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Combined Top Quark Mass Measurement in the Dilepton Channel

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Abstract

We present a measurement of the top mass in dilepton events from a combination of four measurements. In 340-360 pb^{-1} of integrated luminosity, we find

$$M_t = 168.2 \pm 4.5_{stat} \pm 3.7_{syst}.$$

Combining this result with the measurement from Run1 yields

$$M_t = 168.1 \pm 4.1_{stat} \pm 3.8_{syst}.$$

1 Introduction

The mass of the top quark is a free parameter of the Standard Model; it is of great interest due to its exceedingly large value and the constraints it places on the mass of the Higgs boson. At the Tevatron, top quarks are primarily produced in pairs and decay to a W boson and b quark nearly 100% of the time. Of these decays, the "dilepton" channel, which includes events where both W bosons decay to an electron or muon, has small statistics but little background contamination. The measurement of the mass in this channel is an important and direct confirmation that events in excess over background are due to the top quark as described by the Standard Model. If the measurement is consistent with that in other channels, it can be combined to yield greater precision; a significant discrepancy from measurements in other channels could indicate contributions to the sample from new sources.

CDF has measured the top quark mass in dilepton events using a matrix-element method[1] and three template methods, ν_η[2], P_z[3] and ν_ϕ[4]. We study the statistical and systematic correlation between the measurements and present a combination with $\approx 15\%$ greater precision than that from the single most precise measurement.

To combine several correlated measurements of a single quantity, we use an implementation of the BLUE[5] method. This requires treating the sources of uncertainty as Gaussian and evaluating the global matrix of correlations. Section 2 discusses the statistical correlations, Section 3 describes the systematic correlations and Section 4 gives the results. In Section5, we combine the Run2 result with the Run 1 measurement.

2 Statistical Correlation

To measure the statistical correlation between the methods, we study the measured mass in pseudo-experiments which model the expected data sample. Three of the methods (ME,ν_ϕ and P_z) use the DIL selection[6] and one, ν_η, uses the LTRK selection[7]. The pseudo-experiments must model the typical signal and background composition of each selection as well as the overlap between the two.

2.1 Pseudo-experiment Construction

We form pseudo-experiments which model the expected events before any selection is applied; each pseudo-experiment corresponds to 350 pb$^-$1. Candidates for each selection are then identified from this pool. Assuming $N_{DIL} = \epsilon_{DIL}\sigma \int dtL$ and $N_{LTRK} = \epsilon_{LTRK}\sigma \int dtL$, the number of signal events in each pseudo-experiment should correspond to the expected number of signal events. There is significant overlap in the selection of $t\bar{t}$ events, as seen in Figure 1.

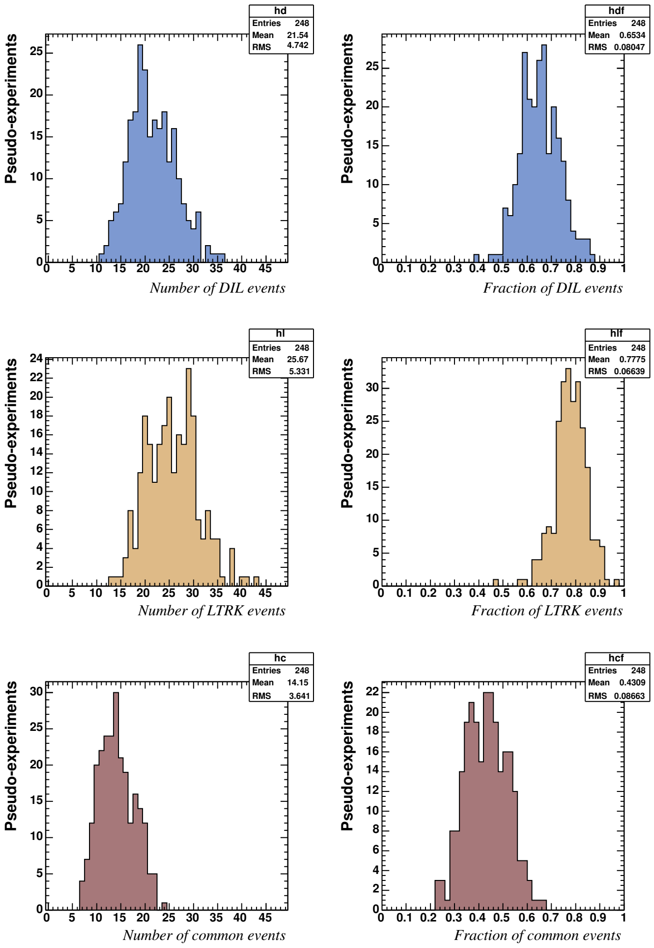

Figure 1: Left column shows the number of $t\bar{t}$ events in pseudo-experiments in DIL, LTRK and the common selection. Right column shows the fraction of events in the union of LTRK and DIL selected by DIL, LTRK and the common selection.

