Beam Interaction with gas filled RF Cavities

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11-30-2007

BNL Workshop

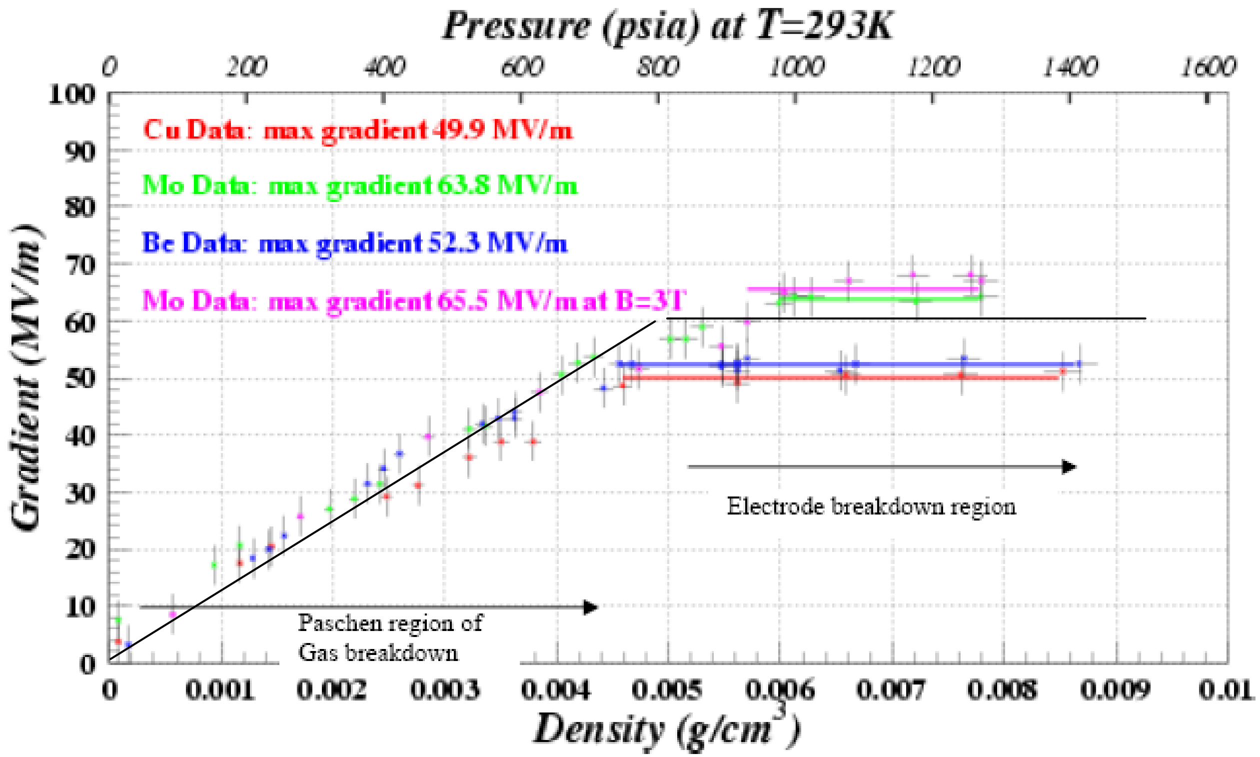
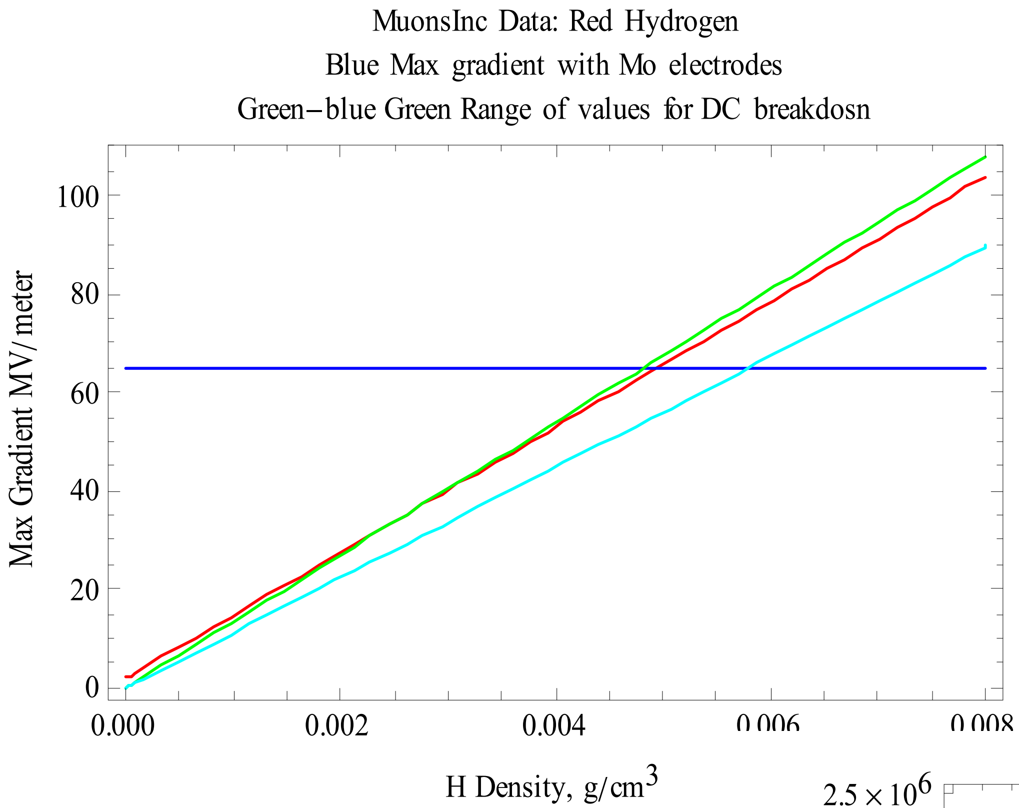


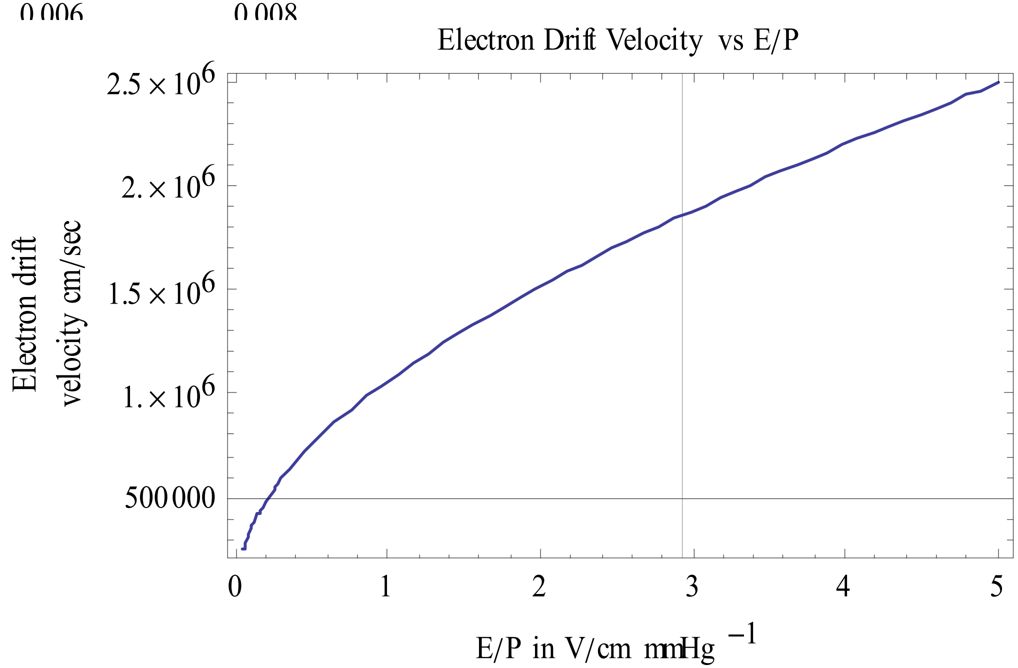
Figure 3: Measurements of the maximum stable TC gradient as a function of hydrogen gas pressure at 800 MHz with no magnetic field for three different electrode materials, copper (red), molybdenum (green), and beryllium (blue). As the pressure increases, the mean free path for ion collisions shortens so that the maximum gradient increases linearly with pressure. At sufficiently high pressure, the maximum gradient is determined by electrode breakdown and has little if any dependence on pressure. Unlike predictions for evacuated cavities, the Cu and Be electrodes behave almost identically while the Mo electrodes allow a maximum stable gradient that is 28% higher. The cavity was also operated in a 3 T solenoidal magnetic field with Mo electrodes (magenta); these data show no dependence on the external magnetic field, achieving the same maximum stable gradient as with no magnetic field.

