temperature at midpoint of the snow layer, and h_s is the depth of the snow layer. The fluxes are all passed from the Flux Coupler, while other values are internal to the ice model.

The new surface temperature T_s is computed by expanding Eq. (3.8) with $T_s = T_p + \Delta T$, where T_p is the surface temperature of the previous timestep:

$$S_1 + H_1 + L\uparrow_1 (T_p) + \Delta T\ 4\sigma(T_p)^3 + \Delta T\ \frac{\partial H_1}{\partial T_s} + k_s\ \frac{T_o - T_p - \Delta T}{h_s/2} = 0\ , \tag{3.9}$$

where the derivative of the sensible and latent heat fluxes to temperature is also passed from the Flux Coupler. While these derivative terms were not included in the original Semtner (1976) energy balance, they were found to be necessary for numerical stability when the fluxes are either rapidly varying in time or are strongly dependent on T_s, i.e., when the derivatives are large. Equation (3.9) is solved for ΔT, and thus the new temperature T_s.

For bare ice (snow-free), the energy balance Eq. (3.8) becomes

$$(1 - I_0)\, S_1 + H_1 + L \uparrow_1 + k_i \, \frac{T_1 - T_s}{h/2} = 0 \; , \tag{3.10}$$

where I_0 is the fraction of solar radiation penetrating into the interior of the ice, k_i is the conductivity of ice, and T_1 is the temperature at the midpoint of the top ice layer. Equation (3.10) is expanded and solved for T_s in the same manner as Eq. (3.8).

f. Penetrating Solar Radiation and Brine Pockets

For snow-free ice surfaces, a fraction of the net solar radiation at the surface is allowed to penetrate the interior of the ice. While the penetrating fraction I_0 is greater for cloudy skies than for clear skies (Ebert and Curry, 1993), $I_0 = 0.30$ is used here, which assumes cloudy conditions approximately 75% of the time.

A fraction of the penetrating radiation is transmitted through the ice vertically, exiting the bottom of the ice into the ocean. This transmitted flux follows Beer's Law

$$S_4 = I_0 \; S_1 \; e^{-k_{sc}\, h_i} \; , \tag{3.11}$$

where S_4 is the solar flux exiting the bottom of the ice and k_{sc} is the bulk shortwave extinction. For thin ice, a significant fraction of the penetrating solar flux is transmitted to the ocean, and for thicker ice, it is entirely absorbed in the ice layers.

The fraction of penetrating solar flux absorbed in the ice, F_{I_0}, computed as

$$F_{I_0} = I_0 \; S_1 - S_4 \; , \tag{3.12}$$

contributes to the internal melting of ice into brine pockets. The brine pockets parameterization is taken from the Semtner (1976) model, in which the absorbed solar flux is stored in a heat reservoir, Q_B, without decreasing the overall ice thickness. This stored heat is later expended in the refreezing of the brine pockets in winter or is lost to the ocean if the entire ice volume in a grid cell melts.

g. Freshwater Flux

When ocean water freezes and becomes ice, some salt settles into the water underneath and the remainder becomes entrapped in pockets of brine within the ice. Likewise, when the ice melts, water and any ambient salt is redeposited into the underlying ocean. The former process acts to increase the salinity of the underlying ocean (a negative freshwater flux), while the latter process acts to freshen the ocean (a positive freshwater flux). This process is parameterized with a method similar to Parkinson (1979) using conservation of salt.

Briefly, by assuming that the ocean mixed layer under the ice is much deeper than the ice thickness, the surface freshwater flux from freezing or melting ice/snow is approximated by:

$$F_4 = (1/S_t)\ \frac{\rho_i}{\rho_w}\ (S_t - S_i)\ \frac{d\,(A_i\,h_i)}{dt}\ , \tag{3.13}$$

where ρ_i/ρ_w is the ratio of sea ice density to sea water density, S_t is the average salinity of ocean water, S_i is a uniform ice salinity, and $d\,(A_i\,h_i)/dt$ represents the change in ice mass over the relevant time period. The term $1/S_t$ represents the conversion from salt flux to a freshwater flux. When snowmelt adds to the freshwater flux, the S_i term is set to zero and the snow depth takes the place of the ice thickness in Eq. (3.13).

h. Surface Albedo

The surface albedo over sea ice is computed in the ice model and passed to the Flux Coupler for its computation of the net solar flux at the surface. The ice model gives the Flux Coupler the surface albedos over two spectral ranges, 0.2-0.7 μm and 0.7-5.0 μm. Only the surface albedo over sea ice is computed in the ice component; the albedo of any open water fraction within a grid cell with sea ice is computed in the ocean component and used in the Flux Coupler to compute the net solar flux.

