UDC 551.513.1:551.511.2

# ON THE INTERANNUAL VARIABILITY OF THE TROPOSPHERIC ENERGY CYCLE AND THE QUASI-BIENNIAL OSCILLATION

### ALVIN J. MILLER

National Meteorological Center, ESSA, Hillcrest Heights, Md.

### ABSTRACT

Geostrophic computations in the wave number domain have been made of the "eddy" kinetic energy, the transfer of kinetic energy between the zonal flow and the eddies, and the internal redistribution of kinetic energy among the eddies at 500 mb. An 11-yr record has been inspected for year-to-year variations and the results related to the quasibiennial oscillation in the lower stratosphere.

The individual wave numbers indicate significant year-to-year variations in their energy cycle that must be considered when only short time periods are under study. While no consistent relationship was found between these variations and the quasi-biennial oscillation, our results suggest that a profitable line of inquiry would be to examine the annual variations of the vertical transfer of kinetic energy by the pressure-work term at the interface of the troposphere and stratosphere.

## **1. INTRODUCTION**

While recent diagnostic studies have increased our knowledge of the tropospheric energy cycle, the evaluation of these endeavors has, in general, been hampered somewhat by our lack of knowledge of the normal interannual variability of the energy cycle and how it pertains to the specific periods under study. Miller et al. (1967) (hereafter referred to as MWT), for example, have shown that marked cyclic variations exist in the tropospheric angular momentum transports which, in turn, are related to the transfer of energy to the zonal mean flow from the perturbation flow (Lorenz, 1955). This suggests that the energy cycle as a whole is subject to an interannual variability that should be considered when data for relatively short time periods are evaluated. At the same time. since Oort (1964), Miller (1967), Muench (1965), and Perry (1967) have shown that the transfer of kinetic energy from the troposphere to the stratosphere is an important component of the energy budget of the latter region, the variability may also be important in consideration of the quasi-biennial oscillation in the lower stratosphere (Newell, 1964).

The purpose of this study is to illustrate the interannual variability in the tropospheric energy cycle and to ascertain its relationship with the quasi-biennial oscillation. As stated in MWT, the daily 500-mb Northern Hemisphere analyses of the National Meteorological Center for the 11-yr period Apr. 1, 1955, through Mar. 31, 1966, have been subjected to zonal harmonic analysis (Saltzman and Fleisher, 1960). Harmonic coefficients of the geostrophic wind components were determined, and the kinetic energy, the conversion of kinetic energy between the zonal flow and the eddies, and the internal redistribution of kinetic energy among the eddies were computed in the wave number domain. Monthly mean values of these quantities form the basis for the present study.

Also as in MWT, a statistical filter in the form of a 12-mo running mean minus a 24-mo running mean was applied to the monthly mean data. Its frequency response is shown in figure 1. While only selected results are presented here, the previous results of MWT (especially at low latitudes) indicate that this filter does not force a biennial oscillation into essentially random data.



FIGURE 1.—Ratio of filtered to unfiltered amplitude versus period for 12-mo minus 24-mo running mean filter. X's represent points spaced too closely for curve-fitting (after Miller et al., 1967).

### 2. LIST OF SYMBOLS

- K(n) kinetic energy per unit mass per wave number p pressure
- $p_1, p_2$  upper and lower pressures bounding region of integration
- $\phi_1, \phi_2$  southern and northern latitudes bounding region of integration
- *a* radius of the earth
- $\lambda$  longitude measured east from Greenwich
- g acceleration of gravity
- $\theta$  potential temperature
- $C_p$  specific heat of air at constant pressure
- R gas constant
- [()] zonal averaging operator defined by

$$[()] = \frac{1}{2\pi} \int_0^{2\pi} () d\lambda$$

u,v eastward and northward components of velocity n wave number

 $\omega$  individual rate of pressure change, dp/dt

Fourier transform pairs:

- Variable  $u \sim z$
- Spectral function U V Z
- $X = \frac{\text{friction force}}{\text{per unit mass}} \text{ in } \lambda \text{-direction}$

$$Y = \frac{\text{friction force}}{\text{per unit mass}} \text{ in } \phi \text{-direction}$$

$$\Phi_{fg} = F(n) G(-n) + F(-n) G(n) \psi_{fg} = F(n-m) G(-n) + F(-n-m) G(n)$$

# 3. BASIC EQUATIONS

Following the development of Perry (1967), the time rate of change of the "eddy" kinetic energy per unit mass of an individual wave number is:

$$\begin{split} \frac{\partial}{\partial t} K(n) &= -\left(\Phi_{u,v}(n) \frac{\cos\phi}{a} \frac{\partial}{\partial\phi} \left(\frac{[u]}{\cos\phi}\right) + \Phi_{v,v}(n) \frac{1}{a} \frac{\partial[v]}{\partial\phi} \\ &+ \Phi_{w,u}(n) \frac{\partial[u]}{\partial p} + \Phi_{w,v}(n) \frac{\partial[v]}{\partial p} - \frac{\tan\phi}{a} \Phi_{u,u}(n)[v]\right) \\ &+ \sum_{\substack{m=-\infty\\ \neq 0}}^{\infty} \left(U(m) \left\{\frac{1}{a\cos\phi} \psi_{u,u_{\lambda}}(m,n) + \frac{1}{a} \psi_{v,u_{\phi}}(m,n) \right. \\ &+ \psi_{w,u_{p}}(m,n) - \frac{\tan\phi}{a} \psi_{u,v}(m,n)\right\} + V(m) \left\{\frac{1}{a\cos\phi} \psi_{u,v_{\lambda}}(m,n) \\ &+ \frac{1}{a} \psi_{v,v_{\phi}}(m,n) + \psi_{w,v_{p}}(m,n) + \frac{\tan\phi}{a} \psi_{u,u}(m,n)\right\}\right) \\ &- \left(\left(\frac{p}{p_{0}}\right)^{R/c_{p}} \frac{R}{p} \Phi_{w,\theta}(n)\right) - \left(\Phi_{u,x}(n) + \Phi_{v,y}(n)\right) \\ &- \frac{\partial}{\partial p} \left(\sum_{\substack{m=-\infty\\ \neq 0}}^{\infty} \left\{U(m)\psi_{w,u}(m,n) + V(m)\psi_{w,v}(m,n)\right\}\right) \\ &- \frac{\partial}{\partial p} \left(g\Phi_{w,z}(n)\right) - \frac{1}{a\cos\phi} \frac{\partial}{\partial \phi} \left(\cos\phi \sum_{\substack{m=-\infty\\ \neq 0}}^{\infty} \left\{U(m)\psi_{v,u}(m,n) + V(m)\psi_{v,v}(m,n)\right\}\right) \end{split}$$

The first and second terms on the right-hand side of equation (1) are hereafter represented symbolically by M(n) and L(n) respectively (Saltzman and Fleisher, 1960).

Now integrating (1) over an arbitrary mass of the atmosphere of unit pressure difference  $\Delta P$  we arrive at:

$$\begin{aligned} \frac{2\pi a^2 \Delta p}{g} \int_{\phi_1}^{\phi_2} \frac{\partial K(n)}{\partial t} \cos \phi \, d\phi &= \frac{2\pi a^2 \Delta p}{g} \int_{\phi_1}^{\phi_2} M(n)^{\mathrm{I}} \cos \phi \, d\phi \\ &\quad + \frac{2\pi a^2 \Delta p}{g} \int_{\phi_1}^{\phi_2} \left( \frac{p}{p_0} \right)^{R_{L_p}} \frac{H}{p} \Phi_{w,\theta}(n) \cos \phi \, d\phi \\ &\quad - \frac{2\pi a^2 \Delta p}{g} \int_{\phi_1}^{\phi_2} \left( \frac{p}{p_0} \right)^{R_{L_p}} \frac{R}{p} \Phi_{w,\theta}(n) \cos \phi \, d\phi \\ &\quad + \frac{2\pi a^2}{g} \int_{\phi_1}^{\phi_2} \left( \Phi_{u,z}^{\mathrm{IV}}(n) + \Phi_{z,y}(n) \right) \cos \phi \, d\phi \\ &\quad + \frac{2\pi a^2}{g} \int_{\phi_1}^{\phi_2} \sum_{\substack{m=-\infty \\ g \neq 0}}^{\infty} \left\{ U(m) \psi_{w,u}^{\mathrm{Va}}(m,n) \\ &\quad + V(m) \psi_{w,v}(m,n) \right\}_{p_1} \cos \phi \, d\phi \\ &\quad - \frac{2\pi a^2}{g} \int_{\phi_1}^{\phi_2} \sum_{\substack{m=-\infty \\ g \neq 0}}^{\infty} \left\{ U(m) \psi_{w,u}^{\mathrm{Vb}}(m,n) \\ &\quad + V(m) \psi_{w,z}(m,n) \right\}_{p_2} \cos \phi \, d\phi \\ &\quad + 2\pi a^2 \int_{\phi_1}^{\phi_2} \left( \Phi_{w,z}(n) \right)_{p_1} \cos \phi \, d\phi - 2\pi a^2 \int_{\phi_1}^{\phi_2} \left( \Phi_{w,z}(n) \right)_{p_2} \cos \phi \, d\phi \\ &\quad + \frac{2\pi a \Delta p}{g} \left( \cos \phi \sum_{\substack{m=-\infty \\ g \neq 0}}^{\infty} \left\{ U(m) \psi_{v,u}(m,n) \\ &\quad + V(m) \psi_{v,v}(m,n) \right\} \right)_{\phi_1} \\ &\quad - \frac{2\pi a \Delta p}{g} \left( \cos \phi \sum_{\substack{m=-\infty \\ g \neq 0}}^{\infty} \left\{ U(m) \psi_{v,u}(m,n) \\ &\quad + V(m) \psi_{v,v}(m,n) \right\} \right)_{\phi_2} \end{aligned}$$
(2)