The black line is at E/P = 13.2 for breakdown in hydrogen. This comes from $U_e = eE\lambda$ and $\lambda = 1/N\sigma$ and the postulate that the physics depends only on the energy of the electrons at the point of collision. A more useful variable is E/N where N is the # molecules.cm^3. E/N for breakdown = 4 $10^{-16.}$



E/P is a measure of

E x distance between electron collisions with the atoms. Doubling E and doubling the pressure should leave the physics the same. This is generally measured in Volts/cm/mmHg



Mobility

$$v = \tfrac{1}{2}\ a\,t\ = \tfrac{1}{2}\ e/m\ E\ t\ = \tfrac{1}{2}\ e/m\ E\ \langle \lambda / Vr \rangle$$

V: drift velocity; Vr random velocity within the swarm. It is not related to kT of the molecules!

$\lambda = 1 / N\sigma$

$v = \tfrac{1}{2}\ e/m\ \langle 1 / (N\sigma Vr) \rangle\ E = \mu E$ and is proportional to E /P or E/N.

For high E, Vr is much higher than given by kT as the electrons absorb energy from the field and then scatter generating hot random electrons. μ is a function of E/P.

deltaZ is given by the following formula:

$$\text{deltaZ} = \int_0^{T/4} \mu[E_0 \,\text{Cos}[\omega\, t] / P]\ E_0\, \text{Cos}[\omega\, t]\, dt$$

Some Cross sections for electrons on hydrogen

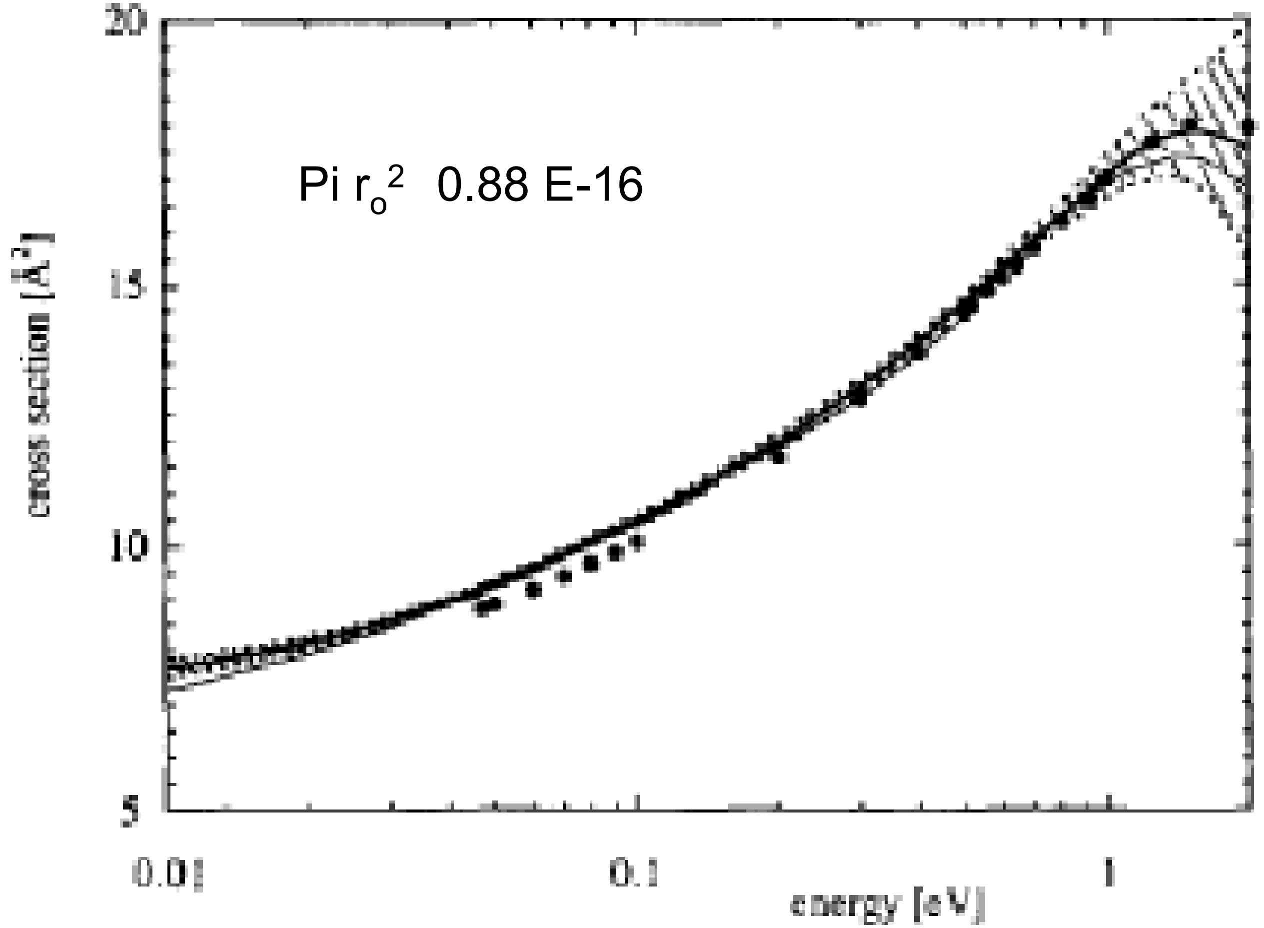
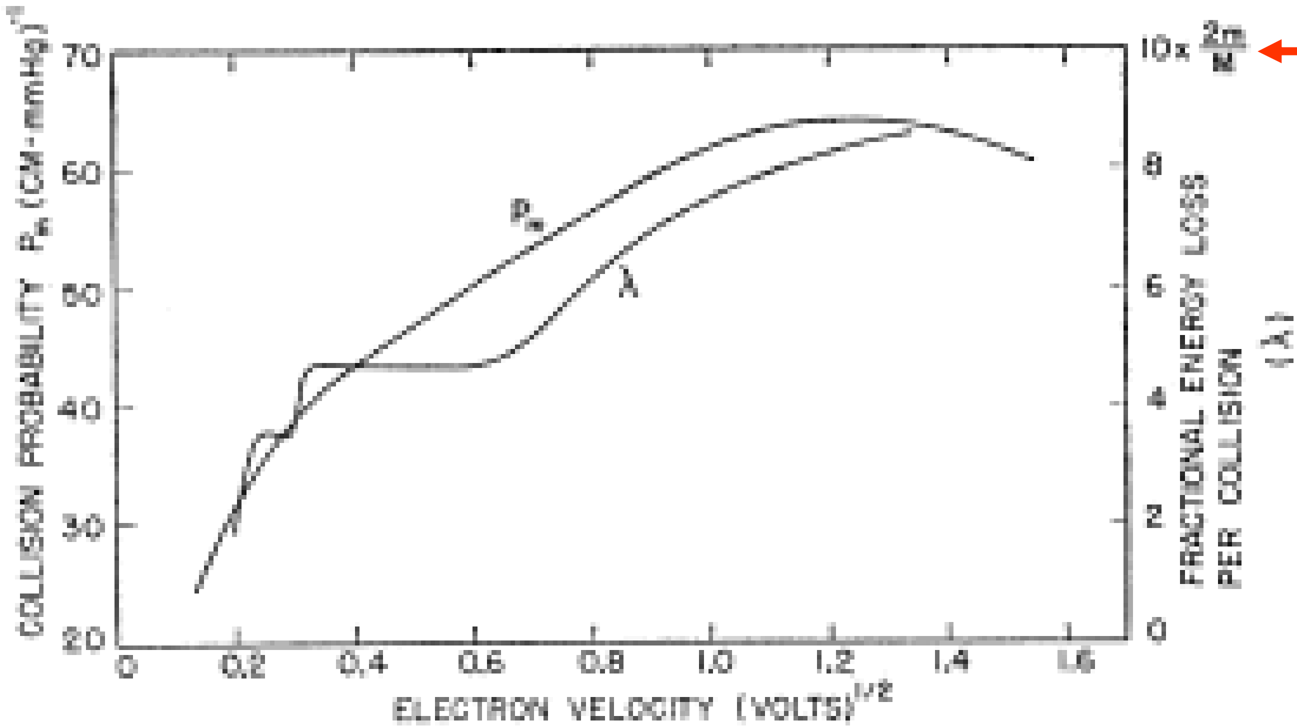


Fig. 12. Elastic momentum transfer cross section of H_2 as obtained in this analysis (thick line) compared with the result of a previous swarm analysis [16] (thin line) and the most rigorous theoretical calculations [25] (dots). The hatched area indicate the limits of accuracy in this analysis for a *local* variation of the cross section.

