MEASURING TRANSVERSITY WITH SPIN-ONE HADRONS

A. Bacchetta

P.J. Mulders

vrije Universiteit amsterdam

- Spin one in correlation functions
- Vector meson decay
- Asymmetries and transversity

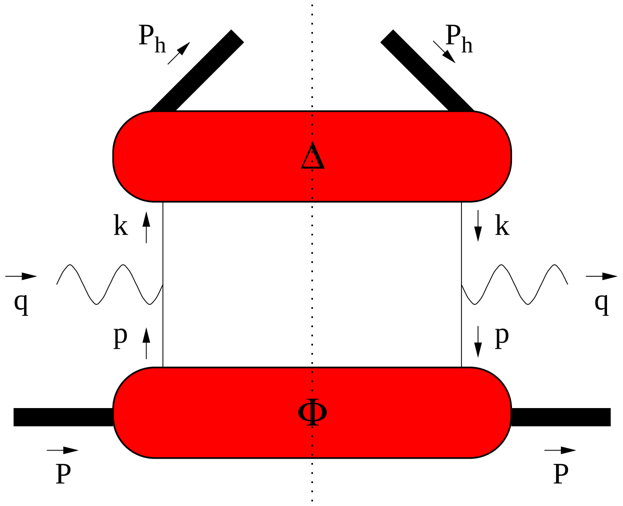
Extent of this analysis

- Leading order in 1/Q
- Tree level
- Including intrinsic transverse momentum
- Including (naive) time-reversal odd contributions

Very essential bibliography

- P. Hoodbhoy, R.L. Jaffe, A. Manohar, NPB **312** (1988) 571
- X. Ji, PRD **49** (1994) 114
- A. B., P.J. Mulders, `hep-ph/0007120`, PRD
- P.J. Mulders, R.D. Tangerman, NPB **461** (1996) 197
- D. Boer, P.J. Mulders, PRD **57** (1998) 5780
- A. B., M. Boglione, A. Henneman, P.J. Mulders, PRL **85** (2000) 712

$$\mathrm{d}\sigma(l + H \to l' + h + X) \propto L_{\mu\nu}\, W^{\mu\nu}$$



$$\boxed{2MW^{\mu\nu} \propto \mathrm{Tr}\left[\Phi(x_B)\,\gamma^\mu\,\Delta(z_h)\,\gamma^\nu\right]}$$

$$x_B = \frac{Q^2}{2\,P\cdot q} \qquad z_h = \frac{2\,P_h\cdot q}{Q^2}$$

with $P_{h\perp}$

$$2MW^{\mu\nu} \propto \int \mathrm{d}^2\boldsymbol{p}_T\,\mathrm{d}^2\boldsymbol{k}_T\,\delta^2(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T) \times \mathrm{Tr}\left[\Phi(x_B, \boldsymbol{p}_T)\,\gamma^\mu\,\Delta(z_h, \boldsymbol{k}_T)\,\gamma^\nu\right]$$

$$\boldsymbol{q}_T = -\frac{P_{h\perp}}{z_h}$$

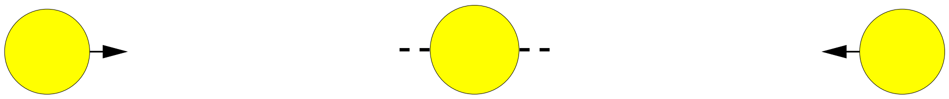
The correlation function can be written as a function of the spin vector S and the spin tensor T of the hadron

$$\Phi_{ij}(p, P, S, T) = \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\, \mathrm{e}^{ip\cdot\xi} \langle P, S, T | \bar{\psi}_j(0)\psi_i(\xi) | P, S, T\rangle$$

or as a matrix in the hadron's helicity space

$$\Phi_{ij;\Lambda\Lambda'}(p, P) = \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\, \mathrm{e}^{ip\cdot\xi} \langle P, \Lambda | \bar{\psi}_j(0)\psi_i(\xi) | P, \Lambda'\rangle$$

where Λ and Λ' are hadron's helicities (1,0,-1)



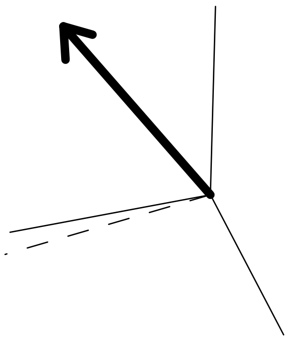
The connection between the two forms is done by means of the spin density matrix

$$\begin{aligned} \Phi(p, P, S, T) &= \rho(S, T)_{\Lambda'\Lambda}\, \Phi_{ij;\Lambda\Lambda'}(p, P) \\ &= \mathrm{Tr}\left[\rho(S, T)\; \Phi(p, P)\right] \end{aligned}$$

