# The Increase in the Primordial $^4$He Yield in the Two-Doublet Four-Neutrino Mixing Scheme

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We assess the effects on Big Bang Nucleosynthesis (BBN) of lepton number generation in the early universe resulting from the two-doublet four-neutrino mass/mixing scheme. It has been argued that this neutrino mass/mixing arrangement gives the most viable fit to the existing data. We study full $4 \times 4$ mixing matrices and show how possible symmetries in these can affect the BBN $^4$He abundance yields. Though there is as yet no consensus on the reliability of BBN calculations with neutrino flavor mixing, we show that, in the case where the sign of the lepton number asymmetry is unpredictable, BBN considerations may pick out specific relationships between mixing angles. In particular, reconciling the observed light element abundances with a $\bar{\nu}_\mu \rightleftharpoons \bar{\nu}_e$ oscillation interpretation of LSND would allow unique new constraints on the neutrino mixing angles in this model.

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## I. INTRODUCTION

In this paper we revisit the long standing issue of how neutrino flavor mixing in the early universe might affect Big Bang Nucleosynthesis (BBN). Though this is an old problem [1–5], with the advent of modern experiments (*e.g.*, solar, atmospheric, and accelerator-based oscillation experiments) we can hope to begin constructing the neutrino mass/mixing matrix. Here we adopt the leading model for this, the two-doublet four-neutrino mass scheme [6,7]. We discuss this mixing in detail, going beyond recent good assessments of this problem [8–10], to examine what BBN considerations *ultimately* may be able to tell us about parameters in this mixing scheme.

Historical attempts [1–5] at constraints on neutrino mixing are based on the argument that an active-sterile neutrino mixing that is too large at (or prior to) the BBN epoch will populate the sterile neutrino sea. The resultant increase in the total energy density of the universe at a given temperature speeds up the Hubble expansion. In turn this leads to a higher weak-freeze-out temperature and, consequently, it could lead to a higher neutron-to-proton ratio at Nuclear Statistical Equilibrium freeze-out [11] (however, the neutron-to-proton ratio is determined not only by the expansion rate, see below). Since essentially all neutrons are incorporated into $^4$He in the early universe, such a mixing yields a higher $^4$He abundance $Y$. This potentially could contradict the observationally inferred primordial $^4$He abundance. From the inferred abundances of $^4$He and D/H there are strong constraints on the increase of the predicted $Y$ from an increased energy density due to sterile neutrino production. These constraints are often translated into a limit on the "effective number of neutrinos" of

$$1.7 \leq N_\nu^{\rm eff} \leq 3.2 \tag{1}$$

at 95% confidence that is strictly limited by the baryon-to-photon ratio, $\eta$, determined by the inferred relative abundance of deuterium (D/H) [12]. This limit is shown in Fig. 1, where we have calculated $Y$ and D/H over a range in $N_\nu^{\rm eff}$ and $\eta$. We have included corrections to $Y$ derived from the zero and finite-temperature radiative, Coulomb and finite-nucleon-mass corrections to the weak rates; order-$\alpha$ quantum-electrodynamic correction to the plasma density, electron mass, and neutrino temperature; incomplete neutrino decoupling; and numerical time-step effects [13]. The observationally-inferred primordial mass fraction we adopt is generous: $0.228 < Y < 0.248$ (95% confidence range) [14]; and the inferred relative abundance D/H we adopt is the well-established D/H $\approx (3.4 \pm 0.5) \times 10^{-5}$ [12]. For arguments and observational evidence on the reliability of the deuterium abundance see Ref. [15]. Similar limits on $N_\nu^{\rm eff}$ have been derived by other groups [16–18]. These limits preclude by a wide margin a fully populated sterile neutrino sea ($N_\nu^{\rm eff} = 4$) in the BBN epoch. The concept of $N_\nu^{\rm eff}$ (an expansion rate measure) as the sole determinant of the $^4$He yield is a misleading and dangerous one.

A few $4 \times 4$ models have been advanced as simultaneous explanations of the atmospheric neutrino data, the solar neutrino data, and the data from LSND [6,7]. One hierarchical scheme would have (near) maximal $\nu_\mu \rightleftharpoons \nu_\tau$ mixing for the atmospheric deficit, a $\nu_e \rightleftharpoons \nu_\mu$ mass splitting that gives either a vacuum or MSW solar solution, and the LSND indication of a large $\delta m^2$ is given by "indirect" mixing through a sterile. This is Scheme I in Fig. 2. In this scheme, the mass states most closely associated with the active neutrinos form a triplet that has a significant mass splitting from the mostly sterile mass state. This scheme is ruled out by BBN because it requires large $\nu_\mu \rightleftharpoons \nu_s$ and $\nu_e \rightleftharpoons \nu_s$ mixing amplitudes to explain the LSND results through the indirect conversion $\nu_\mu \to \nu_s \to \nu_e$ (and the $CP$ conjugate). The required large mixing amplitudes for $\nu_\mu \rightleftharpoons \nu_s$ and $\nu_e \rightleftharpoons \nu_s$ populate the sterile sea in the early universe through direct oscillation production of steriles [2–5]. This mass model is also disfavored by a combined $4 \times 4$ experimental data analysis [7].

An alternative two-doublet hierarchical scheme (Scheme II) has (near) maximal $\nu_\mu \rightleftharpoons \nu_s$ mixing to fit the atmospheric data in Super-Kamiokande, and $\nu_e \rightleftharpoons \nu_\tau$ mixing explaining the solar neutrino puzzle. This option is excluded by BBN since the $\nu_\mu \rightleftharpoons \nu_s$ transformation is too large to be compatible with $^4$He observations [5,19]. (Recently Foot and Volkas [23] have argued that $\nu_\mu \rightleftharpoons \nu_s$ maximal vacuum mixing for SuperK atmospheric neutrinos in a related mass scheme *can* be reconciled with BBN limits.) [1]

Therefore, adopting the previously considered limits from BBN, we are left only with the two-doublet four-neutrino mixing model, Scheme III. The BBN effects of the model in Scheme III were considered in Ref. [8]. In this paper we expand on the analysis of the mixing and suggest how BBN could give potentially stricter limits that those found in Ref. [8]. (We note here that the "inverted scheme," where the $\nu_e/\nu_s$ doublet is more massive than the $\nu_\mu/\nu_\tau$ doublet is not yet completely ruled out by laboratory and astrophysical considerations.)

A new twist was added to the saga of neutrino-mixing in the early universe when it was found that resonant active-sterile neutrino transformation in the BBN epoch can alter neutrino energy spectra and generate a lepton number asymmetry [21,22]. This raises the possibility that an asymmetry in $\nu_e/\bar{\nu}_e$ numbers could be generated. In the mean time, their energy spectra may be modified so that the weak reaction rates themselves may change, resulting in a different neutron-to-proton ratio and a different $^4$He yield $Y$ [23–25]. It has been argued that a positive $\nu_e/\bar{\nu}_e$ asymmetry behaves like a positive chemical potential for $\nu_e$. This would reduce $Y$, and by the same token a negative asymmetry would increase it [23]. However, this argument of a direct asymmetry-$Y$ leverage relation is too naive in the context of active-sterile neutrino mixing, and is in fact incorrect. This is because the process of lepton number generation via resonant active-sterile neutrino mixing potentially has a crucial and unique feature [17,22,26–28]: that the lepton number asymmetry is first damped to essentially zero and then can oscillate chaotically with an increasingly larger amplitude, until it converges to a growing asymptotic value that is either positive or negative.