Fractional energy loss / collision for electrons in Hydrogen

Bekefi & Brown. PR112,159,1958.



0.5 E-3

FIG. 4. Collision probability for momentum transfer P_m, and the fractional energy loss λ per collision, as a function of the electron velocity.

Table I. *Inelastic processes in H_2 contributing in the energy range 0.01 < ε < 3 eV and included in this analysis with their thermal population of the ground state at 298 K*

Process		$\varepsilon_{thr.}$ (eV)	q_{pop} (298 K)
rot:	$0 \to 2$	0.0439	12.9%
	$1 \to 3$	0.0727	65.9%
	$2 \to 4$	0.1008	11.8%
	$3 \to 5$	0.1280	9.0%
	$4 \to 6$	0.1538	0.4%
vib:	$0 \to 1, \Delta j = 0$	0.5159	100%
	$0 \to 1, \Delta j = 2$	0.442 … 0.609	100%
	$0 \to 2, \sum \Delta j$	1.003	100%
	$0 \to 3, \sum \Delta j$	1.461	100%

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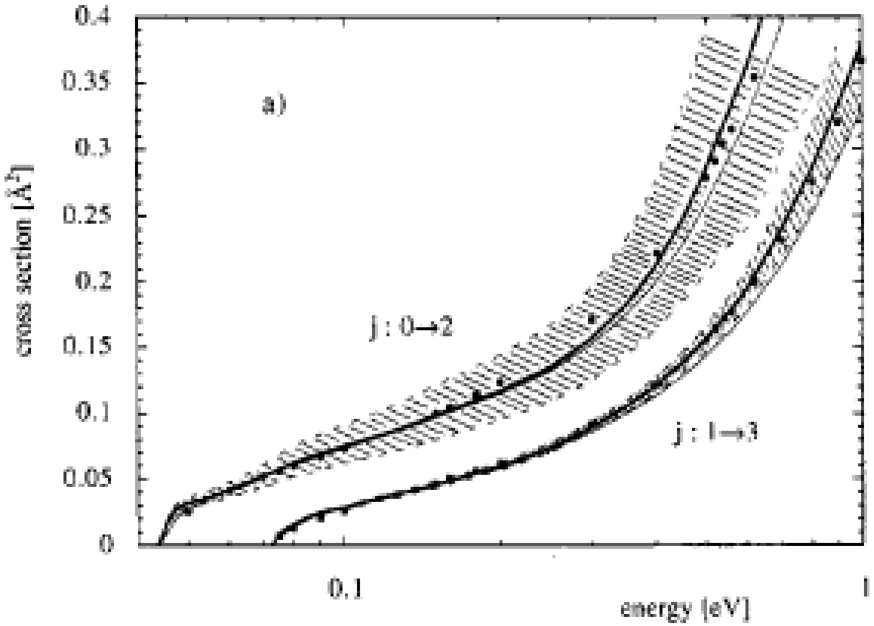
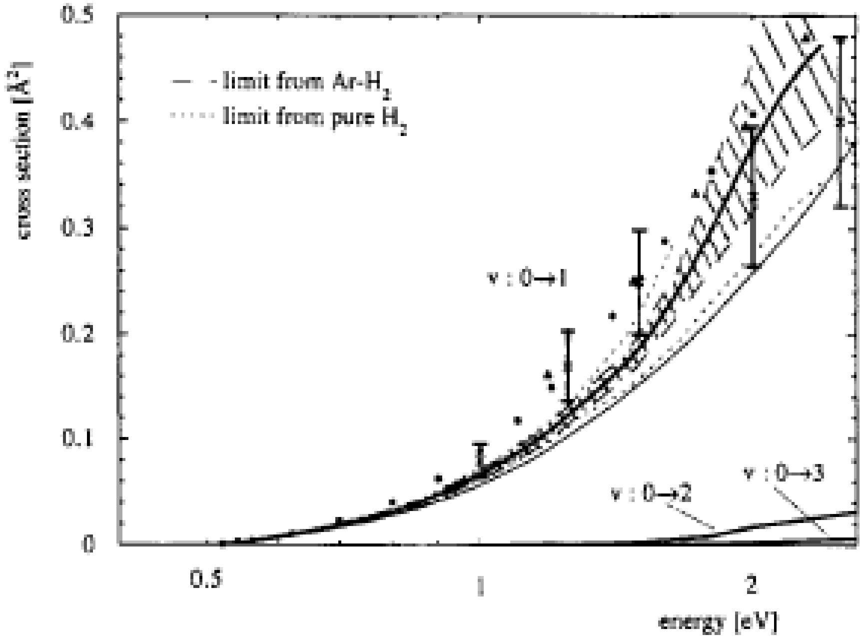
Physica Scripta. Vol. T53, 30–42, 1994

Inelastic cross sections

B. Schmidt, K. Berkhan, B. Go ¨tz and M. Müller





). *11.* Rotationally summed cross section for vibrational excitation of H_2 obtained in this analysis (thick line) and compared with the result of a :vious swarm analysis [16] (thin line), crossed beam experiments unger *et al.* [28] (crosses with error bars) and Ehrhardt *et al.* [29] angles without error bars) and the most rigorous theoretical calculation ;] (dots without error bars). The hatched area indicates the uncertainty its in this analysis for a *local* variation of the cross section. The cross tions for the transitions v: $0 \to 2$ and v: $0 \to 3$ used in the analysis are)wn for completeness and are of negligible influence in this energy range.

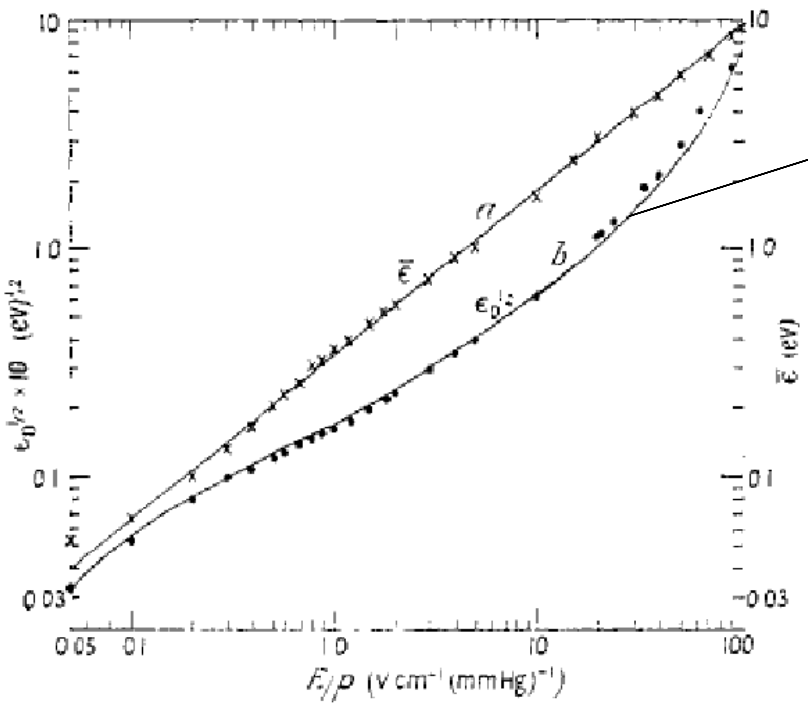


Figure 2. Electron mean energy: crosses, Crompton and Sutton (1952); curve *a*, according to equation (5). Electron drift velocity: points, Bradbury and Nielsen (1936) and Gill and von Engel (1949), curve *b* corresponds to equation (6)

