ow-Frequency Gravitational Wave Searches Using Spacecraft Doppler Tracking

ppler Method

ınal transfer function

ises and their transfer functions

nents to date: some results from GLL/MO/MGS/Pioneer/ULS

i Experiment

ıtus

pected sensitivity

› do better than Cassini with earth-spacecraft Doppler tracking?

J. Anderson
B. Bertotti
F. Estabrook
R. Hellings
L. Iess
M. Tinto
H. Wahlquist

and many engineers and analysts from the JPL technical divisions, the Deep Space Network, and the flight projects

SOME JARGON

DSN	Deep Space Network, the NASA/JPL tracking system with antennas in California, Spain, and Australia
S-band	radio frequency ≈ 2.3 GHz (e.g., Galileo)
X-band	radio frequency ≈ 8.4 GHz (e.g., Mars Observer)
Ka-band	radio frequency ≈ 32 GHz (e.g., Cassini)
$y(t)$	time series of $\Delta f/f$
$S_y(f)$	power spectrum of $y(t)$
$S_\phi(f)$	power spectrum of phase

MORE JARGON

Allan variance	$\sigma_y(\tau)$, a measure of fractional frequency stability, $\Delta f/f$, as a function of integration time $\sigma_y^2(\tau) = \frac{1}{2}\left\langle \left\|\bar{y}(t) - \bar{y}(t+\tau)\right\|^2 \right\rangle$ $\bar{y}(t) = \frac{1}{\tau}\int_t^{t+\tau} y(t')\,dt'$ $\sigma_y^2(\tau) = 4\int_0^\infty S_y(f)\frac{\sin^4(\pi f\tau)}{(\pi f\tau)^2}\,df$ $S_y(f) = S\phi(f)\cdot f^2 f_0^{-2}$
scintillation	variation of phase of radio signals due to refractive index variations by a medium (solar wind, ionosphere, troposphere) between the source and the receiver

JARGON (CONCLUDED)

clock	precision frequency standard
uplink	radio beam transmitted from the earth to a distant spacecraft
downlink	radio beam transmitted from a distant spacecraft to the earth
DSS	Deep Space Station. Followed by a number it designates antennas within the Deep Space Network, as in "DSS 25"

REPRESENTATIVE REFERENCES

Regarding the method:

Estabrook and Wahlquist, *GRG*, 6, 439 (1975)

Wahlquist *GRG*, 19, 1101 (1987)

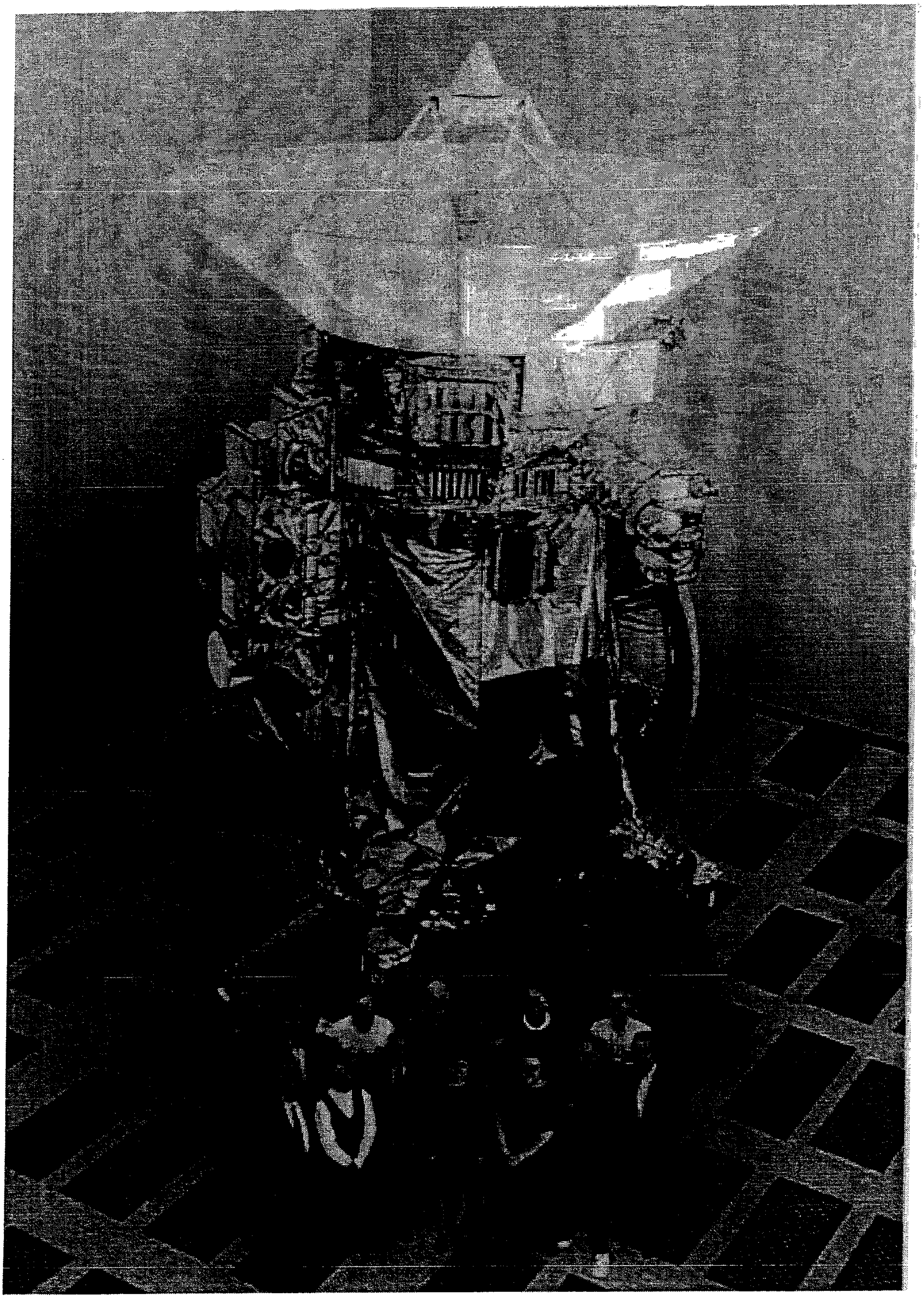
Tinto *Phys. Rev. D.* 53, 5354 (1996)

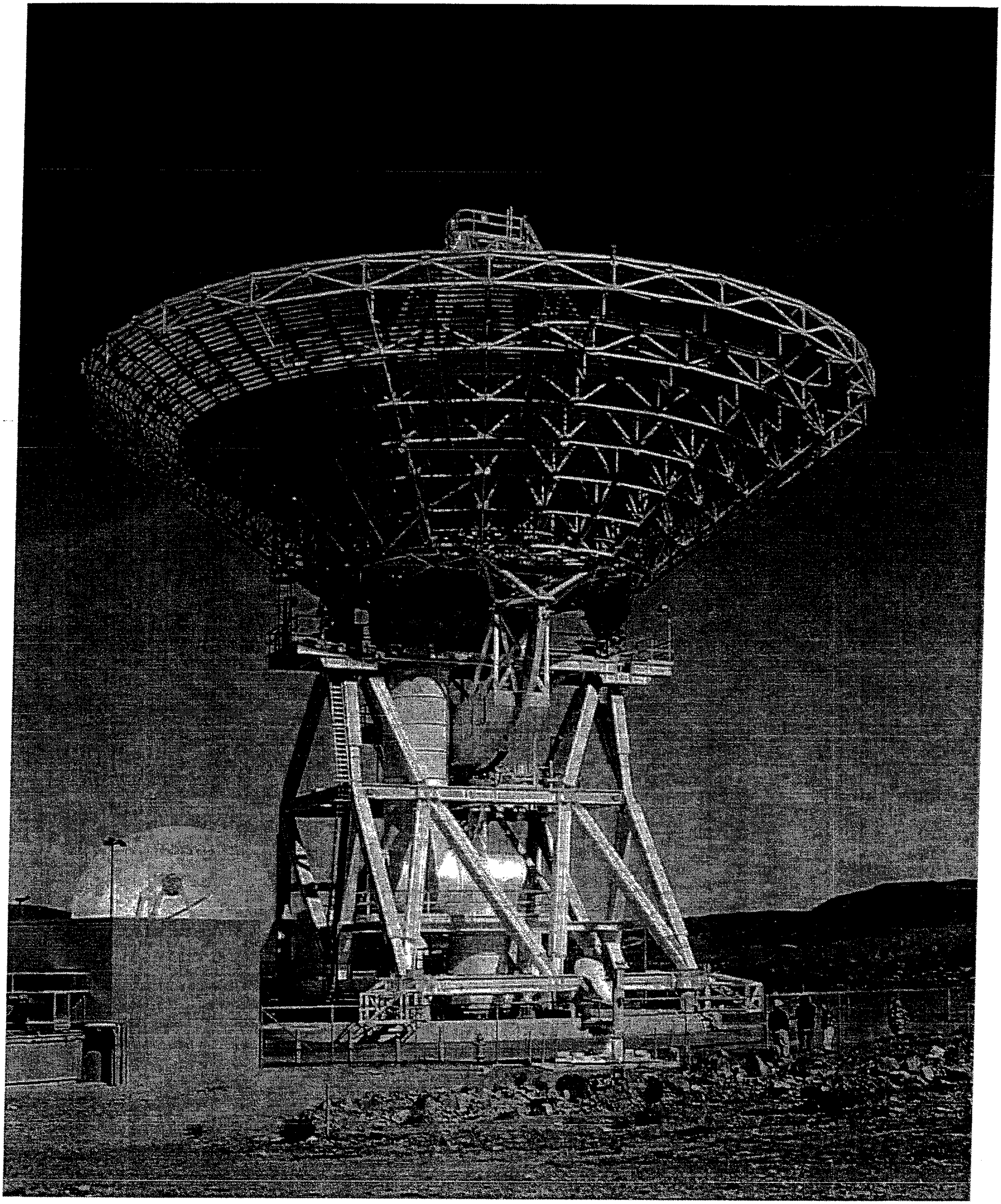
Regarding the noises:

Armstrong, Woo, and Estabrook *Ap. J.* 230, 570 (1979)

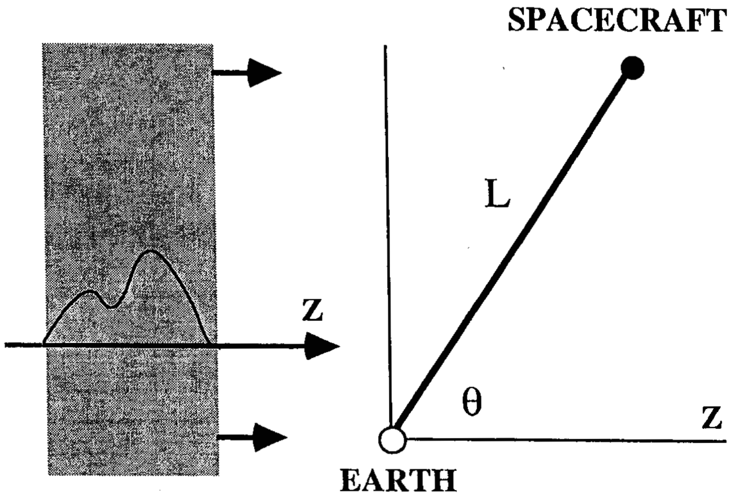
Armstrong *Radio Sci.* 33, 1727 (1998)

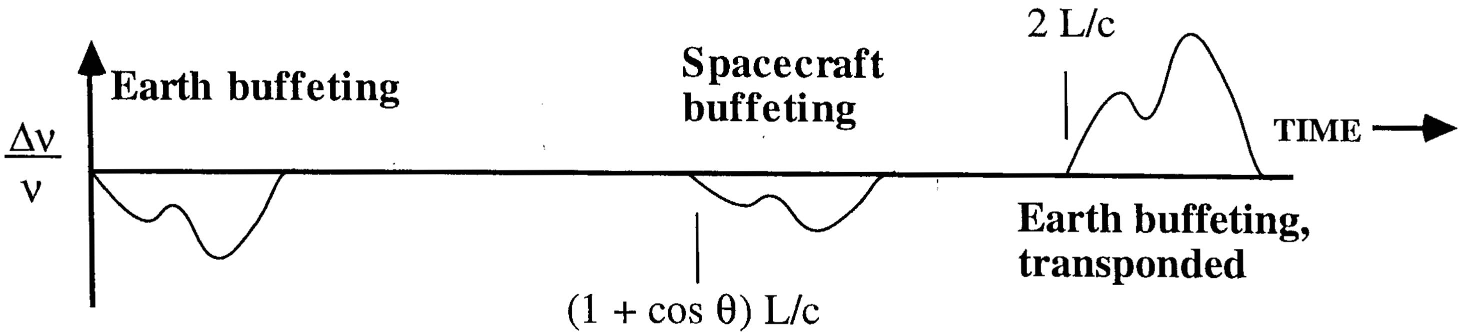
Iess et al. *CQG* 16, 1487 (1999)