However, the existence of "chaoticity" (unpredictability) in the final sign of the lepton asymmetry, $L$, is controversial. Indeed, *all* claims (*e.g.* [21,22]) for lepton number generation in the early universe via neutrino mixing are now at odds with at least one calculation (Dolgov, Hansen, Pastor and Semikoz [29]). Ref. [29] uses a new formulation of the solution of the neutrino energy density matrix evolution that finds a non-chaotic generation of only a small and insignificant lepton asymmetry. Refs. [26] and [27] recalculated the lepton number generation found in Ref. [22] and corroborated a random nature to the sign of the $L$. Ref. [27] showed that certain over-simplifying approximations in Ref. [29] may have unphysically stabilized the evolution of the lepton number. Ref. [28] found a randomness in the sign of $L$, but only for small non-phenomenological mixing angles, much smaller than those that may be involved in neutrino oscillation solutions to the experiments discussed here. The oscillations in lepton number sign seen in Ref. [28] only occur below the precision of their numerical solution. One may call into question then the ability of their numerical formulation to resolve oscillations occuring at larger mixing angles which can be found in the more straight-forward momentum-averaged solution of Refs. [22,26,27]. The momentum-averaged solution is valid at the relevant high-temperatures where the instantaneous approximation to repopulation are good. For the LSND mass scale in Scheme III considered here, $m^2_{\nu_\mu} - m^2_{\nu_e} \approx m^2_{\nu_\mu} - m^2_{\nu_s} \sim 0.2$ to $10\ \mathrm{eV}^2$ and a small effective mixing angle ($\sin^2\theta_{\mathrm{eff}} > 10^{-10}$), the bulk of the active neutrinos undergo resonance at a temperature $T \sim 10$ to $20$ MeV where the instantaneous approximation can be considered valid.

Given the controversy and disagreement among these different calculations it is difficult to take any BBN-derived constraint with confidence. With this caveat in mind, we explore what constraints *might* be possible if the lepton number generation magnitude is as in Refs. [21,22], but where chaoticity in lepton number sign obtains for relevant parameters.

In a chaotic lepton number generation regime, the sign of the lepton number asymmetry is independent of the initial conditions before the amplification begins, and is exponentially sensitive to the parameters involved during the chaotic oscillitory phase [22]. In turn, the sign of the asymmetry cannot correlate over a scale bigger than the particle horizon [19].

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[1] There is still another loop-hole: if there is a pre-existing lepton number asymmetry with a magnitude $\gtrsim 10^{-5}$ during the BBN epoch, the $\nu_\mu \rightleftharpoons \nu_s$ mixing can be suppressed [20]. This assumption must then involve physics that is beyond neutrino mixing.

This causal structure of space-time can make it impossible to obtain a universe with a uniform lepton number asymmetry. Instead, the lepton number generating process gives rise to a universe with numerous lepton number domains with similar lepton number magnitudes, albeit different signs [19]. The size of each domain is less than the horizon size at that epoch ($\sim 10^{10}$ cm at the weak-freeze-out temperature, although the detailed geometric structure of the domains at the BBN epoch depends on the manner in which domain "percolation" occurs). The distribution of domains with different signs is completely random so that in total each sign occupies half of the space.

The overall primordial $^4$He yield in such a universe must be an average of $Y$ over domains with opposite $\nu_e/\bar{\nu}_e$ asymmetries. Interestingly, this implies that the overall $Y$ is always *larger* than that expected when there are no lepton asymmetries. This is because the increase in $Y$ from a negative lepton asymmetry generated by the resonant active-sterile neutrino mixing process is always *bigger* than the decrease in $Y$ from a positive asymmetry generated in the same process [24].

In this paper we examine in detail how this causality effect operates in the context of a *specific* four neutrino mixing scheme. There are some surprises.

We can quantify these arguments for the two-doublet neutrino mixing model (Scheme III). In this model, the $\nu_\mu \rightleftharpoons \nu_e$ mixing that fits the LSND data has a mass-squared-difference $m^2_{\nu_\mu} - m^2_{\nu_e} \sim 0.2$ to 10 eV$^2$ and an effective two-neutrino mixing angle satisfying $\sin^2 2\theta_{e\mu} \sim 10^{-3}$ to $10^{-2}$ [30]; the $\nu_e \rightleftharpoons \nu_s$ mixing that solves the solar neutrino puzzle has either $m^2_{\nu_s} - m^2_{\nu_e} \sim 10^{-5}$ eV$^2$ and an effective two-neutrino mixing angle satisfying $\sin^2 2\theta_{es} \sim 10^{-2}$ for the small mixing angle (SMA) MSW (Mikheyev-Smirnov-Wolfenstein) solution or $|m^2_{\nu_s} - m^2_{\nu_e}| \sim 10^{-10}$ eV$^2$ and $\sin^2 2\theta_{es} \sim 1$ for the vacuum solution [31]; the Super-Kamiokande atmospheric neutrino data are nicely fit by $\nu_\mu \rightleftharpoons \nu_\tau$ mixing with $|m^2_{\nu_\mu} - m^2_{\nu_\tau}| \sim 10^{-3}$ to $10^{-2}$ eV$^2$ and $\sin^2 2\theta_{\mu\tau} \sim 1$ [32]. The hierarchy in masses which emerges from this mixing scheme includes an upper doublet of heavier neutrinos, consisting of (almost) degenerate $\nu_\mu$ and $\nu_\tau$, and a lower doublet of lighter neutrinos, consisting of slightly mixed (in the case of the MSW solution) or (near) maximally mixed (in the case of vacuum mixing) $\nu_e$ and $\nu_s$. The inter-doublet mixing between $\nu_\mu$ and $\nu_e$ is small. So also must be the inter-doublet mixing between $\nu_\mu$ or $\nu_\tau$ and $\nu_s$, so as to avoid conflicts with BBN [3–5].

The large mixing angle (MSW) solution to the solar neutrino problem has been previously disfavored by BBN [3–5]. We consider the SMA MSW active-sterile neutrino mixing solar solution and the vacuum active-sterile neutrino mixing solar solution. In Ref. [31], the vacuum sterile neutrino solar solution was considered to be disfavored using the current combined solar neutrino experiment rate data. However, in Ref. [33], it was argued that if only the solar Super-K spectrum or the seasonal solar data are considered, then the vacuum active-sterile neutrino mixing solution gives a better fit than a vacuum active-active neutrino mixing solution. Since the nature of neutrino mixing of solar neutrinos is still not certain and since BBN considerations may prove to be enlightening, we entertain the possibility of a vacuum active-sterile neutrino mixing solution to the solar neutrino deficit. The vacuum sterile neutrino mixing solution to the solar neutrino problem was also considered in terms of a neutrino mass model represented by the $[\mathrm{SU}(3)]^3$ or $E^6$ groups in Ref. [34].

The interpretation of current experimental results usually is framed in terms of an effective two neutrino mixing scenario (i.e., in terms mass-squared-differences and mixing angles) for each experimental situation. This model is approximately valid in the two-doublet four-neutrino mixing scheme because, as a result of the mass hierarchy, one two-species mixing dominates in each of the above experiments. It is, however, more informative to employ the full $4 \times 4$ mixing matrix in our discussion. In the next section we will briefly review what has been learned about this mixing matrix from the current experiments. We will then proceed to consider BBN $^4$He synthesis in the presence of hierarchical four-neutrino mixing schemes. From the $^4$He yield we infer potential new constraints on the inter-doublet mixing matrix elements between active neutrinos and the sterile neutrinos. In section III, we will summarize our results.

## II. THE HIERARCHICAL FOUR-NEUTRINO SCHEME AND THE PRIMORDIAL $^4$HE ABUNDANCE

We adopt the convention of employing Greek indices to denote flavor eigenstates $\nu_s$, $\nu_e$, $\nu_\mu$ and $\nu_\tau$, and employing Latin indices to denote mass eigenstates $\nu_0$, $\nu_1$, $\nu_2$ and $\nu_3$. The two bases are related by a unitary transformation $U$:

$$\nu_\alpha = \sum_{i=0,3} U_{\alpha i} \nu_i. \tag{2}$$

The mass matrix in the flavor basis is then

$$M_{\alpha\beta} = \sum_{k=0,3} \sum_{l=0,3} U^*_{\alpha k} m_k \delta_{kl} U^\dagger_{l\beta}, \tag{3}$$

where $m_k$ are the mass eigenvalues, and $\delta_{kl}$ are the Kronecker deltas. In the scheme considered here, $m_0, m_1 \ll m_2, m_3$.