MEAN ELECTRON ENERGY IN THE SWARM

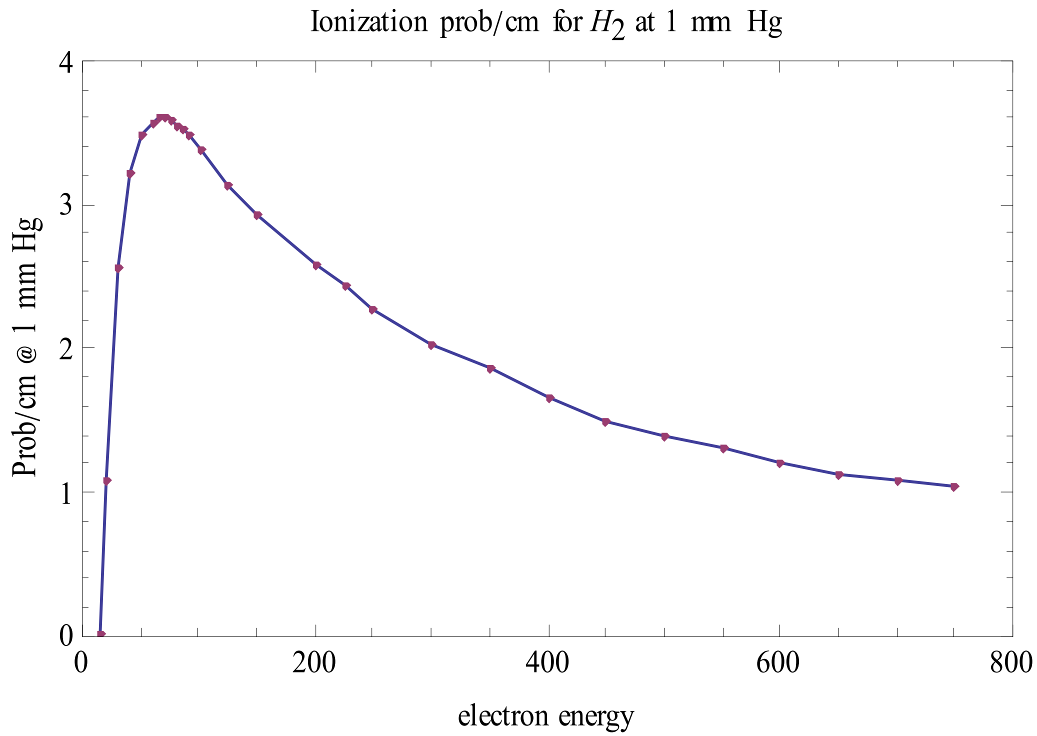
The distribution is roughly gaussian.

Heylen has fit the data with an analytical expression that is valid for:

0.1 < E/P < 100 with an accuracy of less than 16%

This fit has been used for calculating deltaZ and other properties of the beam-cavity interaction

IONIZATION CROSS SECTION FOR e + H_2



P= 200 Bo=13.6` betaPerp= 0.0981

emit= 0.000422 minEmit= 0.000209

Nbeam= $1.\times 10^{11}$ rhoGas= 0.03`

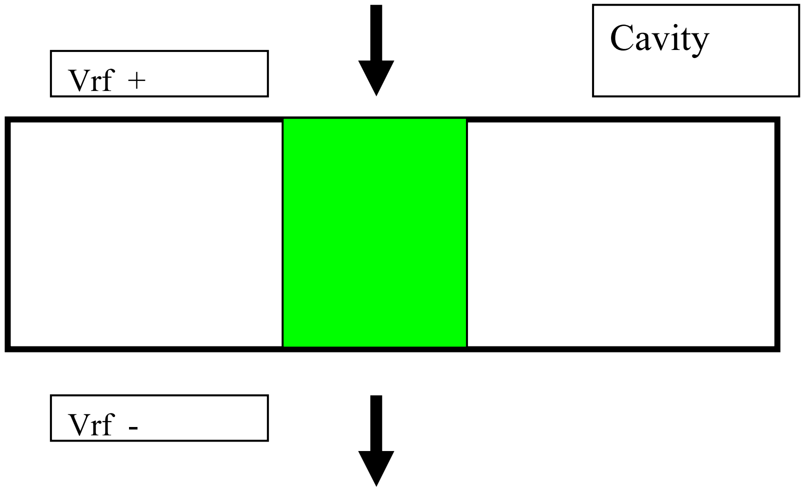
RF Gradient V/m $=25.\times 10^{6}$,Frf Hz= $400.\times 10^{6}$

Example

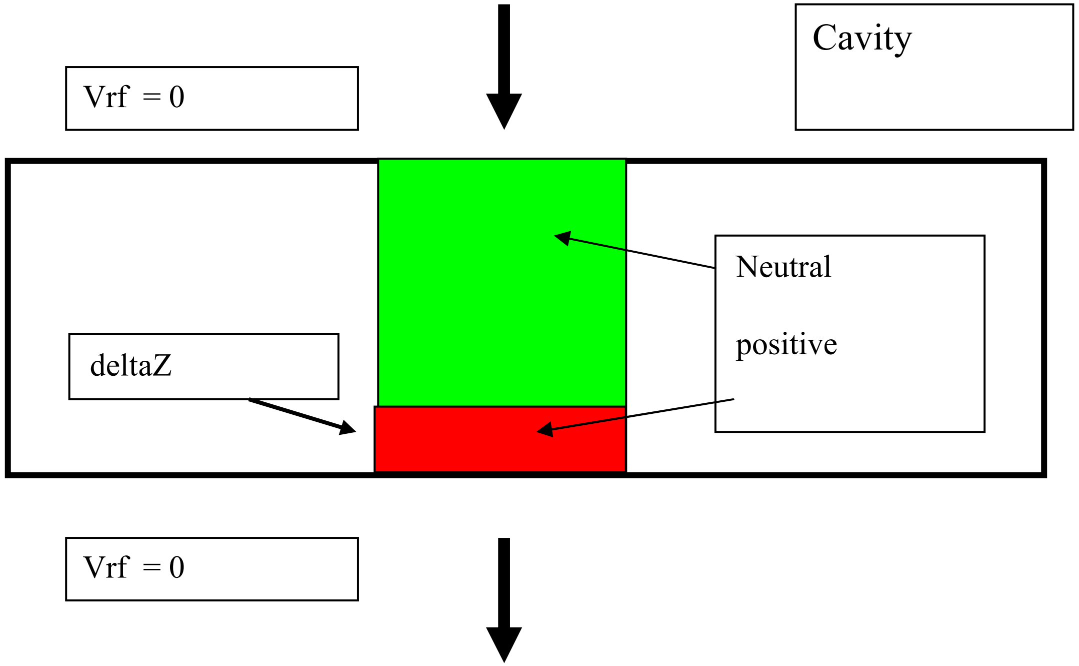
____

1. beamRadius, cm	0.643745
2. H molecule density	9.033×10^{21}
3. av. molecule Spacing in microns	0.000480245
4. muons/cm^2 =	7.68108×10^{10}
5. Averge μ spacing microns	0.0360819
6. Radius 2 eV electron, Bo T field, microns	0.350413
7. spacing between ions along track, microns	2.91667
8. path length to ionize, microns	0.54
9. tforIonization ps	0.522413
10. positive ion density/cm^3	2.63351×10^{14}
11. plasma Frequency	9.14864×10^{11}
12. EoverP, KV/cm/torr =	0.919481
13. Mobility=	0.0187169
14. electron velocity cm/sec=	1.0207×10^{6}
15. deltaZ, cm	0.000977355
16. q/cm^2 cavity,No. electrons	1.38349×10^{11}
17. plasma density x deltaZ/2	1.28694×10^{11}
18. plasma Charge/ Cavity Charge	0.930212
19. deltaW/cm^3 / cycle	0.00686796
20. Qeffective	2.53138
20. E/n (V/cm /Molecules/cm^3	2.78977×10^{-17}
	Null

Beam passes thru cavity at max Vrf as a delta function



After a ¼ cycle the electrons have all drifted upwart by a distance deltaZ. This discharges the top plate and leaves a layer of positive charge against the bottom. The field in this region remains the same and the field outside decreases. A ¼ cycle later the image is reversed.