The spin tensor and density matrix

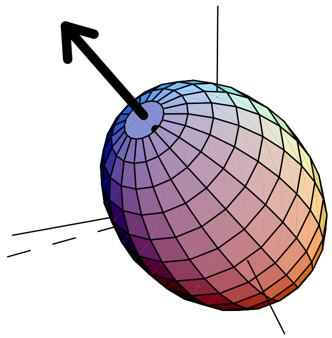
Spin 1/2

$$\boldsymbol{S} = \begin{pmatrix} S_T^x \\ S_T^y \\ S_L \end{pmatrix}$$



Spin 1

$$\boldsymbol{T} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & \frac{4}{3}S_{LL} \end{pmatrix}$$



Density matrix

$$\rho(S) = \frac{1}{2}(\mathbb{1} + S_i\, \sigma_i) \qquad \Big| \qquad \rho(S,T) = \frac{1}{3}(\mathbb{1} + \frac{3}{2}\, S_i\, \Sigma_i + 3\, T_{ij}\, \Sigma_{ij})$$

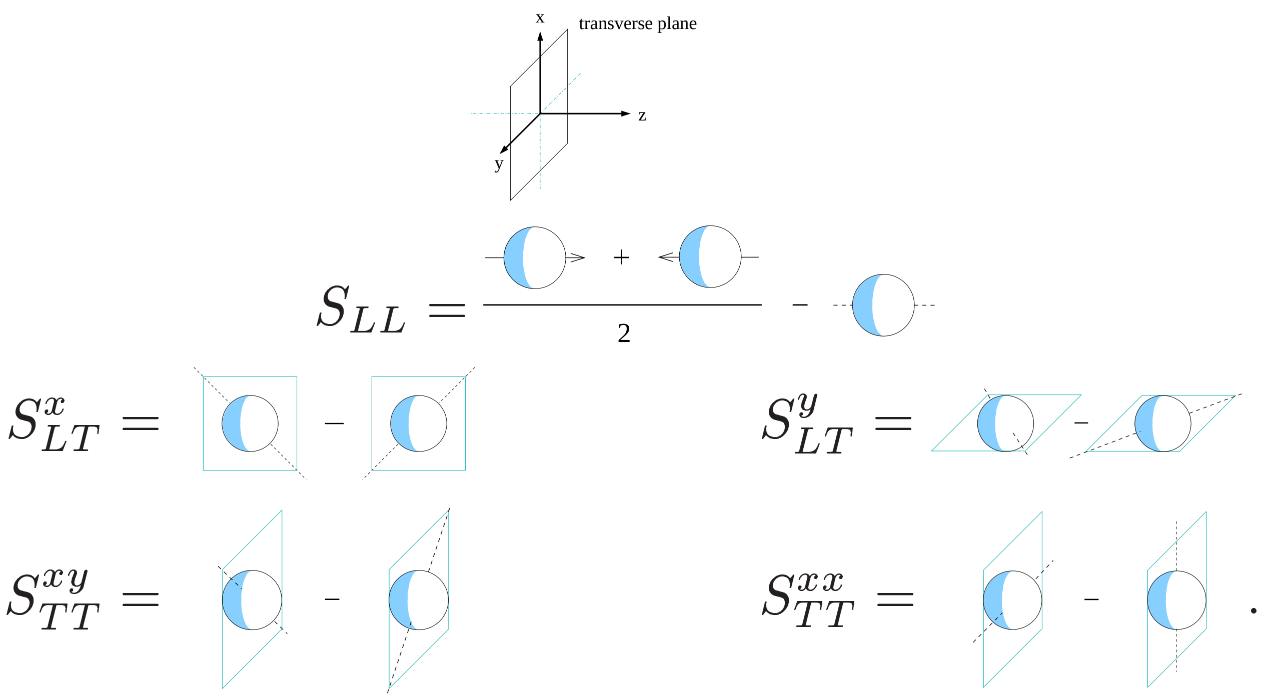
Degree of polarization

$$0 \le \sqrt{\boldsymbol{S}^2} \le 1 \qquad \Big| \qquad 0 \le \sqrt{\frac{3}{4}\boldsymbol{S}^2 + \frac{3}{2}T^{ij}T_{ij}} \le 1$$