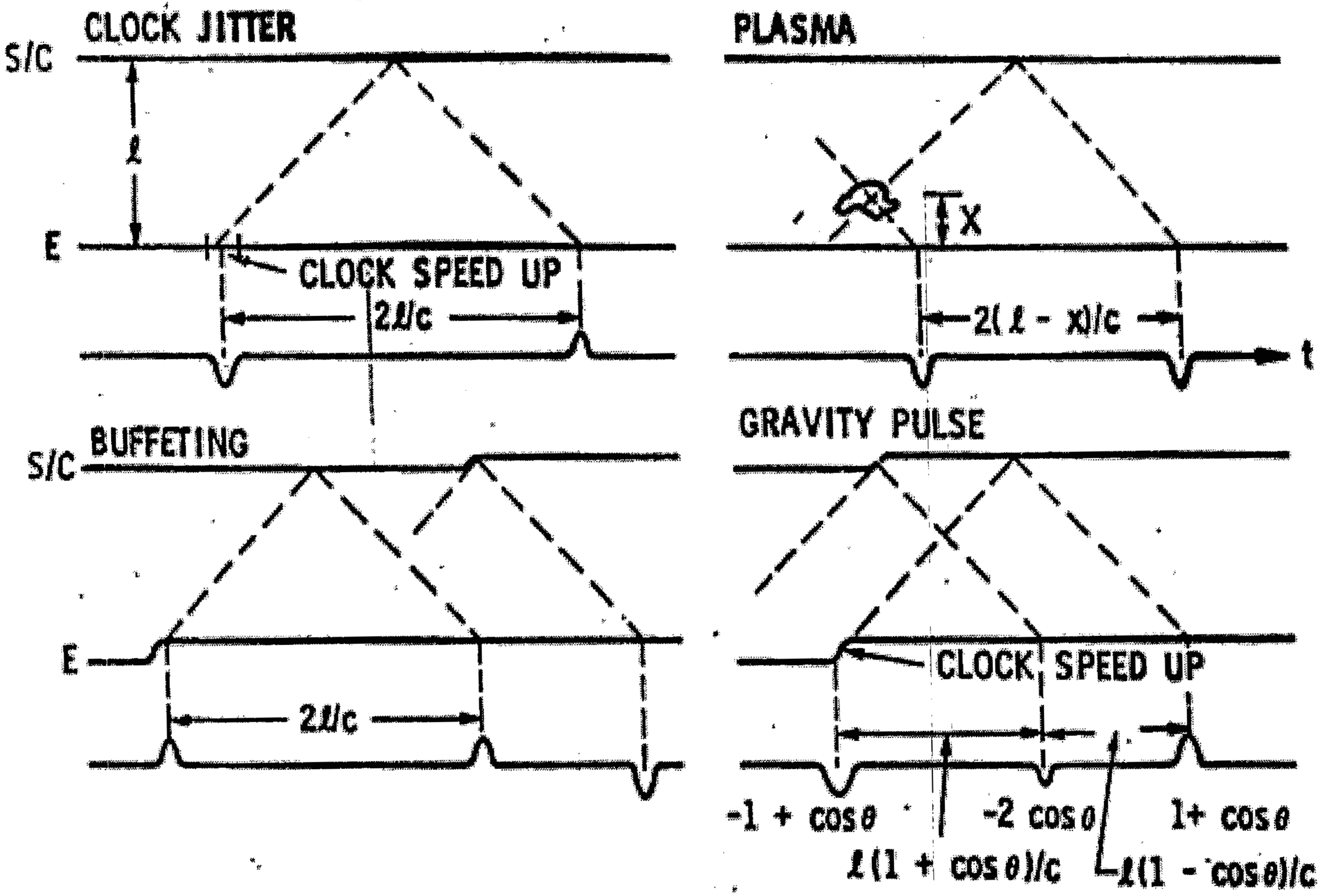
Response of Spacecraft Doppler Tracking to Gravitational Wave





Estabrook and Wahlquist, *Gen. Rel. Grav.* 6, 439 (1975)

DOPPLER SIGNALS CORRESPONDING TO DIFFERENT TYPES OF DISTURBANCE IN THE COMMUNICATION LINK



Estimated Value of S/C's σ_Z^{GWE}

- **Results:**
  - 40 hours of data beginning 2001-DOY-152/T02:00:03.558 (sampling time is 4 s)
  - CAPS articulation motion was active over this time span (articulation frequency ≈ 0.0025 Hz)
  - Estimated value of σ_Z^{GWE} is ≈ 0.071 μm/sec. It is significantly better than the requirement (0.3 μm/sec)

- **Corresponding Allan deviation**

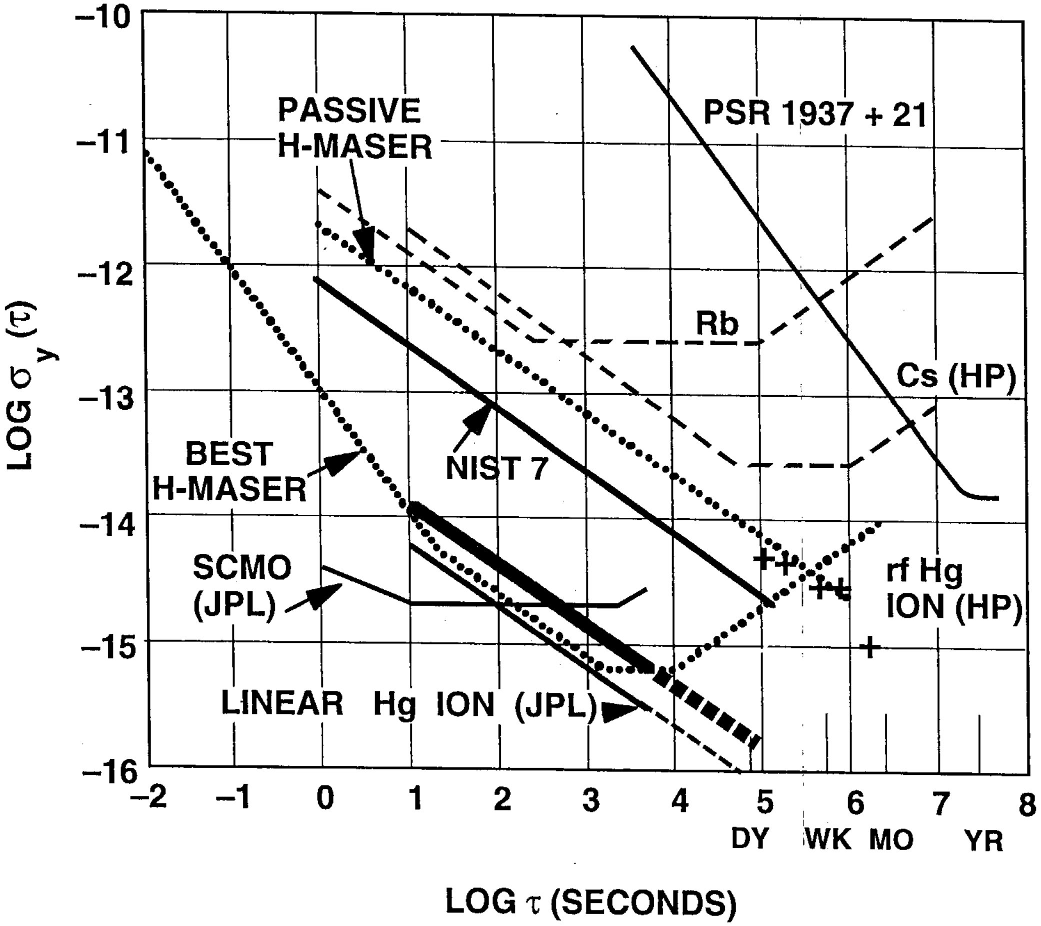
  $\approx 2.3\times10^{-16} \quad < 10^{-15}$ (Requirement)

from: Won, Hanover, Belenky & Lee 10/18/01

MAIN NOISES: FREQUENCY STANDARD STABILITY

- Spacecraft Doppler tracking is not interferometric; coherence is maintained through the frequency and timing system. Thus FTS is fundamental
- Transfer function in two-way Doppler: $\delta(t) - \delta(t - T_2)$
- Cassini era LITS/SCO has excellent stability on integration times 1–10,000 seconds (see Allan deviation plot, due to L. Maleki)

LINEAR ION TRAP STANDARD (LITS)
FRACTIONAL FREQUENCY STABILITY

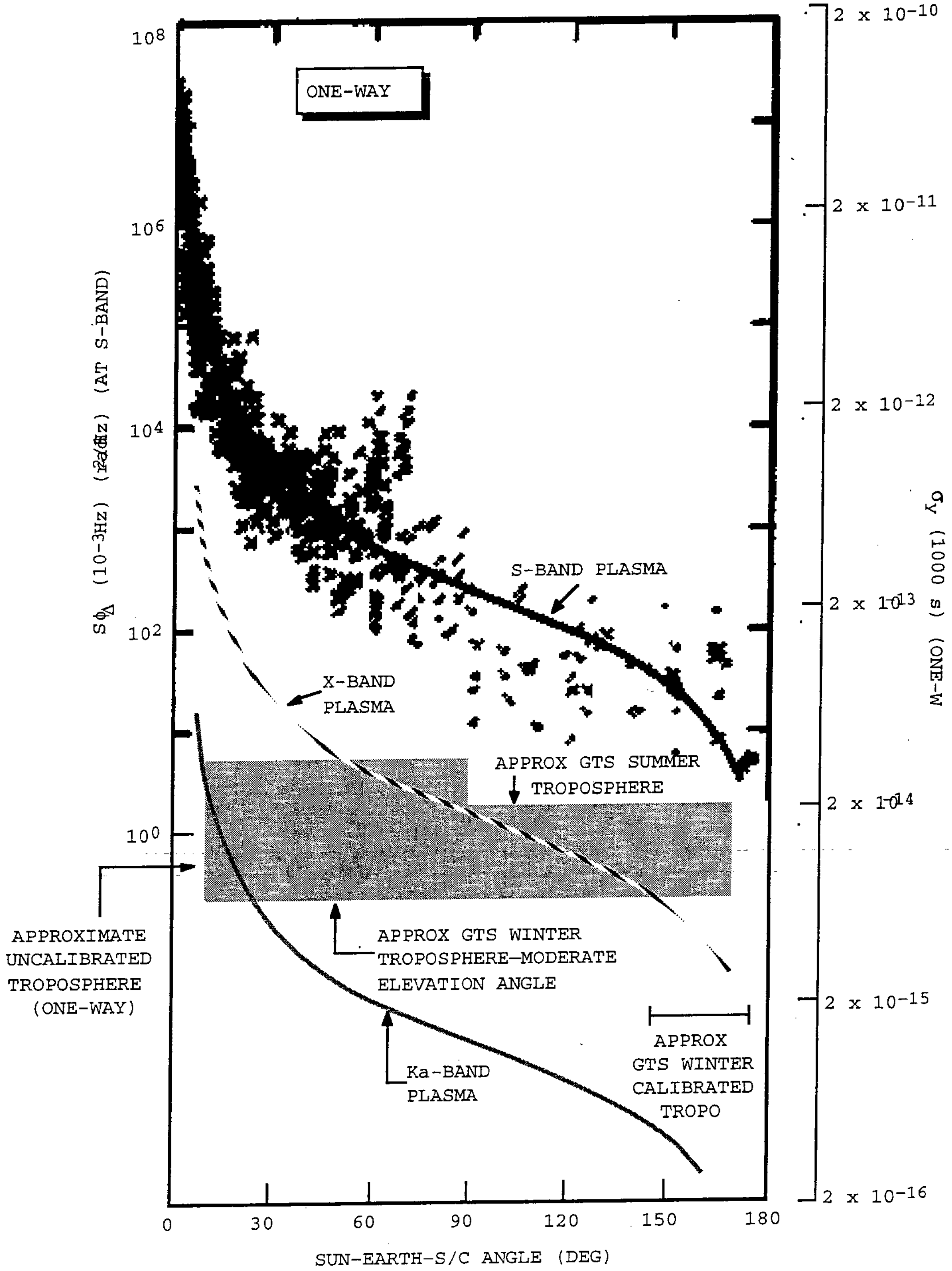


CaJAGWR-14
11/3/00

MAIN NOISES: PLASMA SCINTILLATION

- Dispersive, refractive index fluctuations proportional to λ^2
- Transfer function in two-way Doppler: $\delta(t) + \delta(t - T_2 + 2x/c)$
- Plasma scintillation is dominant noise in S-band observations (even at opposition), but a secondary noise source for Ka-band observations at opposition

INTERPLANETARY PHASE SCINTILLATION



MAIN NOISES: TROPOSPHERIC SCINTILLATION

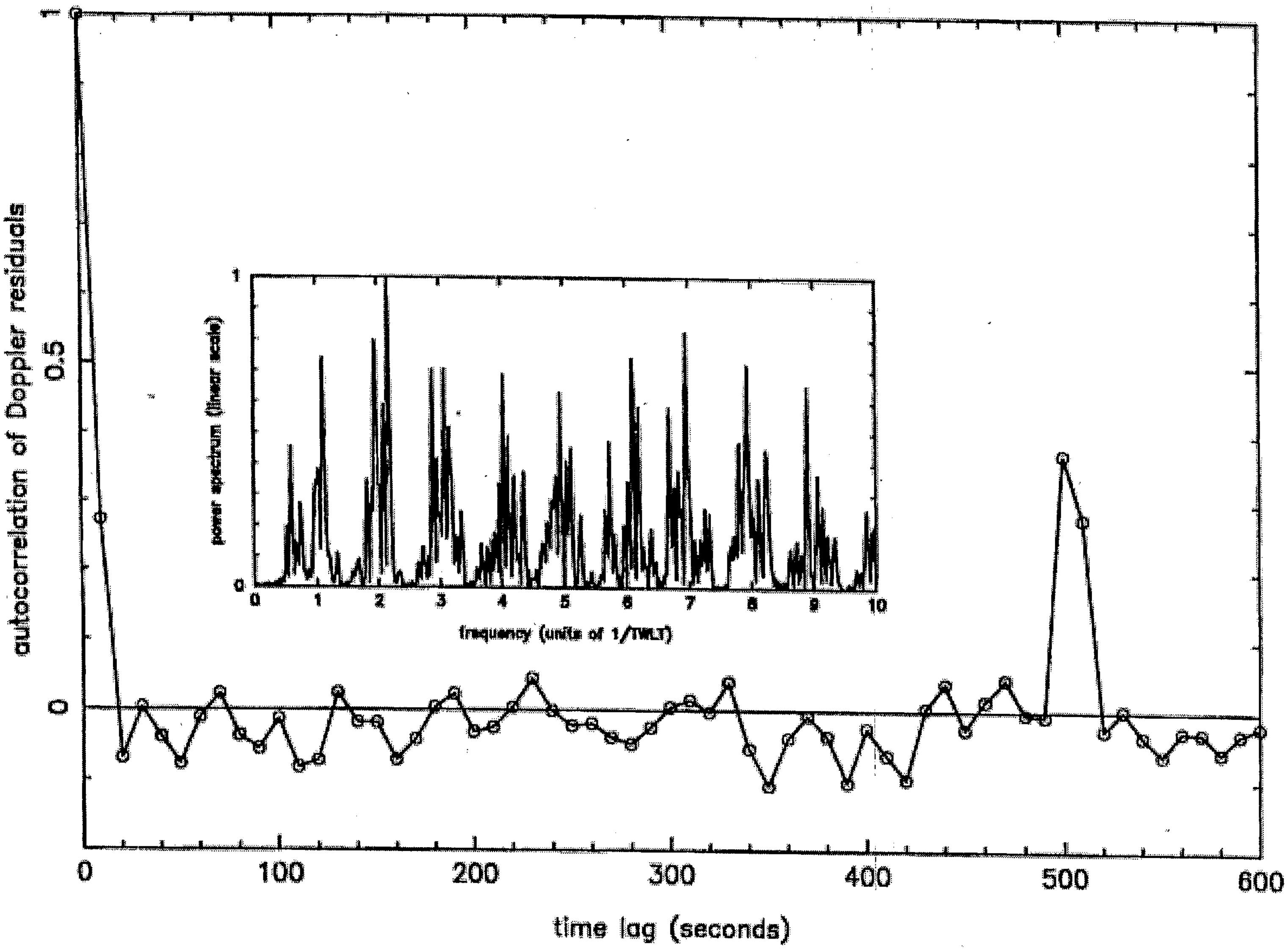
- Refractive index fluctuations at microwave frequencies are dominated by fluctuations of the water vapor along the LOS.