The surface albedo over sea ice is dependent on whether snow cover is present, the depth of snow, and whether the surface is melting or dry, i.e., whether surface temperature T_s is equal to or less than 0°C. Even though the fraction of snow cover (snow concentration) equals unity when any snow exists in a grid cell, the computation of surface albedo assumes a mix of snow-covered and snow-free ice and uses a proxy 'snow fraction.'

Table 3.1 Spectral Albedos for Ice and Snow

	0.2-0.7 μm	0.7-5.0 μm
Snow, dry ($\alpha_{snow,d}$)	0.95	0.70
Snow, melting ($\alpha_{snow,m}$)	0.85	0.55
Sea ice, dry ($\alpha_{ice,d}$)	0.70	0.50
Sea ice, melting ($\alpha_{ice,m}$)	0.50	0.50

The spectral albedos of dry snow and sea ice were taken from the NCAR Community Climate Model version 2 (CCM2) (Hack *et al.*, 1993). The values are in the range between young bare ice and multi-year bare ice in the observations cited by Ebert and Curry (1993). The reduction of snow and ice albedos for melting conditions are based on the one-dimensional sea ice model of Ebert and Curry (1993), which computes a minimum albedo of 0.50 for their specified 10% fraction of melt ponds on sea ice.

For ice with no snow cover, the surface albedo uses only the values for sea ice. For $T_s < 0°$C, the dry ice albedos are used; for $T_s = 0°$C, the melting ice albedos are used.

For snow-covered ice ($h_s > 0$), the surface albedo is computed using the proxy snow fraction A_s,

$$A_s = \frac{h_s}{h_s + 0.10} \; , \tag{3.14}$$

where h_s is the liquid-water equivalent depth of snow. For $T_s < 0°$C, the surface albedo is:

$$\alpha_{sfc} = A_s * \alpha_{snow,d} + (1 - A_s) * \alpha_{ice,d} \; , \tag{3.15}$$

and for $T_s = 0°$C

$$\alpha_{sfc} = A_s * \alpha_{snow,m} + (1 - A_s) * \alpha_{ice,m} \; . \tag{3.16}$$

4. Dynamics

The dynamics of sea ice, in response to atmospheric and oceanic forcing, modifies the polar climate in several significant ways: it can either increase or decrease the momentum exchange between atmosphere and ocean; it controls the build up of ice thickness, the position of the ice-edge, and the creation of open-water leads within the ice pack; and it transports freshwater in the form of sea ice between regions of net ice growth and net melt. The dynamic treatment of sea ice in the CSM ice model is formulated to produce these climatic effects with a simple sea ice rheology.

a. Ice Velocity

Until recently, treatments of sea ice dynamics in global, coupled climate models have assumed sea ice moves in direct proportion to ocean currents or surface winds (e.g., Manabe and Bryan, 1969). These approximations are oversimplified for the Arctic ice pack in particular, for which motion can be strongly influenced by the rigidity of the thicker ice. In general, the ice momentum equation that includes the resistance of thicker ice must be solved to produce realistic velocities.

The sea ice dynamics in CSM is based on the cavitating-fluid approximation of Flato and Hibler (1990), in which ice has a finite resistance to compression but can diverge freely and has no resistance to shear stress. The numerical formulation of the cavitating-fluid model has been taken from Pollard and Thompson (1994), which solves the ice velocities at the corner points of the Arakawa B-grid in spherical coordinates.

The ice momentum equations include the Coriolis force, wind stress, ocean current stress, down-slope acceleration from the ocean surface tilt, and the gradient of accumulated ice pressure due to compression:

$$\begin{aligned} \rho_i\, h_i\, f\, v_i + \tau_\lambda + C_w\ \cos\ \theta\ (u_w - u_i) - C_w\ \sin\ \theta\ (v_w - v_i) \\ - \frac{\rho_i\, h_i\, g}{a\ \cos\,\phi}\ \frac{\partial \eta}{\partial \lambda} - \frac{1}{a\ \cos\,\phi}\ \frac{\partial p^*}{\partial \lambda} = 0 \end{aligned} \tag{4.1}$$