Meaning of spin tensor components

$P\left(m_{(\theta,\varphi)}\right)$ = probability of finding a state with spin-component m along the direction specified by θ and φ

$$
\begin{aligned}
S_{LL} &= \frac{1}{2} P\left(1_{(0,0)}\right) + \frac{1}{2} P\left(-1_{(0,0)}\right) - P\left(0_{(0,0)}\right), \\
S_{LT}^{x} &= P\left(0_{(-\frac{\pi}{4},0)}\right) - P\left(0_{(\frac{\pi}{4},0)}\right), \\
S_{LT}^{y} &= P\left(0_{(-\frac{\pi}{4},\frac{\pi}{2})}\right) - P\left(0_{(\frac{\pi}{4},\frac{\pi}{2})}\right), \\
S_{TT}^{xx} &= P\left(0_{(\frac{\pi}{2},-\frac{\pi}{4})}\right) - P\left(0_{(\frac{\pi}{2},\frac{\pi}{4})}\right), \\
S_{TT}^{xy} &= P\left(0_{(\frac{\pi}{2},\frac{\pi}{2})}\right) - P\left(0_{(\frac{\pi}{2},0)}\right).
\end{aligned}
$$



Decomposition of Φ

Available Dirac structures:

$$\mathbf{1},\ \gamma_5,\ \gamma^\mu,\ \gamma^\mu\gamma_5,\ i\sigma^{\mu\nu}\gamma_5$$

Available Lorentz structures:

$$p,\ P,\ S,\ T$$

General decomposition

Leading order in $\frac{1}{Q}$

$$\Phi(x) =$$
$$f_1(x)\, \not{n}_+ + g_1(x)\gamma_5\, \not{n}_+ S_L + h_1(x)\, \not{n}_+\gamma_\nu\gamma_5 S_T^\nu$$
$$+f_{1LL}(x)\, \not{n}_+\, S_{LL} + h_{1LT}(x)\, \not{n}_+\gamma_\nu\gamma_5\, \varepsilon_\perp^{\nu\rho} S_{LT\,\rho}$$

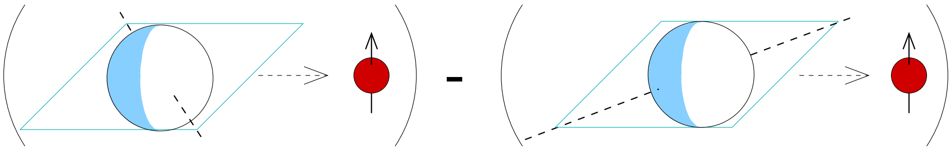
with k_T

$$\Phi(x, \boldsymbol{p}_T) =$$
$$\ldots + h_{1LL}^\perp(x, p_T^2)\, \not{n}_+\, \gamma_\nu\, \frac{p_T^\nu}{M}\, S_{LL}$$
$$+ h'_{1LT}(x, p_T^2)\, \not{n}_+\gamma_\nu\gamma_5\, \varepsilon_\perp^{\nu\rho} S_{LT\,\rho}$$
$$+ h_{1LT}^\perp(x, p_T^2)\, \not{n}_+\, \gamma_\nu\, \frac{p_T^\nu}{M}\, \frac{\boldsymbol{S}_{LT}\cdot\boldsymbol{p}_T}{M}$$
$$+ h'_{1TT}(x, p_T^2)\, \not{n}_+\, \gamma_\nu\, \gamma_5 \varepsilon_T^{\nu\rho} S_{TT\,\rho\sigma} \frac{p_T^\sigma}{M}$$
$$+ i h_{1TT}^\perp(x, p_T^2)\, \not{n}_+\gamma_\nu\, \frac{p_T^\nu}{M}\, \frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_{TT}\cdot\boldsymbol{p}_T}{M^2} + \ldots$$

Decomposition of correlation functions

Without transverse momentum $\Rightarrow$ 5 FUNCTIONS

h_1 χ-odd

$h_{1LT} =$  χ-odd T-odd

With transverse momentum $\Rightarrow$ 18 FUNCTIONS

h_{1LL} χ-odd T-odd

h'_{1LT} χ-odd T-odd

$h^{\perp}_{1LL}$ χ-odd T-odd

h'_{1LL} χ-odd T-odd

$h^{\perp}_{1LL}$ χ-odd T-odd

By merging together the two dimensional parton chirality space with the two dimensional target helicity space, we obtain the full scattering matrix $M = (\Phi\gamma^+)^T$