The full expression for $U$ can be found, for example, in Eq. (8) of Ref. [35]. It has 12 degrees of freedom, parametrized by 6 rotation angles $\theta_{ab}$, and 6 $CP$-violating phases $\phi_{ab}$. The parameters are symmetric in indices $a$ and $b$, which run from 0 to 3 ($a < b$). We follow convention and write $s_{ab} \equiv \sin\theta_{ab} e^{i\phi_{ab}}$ and $c_{ab} \equiv \cos\theta_{ab}$. Because the inter-doublet mixing is small, $|U_{s2}|^2$, $|U_{s3}|^2$, $|U_{e2}|^2$, $|U_{e3}|^2$, $|U_{\mu 0}|^2$, $|U_{\mu 1}|^2$, $|U_{\tau 0}|^2$ and $|U_{\tau 1}|^2$ are small, implying that $|s_{02}|$, $|s_{03}|$, $|s_{12}|$, $|s_{13}|$ are small. In fact, the Bugey result can be translated into a limit $|s_{12}|, |s_{13}| \lesssim 0.1$ [35]. The assumption that $\nu_\mu$ and $\nu_\tau$ are (nearly) maximally mixed yields $c_{23} \sim |s_{23}| \sim 1/\sqrt{2}$. Furthermore, the LSND result suggests that it is likely that $|s_{12}|, |s_{13}| \sim 0.1$ [35]. Finally and obviously, we should have $c_{02} \sim c_{03} \sim c_{12} \sim c_{13} \sim 1$.

To leading order in $s_{02}$, $s_{03}$, $s_{12}$, and $s_{13}$, we have [35]

$$U \approx \begin{pmatrix} c_{01} & s_{01}^* & s_{02}^* & s_{03}^* \\ -s_{01} & c_{01} & s_{12}^* & s_{13}^* \\ \begin{array}{c} -c_{01}(s_{23}^* s_{03} + c_{23}s_{02}) \\ +s_{01}(s_{23}^* s_{13} + c_{23}s_{12}) \end{array} & \begin{array}{c} -s_{01}^*(s_{23}^* s_{03} + c_{23}s_{02}) \\ -c_{01}(s_{23}^* s_{13} + c_{23}s_{12}) \end{array} & c_{23} & s_{23}^* \\ \begin{array}{c} c_{01}(s_{23}s_{02} - c_{23}s_{03}) \\ -s_{01}(s_{23}s_{12} - c_{23}s_{13}) \end{array} & \begin{array}{c} s_{01}^*(s_{23}s_{02} - c_{23}s_{03}) \\ +c_{01}(s_{23}s_{12} - c_{23}s_{13}) \end{array} & -s_{23} & c_{23} \end{pmatrix}. \tag{4}$$

In Ref. [35] this matrix is discussed and is shown to be unitary to second order in the LSND mixing angle.

In a simplifying approximation, one can take all mixing angles except $\theta_{01}$, $\theta_{12}$ and $\theta_{23}$ to be zero, ignore the $CP$ violating phases, and take the Super-K associated mixing to be maximal, $c_{23} \simeq 1/\sqrt{2}$. In this case, the transformation matrix becomes

$$U \approx \begin{pmatrix} c_{01} & s_{01} & 0 & 0 \\ -s_{01} & c_{01} & s_{12} & 0 \\ s_{01}s_{12}/\sqrt{2} & -c_{01}s_{12}/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ -s_{01}s_{12}/\sqrt{2} & c_{01}s_{12}/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \tag{5}$$

which is (approximately) unitary for nearly maximal mixing, $c_{23} \simeq 1/\sqrt{2}$.

We can define a linear row transformation [36,37]

$$|\nu_\mu^*\rangle \equiv \frac{|\nu_\mu\rangle - |\nu_\tau\rangle}{\sqrt{2}}, \tag{6}$$

and

$$|\nu_\tau^*\rangle \equiv \frac{|\nu_\mu\rangle + |\nu_\tau\rangle}{\sqrt{2}}, \tag{7}$$

such that Eq. (5) becomes

$$U' \approx \begin{pmatrix} c_{01} & s_{01} & 0 & 0 \\ -s_{01} & c_{01} & s_{12} & 0 \\ s_{01}s_{12} & -c_{01}s_{12} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{8}$$

and

$$\begin{pmatrix} |\nu_s\rangle \\ |\nu_e\rangle \\ |\nu_\mu^*\rangle \\ |\nu_\tau^*\rangle \end{pmatrix} = U' \begin{pmatrix} |\nu_0\rangle \\ |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}. \tag{9}$$

Here, we can see that the fourth state $|\nu_\tau^*\rangle$ is a mass eigenstate. If there are no lepton asymmetries generated by a mechanism other than neutrino mixing, then the muon- and tau-neutrino flavors see the same matter effects (that is the same thermal and fermion potentials) throughout their evolution. The state $|\nu_\mu^*\rangle$ is mixed with the sterile and electron neutrino, and will undergo resonant MSW transformation under the appropriate conditions. However, $|\nu_\tau^*\rangle$ will pass through resonances unchanged. This reduces the $4 \times 4$ mixing to essentially a $3 \times 3$ evolution, at least as far as MSW resonances are concerned.[2] It should be noted that the mixing matrix discussed here uses the rotation order convention $U = R_{23}R_{13}R_{03}R_{12}R_{02}R_{01}$, while Caldwell, Fuller and Qian use $U_{\rm CFQ} = R_{23}R_{01}R_{12}$ [36]. The different unitary mixing matrices are physically equivalent under the same approximations, and exhibit the same decoupling seen in Eq. (8).

The matter effects present at the epoch of BBN comprise two pieces: a part due to a finite-temperature thermal bath, and a part due to a possible lepton number asymmetry in the active neutrino sectors (the baryon number asymmetry and the associated electron-positron asymmetry is too small to play a significant role in the neutrino mixing). For two-neutrino mixing, the effective matter mass-squared-difference is

$$\delta m^2_{\alpha\beta}{}^{(\rm eff.)} = \left\{ \left(m^2_{\nu_\alpha} - m^2_{\nu_\beta}\right)^2 \sin^2 2\theta_{\alpha\beta} + \left[\left(m^2_{\nu_\alpha} - m^2_{\nu_\beta}\right)\cos 2\theta_{\alpha\beta} + 2EV^T_{\alpha\beta} + 2EV^L_{\alpha\beta}\right]^2 \right\}^{1/2} \tag{10}$$

and the effective mixing angle satisfies

$$\tan\theta^{(\rm eff.)}_{\alpha\beta} = \frac{(m^2_{\nu_\alpha} - m^2_{\nu_\beta})\sin 2\theta_{\alpha\beta}}{(m^2_{\nu_\alpha} - m^2_{\nu_\beta})\cos 2\theta_{\alpha\beta} + 2EV^T_{\alpha\beta} + 2EV^L_{\alpha\beta}}. \tag{11}$$

Here we use $m_{\nu_\alpha}$ to denote the mass eigenstate most closely associated with a neutrino of flavor $\alpha$; $E$ is the neutrino energy; $V^T_{\alpha\beta}$ is the effective potential due to the finite-temperature part of the matter effect, and $V^L_{\alpha\beta}$ is the effective potential due to the lepton number asymmetry. Both potentials vary with temperature $T$.