$$\begin{aligned} -\rho_i\, h_i\, f\, u_i + \tau_\phi + C_w\ \cos\ \theta\ (v_w - v_i) + C_w\ \sin\ \theta\ (u_w - u_i) \\ - \frac{\rho_i\, h_i\, g}{a}\ \frac{\partial \eta}{\partial \phi} - \frac{1}{a}\ \frac{\partial p^*}{\partial\, \phi} = 0\ , \end{aligned} \tag{4.2}$$

where u_i and v_i are the zonal and meridional components of ice velocity, $\vec{U}_i$, respectively, ρ_i is ice density, h_i is ice thickness, f is the Coriolis parameter, τ_λ and τ_ϕ are the components of the wind stress, u_w and v_w are the components of the geostrophic ocean current, C_w is the linear drag coefficient, θ is the turning angle of the geostrophic current to surface current, η is the dynamic height of the ocean, g is gravity, a is Earth's radius, and p^* is the ice pressure.

These equations take the form:

$$-Du_i + Ev_i + X = \frac{1}{a\,\cos\,\phi}\,\frac{\partial p^*}{\partial \lambda}\,, \tag{4.3}$$

$$\begin{aligned}
-Dv_i - Eu_i + Y &= \frac{1}{a}\,\frac{\partial p^*}{\partial\,\phi} \\
D &= C_w\,\cos\,\theta \\
E &= \rho_i\,h_i\,f + C_w\,\sin\,\theta \\
X &= C_w\,\cos\,\theta\,u_w - C_w\,\sin\,\theta\,v_w - \frac{\rho_i\,h_i\,g}{a\,\cos\,\phi}\,\frac{\partial\eta}{\partial\lambda} + \tau_\lambda \\
Y &= C_w\,\cos\,\theta\,v_w + C_w\,\sin\,\theta\,u_w - \frac{\rho_i\,h_i\,g}{a}\,\frac{\partial\eta}{\partial\phi} + \tau_\phi\,,
\end{aligned} \tag{4.4}$$

where X and Y are the terms that do not depend on u_i and v_i. They are computed in the Flux Coupler from the atmosphere and ocean variables and are passed to the ice as the components of $\vec{\tau}_c$.

The equations are solved through an iterative procedure that starts with the free-drift ice velocities, i.e., $p^* = 0$ everywhere, for which Eqs. (4.3) and (4.4) are linear. For grid cells with convergent velocities, $\bigtriangledown \cdot \vec{U}_i < 0$, an ice pressure correction p' is added to p^*, based on the equation modified from Flato and Hibler:

$$-D\,\vec{U}_i' - E\,\vec{k}\,\times\,\vec{U}_i' = \bigtriangledown p'\,, \tag{4.5}$$

where $\vec{U}_i'$ is the velocity correction that will satisfy

$$\bigtriangledown \cdot \left(\vec{U}_i + \vec{U}_i'\right) = 0 \tag{4.6}$$

for grid cells under convergent stresses. The ice velocities are recomputed with the gradient of the updated pressure included, which adjusts the velocities towards the condition in

Eq. (4.6) but may also create convergence in neighboring grid cells. Therefore, the pressure correction procedure is iterated over the entire (hemispheric) grid, until the solution satisfies the criteria at iteration n:

$$max\left(|\vec{U}_i^n| - |\vec{U}_i^{n-1}|\right) < 0.002\ m\ s^{-1}\ . \tag{4.7}$$

The pressure p^* is accumulated in this procedure for grid cells under convergence until the solution is reached or the pressure reaches a threshold yield strength:

$$P_{max} = P^*\ h_i\ \exp\left[-20\ (1 - A_i)\right]\ , \tag{4.8}$$

where P^* is the ice strength parameter and A_i is the ice concentration. For $p^* = P_{max}$, there is no further pressure correction, and the convergent velocities result in the build up of ice volume at a grid point.

For grid cells in which $p^* < P_{max}$, the velocity solution should satisfy Eq. (4.6). In practice, however, there remains a small amount of residual convergence of velocities at these points (on the order of $1 \times 10^{-7} s^{-1}$) even though the solution satisfies Eq. (4.7). Reductions in the solution criteria in Eq. (4.7) do not greatly reduce this residual. In areas of persistently converging wind/ocean stresses, this residual leads to unrealistic ice build up. The effects of the residual convergence on the ice is corrected in the numerical advection scheme.