$$
\begin{pmatrix}
f_1 + g_1 + \frac{f_{1LL}}{2} & 0 & 0 & 0 & \sqrt{2}\,(h_1 + \mathrm{i} h_{1LT}) & 0 \\
0 & f_1 - f_{1LL} & 0 & 0 & 0 & \sqrt{2}\,(h_1 - \mathrm{i} h_{1LT}) \\
0 & 0 & f_1 - g_1 + \frac{f_{1LL}}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & f_1 - g_1 + \frac{f_{1LL}}{2} & 0 & 0 \\
\sqrt{2}\,(h_1 - \mathrm{i} h_{1LT}) & 0 & 0 & 0 & f_1 - f_{1LL} & 0 \\
0 & \sqrt{2}\,(h_1 + \mathrm{i} h_{1LT}) & 0 & 0 & 0 & f_1 + g_1 + \frac{f_{1LL}}{2}
\end{pmatrix}
$$

Alessandro Bacchetta

Two words on spin one targets

When one measures a longitudinal spin asymmetry

$$\frac{d\sigma^{\rightarrow} - d\sigma^{\leftarrow}}{d\sigma^{\rightarrow} + d\sigma^{\leftarrow}} = \frac{g_1}{f_1 + \frac{1}{2} f_{1LL}}$$

then to extract g_1 we need to know also

$$\frac{d\sigma^{\rightarrow} + d\sigma^{\leftarrow} - d\sigma^{0}}{d\sigma^{\rightarrow} + d\sigma^{\leftarrow} + d\sigma^{0}} = \frac{2\, f_{1LL}}{3\, f_1}$$

cf. Hoodboy, Jaffe, Manohar, Nuc. Phys. B 312, 571

$$f_{1LL} \equiv -\frac{2}{3}\, b_1$$

Exactly the same happens with transversely polarized hadrons and transversity.

The only spin-one hadronic target is the deuteron: in the hypothesis of independent scattering on the two nucleons, f_{1LL} is small, but how much ?

The decomposition of Δ can be carried out in the same way. Only some replacements are needed:

x	M		REPLACE	$1/z$	M_h	
P	p			P_h	k	
P^+	$\not{n}_+$			P_h^-	$\not{n}_-$	
f_1	g_1	h_1		D_1	G_1	H_1

We also need to replace

S_{hL}	S_{hT}^i		REPLACE	A_{hL}	A_{hT}^i	
S_{hLL}	S_{hLT}^i	S_{hTT}^{ij}		A_{hLL}	A_{hLT}^i	A_{hTT}^{ij}

since in a cross section we observe

$$\boxed{\Delta(k, P_h, A_h^i, A_h^{ij}) = \mathrm{Tr}\left[R_h^{\text{(decay)}}(A_h^i, A_h^{ij})\ \Delta(k, P_h)\right]}$$

The analyzing powers depend on the details of the final-state hadron decay.

Where to observe transversity?

