

***“Multiscale Analysis and Computation for Flows in Heterogeneous Media”***

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**Summary**

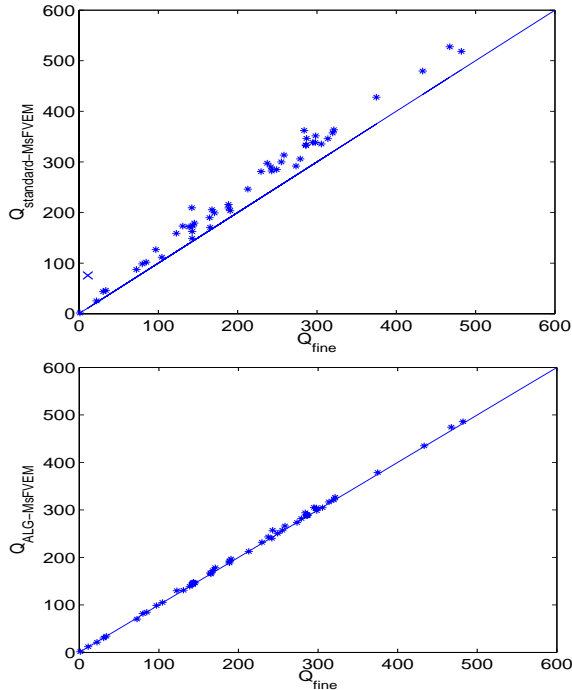
***Our work in this project is aimed at making fundamental advances in multiscale methods for flow and transport in highly heterogeneous porous media. This work is directly applicable to the efficient modeling of flow in subsurface formations such as oil reservoirs and aquifers. A main thrust of this research is to develop a systematic multiscale analysis and efficient coarse-scale models that can capture global effects. We also aim to extend existing multiscale approaches to problems with additional physical effects. A key emphasis is on problems without an apparent scale separation.***

Multiscale solution methods are currently under active investigation for the simulation of subsurface flow in heterogeneous formations. These procedures capture the effects of fine-scale property variations through the calculation of specialized coarse-scale basis functions. Most of the multiscale techniques presented to date employ localization approximations in the calculation of these basis functions. For formations with large-scale features (as commonly occur in nature), however, global effects can be important and these may need to be incorporated into the multiscale representation. Other challenging issues facing multiscale simulations are the extension of existing techniques to problems with additional physics, such as fluid compressibility, capillary effects, etc.

We describe here three of our main accomplishments during our first year of collaboration. Our first accomplishment is the development of iterative multiscale techniques. The main idea of these techniques is to use the multiscale basis

functions iteratively to obtain more accurate solutions. In particular, multiscale solutions generated using basis functions with different oversampling regions are applied to obtain the next iteration. These solutions can be computed efficiently since they rely on local solutions. The improvement in accuracy is due to the fact that each iteration improves the global information that is incorporated into the basis functions. The resulting procedure (referred to as ALG-MsFVEM) is formulated as a finite volume element method and is applied for a number of single and two-phase flow simulations of channelized systems. The level of accuracy of the resulting method is shown to be consistently higher than that of the standard finite volume element multiscale technique based on simple localized basis functions. This is demonstrated in **Fig. 1**, where the global flow rates for standard MsFVEM and ALG-MsFVEM are compared for 50 channelized permeability fields of a benchmark test (SPE 10). Taken in total, this work demonstrates that we are able to incorporate approximate global information

into the multiscale basis functions and that this leads to improvements in the accuracy of the coarse-scale solution. This work also provides a conceptual framework for the efficient use of global information in multiscale methods and suggests how to improve local basis functions via iterations. Our numerical results show that the proposed methods converge within a very few iterations for channelized permeability fields.



**Figure 1.** Comparison of global flow rates for standard MsFVEM (top) and ALG-MsFVEM (bottom).

Our second accomplishment explores the use of special coordinate systems in developing an efficient multiscale method for flow and transport in heterogeneous porous media. The main idea of the new technique is the use of a flow-based coordinate system. Flow-based coordinate systems allow us to simplify the scale interaction and derive the upscaled equations for hyperbolic transport equations. This is due to the fact that two-phase flow and transport properties are smooth

functions in flow-based coordinate systems. For two-phase flow in heterogeneous porous media, the use of a flow-based coordinate system requires limited global information, such as the solution of a single-phase flow problem. In this coordinate system, we propose an efficient upscaling of the transport equation using non-local information along streamlines. The numerical results decisively show that one can achieve accurate upscaling results using a flow-based coordinate system. Moreover, this coordinate system facilitates the incorporation of additional physics into the transport equation.

Our next accomplishment is the development of operator-based multiscale method for compressible two-phase flow. Existing multiscale methods deal with the incompressible flow problem only. However, compressibility will be significant if a gas phase is present. A scalable and extendible operator-based multiscale method (OBMM) is proposed in this work. OBMM is cast as a general algebraic framework of the multiscale method. It is very natural and convenient to incorporate more physics in OBMM for multiscale computations. In OBMM, two multiscale operators are constructed: prolongation and restriction. The prolongation operator can be constructed by assembling basis functions, and the specific form of the restriction operator depends on the coarse-scale discretization formulation (e.g., finite volume or finite element). The coarse-scale pressure equation is obtained algebraically by applying the prolongation and restriction operators on the fine-scale flow equations. The fine-scale pressure can be reconstructed by applying the prolongation operator to the coarse-scale pressure. A conservative fine-scale velocity field is then reconstructed to solve the transport equation. Numerical results convincingly show that the proposed

general framework provides an accurate approximation of the fine-scale solution of compressible flow and transport in highly heterogeneous porous media.

More information about our ongoing research can be found at [http://www.acm.caltech.edu/~hou/doi\\_joint/doi\\_joint.html](http://www.acm.caltech.edu/~hou/doi_joint/doi_joint.html)