- Transfer function in two-way Doppler: $\delta(t) + \delta(t - T_2)$

- Independent measures of the effect available using WVRs (e.g., Keihm *TDA Prog. Rep.* 42-122, 1 (1995))

- Operational X-band data can be approximately decomposed into tropospheric and plasma scintillation; results consistent with Keihm's observations (Armstrong *Radio Sci.* 33, 1727 (1998))

- Cassini-era Advanced Media Calibrations System will calibrate and allow removal of ≈80% of the wet component; dry component + residual wet component will have transfer function $\delta(t) + \delta(t - T_2)$



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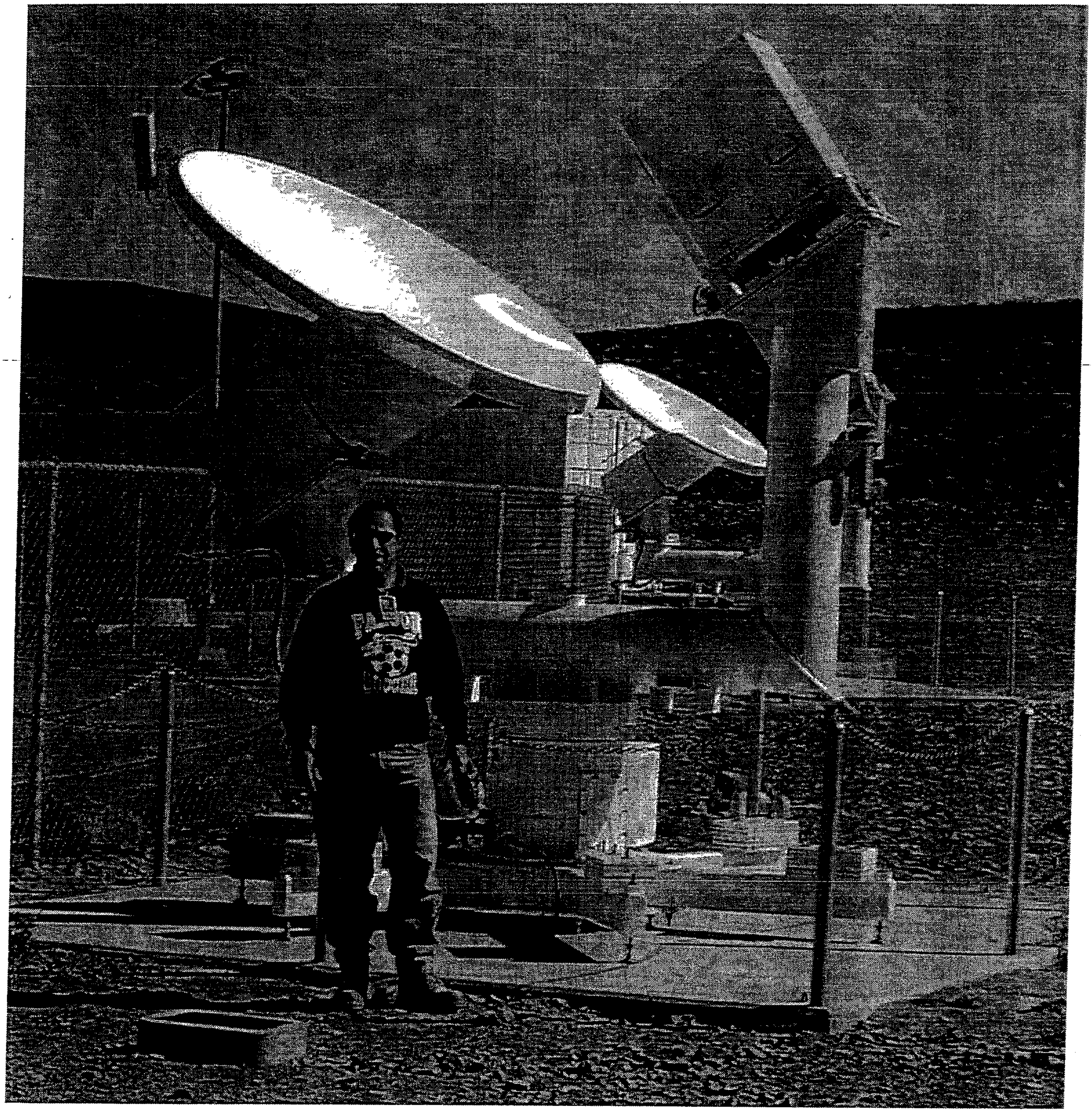
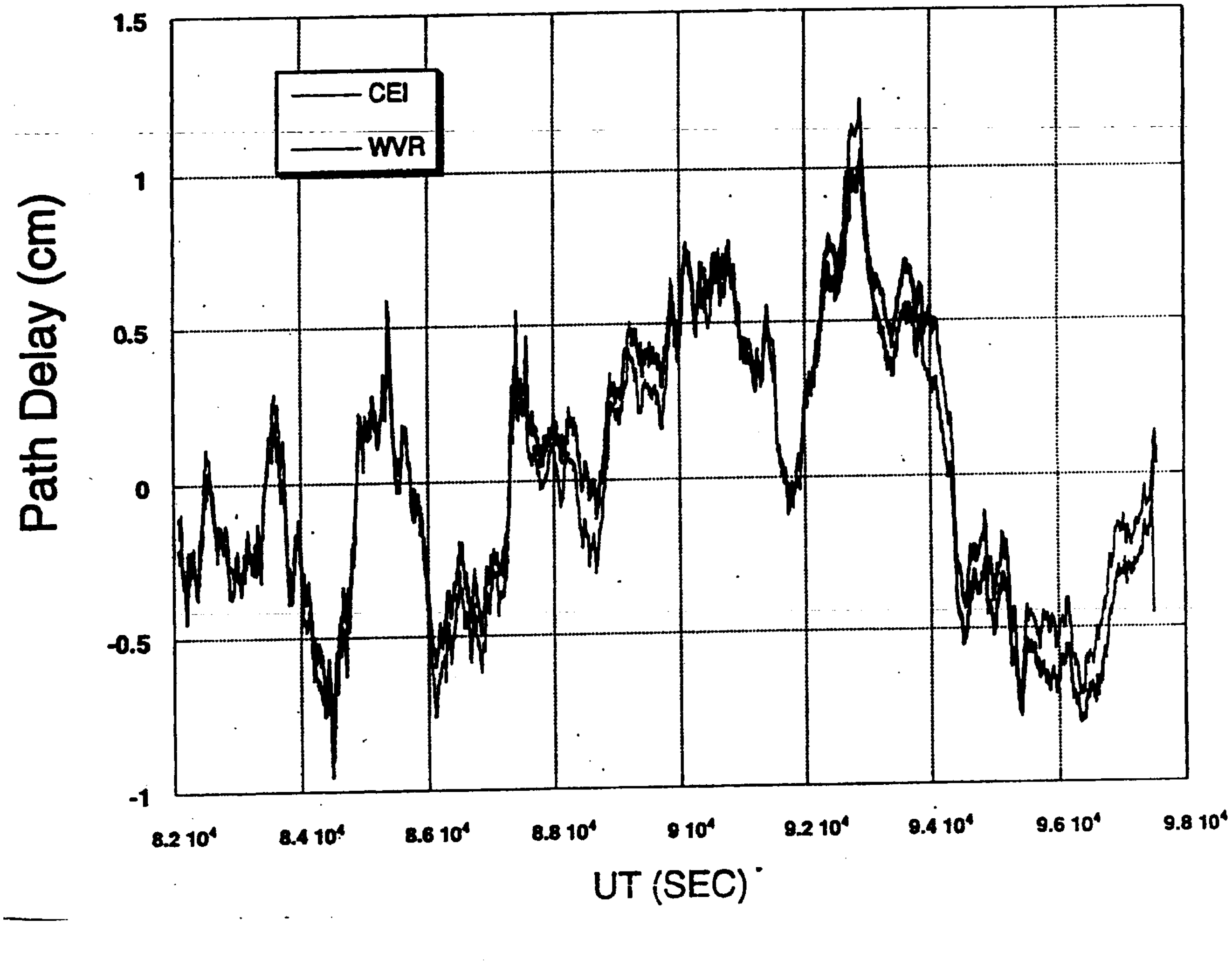
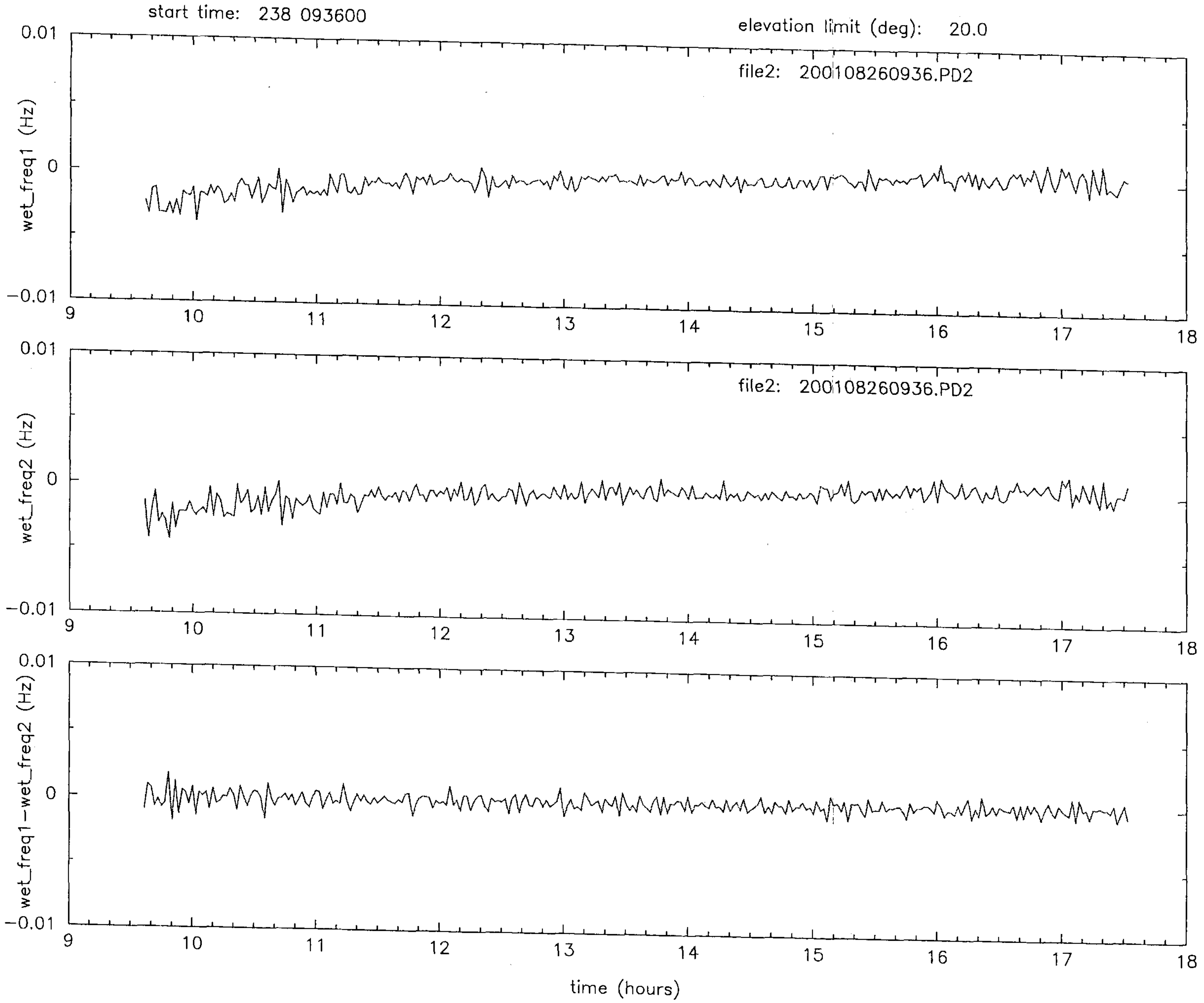
11/3/00