Near the north pole, the ice velocity is solved in the same manner, except that poleward of 86°N, the velocity is filtered toward uniform motion, i.e., keeping the mean and first wavenumber of u_i and v_i. The sea ice grid has a tracer point at 90°N, so there is no singularity in the velocity solution.

b. Advection

The following quantities are advected in the same manner: ice concentration (A_i), ice volume ($h_i\, A_i$), heat content of each ice layer ($T_i\, h_i\, A_i$), brine heat reservoir ($Q_B\, h_i\, A_i$), snow volume ($h_s\, A_i$), and snow heat content ($T_0\, h_s\, A_i$).

The advection scheme has been modified from the flux-form upstream differencing scheme of the Pollard and Thompson (1994) model to account for the effects of the residual convergence of the velocities. The original upstream scheme computed the local change in a quantity f due to advection by a velocity $\vec{U}_i$ as

$$\frac{\partial f}{\partial t} = - \bigtriangledown \cdot (f \, \vec{U}_i)$$

$$\frac{\partial f}{\partial t} = (u_\ell \; f_\ell \; \Delta y - u_r \; f_r \; \Delta y + v_b \; f_b \; \Delta x_b - v_t \; f_t \; \Delta x_t) \, / \Delta A \; , \tag{4.9}$$

where $f_{(\ell,r,b,t)}$ are the upstream values of f on the left, right, bottom (south), and top (north) sides of a grid box, $u, v_{(\ell,r,b,t)}$ are the average velocity components along each side, Δx and Δy are the dimensions of the grid box, and ΔA is the grid box area.

The advection at the grid cell at 90°N is computed by adding the poleward fluxes of the advected quantity f along the northern edges of the adjacent southern grid cells. The grid cell at 90°N is also tested for residual convergence and adjusted accordingly.

In the present model, Eq. (4.9) is rewritten in terms of the divergence of the velocity, i.e.,

$$\begin{aligned} \bigtriangledown \cdot \; \vec{U}_i &= \frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} \\ \frac{\partial u_i}{\partial x} &= (u_r - u_\ell) \; \Delta y / \Delta A \\ \frac{\partial v_i}{\partial y} &= (v_t \; \Delta x_t - v_b \; \Delta x_b) / \Delta A \; , \end{aligned} \tag{4.10}$$

so that Eq. (4.9) becomes

$$\begin{aligned} \frac{\partial f}{\partial t} = &- 2 \bigtriangledown \cdot \vec{U}_i \, \frac{(f_\ell + f_r + f_b + f_t)}{4} + \frac{\partial v_i}{\partial y} \; \frac{(f_\ell + f_r)}{2} + \frac{\partial u_i}{\partial x} \; \frac{(f_b + f_t)}{2} \\ &+ \left\{ \frac{u_\ell + u_r}{2} \; (f_\ell - f_r) \; \Delta y + \frac{v_b \; \Delta x_b + v_t \; \Delta x_t}{2} \; (f_b - f_t) \right\} / \Delta A \; . \end{aligned} \tag{4.11}$$

To correct for the residual convergence in grid cells that should, in reality, be incompressible (because the accumulated ice pressure does not exceed the threshold ice strength P_{max}), the same advection Eq. (4.11) is used, with the condition that

$$\bigtriangledown \cdot \; \vec{U}_i = 0, \;\; \frac{\partial u_i}{\partial x} = - \frac{\partial v_i}{\partial y} \; . \tag{4.12}$$

This eliminates the accumulation of ice (and other advected quantities) by the residual convergence in the incompressible points. The correction, however, is not conservative, and so an equal quantity of ice must be replaced at other grid points.

The replacement or redistribution of ice after the correction of the residual convergence is computed by assuming that the ice converges into the incompressible grid points from all the other compressible or divergent ice points. In fact, the residual convergence would act to create excess open water in divergent points, which also needs to be corrected. Therefore, the advected quantities are adjusted in all of the non-incompressible ice points so that the hemispheric totals of each quantity are conserved before and after advection. The adjustment for the non-incompressible points is

$$f_n'' = f_n' \, \frac{\left(\overline{f} A - \overline{f}_m' \, A_m\right)}{\overline{f}_n' \, A_n} \, , \tag{4.13}$$

where f_n'' is the new non-incompressible grid point value, f_n' is the gridpoint value before adjustment, $\overline{f}$ is the (hemispheric) mean of f over the total ice area A before advection, $\overline{f}_m'$ is the mean over the incompressible points over area A_m after the advection and correction of residual convergence, and $\overline{f}_n'$ is the mean over the non-incompressible points over area A_n after advection.