where:

term I represents the transfer of kinetic energy from the zonal mean flow to the eddies;

term II describes the internal transfer of kinetic energy between wave numbers and is identically equal to zero when summed over wave numbers  $1-\infty$ ;

term III is the conversion from eddy available potential energy to eddy kinetic energy as a function of wave number;

term IV represents the frictional loss;

term Va,b depicts the "vertical" advection of eddy kinetic energy across the upper and lower boundaries;

term VIa,b presents the transfer of eddy kinetic energy across the upper and lower boundaries by the pressurework term (e.g. Miller, 1967);

term VIIa,b delineates the horizontal advection of eddy kinetic energy across the southern and northern boundaries;

term VIIIa,b denotes the horizontal transfer of eddy kinetic energy across southern and northern boundaries by the pressure-work term.

As mentioned above, the only terms of equation (2) that we could investigate in this study were I, II, VIIa,b integrated between  $22.5^{\circ}$ N and  $72.5^{\circ}$ N and the kinetic energy per wave number integrated between  $17.5^{\circ}$ N and  $77.5^{\circ}$ N. Inasmuch as term VIIa, b, was found to be about an order of magnitude less than the other terms, discussion of its interannual variability is reserved till a later section when the relationship of the measured terms to the total kinetic energy budget is discussed.

While calculations were made for individual wave numbers 1-6 (the upper limit of n=6 was determined by the results of MWT) and for the sum of waves 1-15, the ensuing discussion centers on the analysis of the following four components:

A) 
$$\sum_{n=1}^{15} M(n); \sum_{n=1}^{15} K(n)$$
 (fig. 3)

B) 
$$\Sigma$$
 (L+M); K(n) n=1 (fig. 4)

C) 
$$\Sigma$$
 (L+M); K(n) n=2 (fig. 5)

D) 
$$\Sigma$$
 (L+M); K(n) n=6 (fig. 6)

The reasons for isolating these particular components lie in the fact that we were interested not only in determining the variability of the energy cycle in the troposphere but also in seeking a possible relationship of this variability with that of the lower stratosphere.

Term A includes the transfer of kinetic energy from the

mean zonal flow to the eddies and has been shown to vary significantly from year to year (e.g., Julian and Labitzke, 1965, Wiin-Nielsen, Brown, and Drake, 1964, and Wiin-Nielsen, 1967).

Wave number 1 (term B) is considered because of the existence of the dominant wave one pattern in the stratosphere during the winter months (e.g., Staff, Upper Air Branch, 1967). Of course, this presupposes that an energy transfer mechanism between the troposphere and stratosphere exists on this scale, but the recent work of Perry (1967) suggests this to be the case. This point is being investigated further.

Wave number 2 (term C), on the other hand, is included because the results of Saltzman and Teweles (1964) indicate that motion of this scale acts as a source of kinetic energy both for the zonal flow and for the other waves. This, in turn, indicates a strong forced conversion on the scale of the major continents and oceans. At the same time wave two is the dominant mode during the stratospheric sudden-warming phenomenon (Finger and Teweles, 1964).