DOES THE HIGH DENSITY INVALIDATE THE MOBILITY CALCULATION

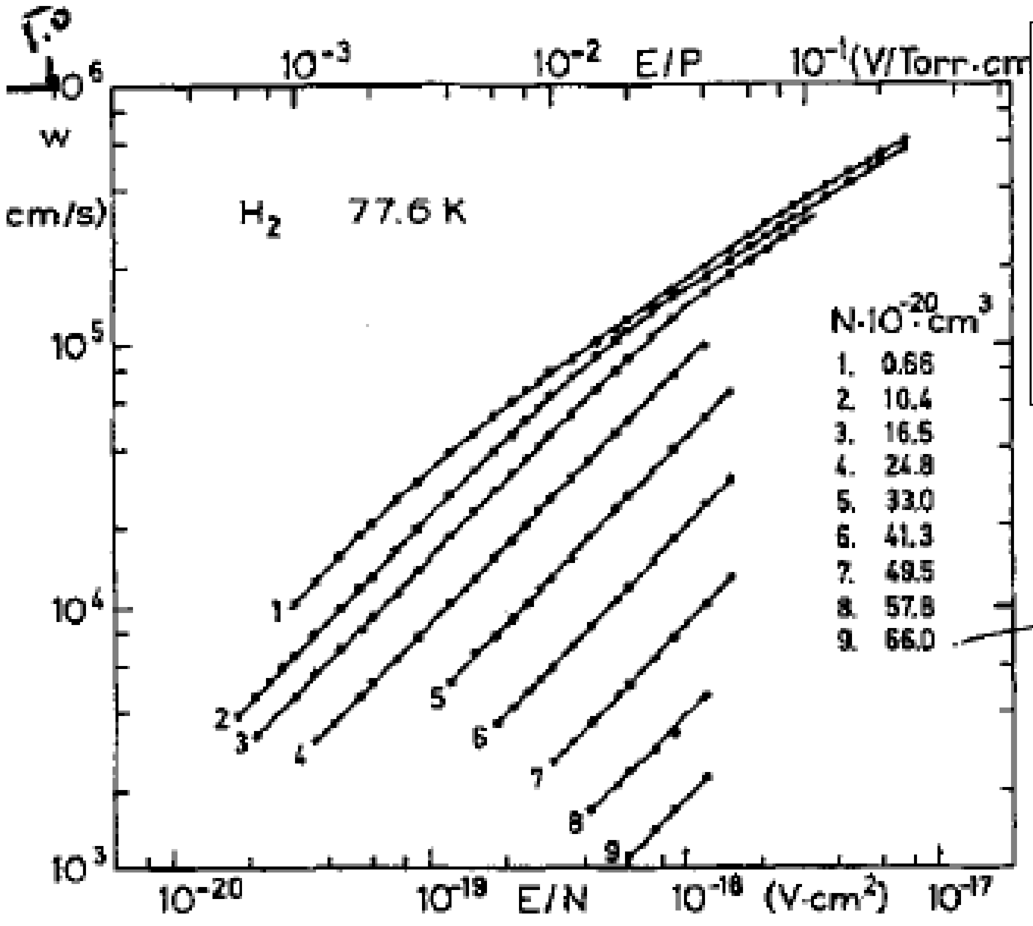


Fig. 2. Electron drift velocity versus E/N in hydrogen at 77.6 K and at nine densities. The points are experimental values. In the scale of E/P is 1 Torr $= 3.3 \times 10^{16}$ cm^{-3}

Conversion from N in the graph at left to gas density in the middle column and equivalent P in torr for the last column.

6.6×10^{19}	0.00022044	1876.39
9.9×10^{20}	0.0033066	28145.8
1.65×10^{21}	0.005511	46909.6
2.48×10^{21}	0.0082832	70506.6
3.3×10^{21}	0.011022	93819.3
4.13×10^{21}	0.0137942	117416.
4.95×10^{21}	0.016533	140729.
5.78×10^{21}	0.0193052	164326.
6.6×10^{21}	0.022044	187639.

I have found no measurements for H in our range

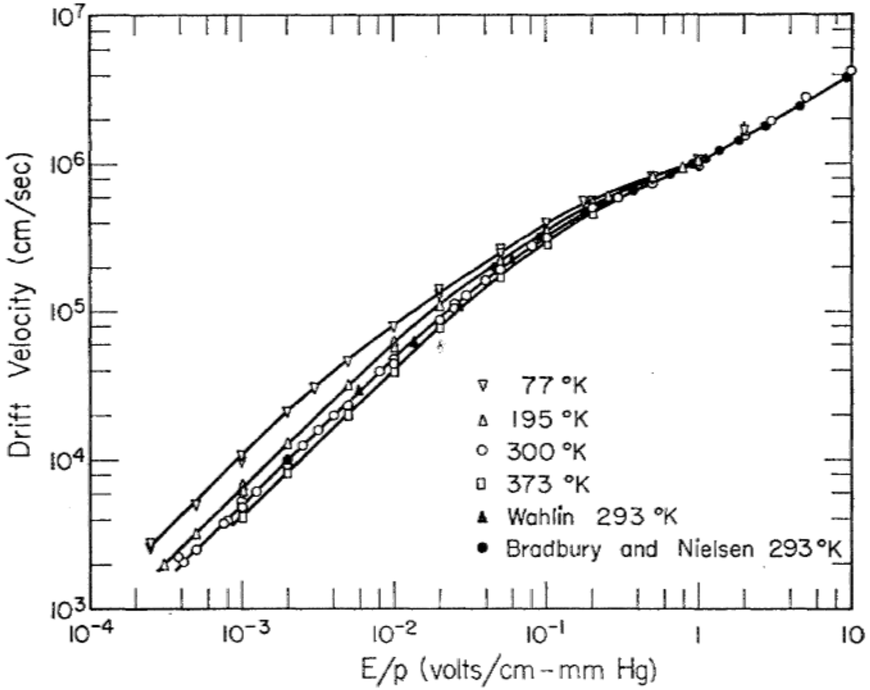


FIG. 7. Electron drift velocity as a function of E/p in hydrogen at 77°K, 195°K, 300°K, and 373°K. For $E/p<3\times10^{-3}$ the electrons are in thermal equilibrium with the gas at each temperature.