We will first consider the case that the lepton number asymmetry is negligible, $2EV^L_{\alpha\beta} \ll 2EV^T_{\alpha\beta}, (m^2_{\nu_\alpha} - m^2_{\nu_\beta})\cos 2\theta_{\alpha\beta}$. Mixings between active neutrino species alone do not modify the $^4$He synthesis because active neutrinos share the same number density distribution in the BBN epoch. (This is not rigorously true because electron/positron annihilation overpopulates $\nu_e/\bar{\nu}_e$ slightly but its impact on the $^4$He yield is less than 0.1% [38].) Mixings between active neutrinos and sterile neutrinos, however, convert active neutrinos and so populate initially unoccupied sterile neutrino states. These mixings can therefore affect the energy spectra of active neutrinos, as well as the $^4$He yield. In particular, MSW resonances, at which the local effective mixing reaches maximal values, can occur between an active neutrino species $\nu_\alpha$ and the sterile neutrino species when $(m^2_{\nu_\alpha} - m^2_{\nu_s})\cos 2\theta_{\alpha s} + 2EV^T_{\alpha s} = 0$. Since

$$V^T_{\alpha s} \approx -A\frac{n_{\nu_\alpha} + n_{\bar{\nu}_\alpha}}{n_\gamma}G_F^2 ET^4, \tag{12}$$

where $A \approx 105(30)$ for $\alpha = e\,(\mu,\tau)$, this resonance condition is met when the temperature of the universe is

$$T_{\rm res} \approx T_0 \left(\frac{E}{T}\right)^{-1/3} \left|\frac{(m^2_{\nu_\alpha} - m^2_{\nu_s})\cos 2\theta_{\alpha s}}{1\text{eV}^2}\right|^{1/6}, \tag{13}$$

where $T_0 \approx 19(22)$ MeV for $\alpha = e\,(\mu,\tau)$.

Therefore, for a two-family mixing between $\nu_\mu^*$ and $\nu_s$, with $m^2_{\nu_\mu^*} - m^2_{\nu_s} \approx m^2_{\nu_\tau^*} - m^2_{\nu_s} \sim 0.2$ to 10 eV$^2$ and a small mixing angle, the bulk of the active neutrinos undergo resonance at a temperature $T \sim 10$ to 20 MeV. This is long before $\nu_\mu^*$ and $\nu_\tau^*$ decouple thermally/chemically from the thermal background. As discussed briefly earlier, the $\nu_\mu^* \rightleftharpoons \nu_s$ mixing potentials behave identically to the $\nu_\tau^* \rightleftharpoons \nu_s$ in the BBN epoch, and they undergo the same scattering and collision evolution. Thus, the approximations giving Eq. (8) effectively "decouple" $\nu_\tau^*$, with only $\nu_\mu^*$ going through resonances. In the following discussion, $\nu_\tau^*$ decoupling can be assumed; however, even if the approximations leading to Eq. (8) are invalid so that angles other than $\theta_{01}, \theta_{12}$ and $\theta_{23}$ are non-zero, the following discussion is still relevant for the standard $\nu_\mu$ and can be extended to $\nu_\tau$.

Resonances may also occur for $\nu_e \rightleftharpoons \nu_s$ mixing with $m^2_{\nu_e} - m^2_{\nu_s} \sim 10^{-10}$ eV$^2$ (a possible vacuum solution for the solar neutrino problem). But the resonance temperature is $\lesssim 0.01$ MeV, which corresponds to an epoch long after the weak-decoupling of neutrinos. Resonant active-sterile neutrino mixing within the lower doublet in this case then cannot influence BBN. Therefore, the BBN constraints on mixing models involving a vacuum solar solution are less stringent.

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[2]Note that Ref. [8] also points out that one "linear combination" of $\nu_\mu$ and $\nu_\tau$ "oscillates" with $\nu_s$ while the other decouples.

In the two-doublet neutrino mixing schemes considered here, the $\nu_\mu^* \rightleftharpoons \nu_s$ channel is essentially decoupled (at or near its resonance temperature) from the other mixings within the four species family. Therefore, we can take the two-neutrino mixing picture as applicable. This is so because while the $\nu_\mu^* \rightleftharpoons \nu_s$ channel is matter-enhanced at its resonance, the other mixings are not, or even are suppressed by the matter effects by a factor of $|(m_{\nu_\alpha}^2 - m_{\nu_\beta}^2)/2EV_{\alpha\beta}^T|^2$ with respect to their vacuum mixing amplitude. For example, the inner-lower-doublet $\nu_e \rightleftharpoons \nu_s$ mixing is suppressed by a factor $\sim 10^9$ for the SMA MSW mixing solar neutrino solution or $\sim 10^{19}$ for the vacuum mixing solar neutrino solution, because $2EV_{es} \sim 8EV_{\mu s} \approx 4(m_{\nu_\mu}^2 - m_{\nu_s}^2)$. Also, since $2EV_{\mu e} = 2E(V_{\mu s} - V_{es}) \approx -6EV_{\mu s} \approx 3(m_{\nu_\mu}^2 - m_{\nu_s}^2) \approx 3(m_{\nu_\mu}^2 - m_{\nu_e}^2)$, the inter-doublet $\nu_\mu^* \rightleftharpoons \nu_e$ mixing is suppressed by a factor $\sim 10$ with respect to its already small vaccuum mixing amplitude ($\lesssim 10^{-2}$). Therefore, it is safe to employ the two-family mixing picture to investigate the $\nu_\mu^* \rightleftharpoons \nu_s$ channel at or near its resonance.

As pointed out in several previous papers, there are two possible consequences of a resonant active-sterile neutrino mixing: (1) the total neutrino energy density at a given temperature increases if neutrino pair production is still effective in replenishing the converted active neutrinos; (2) a lepton number asymmetry may be generated in the active neutrino sector from initial small and negligible statistical fluctuations during the resonant active-to-sterile neutrino conversion process. In either the nonresonant or resonant mixing case, the limit on the total neutrino energy density from the primordial $^4$He abundance puts the following constraint on the parameters of active-sterile neutrino mixing [3]:

$$(m_{\nu_\alpha}^2 - m_{\nu_s}^2)\sin^4 2\theta_{\alpha s} \lesssim \begin{cases} 10^{-9}\,\mathrm{eV}^2 & \text{if } \nu_\alpha = \nu_e; \\ 10^{-7}\,\mathrm{eV}^2 & \text{if } \nu_\alpha = \nu_\mu,\, \nu_\tau. \end{cases} \tag{14}$$

In the resonant case, the increase in the neutrino energy density is significant to BBN when the resonance is adiabatic and occurs before chemical decoupling of the active neutrino (about 5 MeV for $\nu_\mu^*$). The lepton number asymmetry generated from an initially very small asymmetry by the resonant active-sterile neutrino mixing process is significant to BBN if part of the asymmetry resides in the $\nu_e/\bar{\nu}_e$ sector and is of order $\gtrsim 0.01$. In the two-doublet neutrino model, this is achieved by having a resonant $\nu_\mu^* \rightleftharpoons \nu_s$ mixing generate $L_{\nu_\mu^*}$, and having a resonant $\nu_\mu^* \rightleftharpoons \nu_e$ mixing transfer part of $L_{\nu_\mu^*}$ into $L_{\nu_e}$.

In Ref. [24], we calculated the change in the primordial $^4$He abundance due to such a process. In the regime where $10^{-1}$ eV$^2 \lesssim m_{\nu_\mu^*}^2 - m_{\nu_s}^2 \lesssim 10$ eV$^2$ and $\sin^2 2\theta_{\mu^* s} \gtrsim 10^{-10}$, the mixing angles are large enough to generate a lepton number asymmetry [22]. Mixings with smaller mixing angles cannot have a material effect on BBN. In regions of the universe where $L_{\nu_\mu^*}$ and in turn $L_{\nu_e}$ is positive, the n$\to$p rate is enhanced while the p$\to$n rate is reduced. This change in n$\rightleftharpoons$p rates tends to lower the neutron-to-proton ratio and consequently $Y$.

On the other hand, the increased neutrino energy density from the active-sterile neutrino mixing before neutrino chemical decoupling always tends to increase $Y$. The overall result is a decrease in $Y$, as the former effect ($\nu_e/\bar{\nu}_e$ asymmetry) dominates. In places where $L_{\nu_\mu^*}$ and $L_{\nu_e}$ are negative, however, the n$\rightleftharpoons$p rates are changed so as to increase $Y$. This is, of course, in addition to the increase in $Y$ from the energy density effect.

When averaged over the positive domains and the negative domains, the net $Y$ turns out to be consistently larger than that predicted by the standard BBN picture assuming no neutrino mixing. This is because the increase in $Y$ in negative domains is never compensated by the decrease in $Y$ in positive domains, as shown in Fig. 3 [22,19].