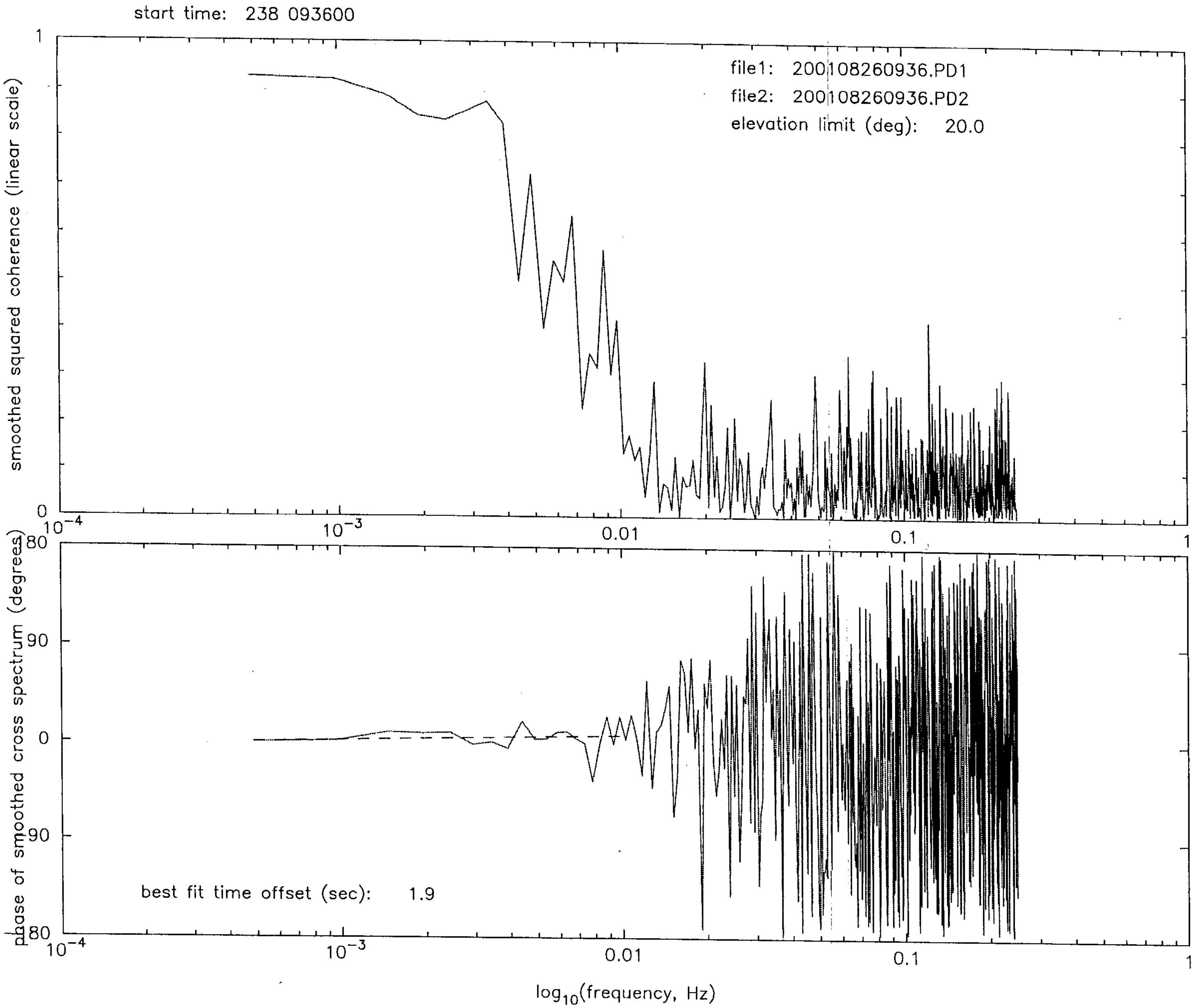


WVR-CEI Comparison
DOY 138, 2000





john 15-Oct-2001 05:49



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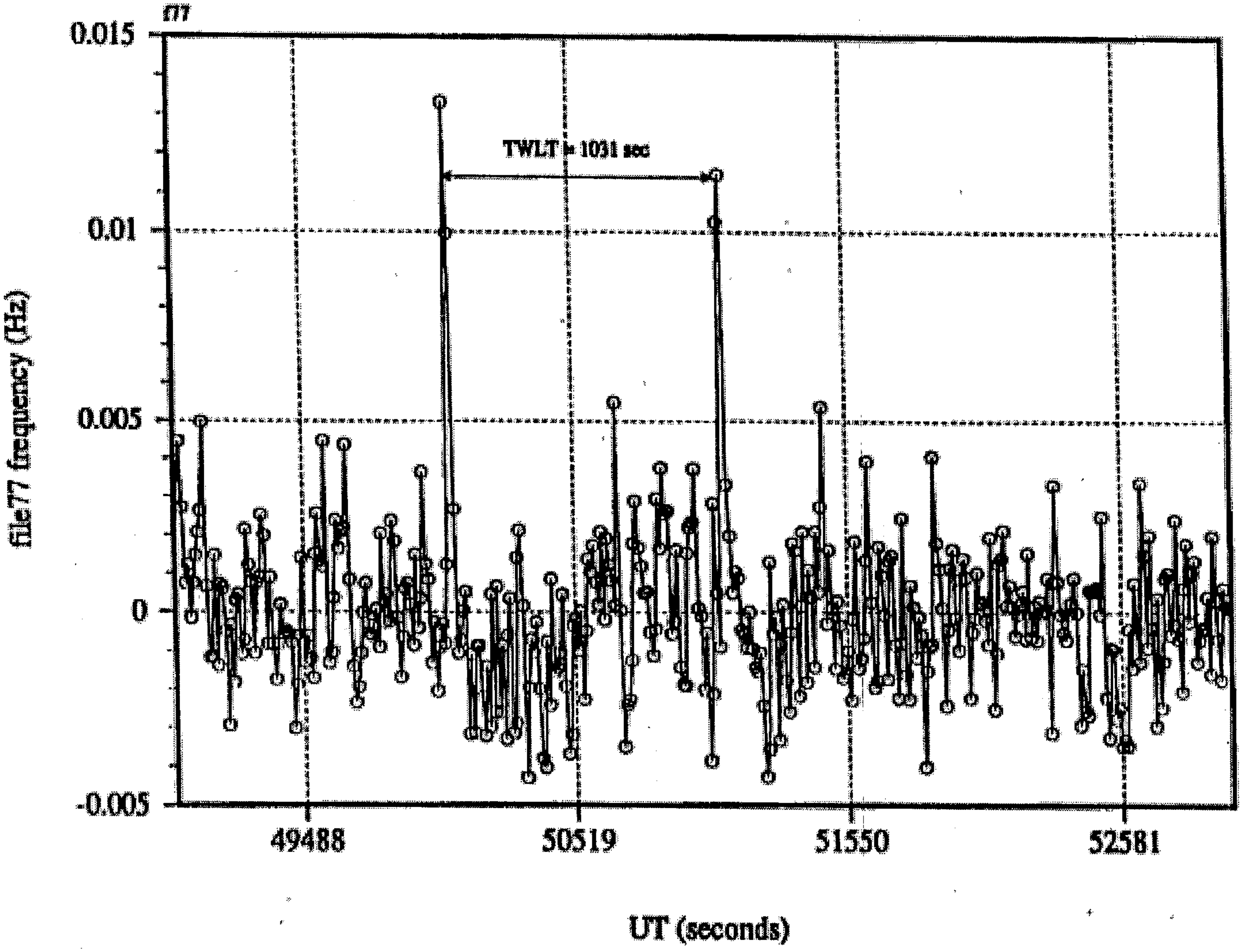
MAIN NOISES: ANTENNA MECHANICAL STABILITY

- Differential measurements (under controlled conditions) indicate $\sigma_y(1000\text{sec}) \approx 1 \times 10^{-15}$ (Otoshi and Franco, *TDA Prog. Report 42-10*, 151 (1992))

- Transfer function in two-way Doppler is $\delta(t) + \delta(t - T_2)$

- Measurements at X-band under <u>operational</u> conditions are confused with tropospheric scintillation and produce only poor limits ($< 1 \times 10^{-14}$ at τ = 1000 sec) (Armstrong *Radio Sci.* 33, 1727 (1998))

- Infrequent large events—almost certainly antenna mechanical—are observed, however (see example) ooj052_mo_089_1254_65_a (file77)



Model of Doppler Time Series

$y(t) = \Delta f/f_o =$ gravity waves
+ propagation noise
+ clock noise
+ systematic effects
+ unmodeled spacecraft motion
+ antenna mechanical noise
+ thermal noise

$$= g(t) * \left\{ \left[(\mu - 1)/2 \right] \delta(t) - \mu \delta \left[t - (1 + \mu) L/c \right] + \left[(1 + \mu)/2 \right] \delta(t - 2L/c) \right\}$$

$$+ \text{propagation}\ (t) * \left\{ \delta(t) + \delta(t - 2L/c) \right\}$$

$$+ \text{antenna mechanical}\ (t) * \left\{ \delta(t) + \delta(t - 2L/c) \right\}$$

$$+ \text{frequency standard}\ (t) * \left\{ \delta(t) - \delta(t - 2L/c) \right\}$$

$$+ \text{thermal}\ (t)$$

$$+ \text{systematic effects}$$

where: $g(t) = \left(1 - \mu^2\right)^{-1} \left\{ n \cdot \left[h_+(t) e_t + h_x(t) e_x \right] \cdot n \right\}$

L = earth-s/c distance; $\mu = k \cdot n$; $*$ = convolution

SUMMARY OF SIGNALS AND NOISE

- Signals: "three-pulse" response in the Doppler of the GW excitation
  - Depends on direction to source and s/c two-way light time
  - Not shift-invariant if direction or distance depend on time-of-observation
  - Bandpass: low-frequency signals attenuated due to pulse cancellation; high frequencies cut off by thermal and clock noise. Typical wave duration for best sensitivity depends on T_2 but ~10-10,000 seconds
  - Unlike LISA and other detectors: antenna size/wavelength ~1 to 100
- Noise sources: various "2-pulse" transfer functions for clock instability, propagation noise (solar wind, ionosphere, troposphere), thermal noise
  - Noise nonstationarity
  - Systematic errors

HOW TO DO A DOPPLER TRACKING EXPERIMENT

- Need two separated test masses—the earth and a spacecraft in cruise as operationally quiet as possible (need to be far from perturbing masses and need to minimize unmodeled motion of the spacecraft)

- Spacecraft should be as close to anti-solar direction as practical (minimize charged particle scintillation due to solar wind)

- Spacecraft-earth separation should be large (maximize band of Fourier frequencies to which the experiment is sensitive)

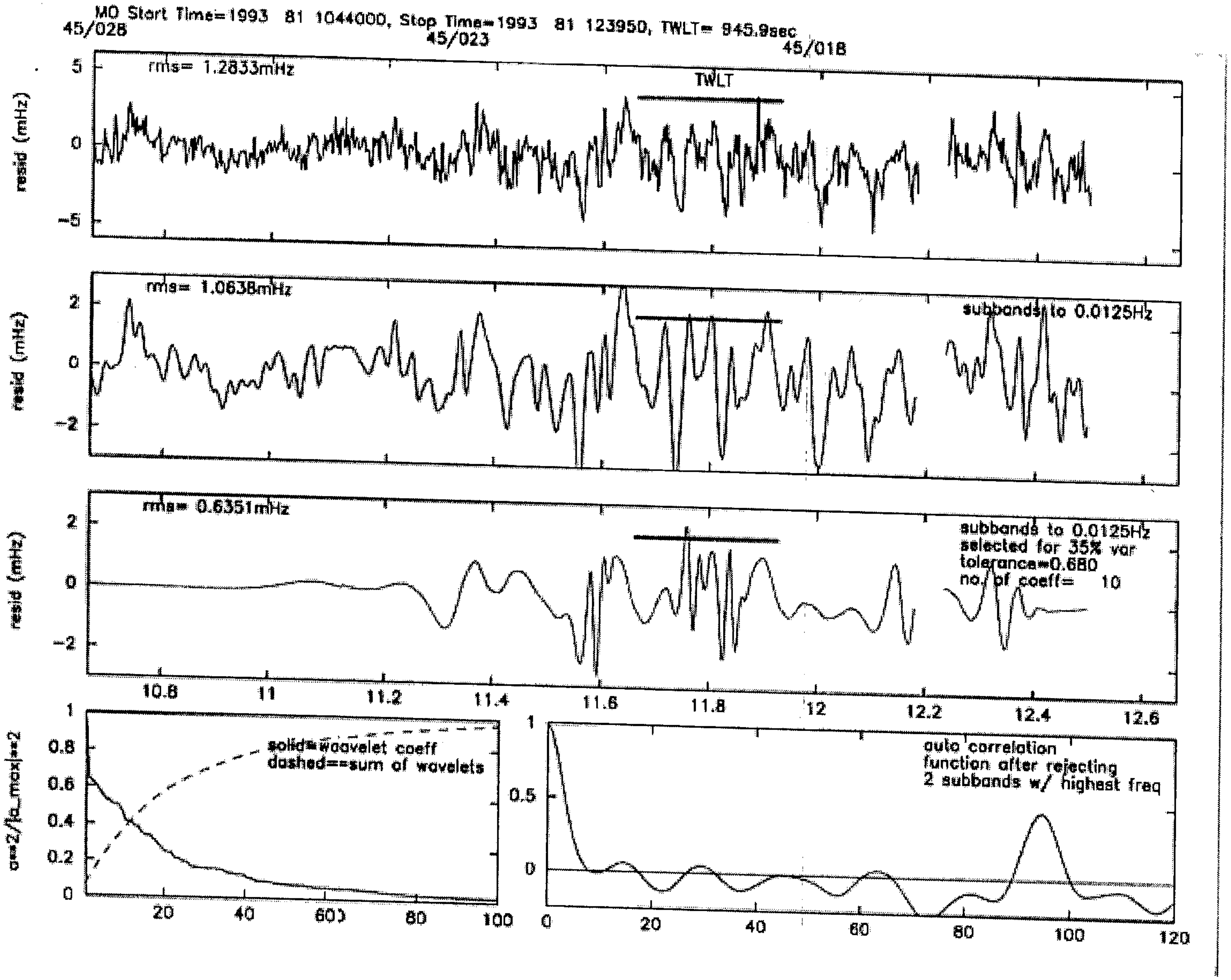
- Highly-stable Doppler system to measure relative velocity of the earth and spacecraft (excellent frequency standard; careful signal distribution, etc.)