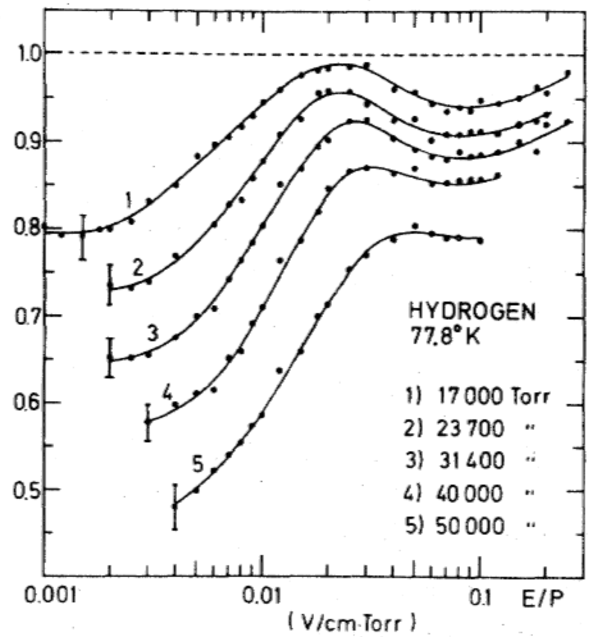
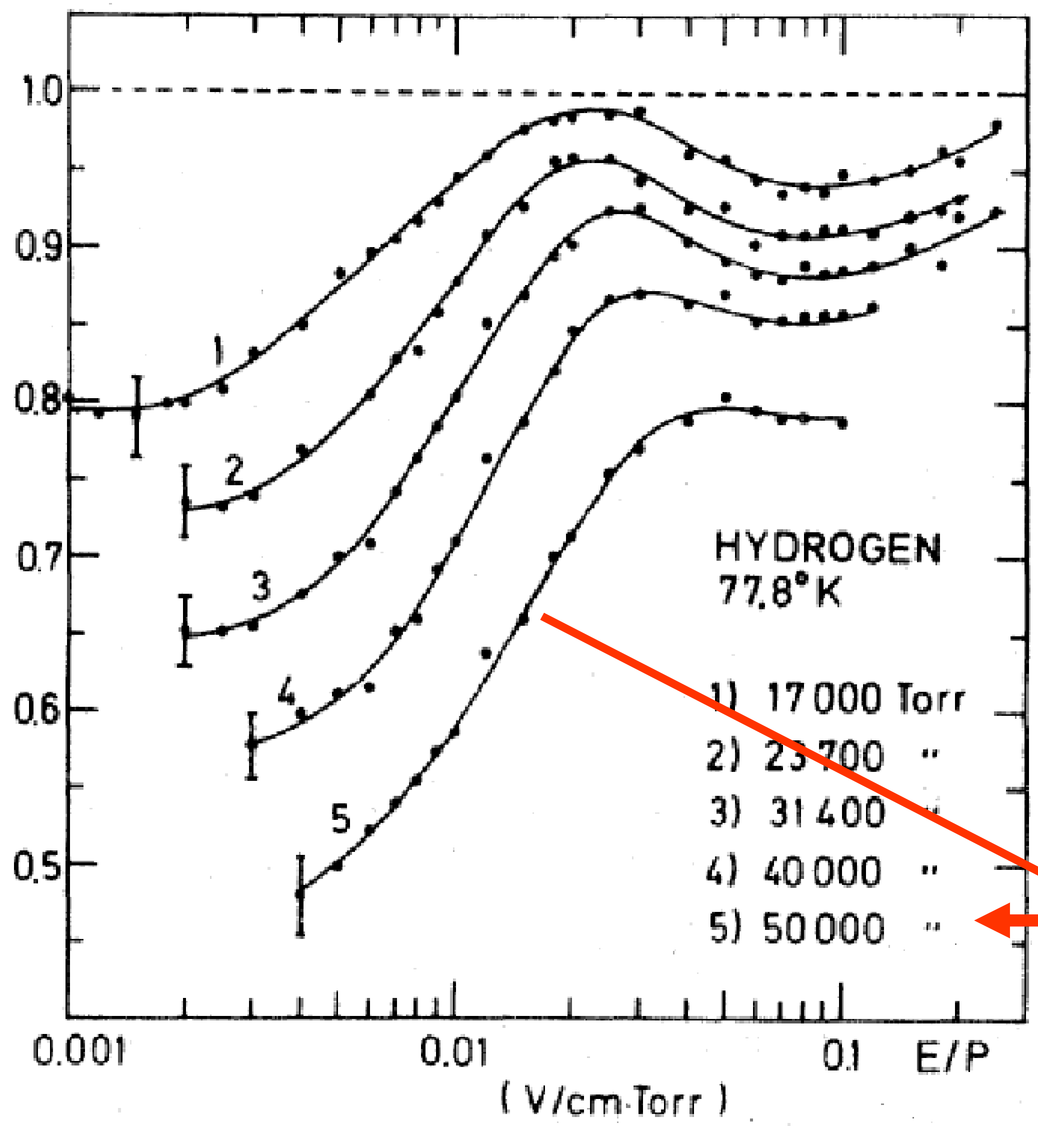


FIG. 1. The quotient q of the drift velocity at high pressures and the drift velocity at low pressure (here 2000 Torr) as a function of E/P.

See Bartels PR 28 1972 Pressure dependence of Electron drift Velocity in Hydrogen at 77.8 K. Pressure is normalized to 273 K



Does the calculation of mobility work at high density? E/P being considered for muons is of order 10.

967 psia

IS RECOMBINATION TAKING PLACE?

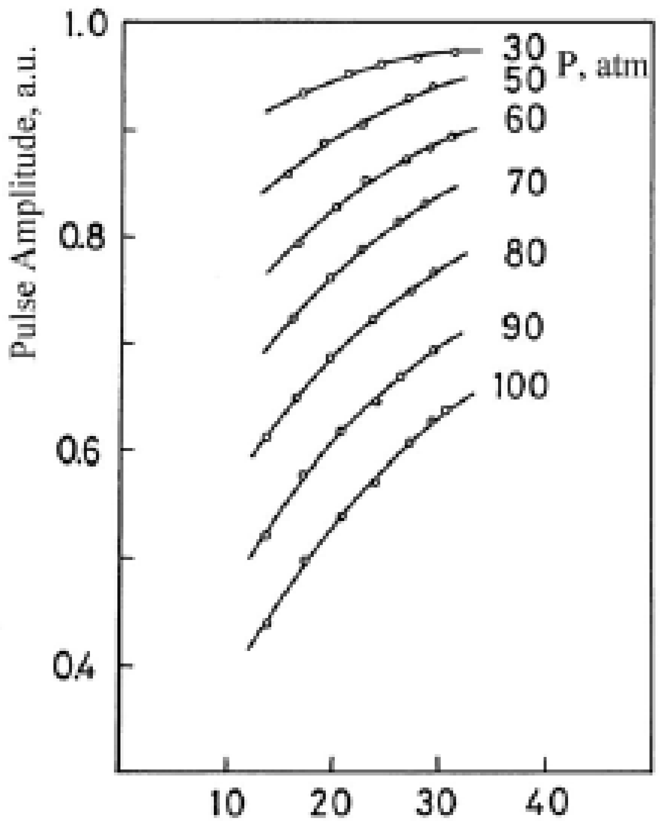


FIG. 5: Dependences of electron-ion recombination on electric field at various gas pressures.

Mobility correct?

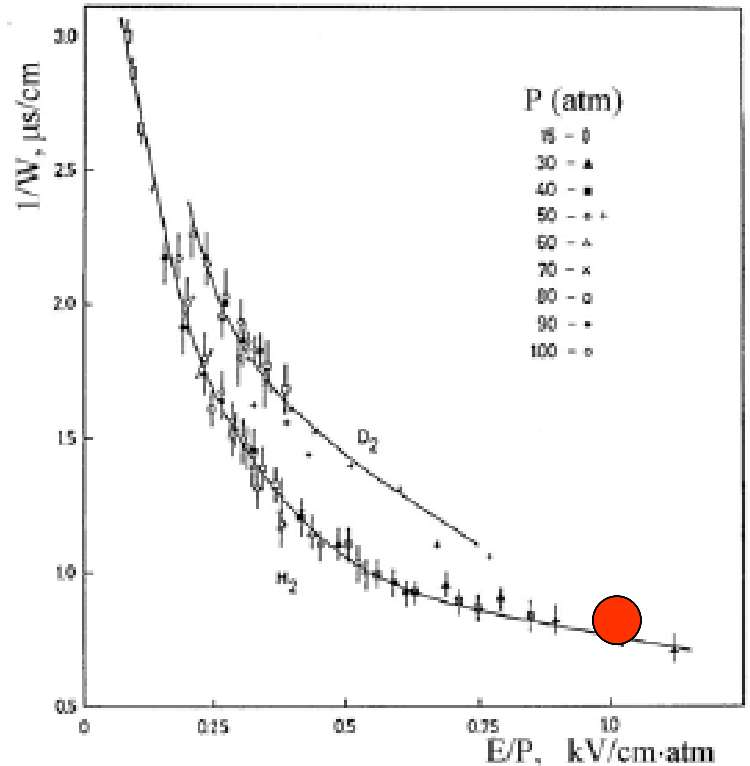


FIG. 6: Electron drift velocities in protium and in deuterium versus ratio of electric field (E) to gas pressure (P).

Data from Kammel at PSI

Fig. 5 shows the charge collected for alpha particles for various gas pressures and drift fields. The peak point in MuonsInc plot is at 600 KV / cm and 54 atm and is far off scale to the right indicating almost complete charge collection.

Fig.6 shows data on the mobility of electrons in high pressure hydrogen. The red point is calculated from the theory of Crompton and Sutton.

EXAMPLE FOR HCC

I got 3 cavities from Yonahara for the HCC that were sized to the beam for 400, 800, and 1600 Mhz. The beam in each case was was 10 bunches of intensity 10 E10 = 10E11 total separated by 2.5 ns. There are 10 of these groups that are coalesced in the collider ring for an intensity of 10 E 12 The following calculations use 10E11 for the total bunch intensity in the cooling channel.

The next slide gives the input data for the three cases.

3 Examples from HCC:

Emittance	4.3 mm rad	1.5 mm rad	.91 mm rad
Freq	400 Mhz	800 Mhz	1600 Mhz
Beta function	.42 m	.26 m	.15 m
Gradient	16 MV/m	16 MV/m	16 MV/m
Beam rms r	27.6 mm	12.8 mm	7.6 mm
Cavity length	5.0 cm	2.5 cm	1.25 cm
Q from plasma	483	553	392
Joules Cav	5.7	.72	.09
Joules gas/cycle	.075	.0082	.0014

H gas density = .017 rgm/cm^3 = 200 atm at 270 deg C.