This result is rather different from that in Ref. [8] and given the controversy in the calculations alluded to above, it is difficult to say which, if either, is correct. Part of the discrepancy may be because the effect of energy density increases on $Y$ were not appropriately taken into account in Ref. [8]. A simple average of the change in $Y$ in our calculation is not beyond observational uncertainty bounds. The average $\Delta Y$ is always less than $\sim 0.001$, or, equivilantly, $N_\nu^{\mathrm{eff}}$ is always less than 3.08. If a definitive, confident solution to lepton number generation by neutrino mixing in the early universe were to show unambiguously that the sign of the neutrino asymmetry for this specific range of neutrino mixing parameters is positive (negative) then the predicted change to $Y$ would follow the lower (upper) curve. The positive lepton number result alone does not exceed the observational bounds of Fig. 1. The observations more greatly constrain increases to $Y$. A negative lepton number result alone therefore creates a $\Delta Y$ that is too large for $\delta m_{\mu^* e}^2 \gtrsim 2.5\,\mathrm{eV}^2$.

The averaged $Y$ is only a lower limit to the actual $Y$ in the two-doublet neutrino model with chaotic lepton number generation. This is because, as first pointed out by Shi and Fuller [19], additional increases in $Y$ may arise from an extra channel for increasing the total neutrino energy density. In the case where the sign of the lepton number generated is undetermined, the extra channel for sterile neutrino production results from the lepton number gradients between domains of regions with opposite lepton number sign (which are sub-horizon scale at the BBN epoch). The domain boundaries can meet the conditions for resonant conversion of not only $\nu_\mu^*$ to $\nu_s$ (and $\bar{\nu}_\mu^*$ to $\bar{\nu}_s$) but also $\nu_e$ to $\nu_s$ (and $\bar{\nu}_e$ to $\bar{\nu}_s$). Therefore, the sterile neutrino sea is not only populated within domains by the resonant neutrino mixing that drives the lepton number generation in the first place, but can also be populated at domain boundaries by the same resonant neutrino mixing as well as other active-sterile neutrino mixings.

To avoid fully thermalizing the sterile neutrinos, therefore, requires that this extra channel of sterile neutrino production be suppressed. In other words, all resonances of active-sterile neutrino conversion have to be non-adiabatic at domain boundaries. This yields another limit for the two-doublet neutrino scheme [19]:

$$\begin{aligned}
&\sin^2 2\theta_{\mu^* s} \lesssim 10^{-10}, && \text{for an MSW solution to the solar neutrino problem;} \\
&\left(m^2_{\nu^*_\mu} - m^2_{\nu_s}\right) \sin^2 2\theta_{\mu^* s} \lesssim 10^{-4}\,\mathrm{eV}^2, && \text{for a vacuum solar neutrino solution.}
\end{aligned} \tag{15}$$

Beyond these limits, $Y$ increases by at least 0.013 due to a fully populated sterile neutrino sea resulting from active-sterile neutrino conversions at domain boundaries.

These limits are summarized in Table 1. Because of the decoupling of various mixings, the above limits can be directly translated into constraints

$$\begin{aligned}
&|s_{02}|,\, |s_{03}| \lesssim 10^{-5}, && \text{for a SMA MSW solar neutrino solution} \\
& && \text{or when } m^2_{\nu^*_\mu} - m^2_{\nu_e} \gtrsim 4\ \mathrm{eV}^2; \\
&|s_{02}|,\, |s_{03}| \lesssim 10^{-2}\left[\left(m^2_{\nu^*_\mu} - m^2_{\nu_s}\right)/1\,\mathrm{eV}^2\right]^{-1/2}, && \text{for a vacuum solar neutrino solution} \\
& && \text{and } m^2_{\nu^*_\mu} - m^2_{\nu_e} \lesssim 4\ \mathrm{eV}^2.
\end{aligned} \tag{16}$$

The constraint on the large mass difference is the result of the fact that for $m^2_{\nu^*_\mu} - m^2_{\nu_e} \gtrsim 4\ \mathrm{eV}^2$, domains of $L_{\nu^*_\mu}$ will facilitate $\nu^*_\mu \rightleftharpoons \nu_s$ population of the sterile sea across domain boundaries. These constraints imply that the inter-doublet mixing elements of $\nu_\mu$ or $\nu_\tau$ with $\nu_s$ ($s_{02}, s_{03}$) are $\sim 10^4$ times smaller than the elements associated with mixing with $\nu_e$ ($s_{12}, s_{13}$), if the solar neutrino problem has its roots in $\nu_e \rightleftharpoons \nu_s$ mixing with $m^2_{\nu_s} - m^2_{\nu_e} \sim 10^{-5}\ \mathrm{eV}^2$ (the SMA MSW solution). However, the mixings between the upper doublet neutrinos and the lower doublet $\nu_e$ and $\nu_s$ are within an order of magnitude if a vacuum "just so" $\nu_e \rightleftharpoons \nu_s$ mixing with $|m^2_{\nu_s} - m^2_{\nu_e}| \sim 10^{-10}\ \mathrm{eV}^2$ explains the solar neutrino data.

Let us now confine our attention to the latter scheme, where $m^2_{\nu^*_\mu} - m^2_{\nu_e}$ is $\lesssim 4\ \mathrm{eV}^2$ and the solar neutrino deficit is explained by (nearly) maximally mixed $\nu_e \rightleftharpoons \nu_s$ vacuum "just so" oscillation with $\delta m^2_{es} \sim 10^{-10}\ \mathrm{eV}^2$. This scheme can produce a significant increase in $Y$ when the values of the inter-doublet mass splitting are in the range that is required to explain the LSND results. The primordial $^4$He yield in this scenario is sensitive to the relative level of mixing for $\nu_{\mu,\tau} \rightleftharpoons \nu_e$ and for $\nu_{\mu,\tau} \rightleftharpoons \nu_s$. For example, we can define a factor

$$\begin{aligned}
F &= \frac{\sin^2 2\theta_{\mu e}}{\sin^2 2\theta_{\mu s}} \\
&= \frac{|s_{12}c_{23} + s_{13}s^*_{23}|^2}{|s_{02}c_{23} + s_{03}s^*_{23}|^2},
\end{aligned} \tag{17}$$

where $\theta_{\mu e}$ and $\theta_{\mu s}$ are the effective two-neutrino vacuum mixing angles (to be constructed from the matrix elements in Eq. (4)) corresponding to $\nu_\mu \rightleftharpoons \nu_e$ and $\nu_\mu \rightleftharpoons \nu_s$, respectively. Figure 4 then shows limits from the observed $^4$He abundance for three cases: (1) $F = 1$; (2) $F = 10$; and (3) $F = 100$. More specifically, if the atmospheric neutrino problem solution is maximal $\nu_\mu \rightleftharpoons \nu_\tau$ mixing (and non-$CP$ violating), then $F$ is just the ratio of inter-doublet mixing angles

$$F \simeq \frac{|s_{12} + s_{13}|^2}{|s_{02} + s_{03}|^2}. \tag{18}$$

It is noteable that a novel solution to r-process nucleosynthesis in Type II supernovae by Caldwell, Fuller and Qian [36] involves a $4 \times 4$ model where only $s_{12}$ is non-vanishing in the above expression for $F$. In this case, effectively $F \to \infty$, indicating an exceptionally high degree of "symmetry." (Here by "symmetry" we mean that only one mixing angle governs the inter-doublet mixing, not four; however, this is very asymmetric as far as $F$ is concerned!) Both this solution to r-process nucleosynthesis and the domain-conversion-based BBN considerations discussed in this paper favor a large $F$. The mixing matrix in Ref. [36], which is related to that shown in Eq. 8, exhibits the symmetry that allows a decoupling of a neutrino state ($|\nu^*_\tau\rangle$) from MSW resonant evolution. For mass Scheme III to both account for the LSND signal (whose two-neutrino parameter space is shown in Fig. 4 [39]) and be consistent with domain-conversion-driven BBN effects, the difference in magnitude of the inter-doublet mixing angles must be large (a large value of $F$).