- Ground system and spacecraft telemetry (correct for or veto data based on known systematics of the apparatus)

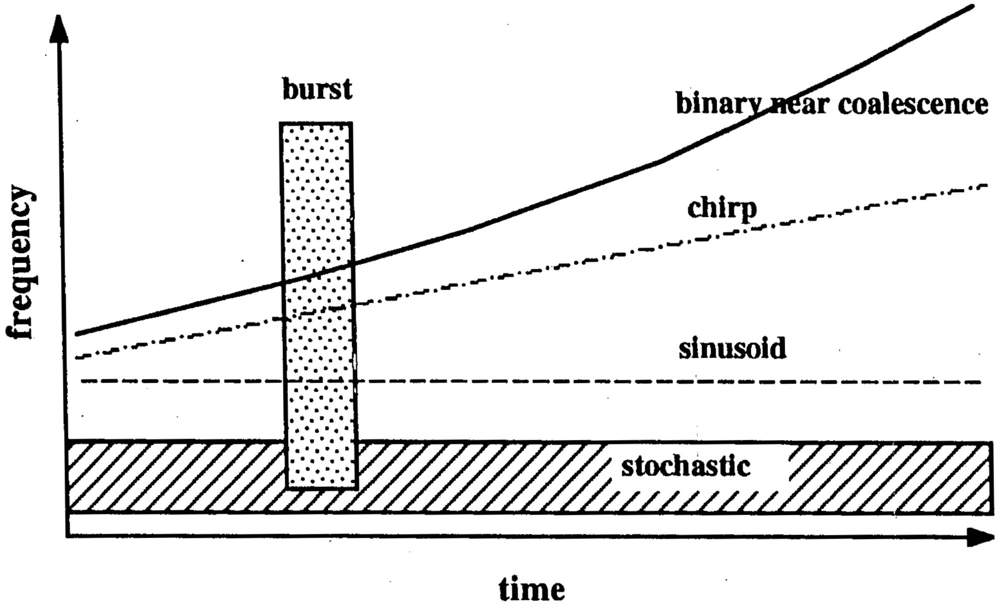
- Good weather and media calibration data

OBSERVATIONS TO DATE

1980	Voyager	Hellings et al. (1981) (few passes; bursts)
1981	Pioneer 10	Anderson et al. (1984) (3 passes, long T_2; no GW from Geminga)
1983	Pioneer 11	Armstrong, Estabrook, Wahlquist (1987) (broadband search for periodic waves)
1988	Pioneer 10	Anderson et al. (1993) (10 days; chirps and coalescing binaries)
1992	Ulysses	Bertotti et al. (1995) (1 month; sinusoids and chirps)
1993	MO/GLL/ULS	jGWE collaboration (19 days; X-band on MO; only LF coincidence experiment)
1994-5	Galileo	Estabrook et al. (40 days; long T_2)
1997	Mars Global Surveyor	Armstrong et al. (3 weeks; X-band)


MO Start Time=1993 81 1044000, Stop Time=1993 81 123950, TWLT= 945.9sec
45/028
45/023
45/018
rms= 1.2833mHz
TWLT
resid (mHz)
5
0
−5
rms= 1.0638mHz
subbands to 0.0125Hz
resid (mHz)
2
0
−2
rms= 0.6351mHz
subbands to 0.0125Hz
selected for 35% var
tolerance=0.680
no. of coeff= 10
resid (mHz)
2
0
−2
10.8
11
11.2
11.4
11.6
11.8
12
12.2
12.4
12.6
a**2/|a_max|**2
1
0.8
0.6
0.4
0.2
0
solid=wavelet coeff
dashed==sum of wavelets
20
40
60
80
100
auto correlation
function after rejecting
2 subbands w/ highest freq
1
0.5
0
0
20
40
60
80
100
120

Frequency-Time for Various Waveforms



- Spectrum localizes in frequency to good effect (minimal noise strip)

- Generalizations (e.g., chirp analysis) can localize in other regions of this phase space

Sinusoids

- If phase is unknown, use power spectral analysis. Appropriate if change in signal frequency < 1/integration time

- In absence of a signal, real and imaginary parts of Fourier transform are gaussian and uncorrelated, thus sum-of-squares = power is exponentially distributed:

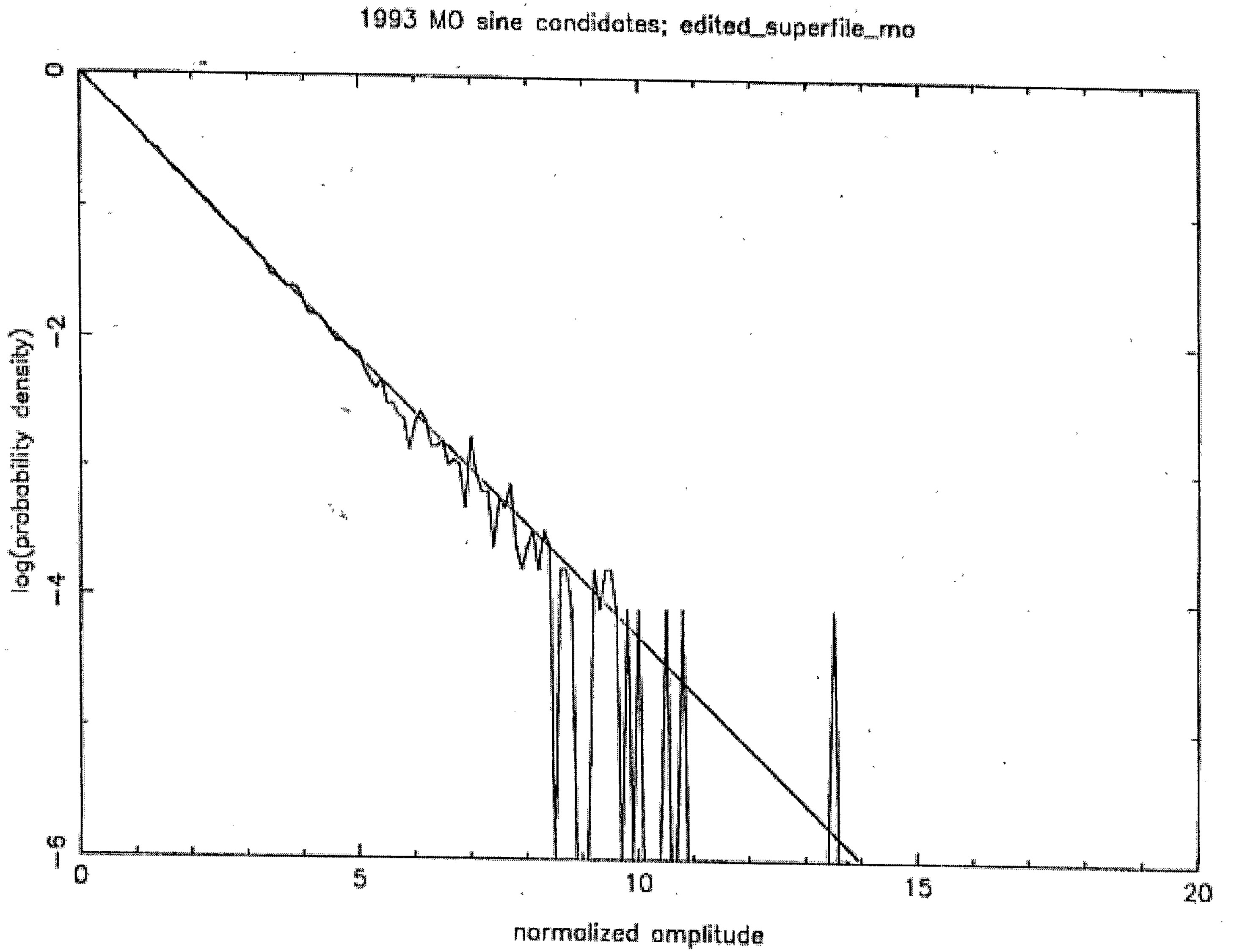
$$p(x) = \exp(-x)$$

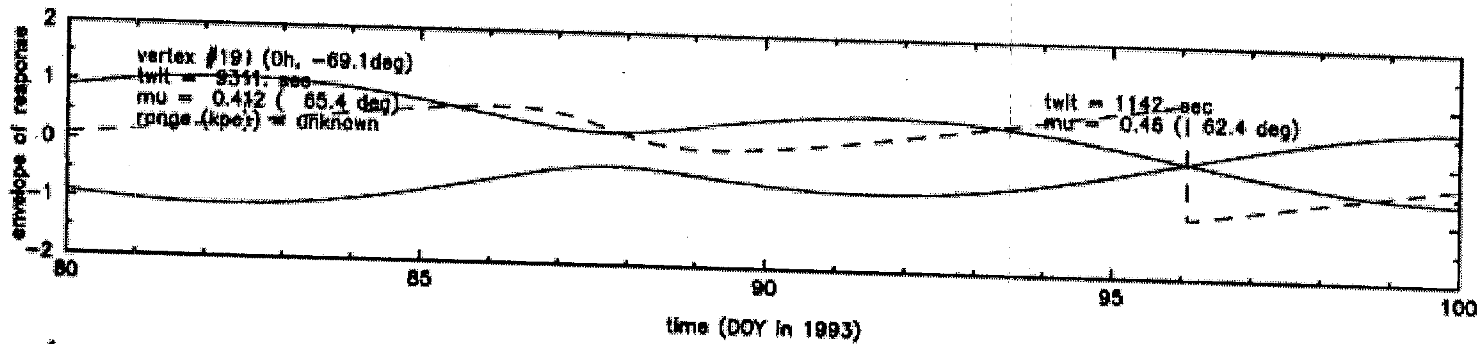
- In presence of a signal of amplitude "c", pdf of power is "Rice-squared":

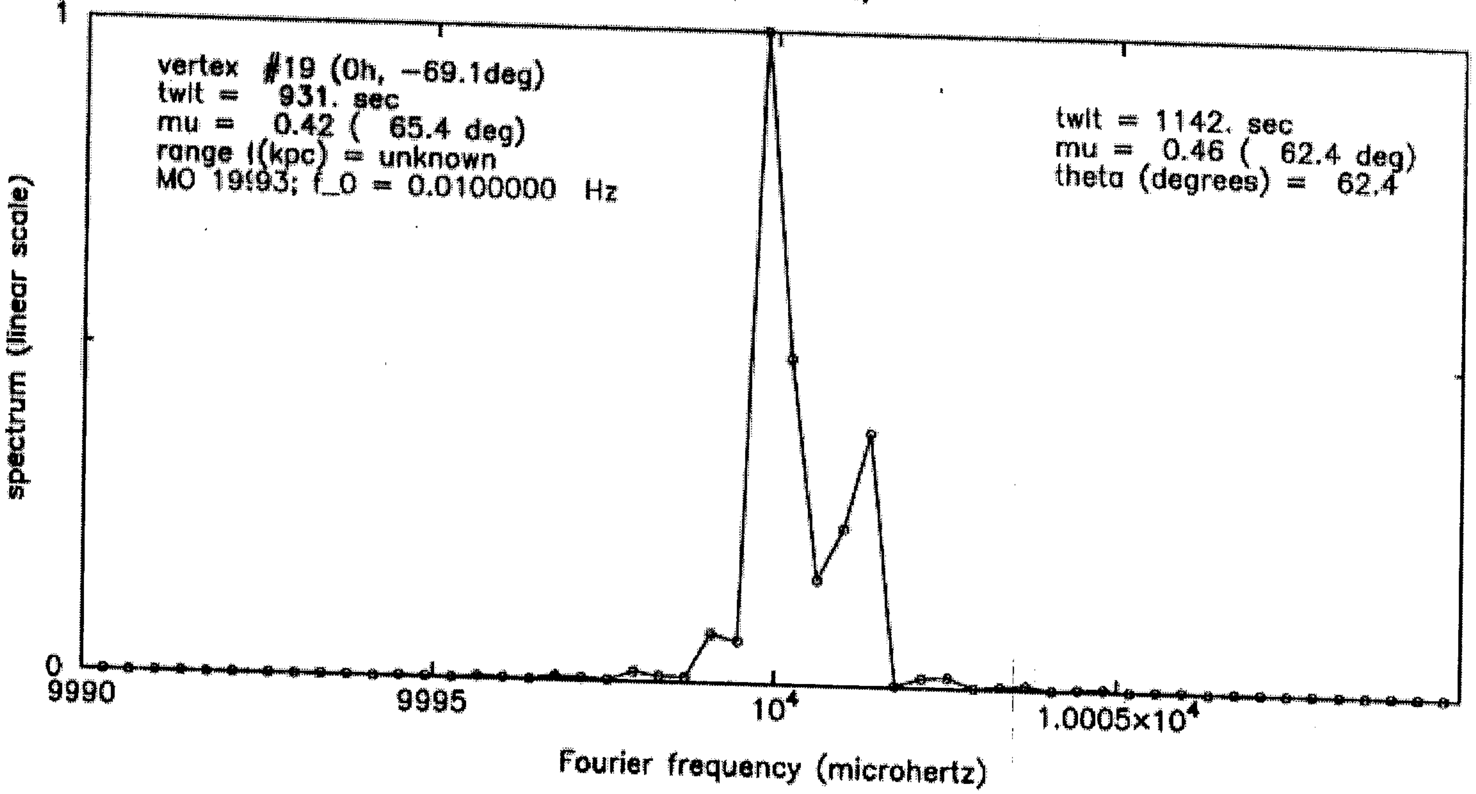
$$p(x) = \exp(-(x + c^2))\ I_0(2c/x^{0.5})$$