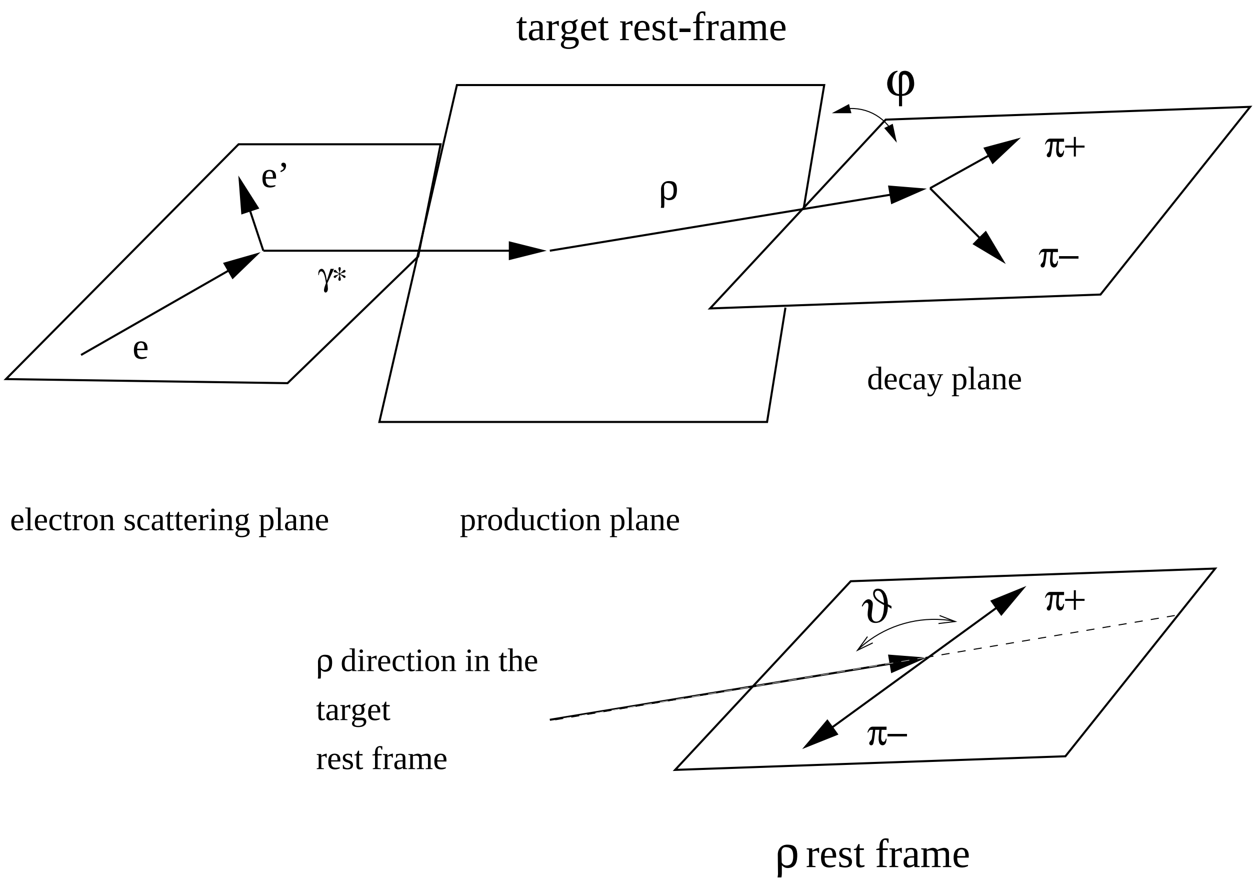
$$R_h^{\text{(decay)}}(A_h) = \frac{1}{4\pi}\left(\mathbf{1} + \frac{3}{2}\,\Sigma_i\, A_h^i + 3\,\Sigma_{ij}\, A_h^{ij}\right)$$

For $\rho \to \pi^+\,\pi^-$

$$\begin{aligned} A_h^i &= 0 \\ {A_h}^{ij} &= \frac{1}{3}\delta^{ij} - \hat{p}^i_{\text{cm}}\hat{p}^j_{\text{cm}}. \end{aligned}$$

