Title: Pythagorean Theorem and Its Converse

Brief Overview:

The formula for the Pythagorean Theorem is $a^2 + b^2 = c^2$. This lesson is designed to enable students to prove the theorem and its converse geometrically and to solve problems involving right triangles.

Link to Standards:

- **Problem Solving** Students will demonstrate their ability to solve mathematical problems through the use of the Pythagorean Theorem.
- **Communication** Students will demonstrate their ability to communicate their hypotheses and their conclusions both verbally and in writing.
- **Reasoning** Students will use and investigate various methods to prove the Pythagorean Theorem and its Converse.
- **Algebra** Students will demonstrate their ability to solve equations using the Pythagorean Theorem.
- **Geometry** Students will use geometry vocabulary and concepts to prove the Pythagorean Theorem and its Converse.

Grade/Level:

Grades 9–12, Geometry

Duration/Length:

This lesson will take 2 or 3 periods (90 min.).

Prerequisite Knowledge:

Students should have working knowledge of the following:

- The Pythagorean Theorem
- Solving equations involving exponents and radicals
- The area formula for polygons
- *The Geometer's Sketchpad*

Objectives:

Students will:

- work cooperatively in pairs.
- state the Pythagorean Theorem and its converse.
- prove the theorem using *The Geometer's Sketchpad*.
- solve problems using the Pythagorean Theorem and its Converse.
- recognize Pythagorean primitive triples and their multiples.

Materials/Resources/Printed Materials:

- *The Geometer's Sketchpad* Software
- Bennet, Dan. **Exploring Geometry with *the Geometer's Sketchpad*.** Key Curriculum Press, 1992.
- Dolciani, Mary. **Algebra Structure and Method Book 1.** Houghton Mifflin, 1986.
- Jurgensen, R., & Brown, R. **Geometry.** Houghton Mifflin, 1992.
- Serra, Michael. **Discovering Geometry.** Key Curriculum Press, 1993.
- Student Worksheets

Development/Procedures:

Day 1:

1. Introduce the lesson with a discussion of the Pythagorean Theorem. Have students state the algebraic formula $a^2 + b^2 = c^2$ and its application to a right triangle with legs of length a and b and hypotenuse of length c. Have them name a set of values that satisfy this equation.

2. Students will work in pairs on the computer on Investigation #1.

3. For a standard class, have students complete the "Pythagorean Theorem Investigation" on p. 227 in **Exploring Geometry with *The Geometer's Sketchpad***. This comprises investigation 2a. "Dissection Proof of the Pythagorean Theorem" on p. 233 of the same book could be used for an honors class or an extension in the standard geometry class; this comprises investigation 2b. Another suggestion is to divide the class into two groups and have each group work on one of the investigations. In this case, have each group explain their investigation.

Day 2:

1. Students will work on investigation #3 to test the Converse of the Pythagorean Theorem.

2. Students will continue with an investigation found on p. 235 of **Exploring Geometry with *The Geometer's Sketchpad***.

Additional worksheets can be used as either follow up or enrichment activities.

Evaluation:

The teacher will circulate around the classroom to make certain all students are on task. Student work done using *The Geometer's Sketchpad* will be assessed through the worksheets and additional activities.

Extension/Follow Up:

- Have students design a proof of the Pythagorean Theorem using a regular polygon other than a square on each of the sides of the right triangle. Evaluation of the project can be assessed by demonstration and observation or printed and collected.

- Write a conjecture for finding the length in the third dimension. Are there any Pythagorean quadruples in the third dimension?

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Investigation I: Does the Pythagorean Theorem Work?

In this investigation you will construct a right triangle and demonstrate that if **a** and **b** are the legs of the triangle and **c** is the hypotenuse, then $a^2 + b^2 = c^2$.

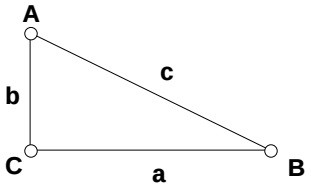
Start with a blank sketch and construct a right triangle. Label the vertices A, B, C, with C being the vertex of the right angle, and the sides opposite the angles a, b, and c respectively.

Investigate

Measure the length of the legs and the hypotenuse.

