Beam Loading

- Long train of bunches
- Bunches in front extract energy from linac
  - Lower gradient
  - Increase phase
- Effect on later bunches
  - Bunch placed correctly ignoring beam loading
  - Bunch doesn't gain enough energy
  - If it gained enough energy, it would arrive at the same RF phase
  - Non-isochronous arc: bunch arrives in next linac late, sees higher gradient.
  - Gains excess energy

- Beam dynamics
  - If bunch gained energy of reference bunch, it would arrive at same phase each time
  - There is (?) a phase which gives bunch the reference energy
  - Thus, fixed point at reference energy, but different time
  - Bunch not placed at that fixed point:
    - Oscillates about fixed point
    - Nonlinearity: filaments to larger emittance
  - Different RF bucket
    - Closer to crest, lower gradient: smaller area
    - Different matched ellipse
    - Offset of fixed point
  - Too much gradient lost: cannot gain back reference energy

Hamiltonain Formulation

- Write down Hamiltonian

$$-\frac{1}{2}A_{56}\Delta^2 + \frac{q\bar{v}}{\omega}\sin(\omega\tau + \bar{\phi}) - \frac{qv}{\omega}(\omega\tau\cos\phi + \sin\phi)$$

  - v is unloaded gradient, ϕ is unloaded phase
  - $\bar{v}$ is loaded gradient, $\bar{\phi}$ is loaded phase

- Fixed point:

$$\Delta = 0 \qquad \bar{v}\cos(\omega\tau + \bar{\phi}) = v\cos\phi$$

- Matched aspect ratio

$$\frac{\sigma_E^2}{\sigma_\tau^2} = \frac{q\omega\sqrt{\bar{v}^2 - v^2\cos^2\phi}}{A_{56}}$$

Compute Results

- $\bar{v}\cos\bar{\phi} - v\cos\phi \equiv \Delta(v\cos\phi)$
- $\bar{v} - v \equiv \Delta v$
- Assume small Δv
- Assume bunch has correct energy, time, aspect ratio for unloaded reference bunch
- $\bar{v} > v\cos\phi$ required for oscillation
    - Otherwise, energy drifts monotonically
- Energy amplitude of oscillation

$$\frac{q\Delta(v\cos\phi)}{\sqrt{\omega A_{56} q v \sin\phi}}$$

- Emittance blowup

$$\frac{\epsilon_L}{8}\left(\frac{\Delta v}{v}\right)^2 \csc^4\phi + \frac{1}{2\omega^2}\sqrt{\frac{q\omega v\sin\phi}{A_{56}}}\left[\frac{\Delta(v\cos\phi)}{v\sin\phi}\right]^2$$

    - First term: mismatch
    - Second term: filamentation
    - Doesn't occur immediately
- Decrease in bucket area

Beam Loading in Linac

$$\Delta(v \cos\phi) = \frac{q\omega r_s}{2Q} \qquad \Delta v = \frac{q\omega r_s}{2Q}\cos\phi$$

- Condition to get oscillations

$$v > \frac{q r_s \omega}{2Q}\csc^2\phi\left[\cos\phi + \sqrt{\cos(2\phi)}\right]$$

  - Always OK when $\phi > 45°$
  - Easier at higher ϕ
- Energy oscillation amplitude, relative to RMS energy spread

$$\frac{q r_s}{2Q\sigma_\tau v \sin\phi}$$

  - Larger for short bunch: fixed point further outside distribution
  - Larger oscillation closer to crest
- Emittance growth

$$\frac{1}{2}\left(\frac{q r_s}{2Q\sigma_\tau v \sin\phi}\right)^2\left[1 + \left(\frac{\omega\sigma_\tau \cot\phi}{2}\right)^2\right]$$

Comments

- Slightly larger in real life:
  - System discrete
  - Performs much of oscillation before arc corrects it
- Large oscillations become an issue before you run out of gradient
- Design of loaded RLAs:
  - Design for middle of train: half the charge for errors
  - Keep matched aspect ratio of reference bunch same for each turn
    - Keep bucket area constant also
    - Result: phase same for each turn
    - Adjust A_{56} for arcs
    - Later passes have lower synchrotron tune
  - Worst beam loading on last turn
  - Ensure that there is sufficient area in bucket for bunch at end of train on last turn

Numbers

- Oscillation amplitudes from simulation, not formulas
- Low charge (2×10^{12}): Fermilab study

$p_{\min}$ GeV/c	$p_{\max}$ GeV/c	f MHz	n	σ_E MeV	ΔE MeV	$\Delta\epsilon_L/\epsilon_L$ %
12	50	200	5	121	21	1.7
12	50	200	8	104	13	1.2
12	50	200	15	85	8	0.9
12	50	400	5	195	44	2.5
12	50	400	8	168	28	1.7
12	50	400	15	139	17	1.1
12	50	800	5	318	97	4.4
12	50	800	8	276	65	2.8
12	50	800	15	234	40	1.7

  Beam loading not an issue even at 800 MHz
- High charge (1.8×10^{13})

$p_{\min}$ GeV/c	$p_{\max}$ GeV/c	f MHz	n	σ_E MeV	ΔE MeV	$\Delta\epsilon_L/\epsilon_L$ %
12	50	200	5	175	260	101
12	50	200	10	154	159	51
12	50	400	5	328	669	207

  Beam loading a major problem!
  - More turns helps: less energy offset before oscillation begins

Correction

- Put each bunch at its fixed point
  - Slightly different frequency in acceleration than in bunching/cooling.
  - Timing
  - Can only fix on average
- Less current in the bunch train
  - E.g.: 6 bunches from AGS
    - Ramp and put into storage ring at top energy
    - Accelerating next set while storing
    - Send individually to second ring to phase rotate
    - More switchyards, more opportunities for activation
  - Increases average power in acceleration, cooling
    - Same stored energy must be supplied and dumped, regardless of charge
    - Higher rep rate, more energy delivered per second