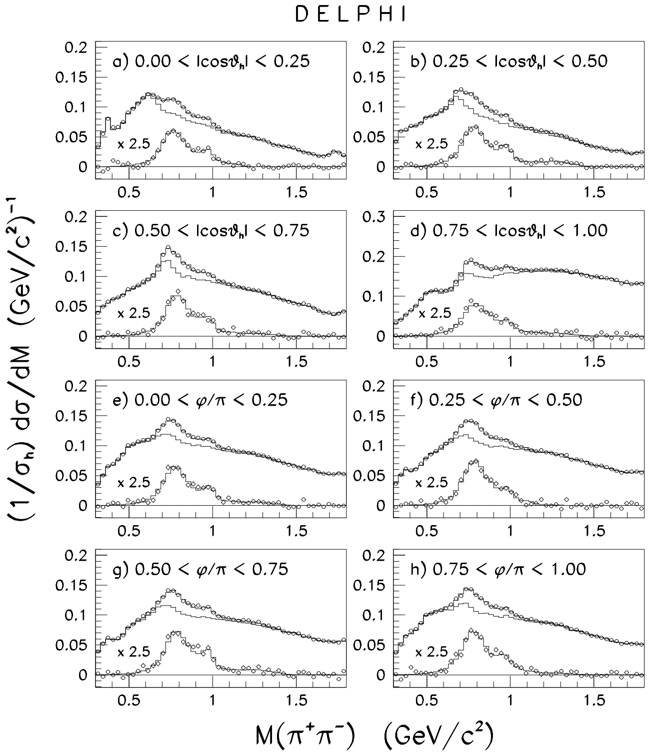
direction of pion in ρ rest-frame

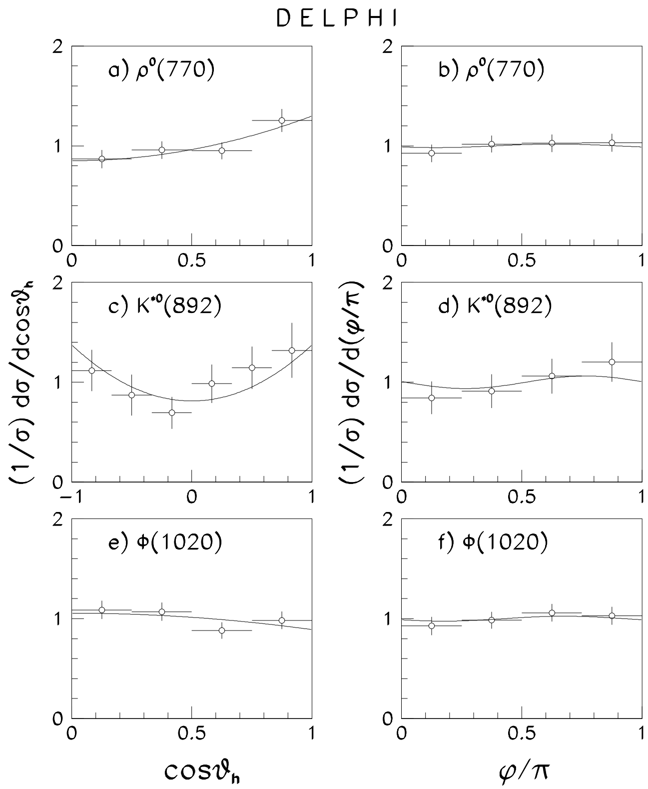
Decay of a self-analyzing spin-1 hadron



$$
\begin{aligned}
A_{LL} &= -\tfrac{1}{2}\left(\cos^2\theta + \cos 2\theta\right) \\
A_{LT}^{x} &= -\sin 2\theta \,\cos\varphi \\
A_{LT}^{y} &= -\sin 2\theta \,\sin\varphi \\
A_{TT}^{xx} &= -\sin^2\theta \,\cos 2\varphi \\
A_{TT}^{xy} &= -\sin^2\theta \,\sin 2\varphi
\end{aligned}
$$

Delphi collaboration Phys. Lett. B 406, 271





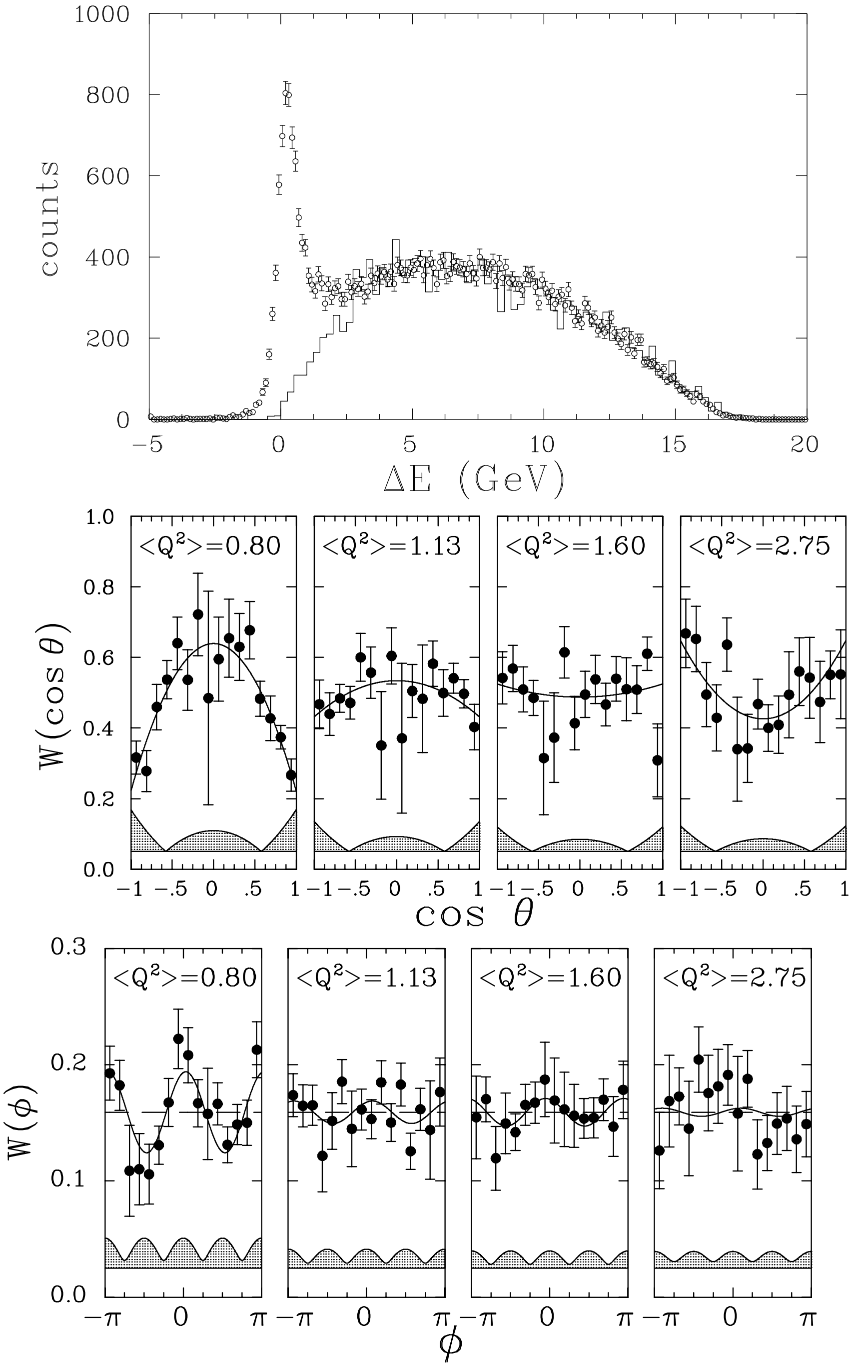
Measurement of decay distribution seems to be feasable