Wave number 6 (term D) is included as a typical example of the "medium" wavelength, baroclinic waves that are sources of kinetic energy (e.g., Saltzman and Teweles, 1964) for both the zonal flow and for the other waves.

By emphasizing these particular components we do not mean to imply that the other wave numbers do not exhibit substantial interannual variability. Rather, it is simply that waves 3-5 generally indicated less coherence with the stratospheric quasi-biennial cycle (hereafter referred as SQB) than those presented and their depiction was deleted for the sake of brevity.

# 4. DISCUSSION

Inasmuch as a major objective of this study is to compare the tropospheric energy variability with the quasi-



FIGURE 2.—Monthly averaged zonal winds (m sec<sup>-1</sup>) at 10 mb for Balboa, Canal Zone, and Ascension Island. Positive winds are from the west (after Quiroz and Miller, 1967).



FIGURE 3.—Twelve-month minus 24-mo running mean of the monthly average kinetic energy exchange,  $\sum_{n=1}^{15} M(n)$  (solid line), and of the monthly average kinetic energy,  $\sum_{n=1}^{15} K(n)$  (dashed line), summed over wave numbers 1-15. Units: M(n),  $10^{14}$  ergs (sec-mb)<sup>-1</sup>; K(n),  $10^{20}$  ergs (mb)<sup>-1</sup>.

biennial oscillation as observed in the lower stratosphere, it is convenient to describe the latter feature first. Figure 2 presents the monthly mean values of the 10-mb ( $\approx$  30 km) zonal wind component from January 1958 through August 1966 at Balboa, Canal Zone (08°56'N, 79°34'W), and Ascension Island (07°59'S, 14°25'W). As the details of this diagram have been discussed by Quiroz and Miller (1967), suffice it to say at this point that through about 1961 the cyclic behavior of the zonal wind field was fairly regular with easterlies generally propagating downward in the evennumbered years and westerlies in the odd-numbered years (Reed, 1965). In 1962, however, there began a disruption of the cyclic behavior leading to an extended period of westerly winds at 10 mb (fig. 2) and continued easterly flow at lower altitudes (Reed, 1965). It was not until late 1963 that the westerly winds propagated downward, after which the cyclic nature was restored.

Figure 3 presents the filtered time series of the sum over wave numbers 1-15 of the transfer of kinetic energy from the zonal flow to the eddies and of the eddy kinetic energy itself (positive value of M(n) implies a positive value of  $\partial K(n)/\partial t$ ).

While large year-to-year variations are apparent in both time series, there does not seem to be any definite relationship with the quasi-biennial cycle (fig. 2). For example, in 1959 and 1960 the perturbations in the M and K terms



FIGURE 4.—Twelve-month minus 24-mo running mean of the monthly average kinetic energy exchange,  $\Sigma(L+M)$  (solid line), and of the monthly average kinetic energy, K(n) (dashed line), for wave number 1. Units:  $\Sigma(L+M)$ ,  $10^{14}$  ergs (sec-mb)<sup>-1</sup>; K(n),  $10^{20}$  ergs (mb)<sup>-1</sup>.

are in phase with SQB, but the situation is confused before and after these times.

Also of interest is the fact that while the total kinetic energy variation is generally in phase with the zonal-eddy exchange, the magnitude of its change is generally too small for the two terms to balance,

e.g., 
$$\frac{600 \times 10^{20} \text{ ergs/mb}}{6 \text{ mo}} = 38.58 \times 10^{14} \text{ ergs}}{\text{mb-sec}}$$

Indeed, in some instances the variations are even oppositely directed. The magnitude of the perturbations of the M term is only about 15 percent of the yearly average determined by Saltzman and Teweles, but as will be shown below this is due mainly to the fact that the Mterms, as measured by Saltzman and Teweles for each wave number, are in the same direction and hence additive. For an individual wave number, the amplitude of the oscillation is of the same order of magnitude as Saltzman and Teweles' yearly mean values (as certain previous investigators' results are presented in units of ergs/cm<sup>2</sup>-sec-mb, it is noted for purposes of comparison that the area of integration in this study, 22.5-72.5°N, is  $\simeq 1.46 \times 10^{18}$  cm<sup>2</sup>).