The same prescription cannot in general be applied to include background events in these psuedo-experiments, as the statistics of the background samples represent considerably smaller integrated luminosity. In addition, the expected contribution of $Z \rightarrow ll$ to the collected sample is estimated to be larger than the simple prediction from simulated events, breaking the relation $N = \epsilon\sigma \int dtL$. Finally, the DIL selection models the fake-lepton contribution using events from the data, while the LTRK selection uses simulated events.

For backgrounds other than fake leptons, we model the contributions of background to the DIL pseudo-experiment by selecting $< N_{DIL} >$ events from the pool of events accepted by DIL.

The contribution to the LTRK pseudo-experiment is constructed as the union of those events in the DIL psuedo-experiment which pass the LTRK selection, and $< N_{LTRK} - N_{LTRK \cup DIL} >$ from the pool of events accepted only by LTRK. $N_{LTRK \cup DIL}$ is the number of events expected in common, which is estimated from the number events selected in common in simulated events. Figure 2 shows the typical number of events from $Z \to ee$ events in DIL and LTRK pseudo-experiments and the common selection.

Fake backgrounds are assumed to be uncorrelated; studies in simulated events suggest that there is very little overlap in the category of fake events which would provide a contribution to both samples.

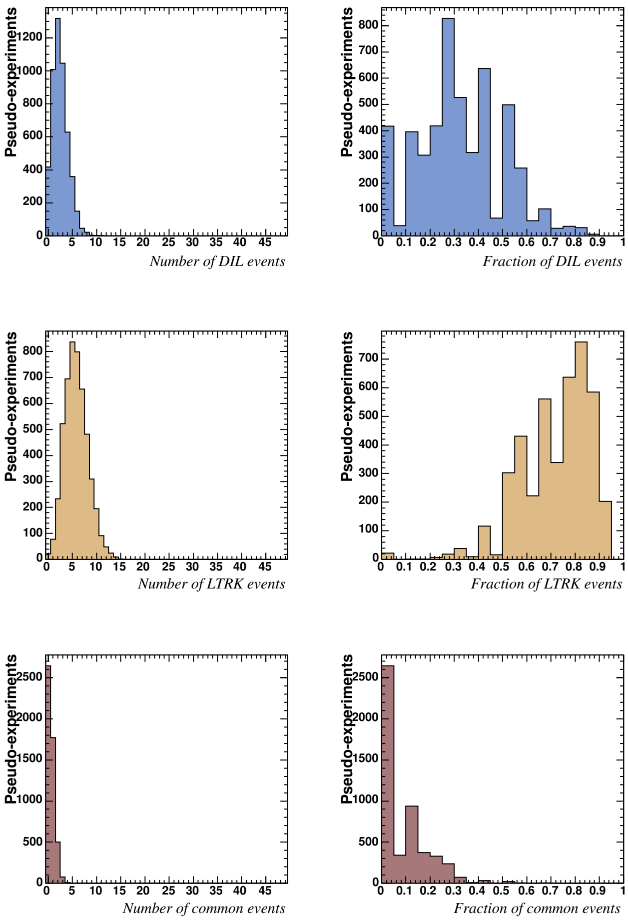

Figure 2: Left column shows the number of $Z \to ee$ events in pseudo-experiments in DIL, LTRK and the common selection. Right column shows the fraction of events in the union of LTRK and DIL selected by DIL, LTRK and the common selection.

2.2 Measured Correlations

In pseudo-experiments as constructed above, we measure the pair-wise correlation coefficient ρ,

$$\rho_{xy} = \frac{N\sum xy - \sum x \sum y}{\sqrt{(N\sum x^2 - (\sum x)^2)}\sqrt{(N\sum y^2 - (\sum y)^2)}}$$

between the residuals $M_t^{res} = M_t^{meas} - M_t^{true}$ from two methods. Figures 3-5 show distributions of residuals from the methods in samples with $M_t = 165, 170, 175$ GeV and Table 2.3 compiles the correlation coefficients. Figure 6 shows the distribution for all pseudo-experiments, and Table 2.3 shows the average correlations.

2.2.1 Validation and Uncertainty

We validate our calculation by using three toy measurements with known correlations, $\rho_{12} = 0.35, \rho_{23} = 0.10, \rho_{13} = 0.25$. Figure 7 shows that the input correlations are correctly extracted.