- Since frequency is unknown, and since Fourier bins are approximately independent, joint pdf of power in "n" Fourier bins is product of individual bin pdf's--this can be used to set confidence limits for broadband observations (Armstrong, Estabrook, Wahlquist 1987)

- Examples follow


1993 MO sine candidates; edited_superfile_mo
log(probability density)
0
−2
−4
−6
0
5
10
15
20
normalized amplitude


vertex #19 (0h, −69.1deg)
twit = 931. sec
mu = 0.42 (65.4 deg)
range (kpc) = unknown
twit = 1142. sec
mu = 0.46 (62.4 deg)
envelope of response
2
1
0
−1
−2
80
85
90
95
100
time (DOY in 1993)


vertex #19 (0h, −69.1deg)
twit = 931. sec
mu = 0.42 (65.4 deg)
range ((kpc) = unknown
MO 1993; f_0 = 0.0100000 Hz
twit = 1142. sec
mu = 0.46 (62.4 deg)
theta (degrees) = 62.4
spectrum (linear scale)
1
0
9990
9995
10^4
1.0005×10^4
Fourier frequency (microhertz)

Other Ideas We Have Tried

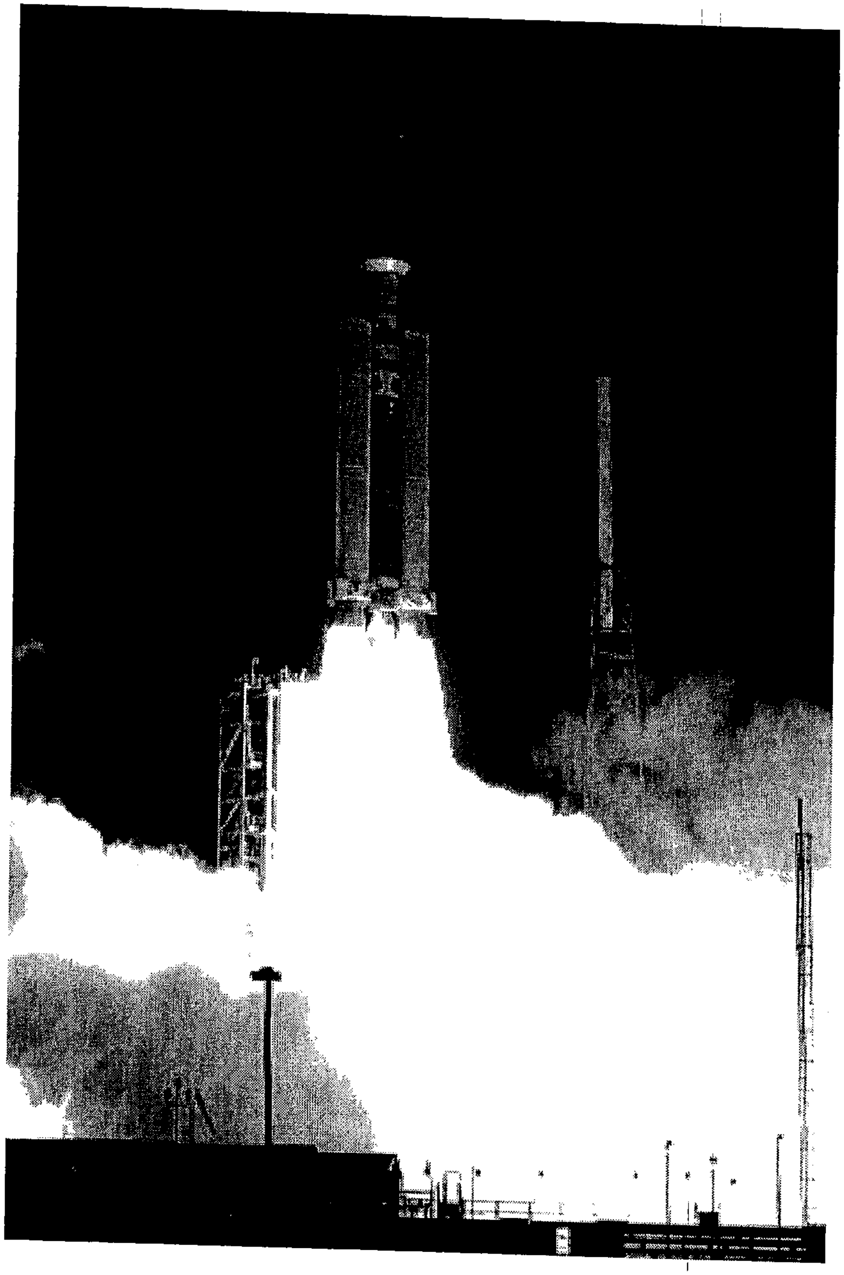
- **Wavelet transforms**
  - **A time-frequency localization procedure**
  - **To date for denoising only--attempt to filter out the "high frequency noise" while retaining the "edges" of bursts. Useful particularly with Galileo (low-gain antenna means lower signal-to-thermal-noise)**

- **Karhunen-Loeve expansion**
  - **Let the data determine their own basis: basis functions derived from the autocovariance matrix**
  - **Problem: in simulations at low SNR (the practical case), modes found are never the physical modes. Disappointing.**

Other Ideas We Have Tried (Continued)

- Bispectral analysis
  - Fourier decomposition of third statistical moment--look for non-gaussian component to time series
  - Problem: difficult to estimate accurately at level of any putative non-gaussian signals in Doppler data

- Multi-taper spectral analysis
  - Another way of partitioning spectrum into a continuum + "lines"
  - Achieved recent notoriety with claim of spectral lines in flux of keV electrons observed in solar wind (Thomson et al. 1995)
  - Main advantage over what we have done to date is when lines are present with a choppy continuum--if continuum is smooth, it reduces to what we already do.
  - A "neat idea", but data gaps may be a problem



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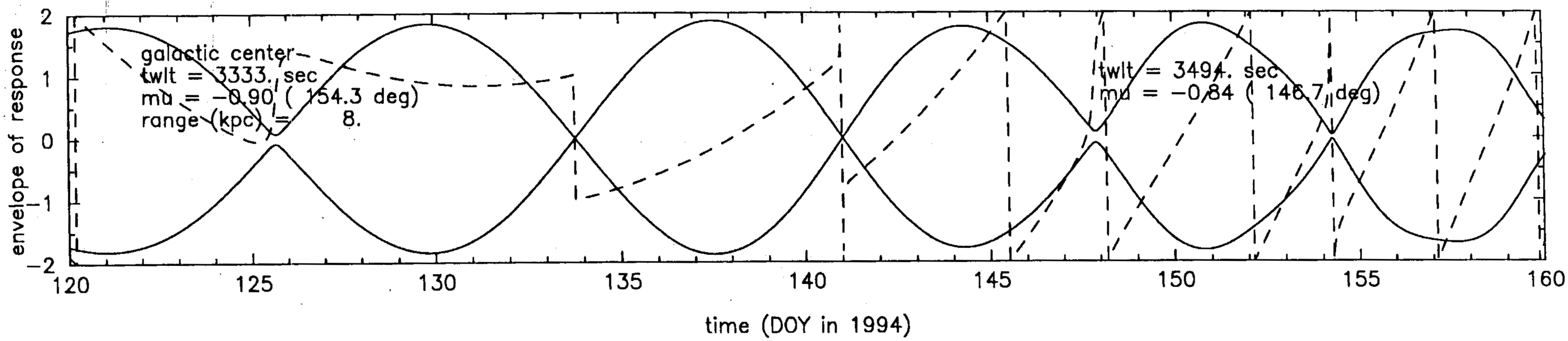
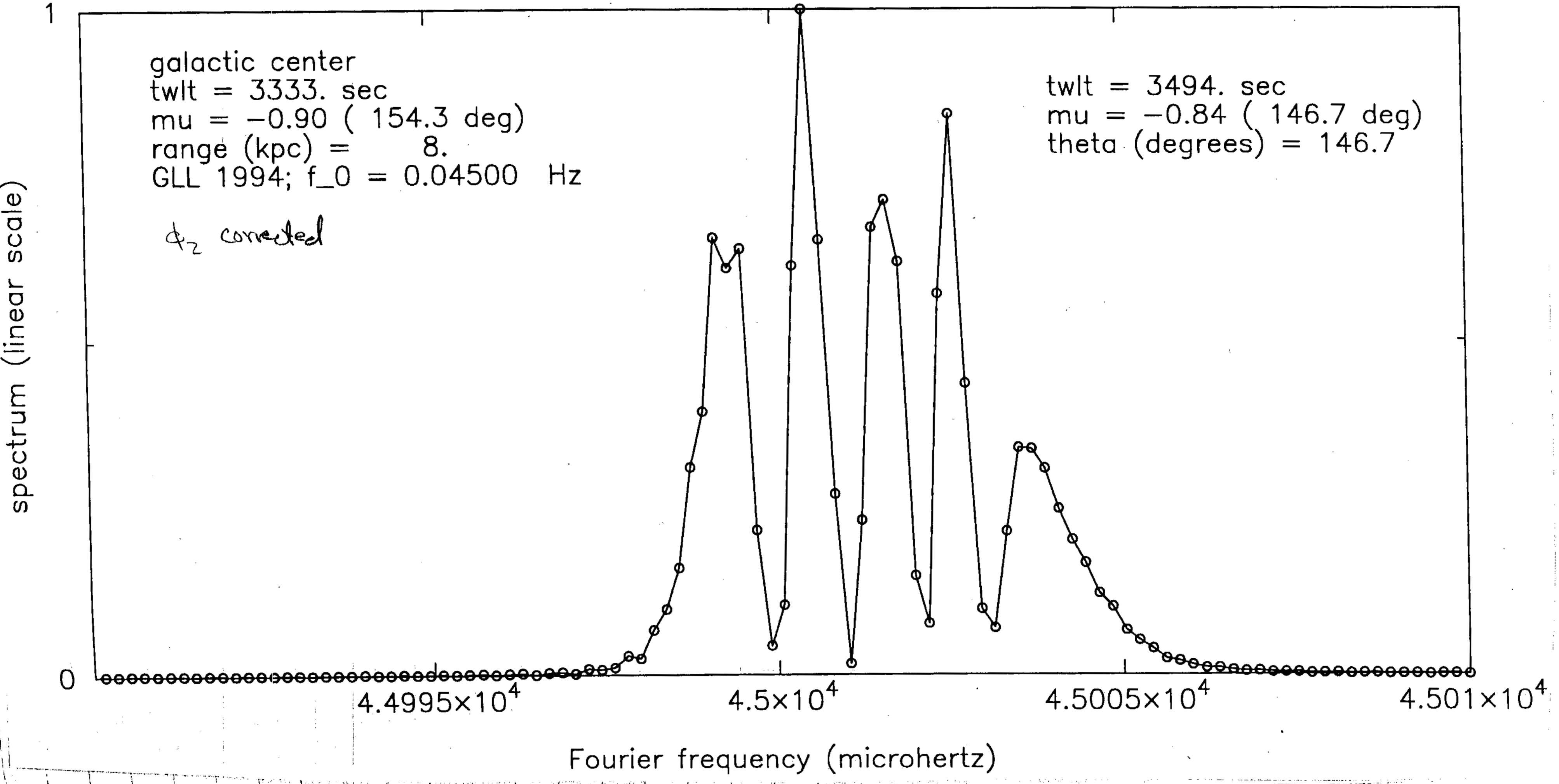
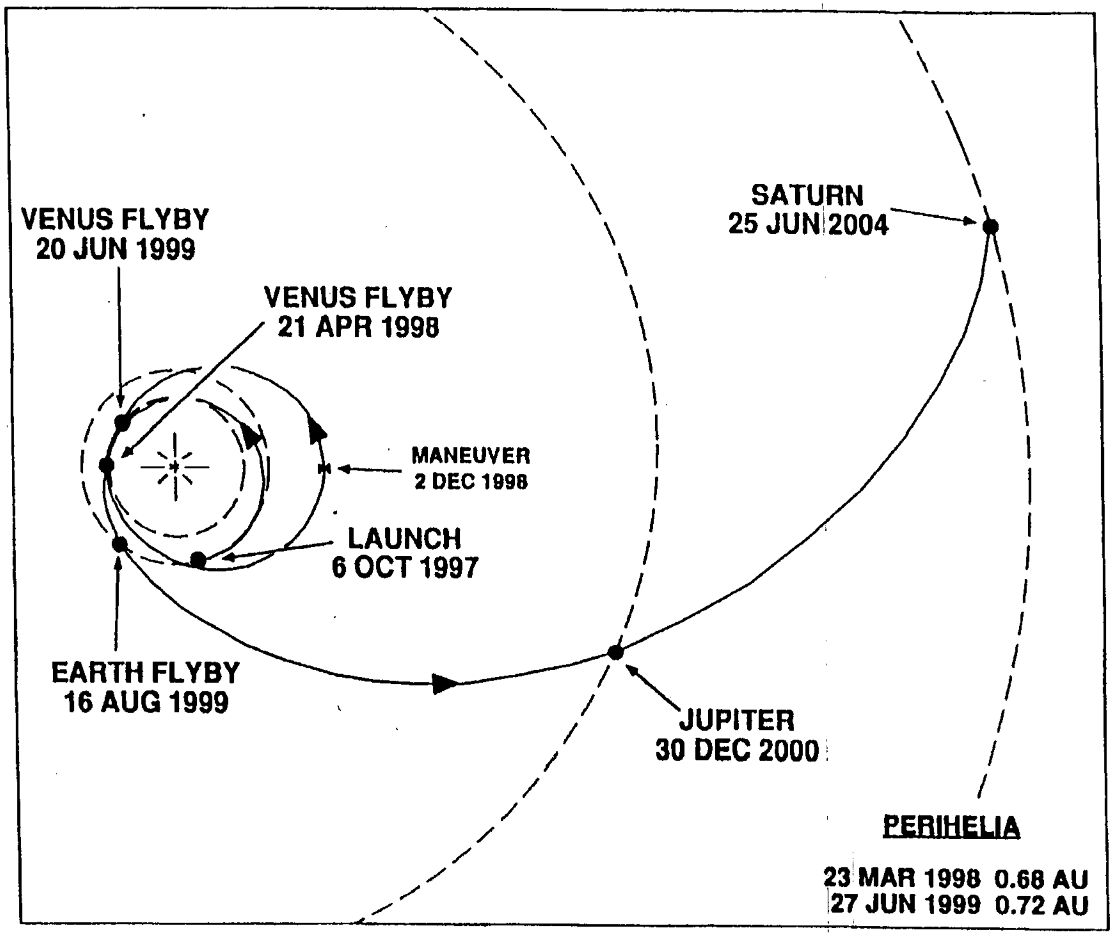