Beam pulse 10 e10 per bunch 10 bunches = 10 e11 total

800 Mhz cavity radius 15.6 cm. Scaled with Freq.

400 Mhz

P= 250 Bo=4.` betaPerp= 0.417

emit= 0.00182 minEmit= 0.000683

Nbeam= $1. \times 10^{11}$ rhoGas= 0.017`

RF Gradient V/m =$16. \times 10^{6}$,Frf Hz= $400. \times 10^{6}$

1. beamRadius, cm	2.75196
2. H molecule density	5.1187×10^{21}
3. av. molecule Spacing in microns	0.000580344
4. muons/cm^2 =	4.20305×10^{9}
5. Averge μ spacing microns	0.154247
6. Radius 2 eV electron, Bo T field, microns	1.1914
7. spacing between ions along track, microns	5.14706
8. electron path length to ionize, microns	0.960625
9. tforIonization ps	0.81627
10. positive ion density/cm^3	8.16593×10^{12}
11. plasma Frequency	1.61099×10^{11}
12. EoverP, KV/cm/torr =	1.03847
13. Mobility=	0.017612
14. electron velocity cm/sec=	1.08474×10^{6}
15. deltaZ, microns	10.3785
16. q/cm^2 cavity,No. electrons	8.85433×10^{10}
17. plasma density x deltaZ/2	4.23749×10^{9}
18. plasma Charge/ Cavity Charge	0.0478579
19. deltaW/cm^3 / cycle	0.0003158
20. Qeffective	22.5494
20. E/n (V/cm /Molecules/cm^3	3.1508×10^{-17}
	Null

800 Mhz

P= 250 Bo=6.4` betaPerp= 0.261

emit= 0.000634 minEmit= 0.000427

Nbeam= $1.\times 10^{11}$ rhoGas= 0.017`

RF Gradient V/m $=16.\times 10^{6}$,Frf Hz= $800.\times 10^{6}$

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1. beamRadius, cm	1.28497
2. H molecule density	5.1187×10^{21}
3. av. molecule Spacing in microns	0.000580344
4. muons/cm^2 =	1.9278×10^{10}
5. Averge μ spacing microns	0.0720226
6. Radius 2 eV electron, Bo T field, microns	0.744628
7. spacing between ions along track, microns	5.14706
8. electron path length to ionize, microns	0.960625
9. tforIonization ps	0.81627
10. positive ion density/cm^3	3.74544×10^{13}
11. plasma Frequency	3.45017×10^{11}
12. EoverP, KV/cm/torr =	1.03847
13. Mobility=	0.017612
14. electron velocity cm/sec=	1.08474×10^{6}
15. deltaZ, microns	5.18923
16. q/cm^2 cavity,No. electrons	8.85433×10^{10}
17. plasma density x deltaZ/2	9.71798×10^{9}
18. plasma Charge/ Cavity Charge	0.109754
19. deltaW/cm^3 / cycle	0.000724234
20. Qeffective	9.83257
20. E/n (V/cm /Molecules/cm^3	3.1508×10^{-17}
	Null

1600 Mhz

P= 250 Bo=11.` betaPerp= 0.152

emit= 0.000384 minEmit= 0.000248

Nbeam= $1.\times 10^{11}$ rhoGas= 0.017`

RF Gradient V/m =$16.\times 10^{6}$,Frf Hz= 1.6×10^{9}

____

1. beamRadius, cm	0.763419
2. H molecule density	5.1187×10^{21}
3. av. molecule Spacing in microns	0.000580344
4. muons/cm^2 =	5.46166×10^{10}
5. Averge μ spacing microns	0.0427895
6. Radius 2 eV electron, Bo T field, microns	0.433238
7. spacing between ions along track, microns	5.14706
8. electron path length to ionize, microns	0.960625
9. tforIonization ps	0.81627
10. positive ion density/cm^3	1.06112×10^{14}
11. plasma Frequency	5.80726×10^{11}
12. EoverP, KV/cm/torr =	1.03847
13. Mobility=	0.017612
14. electron velocity cm/sec=	1.08474×10^{6}
15. deltaZ, microns	2.59462
16. q/cm^2 cavity,No. electrons	8.85433×10^{10}
17. plasma density x deltaZ/2	1.3766×10^{10}
18. plasma Charge/ Cavity Charge	0.155472
19. deltaW/cm^3 / cycle	0.00102591
20. Qeffective	6.9412
20. E/n (V/cm /Molecules/cm^3	3.1508×10^{-17}
	Null

Recombination

$dN/dt = C \ N^2$

$N(t) = No / (1 + No\ C\ t)$ Not exponential! Very long tail. C is between 10^{-10} and 10^{-8} and No between 10^{11} and 10^{13}.

This gives a ½ life = 1 /(No C) or 1/10 sec to 10 micro sec.

Two different species…for instance if N_2 is SF_6:

$dN_1 /dt = C \ N_1 N_2$ If $N_2 >> N_1$ then this is an exponential decay of N_1.

B field and diffusion

The coefficient of diffusion perpendicular to the field is:

$$D(B) = D(B{=}0)\ \omega_c^2 / [\omega_c^2 + \omega_B^2] \quad \text{where}$$

ω_c is the electron collision frequency and ω_B is the cyclotron freq.

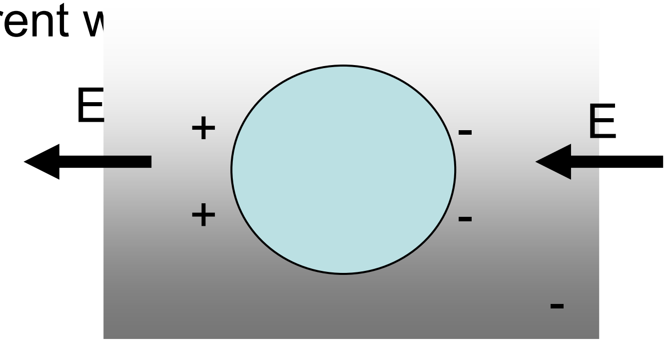
ω_c is of the order of 10^{15} and ω_B is of order 10^{12}

The magnet field does not confine things!

Interesting Problem: Sweep field to clean out ions

1. If all of the electrons were swept out leaving only the positive ions, the radial field would be of the order of 10^9 volts/meter! A sweep field might be 100 V/cm and will be neutralized inside the plasma. See figure below. It isn't clear whether the sweep field or diffusion will predominate in the expansion of the charges. At these high densities, at this low E field the electron velocity is of the order of 1000 cm/sec.

2. 10 e11 muons going thur a 2.5 cm cavity with hydrogen density .017 grm/cm^3 generates a total charge of 90 micro Coulombs. If this could be swept out in 1 ms the current w



s = E eo ; For instance 100 KV/meter requires a surface charge of 88 pC

A factor of 10^6 less than the total charge in the ions.

MTA Test

Consider 800 Mhz cavity in linac beam:

1. Bunch spacing 5 ns and 0.13 ns wide with about 10 e9/ bunch or less. So the bunches are delta functions crossing every 4th cycle. We assume that the cavity is synched to the linac. The first couple of bunches may be smaller as the batch gets gated on. There can be up to 1000 bunches for a total to 1.7 10e12 protons with βγ=1 for a dE/dx= 6 MeV/gm/cm^2 or 1.5 times min. So each bunch is equivalent to

2.5 10e9 mip. Using the numbers from slide 2 of 8.2 10e-3 joules loss/cycle for 10e11 mip we get for 4 cycles and 1 bunch:

$$4 \ \ 2.5 \ 10^{9} \ \ \ 8.2 \ \ 10^{-3} \ / \ 10^{11} = 8.2 \ 10^{-4} \text{ joules/4 cycles (or bunch)}$$

Energy stoired in cavity = .72 joules

So the energy loss/bunch is like the single bunches of 10^{10} particles but spread over 4 cycles. So one will be able to measure the loading of the cavity by the beam by measuring the loading of the cavity which increases linearly thru the linac pulse.

Maybe one wants to turn off power to cavity before beam hits and watch it decay in order to measure loading.

Questions and things to measure

1. What is the mobility at high pressures?

2. Can we get an estimate of the recombination rate? How long ions hang around? Does it matter?

3. What is the recombination time with a little SF_6 added?
   $1/n1\ dn1/dt = R\ n_2$ If the right side is 10^9 and n_2 is 1% hydrogen (X_0 SF6 is 1/150 X_H) $n_2 = 10^{22}/100$ Then $R = 10^{-11}$
   R seems to be in range I can find for various processes.