If one were to entertain models where $F$ was not large (*i.e.*, comparable values for $\theta_{12}, \theta_{13}, \theta_{02}$ and $\theta_{03}$), then already we can see from Fig. 4 that the LSND data compatible with smaller $F$ all lies near $m^2_\mu - m^2_e \approx 1\ \mathrm{eV}^2$ and

$\sin^2 2\theta_{\mu e} \approx 10^{-3}$. Concommitantly, future experiments such as BooNE [40] might indicate mixing parameters which fall outside this limit. Assuming our BBN domain-conversion-driven considerations are correct, this would argue strongly for large $F$.

This would be a remarkable outcome. Naively, one might assume that, *e.g.*, the BooNE experiment measures only the effective two-neutrino mixing $\theta_{\mu e}$. However, when combined with other experiments and the BBN physics adopted here, it is evident that many other mixing matrix elements (those in $F$) are also probed.

## III. SUMMARY

We have explored the possible change in the primordial $^4$He abundance in the currently favored two-doublet four-neutrino ($\nu_\tau$-$\nu_\mu/\nu_e$-$\nu_s$) mixing scheme proposed to simultaneously explain the current neutrino experiments. Though definitive calculations of matter-enhanced neutrino conversion effects are elusive at present (in the eyes of some), here we have adopted the set of calculations which could give the tightest constraints on the neutrino mass and mixing matrix. We do this in the spirit of determining what may be possible. When we analyze the BBN effects in the context of full four-neutrino mixing we find some remarkable hints. Namely, we find that putative limits on the matrix elements governing $\nu_{\mu,\tau} \rightleftharpoons \nu_e$ and $\nu_{\mu,\tau} \rightleftharpoons \nu_s$ could be very restrictive.

We have found that these limits strongly depend on the mixing between $\nu_e$ and $\nu_s$: the limits are exceptionally strong if the $\nu_e \rightleftharpoons \nu_s$ mixing parameters are in the range of the SMA MSW solution of the solar neutrino problem; but they are much less so, allowing the $\nu_\mu$, $\nu_\tau \rightleftharpoons \nu_s$ mixing to be at the same level as the $\nu_\mu$, $\nu_\tau \rightleftharpoons \nu_e$ mixing, if the $\nu_e \rightleftharpoons \nu_s$ mixing parameters lie at the vacuum "just so" solution region the solar neutrino problem. (Note: Since the submission of this paper there has been recent evidence from Super-Kamiokande that disfavors all sterile neutrino solutions to the solar neutrino problem [41]. This emphasizes BBN constraints on any four-neutrino models.) In addition, we have found that if $m^2_{\nu_\mu} - m^2_{\nu_s} \approx m^2_{\nu_\mu} - m^2_{\nu_e} \gtrsim 4$ eV$^2$, the $\nu_\mu$, $\nu_\tau \rightleftharpoons \nu_s$ mixing should also be extremely small.

Therefore, the mixing structures in these different cases of the hierarchical four-neutrino schemes can be very disparate. Potentially, unless the inter-doublet active-active and active-sterile mixings are very asymmetric, BBN considerations demand a vacuum "just so" solution to the solar neutrino problem and an LSND solution with $m^2_\mu - m^2_e \approx 1$ eV$^2$ and $\sin^2 2\theta_{\mu e} \approx 10^{-3}$ in the two-doublet hierarchical mass scheme. Alternatively, future $\nu_\mu \rightleftharpoons \nu_e$ experiments such as BooNE could be able to place significant constraints on functions of $\theta_{12}, \theta_{13}, \theta_{02}$ and $\theta_{03}$. This is a tantalizing result. Realizing it depends on the veracity of the particular BBN calculations we have adopted, but it is clear that the stakes are high.

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[1] A. Dolgov, Sov. J. Nucl. Phys. **33**, 700 (1981); M. Khlopov and S. Pectov, Phys. Lett. B **99**, 117 (1981) [Erratum: **100**, 520 (1981)]; D. Fargion and M. Shepkin, Phys. Lett. B **146**, 46 (1984); P. G. Langacker, Univ. Penn. Preprint UPR0401T (unpublished).
[2] R. Barbieri and A. Dolgov, Phys. Lett. B **349**, 743 (1991).
[3] K. Enqvist, K. Kainulainen and J. Maalampi, Nucl. Phys. **B349**, 754 (1991); K. Enqvist, K. Kainulainen, and M. Thomson, Nucl. Phys. **B373**, 498 (1992).
[4] J. M. Cline, Phys. Rev. Lett. **68**, 3137 (1992).
[5] X. Shi, D. N. Schramm, and B. D. Fields, Phys. Rev. D **48**, 2568 (1993).
[6] D. O. Caldwell, Int. J. Mod. Phys. A **13**, 4409 (1998), and references therein; J. Petoniemi and J. W. Valle, Nucl. Phys. **B406**, 409 (1993); D. O. Caldwell and R. N. Mohapatra, Phys. Rev. D **48**, 3259 (1993).
[7] S. M. Bilenky, C. Giunti and W. Grimus, Eur. Phys. J. C **1**, 247 (1998).
[8] N. Bell, R. Foot and R. R. Volkas, Phys. Rev. D **58**, 105010 (1998).
[9] N. Okada and O. Yasuda, Int. J. Mod. Phys. A **12**, 3669 (1997).
[10] S. M. Bilenky, C. Giunti, W. Grimus and T. Schwetz, Astropart. Phys. **11**, 413 (1999).
[11] G. Steigman, D. N. Schramm and J. E.Gunn, Phys. Lett B **66**, 262 (1977).
[12] S. Burles and D. Tytler, Astrophys. J. **499**, 699 (1998); Astrophys. J. **507**, 732 (1998); D. Tytler, X.-M. Fan and S. Burles, Nature **381**, 207 (1996).

[13] R. E. Lopez and M. S. Turner, Phys. Rev. D **59**, 103502 (1999); S. Esposito, G. Mangano, G. Miele and O. Pisanti, Nucl. Phys. **B568**, 421 (2000).
[14] K. A. Olive, G. Steigman and T. P. Walker, Physics Reports (to be published), astro-ph/9905320.
[15] S. Burles, D. Kirkman and D. Tytler, Astrophys. J. **519**, 18 (1999); D. Kirkman, D. Tytler, S. Burles, D. Lubin and J. O'Meara, in American Astronomical Society Meeting 194 #30.01; D. Tytler, *et al.*, Astron. J. **117**, 63 (1999); S. A. Levshakov, W. H. Kegel and F. Takahara, Astron. Astrophys. **336**, L29 (1998).
[16] C. Y. Cardall and G. M. Fuller, Astrophys. J. **472**, 435 (1996); G. M. Fuller and C. Y. Cardall, Nucl. Phys. Proc. Suppl. **B51**, 259 (1996).
[17] X. Shi, and G. M. Fuller. Phys. Rev. D **59**, 063006 (1999).
[18] S. Burles, K. M. Nollett, J. N. Truran and M. S. Turner, Phys. Rev. Lett. **82**, 4176 (1999).
[19] X. Shi and G. M. Fuller, Phys. Rev. Lett. **83**, 3120 (1999).
[20] R. Foot and R. R. Volkas, Phys. Rev. Lett., **75**, 4350 (1995).
[21] R. Foot, M. J. Thomson, and R. R. Volkas, Phys. Rev. D **53**, 5349 (1996).
[22] X. Shi, Phys. Rev. D **54**, 2753 (1996).
[23] R. Foot and R. R. Volkas, Phys. Rev. D **56**, 6653 (1997) [Erratum: **59**, 029901 (1999)].
[24] X. Shi, G. M. Fuller and K. Abazajian, Phys. Rev. D **60**, 063002 (1999).
[25] K. Abazajian, X. Shi and G. M. Fuller, astro-ph/9909320 (unpublished).
[26] K. Enqvist, K. Kainulainen and A. Sorri, Phys. Lett. B **464**, 199 (1999).
[27] A. Sorri, Phys. Lett. B **477**, 201 (2000).
[28] P. Di Bari and R. Foot, Phys. Rev. D **61**, 105012 (2000).
[29] A. D. Dolgov, S. H. Hansen, S. Pastor and D. V. Semikoz, Astropart. Phys. **14**, 79 (2000).
[30] C. Athanassopoulos *et al.* (LSND Collaboration), Phys. Rev. C **54**, 2685 (1996).
[31] J. N. Bahcall, P. I. Krastev, A. Yu. Smirnov, Phys. Rev. D **58**, 096016 (1998).
[32] Y. Fukuda, *et al.*, Phys. Rev. Lett., **81**, 1562 (1998).
[33] S. Goswami, D. Majumdar and A. Raychadhuri, hep-ph/9909453 (unpublished).
[34] Z. Chacko and R. N. Mohapatra, Phys. Rev. D **61**, 053002 (2000).
[35] V. Barger, Y.-B. Dai, K. Whisnant, and B.-L. Wang, Phys. Rev. D **59**, 113010 (1999), and references therein.
[36] D. O. Caldwell, G. M. Fuller and Y.-Z. Qian, Phys. Rev. D **61**, 123005 (2000).
[37] A. B. Balantekin and G. M. Fuller, Phys. Lett. B **471**, 195 (1999).
[38] B. D. Fields, S. Dodelson and M. S. Turner, Phys. Rev. D **47**, 4309 (1993).
[39] K. Eitel, New J. Phys. **2**, 1 (2000).
[40] E. Church, *et al.*, LA-UR-97-2120, June 1997, nucl-ex/9706011 (unpublished).
[41] Y. Suzuki, in XIX International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, June 16-21, 2000.