The number of pseudo-experiments for the study in full simulation is relatively small (≈ 250) and limited by the size of the simulated $t\bar{t}$ samples. To estimate the error on ρ due to limited statistics, we build pseudo-experiments as above, but with limited statistics. Figure 8 shows that the distribution of extracted ρ peaks at the correct value; the width of the distributions is ≈ 0.05, which is a measure of the uncertainty.

2.3 Psuedo-experiments for combination

Using the measured correlations, we extracted a combined result for all available pseudo-experiments. The pull distribution of these results, see Figure 9, shows that the statistical error estimation is unbiased, suggesting that the measured correlations are reliable.

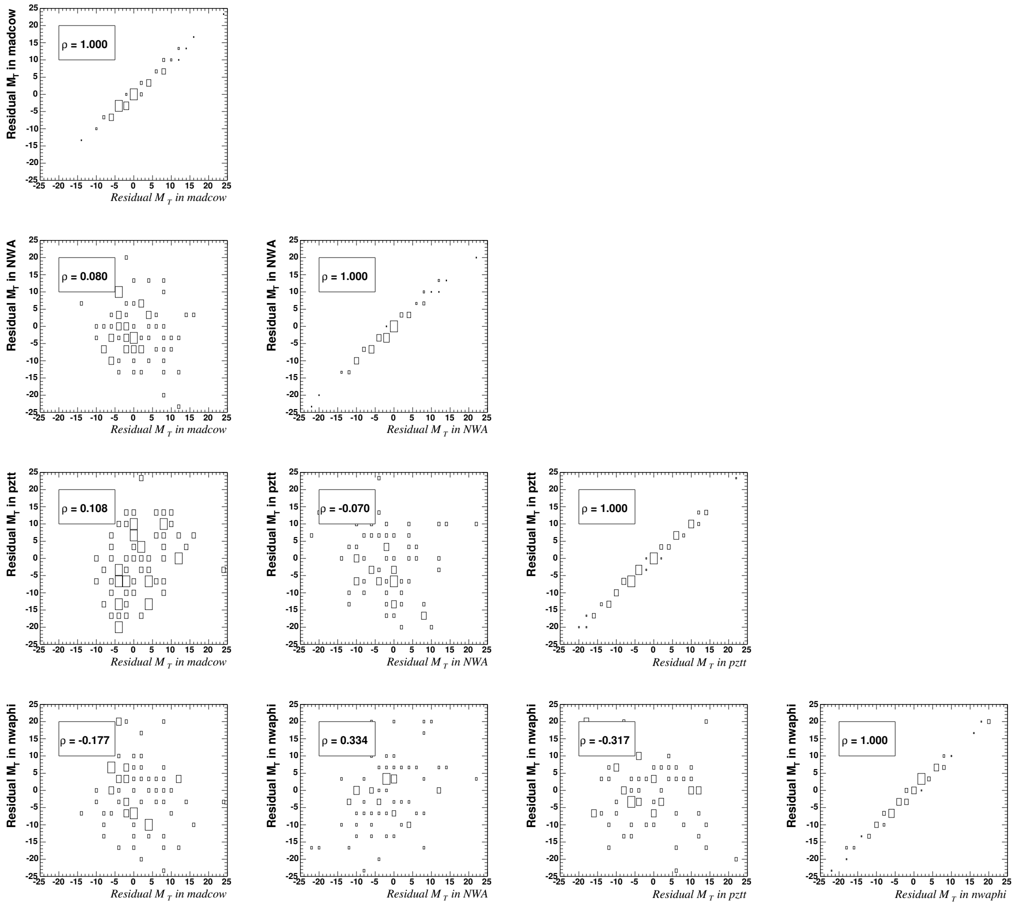

Figure 3: Correlation in mass residual in pseudo-experiments of $t\bar{t}$ events with $M_t = 165$ GeV$/c^2$.