ϕ_2 corrected

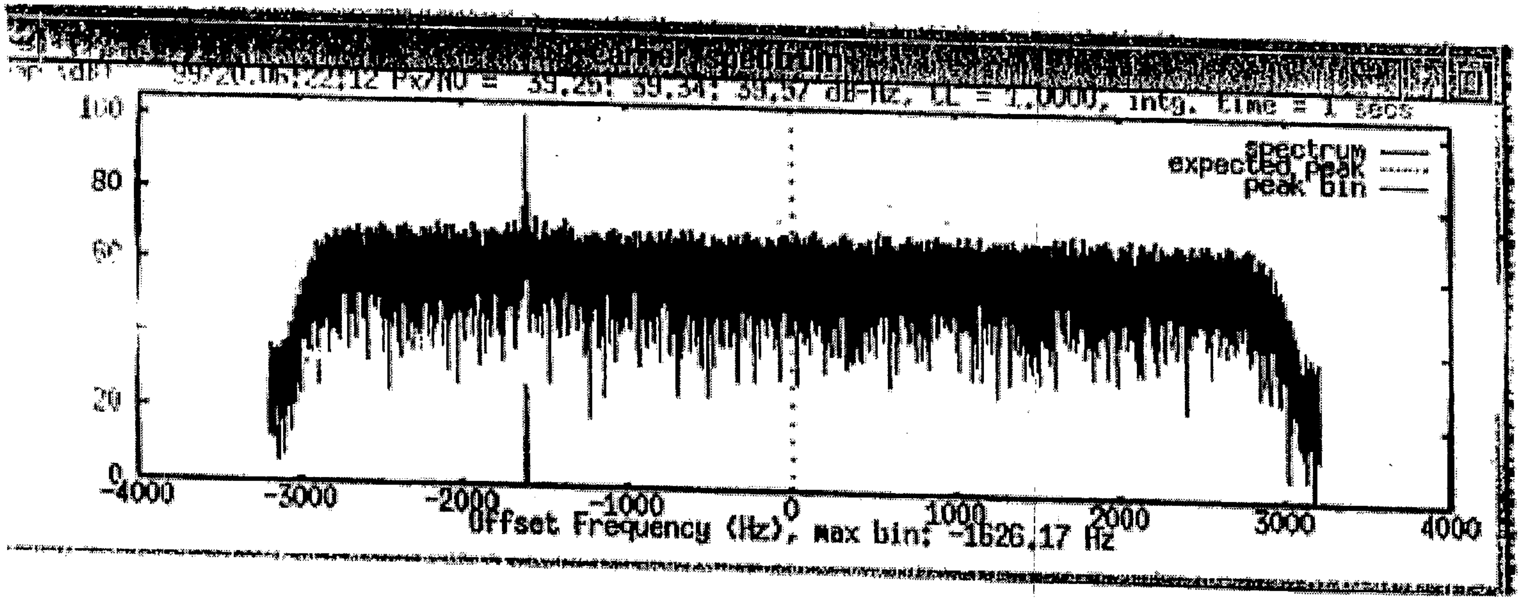
CASSINI OCT 1997 VVEJGA
INTERPLANETARY TRAJECTORY

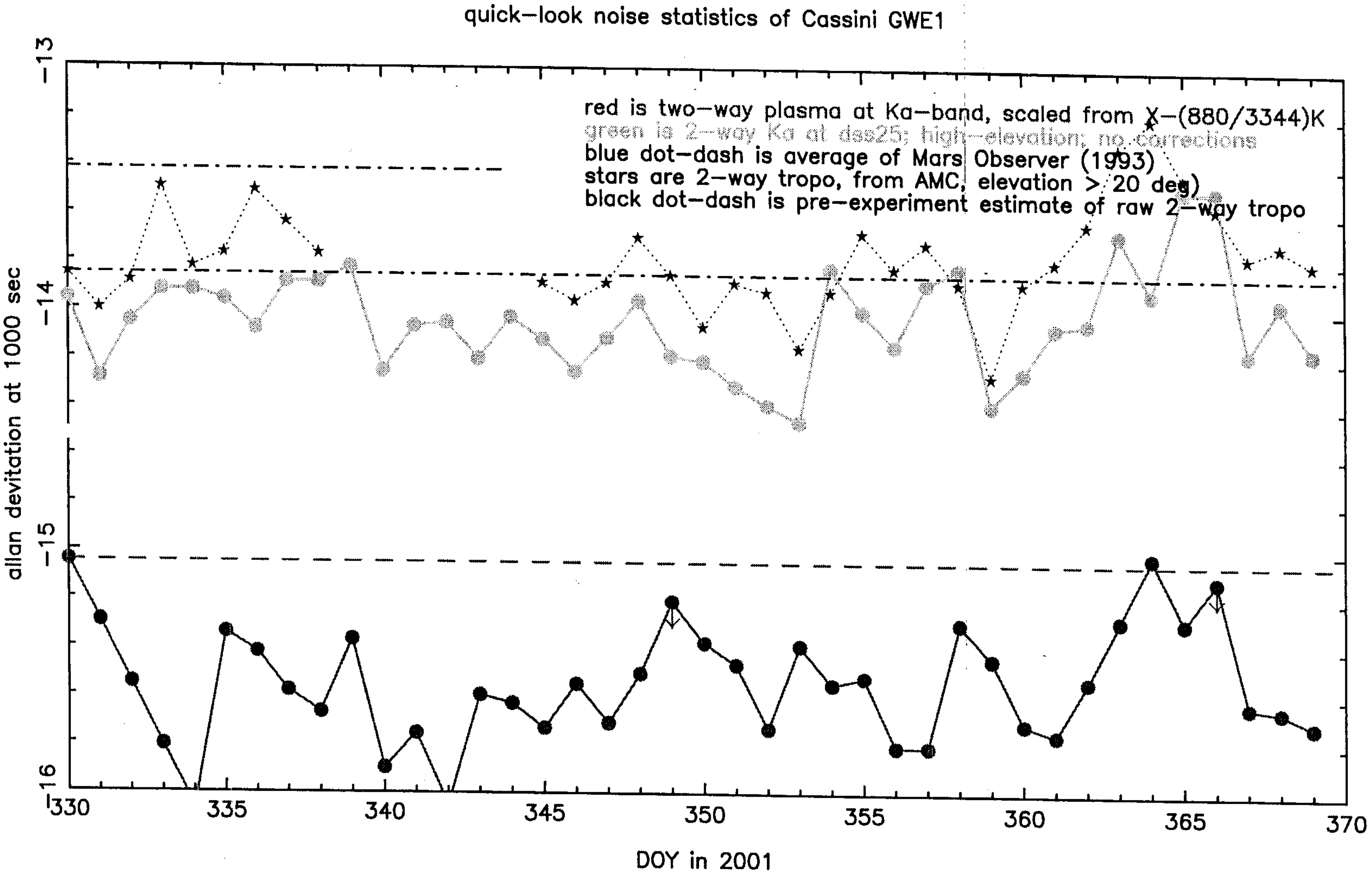


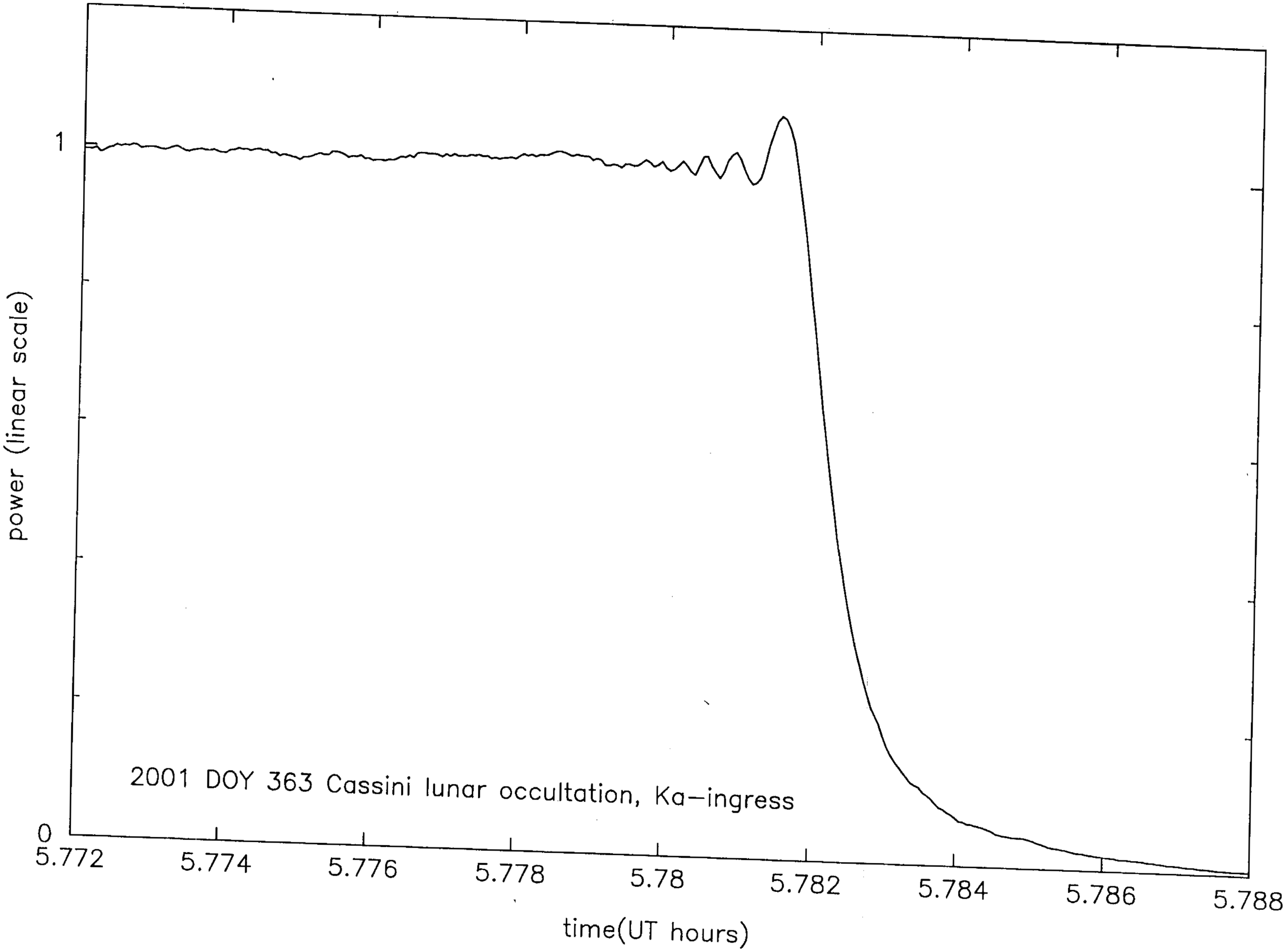
13 JUL 1992

FIRST EVER RECEIPT OF 2-WAY COHERENT KA-BAND SIGNAL

DOY 20, 1999
(10:22 PM PST, 1/19/99)




quick-look noise statistics of Cassini GWE1
red is two-way plasma at Ka-band, scaled from X-(880/3344)K
green is 2-way Ka at dss25; high-elevation; no corrections
blue dot-dash is average of Mars Observer (1993)
stars are 2-way tropo, from AMC, elevation > 20 deg)
black dot-dash is pre-experiment estimate of raw 2-way tropo
allan devitation at 1000 sec
-13
-14
-15
-16
330
335
340
345
350
355
360
365
370
DOY in 2001