Figure 4 presents the filtered time series of the  $\Sigma(L+M)$ and K terms for n=1. The former quantity was plotted as the sum of the terms rather than individually, because earlier separate analyses (not shown here) indicated that phase differences between them were such that meaningful interpretation with respect to the total energy variability





FIGURE 5.—Same as figure 4 for wave number 2.

could be made only by considering the two terms together. As a general rule, the value of L(n) tended to be several times larger than M(n) (Saltzman and Teweles, 1964).

As in figure 3, there are significant year-to-year variations in the energy transfer terms (note particularly the major peaks in 1960 and 1961), but also as in the previous example, there is little definite relationship with the SQB. The value K(1), in general, follows the variations of  $\Sigma(L+M)$ , but the changes are again too small for a balance to occur.

The variability of  $\Sigma(L+M)$  for wave number 2 as depicted in figure 5, on the other hand, is, up to a point, far more regular in nature and displays an interesting relationship with K(2). From 1956 through the early part of 1962, significant variations of  $\Sigma(L+M)$  occur whose amplitude is about one-half that of the yearly mean (Saltzman and Teweles, 1964) and which exhibit a marked negative phase relationship with the SQB. From 1962 through 1965, while large interannual variations still persist, the phase relationship is unclear, and a longer record is required before one can determine if the previous phase relationship reestablishes itself.

At the same time, it is interesting to note that from 1956 through early 1959, the K(2) and  $\Sigma(L+M)$  terms are opposite in phase, which means that other terms in equation (2) are overcompensating for the  $\Sigma(L+M)$  terms. While the variations of the two terms become more coherent after 1959, the changes of K(2) are still too small for balance to occur.



FIGURE 6.—Same as figure 4 for wave number 6.

The results for wave number 6 are presented in figure 6, and in contrast to the results for wave 2 indicate a generally positive phase relationship with the SQB. The major discrepancies, which occur in 1962 and 1964-65, are due to the extended positive variations in the energy terms. Once again, however, we must await an extended data sample to determine if the phase relationship becomes coherent again. The value of K(6) is generally small and shows little association with the  $\Sigma(L+M)$  terms.

# 5. FINAL REMARKS

While we were able to measure only three of the eight forcing function terms in the eddy kinetic energy balance equation (2), qualifying remarks can be made about several of those remaining. Term V, for example, has been estimated by Jensen (1961) and Miller (1967) and determined to be about an order of magnitude smaller than term VI. Therefore, if we assume that this magnitude difference is not a result of a cancellation effect between the different wave numbers for the former term, then term V may properly be neglected. Term VII, on the other hand, under the geostrophic approximation is identically equal to zero. Therefore, since we have already remarked on the relatively small magnitudes of term VII and the time rate of change of the kinetic energy, approximate balance must occur between terms I, II, III, IV, and VI. Unfortunately, the measurement of terms III, IV, and VI was beyond our capability at this time and we must confine our discussion to their sum balanced against the sum of the measured terms, I and II. Our results indicate that while the variation of total exchange of kinetic energy is generally about 15 percent of the yearly average, this is a result of the additive nature of the yearly mean values. The individual wave numbers indicate significant year-to-year variations that must be considered when only short time periods are under study. Similarly, the sum of the frictional dissipation conversion from potential to kinetic energy and pressure work terms must approximately balance the measured terms and must, therefore, exhibit similar interannual variations.

We, of course, cannot state how these terms vary individually and this leads to the second purpose of this study: to determine if a relationship exists between the energetics of the troposphere and the stratospheric quasibiennial cycle. Unfortunately, the results of our analysis were inconclusive on this point, possibly because of the lack of vertical resolution. At the same time, however, the fact that large interannual variations in the energy cycle at 500 mb were found to exist suggests that a profitable line of inquiry would be to examine the annual variations of the pressure-work term as a function of wave number at the interface of the troposphere and stratosphere. If, at the same time, we consider that the scale of motion in the stratosphere is generally constrained to the lower wave numbers (Perry, 1967, Teweles, 1963), then our results suggest that particular emphasis should be placed on an examination of the energy variation of wave number 2. The feasibility of such an endeavor is being investigated.