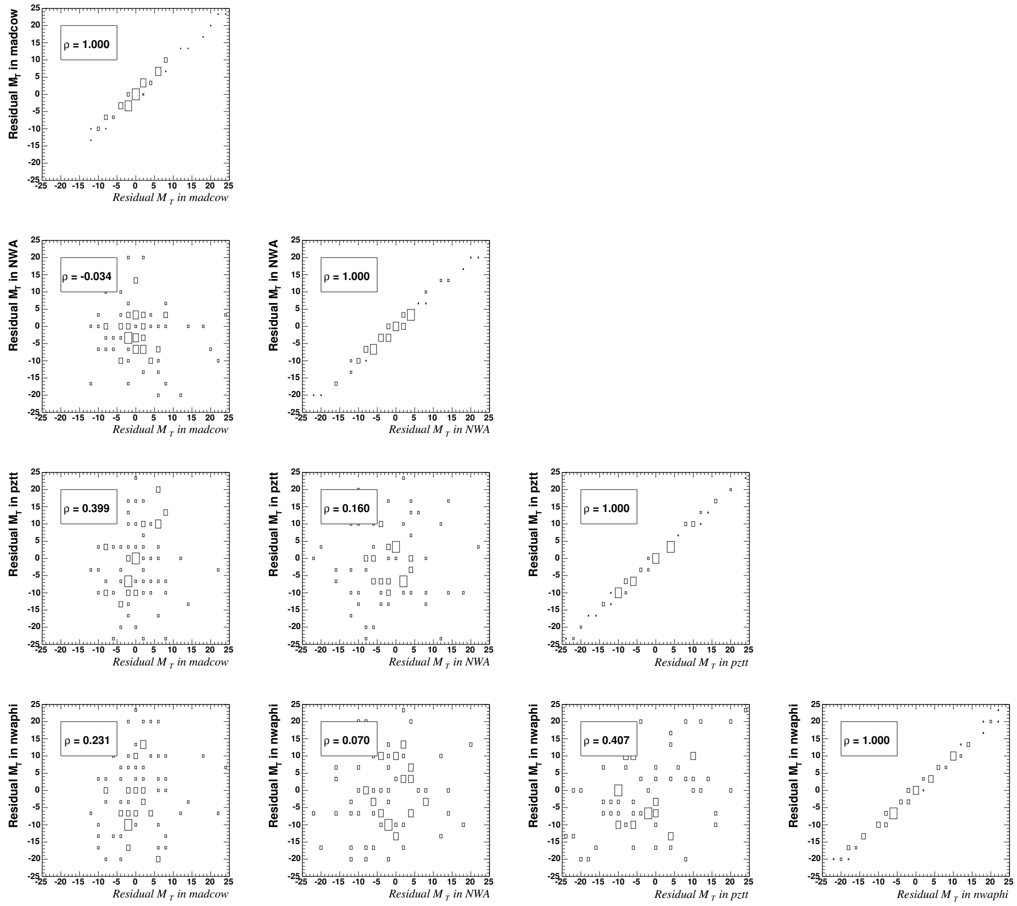

Figure 4: Correlation in mass residual in pseudo-experiments of $t\bar{t}$ events with $M_t = 170$ GeV$/c^2$.