4. Need measurements for narrow beam and high intensity as ex end of cooling change.

5. Open cell cavities are different. There is a column of plasma a end electrodes. B keeps the plasma from diffusing in radial di very fast so at peak of cycle there are blobs of positive charge between cells.

6. The linac has 10 μSec pulse. Is there any way to get single bunches?
7. A small light pipe leading out of the cavity to observe light could be useful.

Questions and things to measure

1. What is the mobility at high pressures?
2. Can we get an estimate of the recombination rate? How long do the ions hang around? Does it matter?
3. What is the recombination time with a little SF_6 added?
   $1/n1 \; dn1/dt = R \; n_2$ If the right side is 10^9 and n_2 is 1% of the hydrogen (X_0 SF6 is 1/150 X_H) $n_2 = 10^{22}/100$ Then $R = 10^{-11}$ This R seems to be in range I can find for various processes.
4. Need measurements for narrow beam and high intensity as exist at end of cooling change.
5. Open cell cavities are different. There is a column of plasma and no end electrodes. B keeps the plasma from diffusing in radial direction very fast so at peak of cycle there are blobs of positive charge between cells.

6. The linac has 10 μSec pulse. Is there any way to get single bunches?
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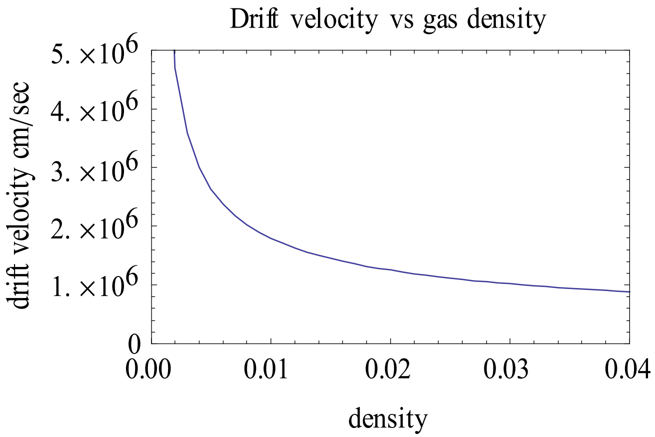
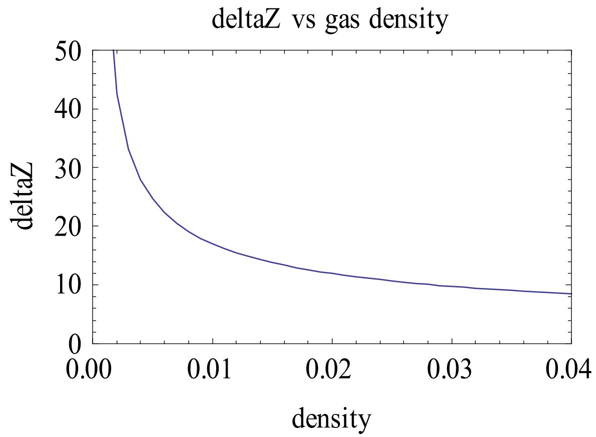
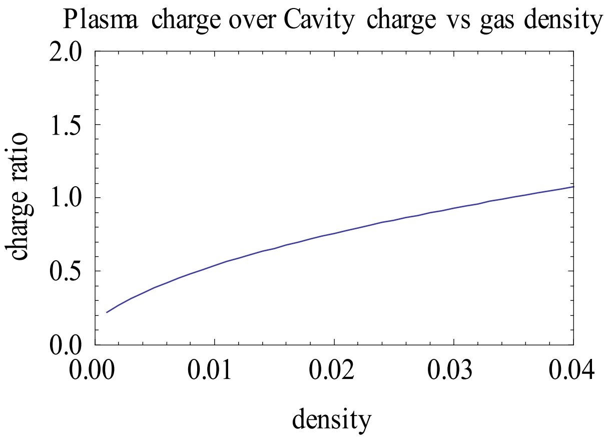
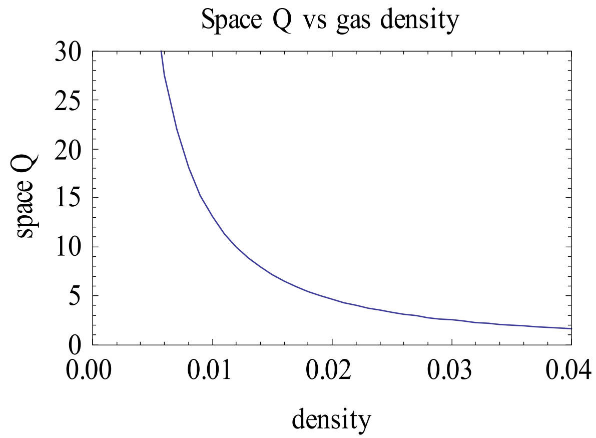
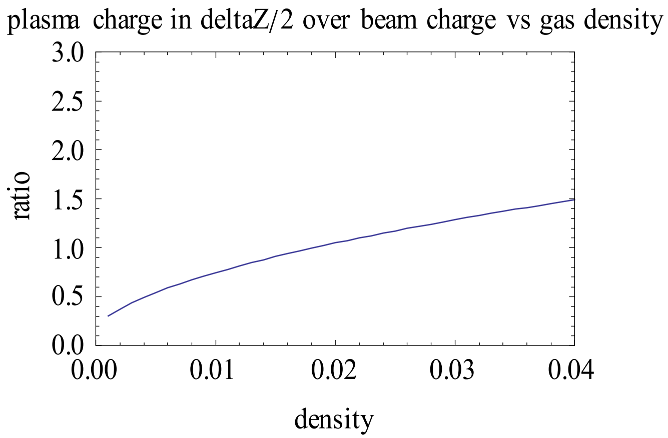


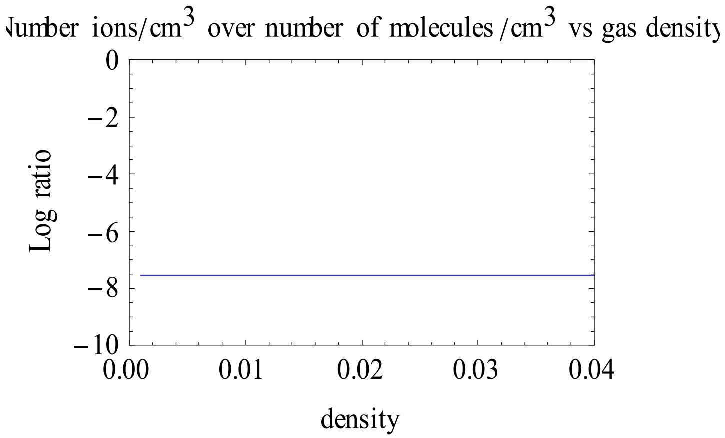


See “Example” for conditions other than density.









400 Mhz

Initial INPUT DATA	*********	muon p	250
B Field T.	4.	Normalized Emit,m-rad	0.0043
emittance, m-rad	0.00181632	muons/pulse	$1.\times 10^{11}$
Focus beta, m	0.416959	Min. channel emit	0.000683081
H gas density grm/cm^3	0.017	RF cavity Grad MV/meter	1.6×10^{7}
Cavity Freq. Mhz	$4.\times 10^{8}$	Cavity (rc,lc) m	0.312 0.05

800 Mhz

Initial INPUT DATA	*********	muon p	250
B Field T.	6.4	Normalized Emit,m-rad	0.0015
emittance, m-rad	0.0006336	muons/pulse	$1.\times 10^{11}$
Focus beta, m	0.260599	Min. channel emit	0.000426926
H gas density grm/cm^3	0.017	RF cavity Grad MV/meter	1.6×10^{7}
Cavity Freq. Mhz	$8.\times 10^{8}$	Cavity (rc,lc), m	0.156 0.025

1600 Mhz

Initial INPUT DATA	*********	muon p	250
B Field T.	11.	Normalized Emit,m-rad	0.00091
emittance, m-rad	0.000384384	muons/pulse	$1.\times 10^{11}$
Focus beta, m	0.151621	Min. channel emit	0.000248393
H gas density grm/cm^3	0.017	RF cavity Grad MV/meter	1.6×10^{7}
Cavity Freq. Mhz	1.6×10^{9}	Cavity (rc,lc), m	0.078 0.0125