### ACKNOWLEDGMENTS

The original work in setting up the numerical formulation and machine programs for the wave number calculations was performed by Prof. Aaron Fleisher of M.I.T. and Dr. Barry Saltzman of Travelers Research Center.

Support for this study has been provided by the National Aeronautics and Space Administration under its EXAMETNET program and by the U.S. Naval Air Systems Command.

#### REFERENCES

- Finger, F. G., and Teweles, S., "The Mid-Winter 1963 Stratospheric Warming and Circulation Change," Journal of Applied Meteorology, Vol. 3, No. 1, Feb. 1964, pp. 1-15.
- Jensen, C. E., "Energy Transformation and Vertical Flux Processes Over the Northern Hemisphere," Journal of Geophysical Research, Vol. 66, No. 4, Apr. 1961, pp. 1145-1156.

- Julian, P. R., and Labitzke, K. B., "A Study of Atmospheric Energetics During the January-February 1963 Stratospheric Warming," Journal of the Atmospheric Sciences, Vol. 22, No. 6, Nov. 1965, pp. 597-610.
- Lorenz, E. N., "Available Potential Energy and the Maintenance of the General Circulation," *Tellus*, Vol. 7, No. 2, May 1955, pp. 157-167.
- Miller, A. J., "Note on Vertical Motion in the Lower Stratosphere," Beiträge zur Physik der Atmosphäre, Frankfurt, Vol. 40, No. ½, 1967, pp. 29-48.
- Miller, A. J., Woolf, H. M., and Teweles, S., "Quasi-Biennial Cycles in Angular Momentum Transports at 500 Mb.," Journal of the Atmospheric Sciences, Vol. 24, No. 3, May 1967, pp. 298– 304.
- Muench, H. S., "On the Dynamics of the Wintertime Stratosphere Circulation," Journal of the Atmospheric Sciences, Vol. 22, No. 4, July 1965, pp. 349-360.
- Newell, R. E., "A Note on the 26-Month Oscillation," Journal of the Atmospheric Sciences, Vol. 21, No. 3, May 1964, pp. 320-321.
- Oort, A. H., "On the Energetics of the Mean and Eddy Circulations in the Lower Stratosphere," *Tellus*, Vol. 16, No. 3, Aug. 1964, pp. 309-327.
- Perry, J. S., "Long-Wave Energy Processes in the 1963 Sudden Stratospheric Warming," Journal of the Atmospheric Sciences, Vol. 24, No. 5, Sept. 1967, pp. 539-550.
- Quiroz, R. S., and Miller, A. J., "Note on the Semi-Annual Wind Variation in the Equatorial Stratosphere," Monthly Weather Review, Vol. 95, No. 9, Sept. 1967, pp. 635-641.
- Reed, R. J., "The Present Status of the 26-Month Oscillation," Bulletin of the American Metreorological Society, Vol. 46, No. 7, July 1965, pp. 374-387.
- Saltzman, B., and Fleisher, A. F., "Spectrum of Kinetic Energy Transfer Due to Large-Scale Horizontal Reynolds Stresses," *Tellus*, Vol. 12, No. 1, Feb. 1960, pp. 110-111.
- Saltzman, B., and Teweles, S., "Further Statistics on the Exchange of Kinetic Energy Between Harmonic Components of the Atmospheric Flow," *Tellus*, Vol. 16, No. 4, Nov. 1964, pp. 432-435.
- Staff, Upper Air Branch, National Meteorological Center, "Weekly Synoptic Analyses 5-, 2-, and 0.4-Mb. Surfaces for 1965," ESSA Technical Report WB-3, U.S. Department of Commerce, Washington, D.C., Aug. 1967, 173 pp.
- Teweles, S., "Spectral Aspects of the Stratospheric Circulation During the IGY," *Planetary Circulations Project Report* No. 8, Department of Meteorology, Massachusetts Institute of Technology, Cambridge, Jan. 1963, 191 pp.
- Wiin-Nielsen, A., "On the Annual Variation and Spectral Distribution of Atmospheric Energy," *Tellus*, Vol. 19, No. 4, Nov. 1967, pp. 540-559.
- Wiin-Nielsen, A., Brown, J. A., and Drake, M., "Further Studies of Energy Exchange Between the Zonal Flow and the Eddies," *Tellus*, Vol. 16, No. 2, May 1964, pp. 168–180.

[Received June 25, 1968; revised August 14, 1968]