TABLE I. BBN Limits on the inter-doublet active-sterile mixing in the two-doublet neutrino scheme.

| $m^2_{\nu_\mu} - m^2_{\nu_e}$ | Type of Solar Neutrino Solution | Limit on Effective $\sin^2 2\theta_{\mu s}$ and $\sin^2 2\theta_{\tau s}$ |
|---|---|---|
| $\gtrsim 4$ eV$^2$ | SMA MSW<br>Vacuum | $\lesssim 10^{-10}$ |
| $\lesssim 4$ eV$^2$ | MSW | $\lesssim 10^{-10}$ |
| | Vacuum | $\lesssim 10^{-4}\,\mathrm{eV}^2\,\left(m^2_{\nu_\mu} - m^2_{\nu_e}\right)^{-1}$ |

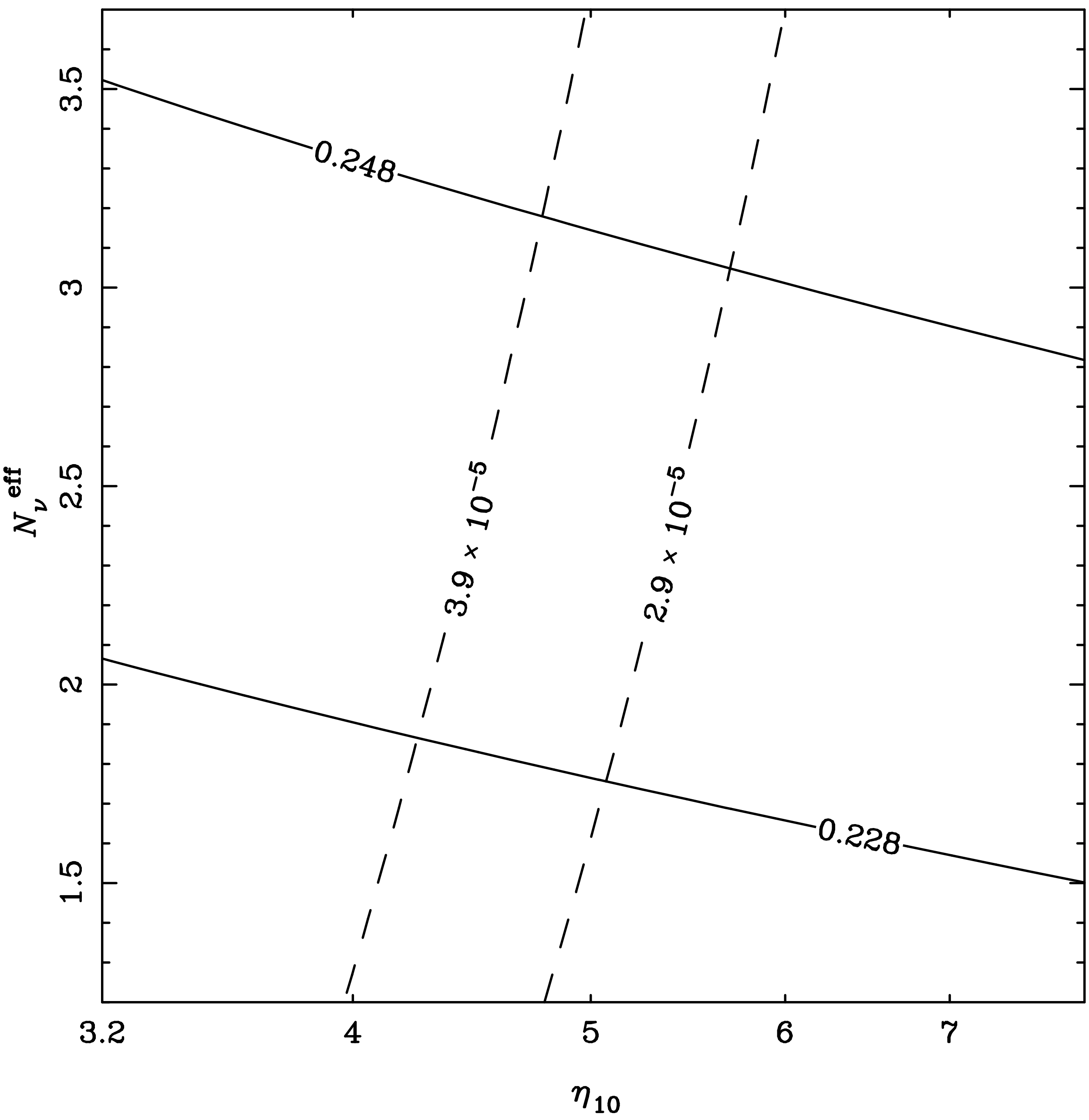


FIG. 1. The limits in the effective number of light neutrinos, $N_\nu^{\rm eff}$, in BBN for varying baryon-to-photon ratio, $\eta_{10} \equiv \eta\ 10^{10}$. The solid contours are 95% confidence limits on the inferred $^4$He mass fraction, $Y$ [14], and the dashed contours are 95% confidence limits on the inferred relative abundance of $D/H$ [12].

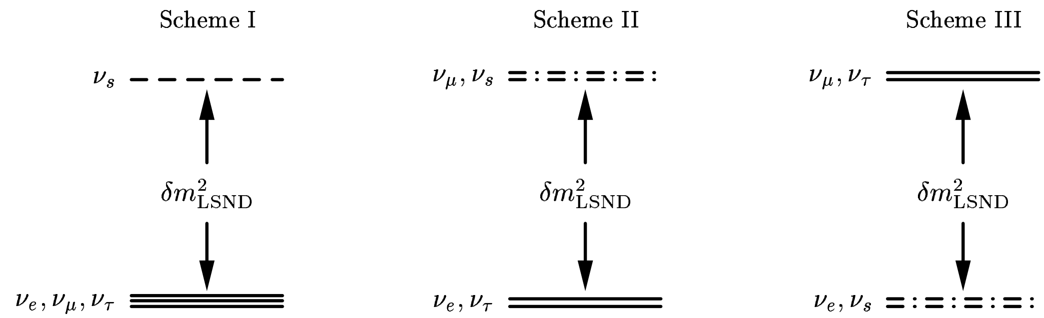


FIG. 2. The three general mass heirarchies discussed. In all cases, the mass splittings correspond to $\delta m^2_{\rm LSND} \sim 1\,{\rm eV}^2, \delta m^2_{\rm atm} \sim 10^{-3}\,{\rm eV}^2$, and $\delta m^2_{\rm solar} \sim 10^{-5}(10^{-10})\,{\rm eV}^2$ for the MSW (vacuum) solar solutions. Scheme I, its mirror ($m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau} > m_s$) and Scheme II (and its mirror) have previously been ruled out by BBN. In this paper, we consider constraints on mass heirarchies and mixings in Scheme III.