Calculate $a^2 + b^2$ and c^2 .

Does $a^2 + b^2 = c^2$? _____



Create a table for the values of $a^2 + b^2$ and c^2 .
Select point A and drag it.
Add these measurements to your table by double clicking on the table.
Repeat these two steps for points B and C.

What do you notice about the sum of the squares of the legs and the square of the hypotenuse when the measurements change?

_____

Does the Pythagorean Theorem hold true in each case? _____

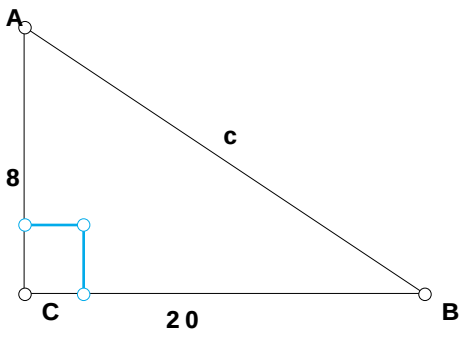
If so, why or why not? _____

_____

WORKSHEET FOR INVESTIGATIONS 1 and 2

I. Find the measure of the missing side. Show all work!!

a.



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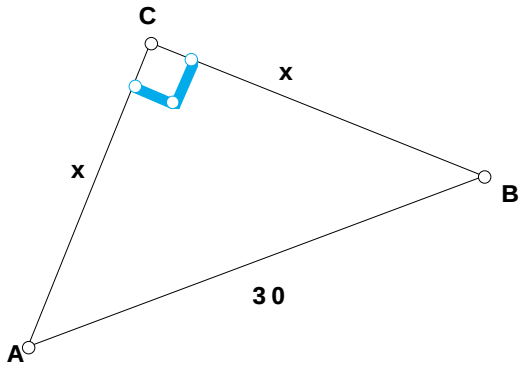
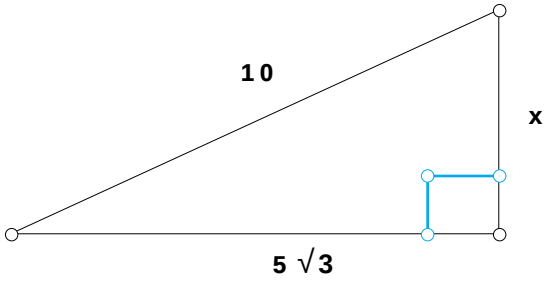


c.



II. Solve the following problems. Show all work!!

a. Given a right triangle with legs 5cm and 12cm, find the length of the hypotenuse.

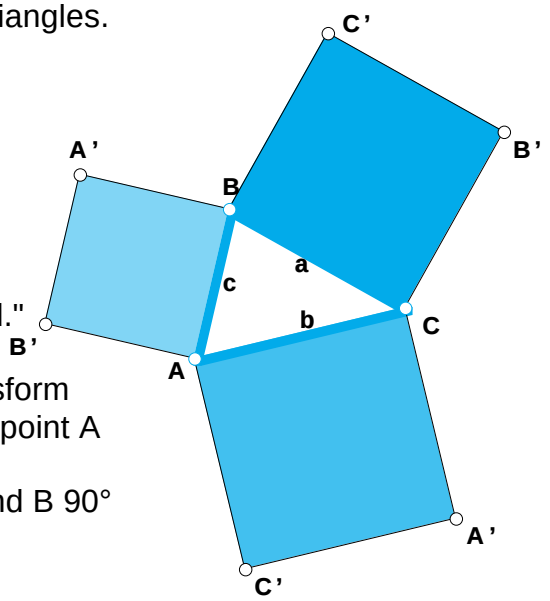
b. Given an isosceles right triangle with side 6m, what is the length of the hypotenuse?

c. If the length of the diagonal of a square is 5yds, find the length of each side of the square. Also find the area of the square.

d. If the length of the hypotenuse is $9\sqrt{3}$cm and one leg measures $8\sqrt{2}$cm, find the length of the other leg. Also, find the area of the right triangle.

Investigation 3: Is the Converse True?

In this investigation you'll create a script for constructing a square, then construct squares on the sides of a non-right triangle. You will test whether or not the Pythagorean Theorem applies to other triangles.