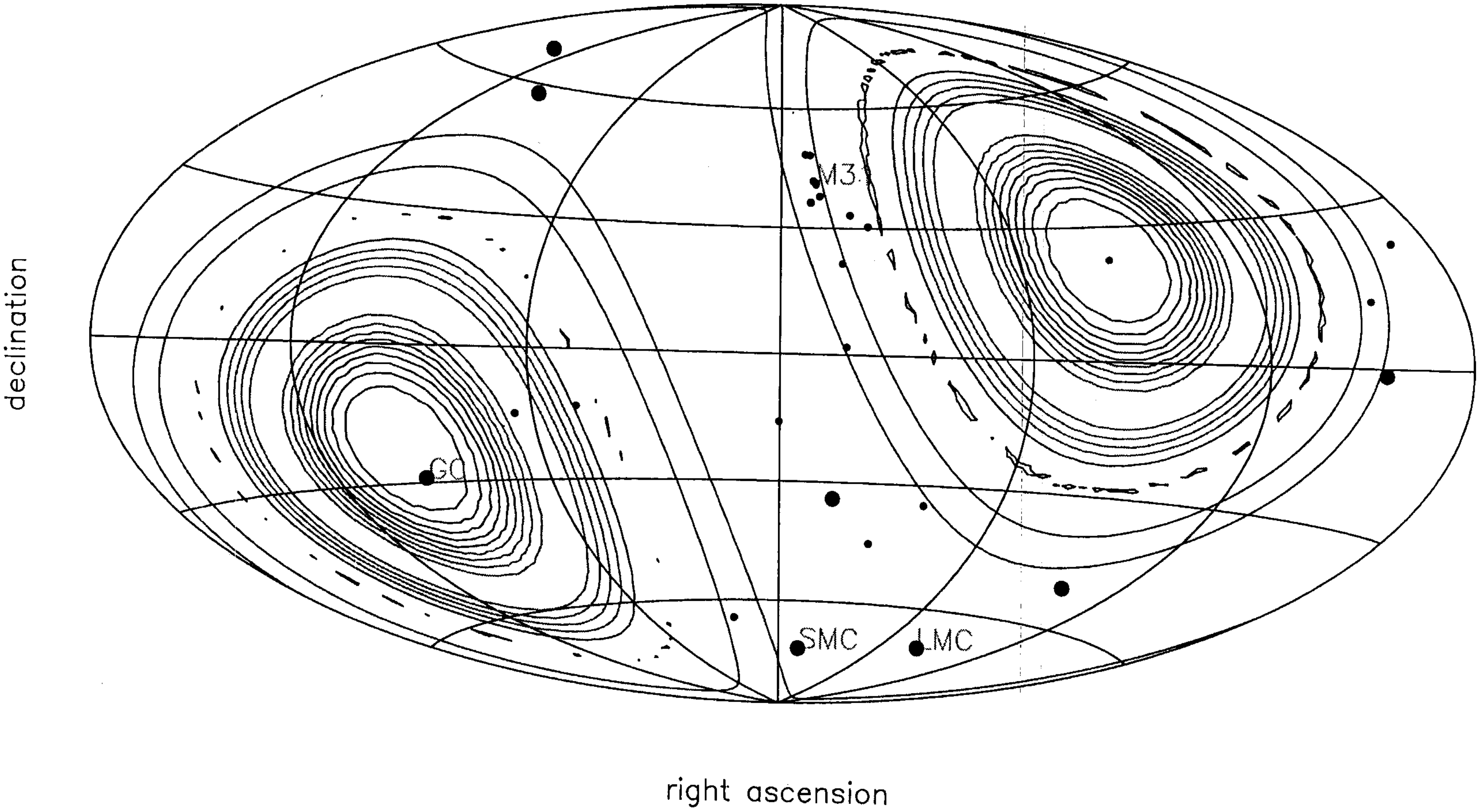


1
0
power (linear scale)
5.772
5.774
5.776
5.778
5.78
5.782
5.784
5.786
5.788
time(UT hours)
2001 DOY 363 Cassini lunar occultation, Ka–ingress

Discussion

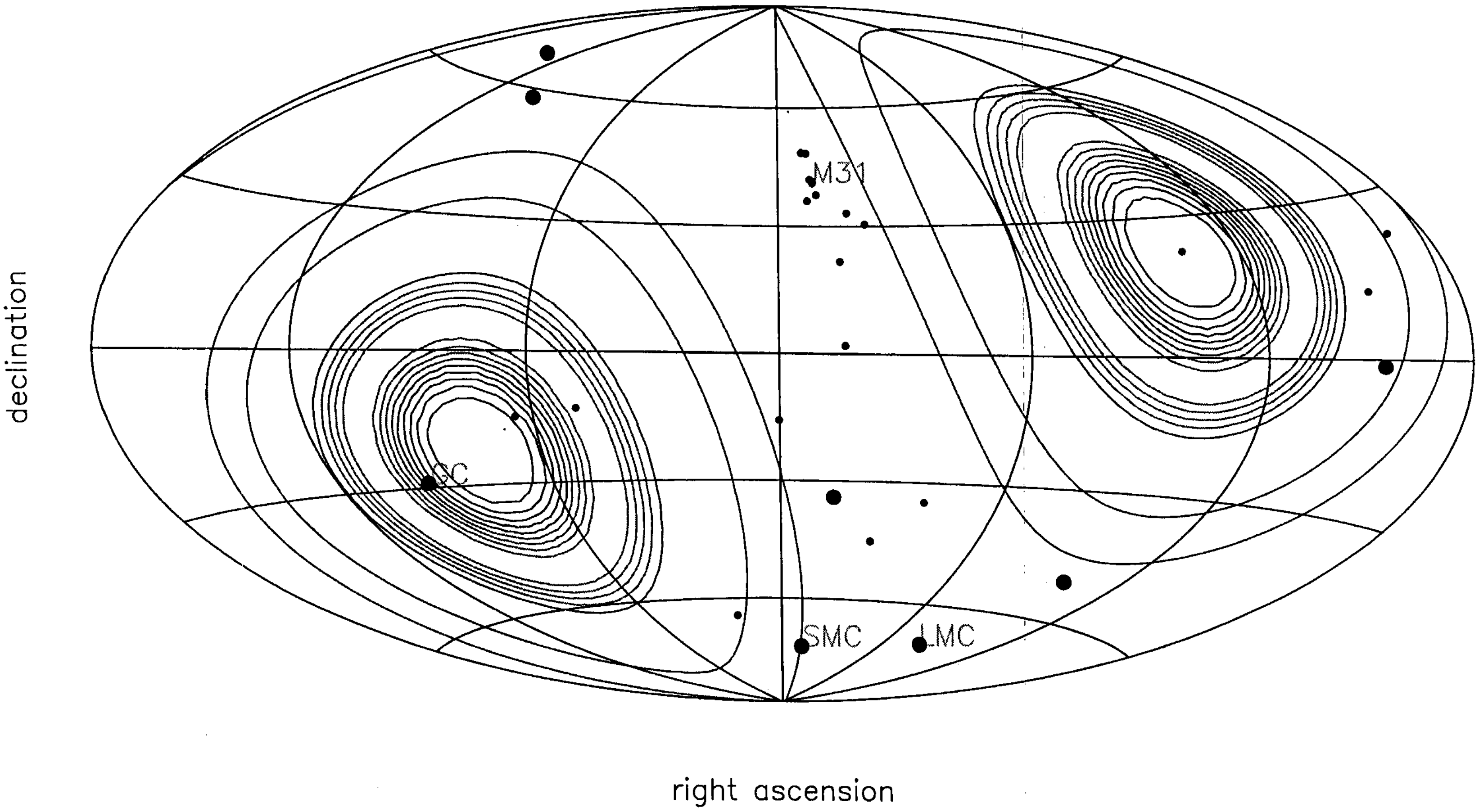
- Ka-band up/down, as it had to, knocked the plasma scintillation noise out of the error budget
  - X – (880/3344)K independently estimates the downlink plasma
  - Two-way Ka-band plasma consistent with pre-experiment expectation
- 2-way Ka-band (uncorrected, selected for high elevation angles) limited by nondispersive process (e.g. some combination of troposphere, antenna mechanical, FTS, KaT instability, s/c motion noise,...)
  - Level is consistent with independent estimate of the troposphere, therefore should be able to correct for this to about the target sensitivity level
  - Potential problem: significant fraction of AMC data flagged—liquid water in AMC beam may degrade AMC correction of the data

relative energy response for Cassini 2001 December 16
circular-pol: sin(2 π (0.001 Hz) t)*exp(−t/1000 sec)

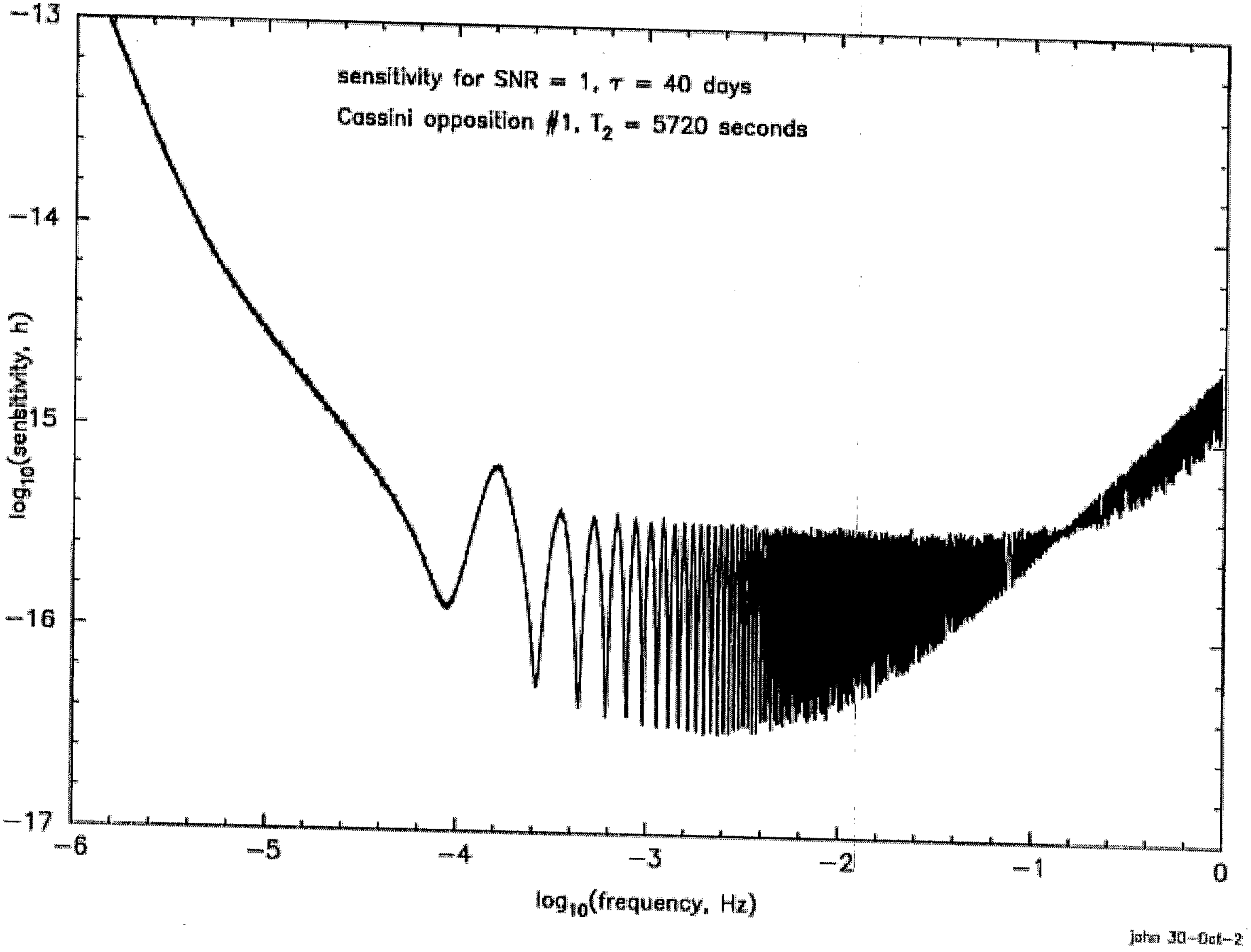


hammer–aitoff equal–area projection (center of plot is RA = 0, dec = 0)

relative energy response for Cassini 2004 January 4
circular–pol: sin(2 π (0.001 Hz) t)*exp(–t/1000 sec)



hammer–aitoff equal–area projection (center of plot is RA = 0, dec = 0)


sensitivity for SNR = 1, τ = 40 days
Cassini opposition #1, T2 = 5720 seconds
log10(sensitivity, h)
log10(frequency, Hz)
−13
−14
−15
−16
−17
−6
−5
−4
−3
−2
−1
0
john 30-Oct-2

Can We Do Better than Cassini?

Problems Are	Possible Fixes
tropospheric scintillation	better calibrations and/or Estabrook/Hellings idea
plasma scintillation	higher radio frequency and/or Cassini-style multi-frequency links
antenna mechanical noise	look in nulls of transfer function (?)
frequency standard noise	30X better clocks are "straightforward"
spacecraft position noise	very careful design (?)

Conclusion: *Maybe* 10-fold improvement—to $\sim 3 \times 10^{-18}$ for periodic sources at selected Fourier frequencies—is possible using spacecraft Doppler tracking from an Earth-based station. However the cost to achieve this would be very high.

Concluding Ideas

- Doppler tracking of Cassini can be used as a broadband gravity wave detector
  - Apparatus is large compared with GW wavelength; thus detector properly described in terms of three pulses GW response
  - Low-frequency band edge is ≈1/(two-way-light-time) set by pulse-overlap
  - High-frequency band edge is ~10^{-1} to 10^{-2} Hz, set by combination of downlink SNR, FTS, ability to calibrate troposphere
  - Not an interferometer: coherence maintained by excellent frequency standard on the ground
- Main noise sources
  - FTS stability
  - Plasma scintillation (dominates S-band; secondary at Ka-band)
  - Tropospheric scintillation (nondispersive)
  - Antenna mechanical stability

Concluding Ideas (continued)

- Signals and noises enter with different transfer functions—a *very* useful discriminator
- Cassini experiment will be ≈10-fold more sensitive than previous observations
    - Ka-band lowers plasma noise at opposition to below troposphere noise
    - Sophisticated tropospheric scintillation calibration
    - Sensitivity:
        - $\approx 3 \times 10^{-15}$ for bursts (i.e., $\sigma_y(\tau \approx 1000 \text{ sec})$)
        - $\Omega \lesssim 10^{-2}$ for backgrounds ($f_c \approx 10^{-4}$ Hz)
        - $\approx 3 \times 10^{-17}$ for periodic waves (at selected Fourier frequencies); $\approx 1.5 \times 10^{-16}$ averaged over the band