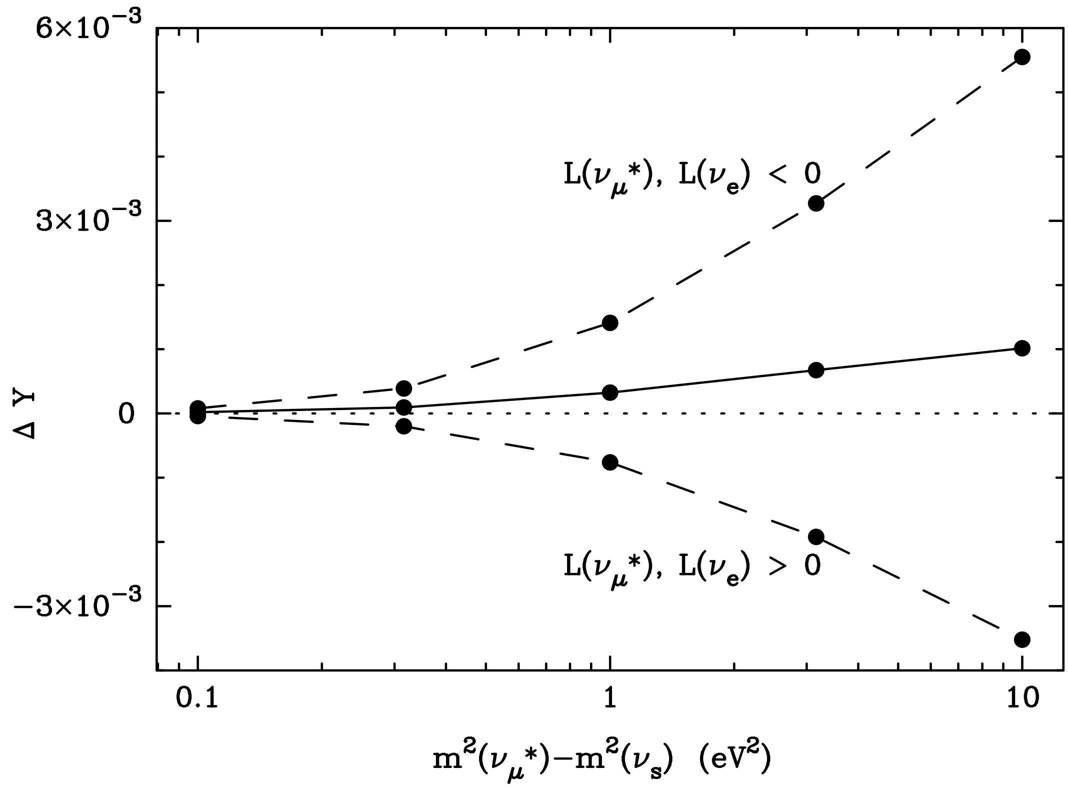


FIG. 3. The increase in the primordial $^4$He yield in the two-doublet mass scheme, as a function of the inter-doublet mass-squared-difference. The mixing amplitude between $\nu_s$ and the $\nu_\mu$-$\nu_\tau$ doublet is assumed to be not too small, $\gtrsim 10^{-10}$. The dashed curve: the increase in $Y$ in individual domains. The solid curve: the increase in $Y$ averaged over positive and negative lepton number domains.

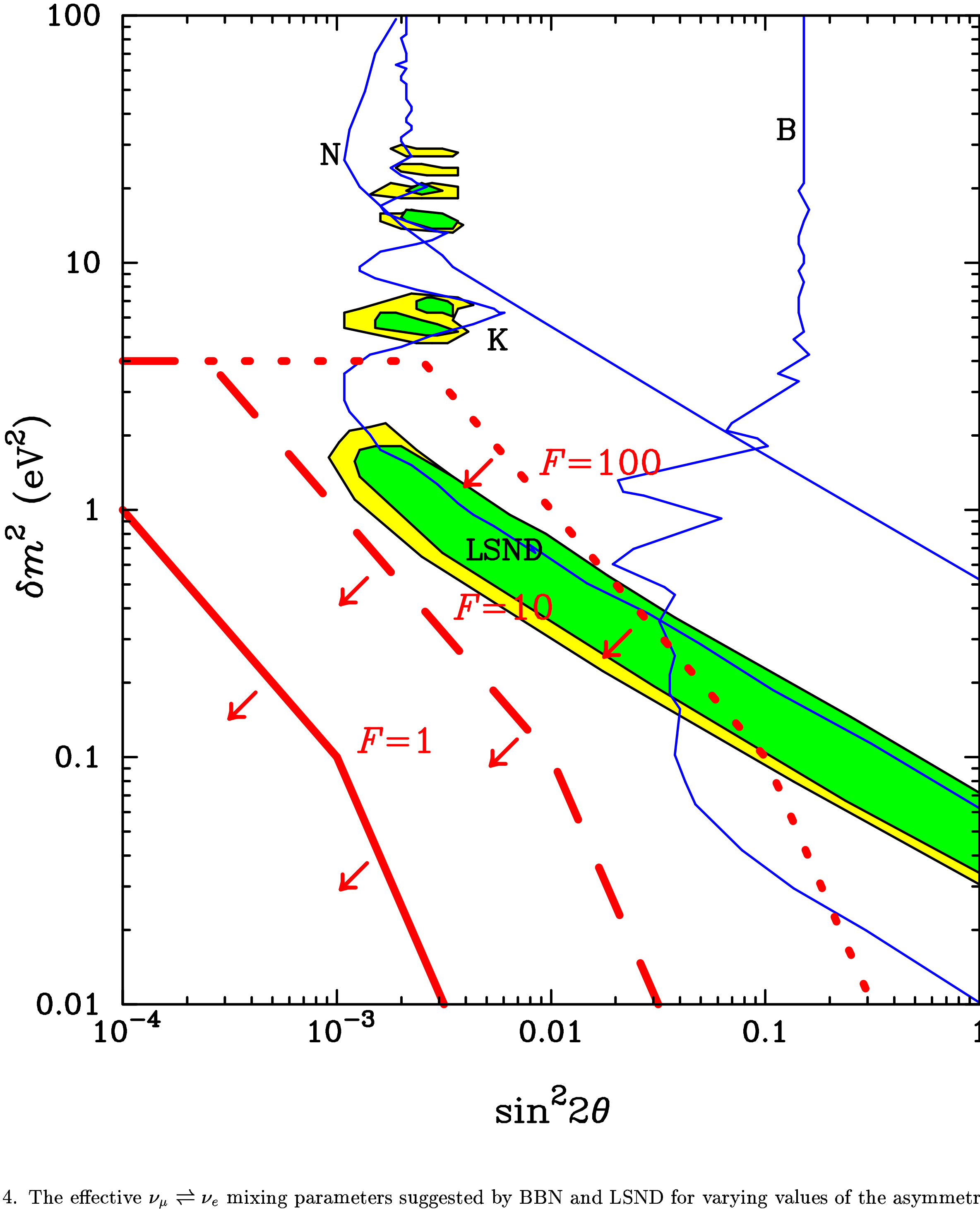


FIG. 4. The effective $\nu_\mu \rightleftharpoons \nu_e$ mixing parameters suggested by BBN and LSND for varying values of the asymmetry factor, $F$. Shown are the 90% and 95% $CL$ limits for LSND. Curves labelled K, B, and N are the 90% $CL$ limits from KARMEN2, Bugey, and NOMAD, respectively. Experimental confidence regions are adapted from Ref. [39].