Measurements in $e^+ e^-$ annihilation

Measurements in diffractive production

Hermes collaboration `hep-ex/0002016`

(Zeus collaboration Eur. Phys. J. C 12, 393)



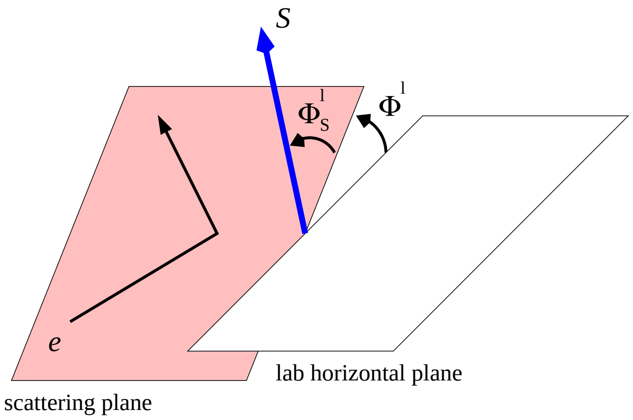
Measurement of decay distribution seems to be feasable

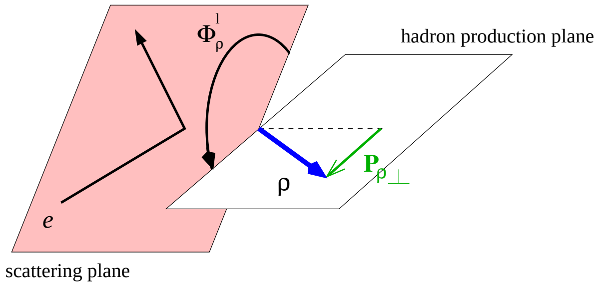
Azimutal asymmetries

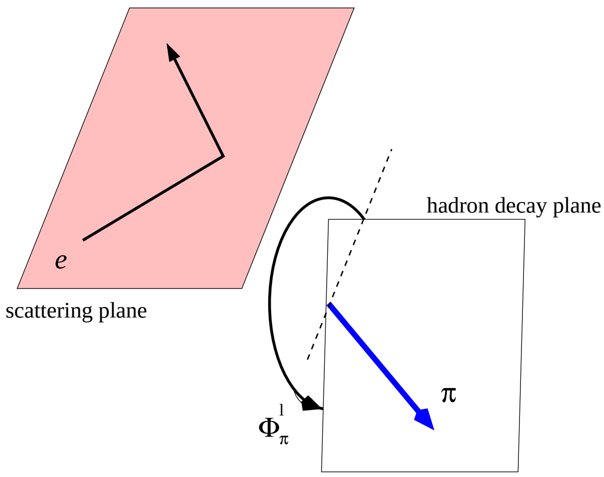
$$\langle W \rangle_{UT}(x_B, y, z_\rho) =$$

$$\int \mathrm{d}\phi^\ell \, \mathrm{d}^2 \boldsymbol{P}_{\rho\perp} \; W \left(\frac{\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow}{\mathrm{d}\phi^\ell \, \mathrm{d}x_B \, \mathrm{d}z_\rho \, \mathrm{d}y \, \mathrm{d}^2 \boldsymbol{P}_{\rho\perp}} \right)$$

$$W = W\left(\frac{|P_{\rho\perp}|}{z}, \phi_\rho^\ell, \phi_S^\ell, \phi_\pi^\ell\right)$$