This adjustment replaces the concentration, volume, and heat (which were removed from the incompressible points) uniformly over all the non-incompressible ice points in each hemisphere. However, it excludes the regions of oceans that are specified as *marginal seas* by the ocean model (NCAR Oceanography Section, 1996), for example, the Hudson Bay, the Baltic Sea, and the Black Sea.

While this redistribution was needed to correct for the residual convergence in the numerical solution of velocities, it is expected that the redistribution step will be unnecessary in future formulations of the ice dynamics, where all of the stresses are more accurately taken into account.

5. Overview of the Sea Ice Model Code

The purpose of this section is to provide an overview of the CSM Sea Ice Model code. It is meant to give the potential user a feel for the structure and flow of the model and its relationship to the Flux Coupler. A description of the code can be found in the *User's Guide to the NCAR CSM Sea Ice Model* (Bettge, 1996).

Under normal conditions the sea ice model simply responds to the forcing that is provided by the Flux Coupler. As such, the sea ice model has no internal calendar. Aside from the namelist input variables, only two datasets are necessary for the ice model to successfully start its integration—a dataset that contains the mask pertinent to the ocean grid and a restart dataset that contains the current state of the ice. The sea ice model may be integrated under various other conditions, such as with climatological sea surface temperatures that are used to specify sea ice concentration, or with forcing from a previous CSM model integration, eliminating the need for any interaction with the Flux Coupler. Refer to the *User's Guide to the NCAR CSM Sea Ice Model* (Bettge, 1996) for further details.

The basic structure of the CSM Sea Ice Model code is illustrated in Fig. 5.1. There are four basic functions that occur during the integration of the sea ice model: initializing the model (reading the namelist input variable information, a land/ocean/marginal sea mask, and a restart dataset); exchanging of information with the Flux Coupler; advancing the sea ice state variables over one timestep, and saving the model state (either as a history dataset or a restart dataset). Figure 5.1 summarizes these functions in conceptual form and provides the names of the important subprograms within which these functions are performed.

Appendix A

List of Physical Constants and Parameters

L_f	= latent heat of fusion	$= 334 \times 10^3\ J\ Kg^{-1}$
Q_i	= heat of fusion of sea ice	$= 302 \times 10^6\ J\ m^{-3}$
ρ_w	= density of sea water	$= 1026\ Kg\ m^{-3}$
ρ_s	= density of snow	$= 330\ Kg\ m^{-3}$
ρ_i	= density of sea ice	$= 905\ Kg\ m^{-3}$
c_{pi}	= specific heat of sea ice	$= 2100\ J\ Kg^{-1}\ K^{-1}$
c_{ps}	= specific heat of snow	$= 2090\ J\ Kg^{-1}\ K^{-1}$
T_{melt}	= melting point of sea ice	$= 0.0°\mathrm{C}$
T_{freeze}	= freezing point of sea water	$= -1.8°\mathrm{C}$
S_t	= ocean salinity	$= 34.7\ ^\circ/_{\circ\circ}$
S_i	= sea ice salinity	$= 4\ ^\circ/_{\circ\circ}$
k_s	= conductivity of snow	$= 0.31\ W\ m^{-1}\ K^{-1}$
k_i	= conductivity of ice	$= 2.03\ W\ m^{-1}\ K^{-1}$
g	= acceleration of gravity	$= 9.80616\ m\ s^{-2}$
a	= mean radius of the earth	$= 6.37122\ \times 10^6\ m$
σ	= Stefan-Boltzmann constant	$= 5.67 \times 10^{-8}\ W\ m^{-2}\ {}^\circ K^{-4}$
I_0	= fraction of solar radiation penetrating into sea ice	$= 0.30$
k_{sc}	= bulk shortwave extinction	$= 1.5\ m^{-1}$
C_w	= linear drag coefficient	$= 0.6524\ \ Kg\ m^{-2}\ s^{-1}$

List of Physical Constants and Parameters (Cont'd)

A_{max}	= maximum sea ice concentration	
	Northern Hemisphere	= 0.99
	Southern Hemisphere	= 0.96
θ	= turning angle of geostrophic ocean current to surface ocean current	= 25°
P^*	= ice strength parameter	= $2.0 \times 10^4\ N\ m^{-2}$

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