Sketch

Record a script for constructing a square:

Step 1: Open a new script and click "record."
Step 2: Construct AB.
Step 3: Mark point B as center in the Transform menu, and rotate the segment and point A 90° about point B.
Step 4:: Mark A' as center and rotate BA' and B 90° about A'.
Step 5: Construct AB'. Stop your script.

Start with a blank sketch and construct a non-right triangle (not too large and approximately in the center of your screen). Label the vertices A, B, C and the segments a, b, c respectively .

Investigate

Play your square script on the two endpoints of each side of your triangle. If your script constructs the square to fall into the triangle, undo and select the points in the opposite order. Construct the interiors of the squares using different colors.

Measure the areas of the squares and <ACB. Find $a^2 + b^2$.
Create a table showing m<ACB, $a^2 + b^2$, and c^2.

1. a) Does $a^2 + b^2 = c^2$? _____
   b) What is the m<ACB? _____

Now drag point A to change m<ACB. Add the new measurements to your table by double clicking on your table.

2. a) Does $a^2 + b^2 = c^2$? _____
   b) What is the m<ACB? _____

Drag point A in another direction. Add these measurements to your table.

3. a) Does the Pythagorean Theorem hold in this case? _____
   b) What is the m<ACB? _____

Drag point A until $a^2 + b^2$ does equal c^2.

4. What is the m<ACB? _____

5. When $a^2 + b^2 = c^2$, what kind of triangle do you have? _____

What you have just discovered is the **Converse of the Pythagorean Theorem.** State the converse in your own words below.

_____

_____

_____

Exercise Set

1. Use the Converse of the Pythagorean Theorem to determine whether each triangle is a right triangle.

   a. 15, 8, 17

   b. 12, 36, 35

   c. √3, √2, √5

2. A triangular plot of land has boundary lines 45 meters, 60 meters, and 70 meters long. The 60 meter boundary line runs north-south. Is there a boundary line for the property that runs due east-west?

PYTHAGOREAN WORKSHEET

1. A rope 17 m long is attached to the top of a flagpole. The rope is able to reach a point on the ground 8 m from the base of the pole. Find the height of the flagpole.

2. The base of an isosceles triangle is 16 cm long. The equal sides are each 22 cm long. Find the altitude of the triangle.

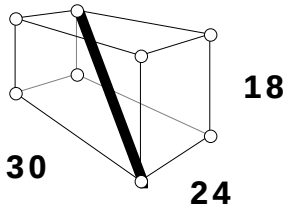
3. The dimensions of a rectangular doorway are 200 cm by 80 cm. Can a circular mirror with a diameter of 220 cm be carried through the doorway?

4. At Martian high noon, Dr. Rhonda Bend leaves the Martian U.S. Research Station traveling due east at 60 km/hr. One hour later Professor I.M. Bryte takes off from the station heading north straight for the polar ice cap at 50 km/hr. How far apart will the doctor and the professor be at 3 P.M. Martian time? Express your answer to the nearest kilometer.

5. A flagpole has cracked 9 feet from the ground and fallen as if hinged. The top of the flagpole hit the ground 12 feet from the base. How tall was the flagpole before it fell?

6. Ellen is standing on a dock 1.5 m above the water. She is pulling in a boat that is attached to the end of a 3.9 m rope. If she pulls in 1.2 m of rope, how far did she move the boat?

7. What is the longest stick that can be placed inside a box with inside dimensions of 24 inches, 30inches, and 18 inches?



Answers for Worksheets

Investigations 1 and 2

I. a. $4\sqrt{29}$ or 21.54
   b. $15\sqrt{2}$ or 21.21
   c. 5

II. a. 13 cm
   b. $6\sqrt{2}$ or 8.49 cm
   c. side = $(5\sqrt{2})/2$ or 3.54 yds; area = 12.5 sq. yds
   d. leg = $\sqrt{115}$ or 10.72 cm; area = $4\sqrt{230}$ or 242.65 sq. cm

Pythagorean Worksheet

1. 15 m
2. $2\sqrt{105}$ or 20.49 cm
3. No
4. $20\sqrt{106}$ or 205.91 km
5. 24 feet
6. $30\sqrt{2}$ or 42.43 inches