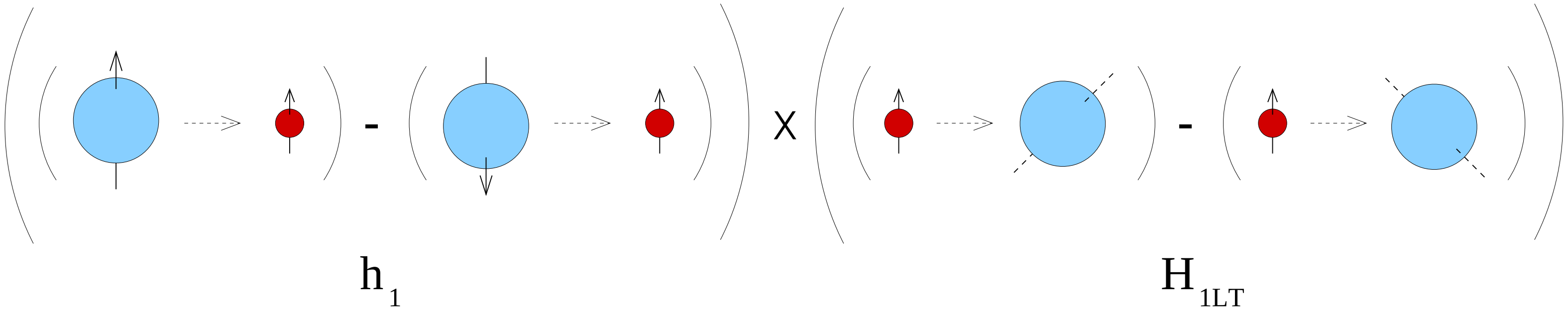




Asymmetries including transversity

$$\left\langle \sin\left(\phi_\pi^\ell + \phi_S^\ell\right)\right\rangle_{UT} \propto \sin|2\theta_\pi| \sum_a e_a^2\, h_1^a(x_B)\, H_{1LT}^a(z_\rho)$$

χ-odd T-odd



cf. X. Ji, Phys. Rev. D 49, 114

$$H_{1LT} \equiv \hat{h}_{\bar{1}}$$

Asymmetries including transversity

$$\left\langle \frac{|P_{\rho\perp}|}{2\,z\,M_h}\ \sin\left(2\phi_\pi^\ell + \phi_S^\ell - \phi_\rho^\ell\right)\right\rangle_{UT} \propto \sin^2\theta_\pi \sum_a e_a^2\, h_1^a(x_B)\, H_{1TT}^{(1)a}(z_\rho)$$

$$\left\langle \frac{|P_{\rho\perp}|}{2\,z\,M_h}\ \sin\left(\phi_S^\ell + \phi_\rho^\ell\right)\right\rangle_{UT} \propto \frac{1}{2}\left(1-3\cos\theta_\pi\right) \sum_a e_a^2\, h_1^a(x_B)\, H_{1LL}^{\perp(1)a}(z_\rho)$$

$$\left\langle \frac{|P_{\rho\perp}|^2}{2\,z^2\,M_h^2}\ \sin\left(\phi_\pi^\ell - \phi_S^\ell - 2\phi_\rho^\ell\right)\right\rangle_{UT} \propto |\sin 2\theta_\pi| \sum_a e_a^2\, h_1^a(x_B)\, H_{1LT}^{\perp(2)a}(z_\rho)$$

$$\left\langle \frac{|P_{\rho\perp}|^3}{4\,z^3\,M_h^3}\ \sin\left(2\phi_\pi^\ell - \phi_S^\ell - 3\phi_\rho^\ell\right)\right\rangle_{UT} \propto \sin^2\theta_\pi \sum_a e_a^2\, h_1^a(x_B)\, H_{1TT}^{\perp(3)a}(z_\rho)$$

Conclusions and open questions

The variety of possibilities in this process includes **5 observables** at leading order in $\frac{1}{Q}$ to measure the transversity distribution h_1.

This process allows the first chance of observing a **time-reversal odd fragmentation function** at first order in $\frac{1}{Q}$ and without including intrinsic transverse momentum effects.

? Are there reasons to believe that these T-odd functions are **different from zero**? Is it possible to provide simple models of them?

? What is the relation with the more general case of **two-pion production**?