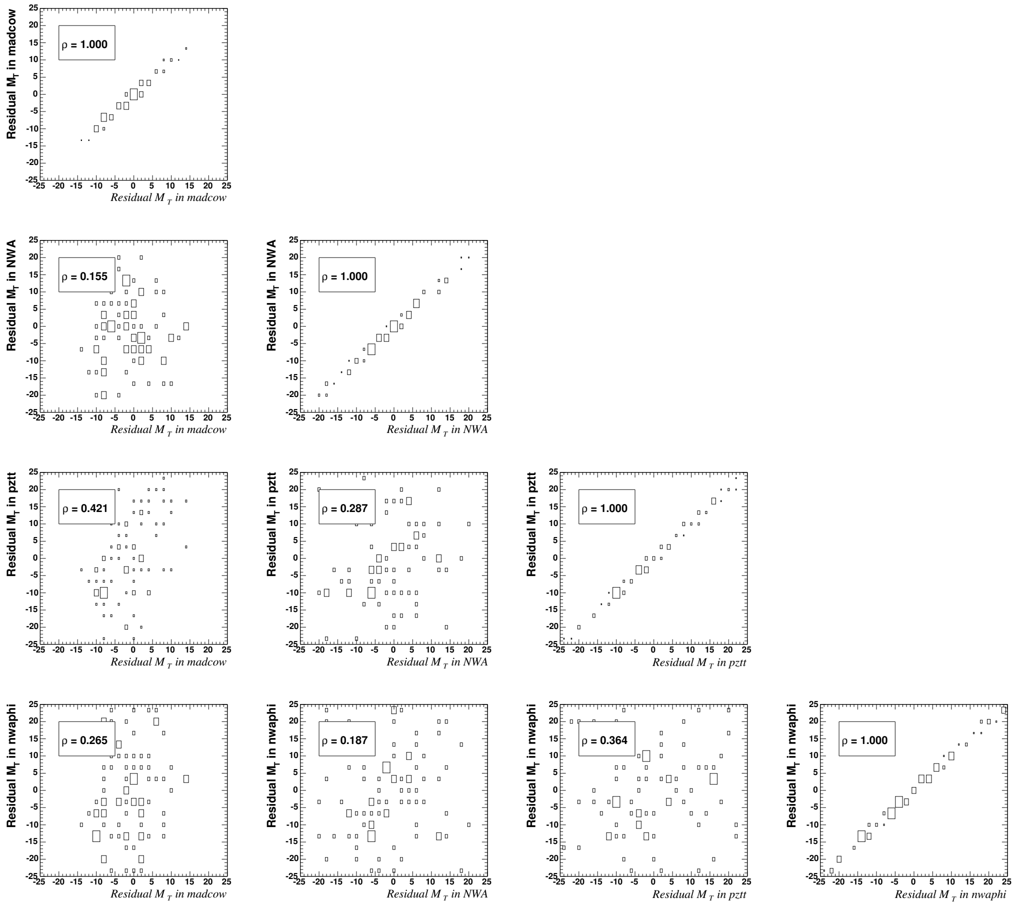

Figure 5: Correlation in mass residual in pseudo-experiments of $t\bar{t}$ events with $M_t = 175$ GeV$/c^2$.

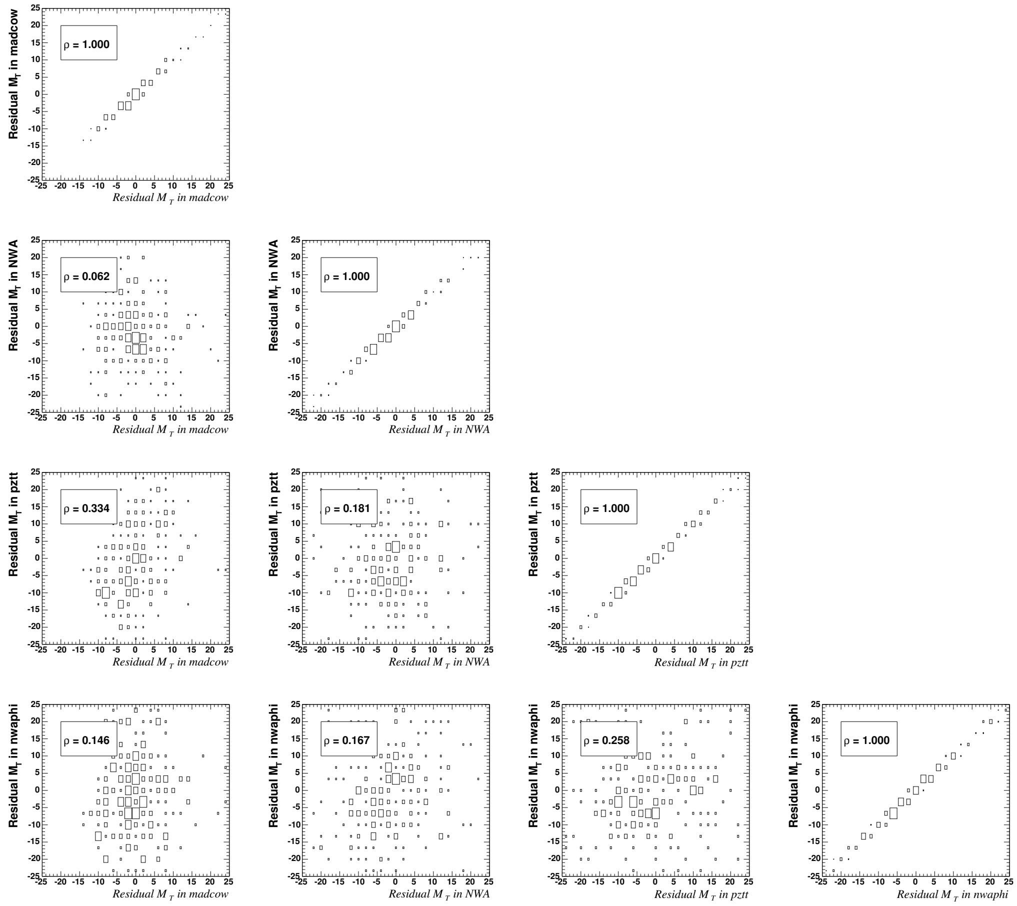

Figure 6: Correlation in mass residual in pseudo-experiments of $t\bar{t}$ events with $M_t = 165 - 175$ GeV$/c^2$.

M_t	ME	ν_η	P_z	ν_ϕ
165	1.0			
	0.08	1.0		
	.11	-0.07	1.0	
	-.18	0.33	-0.32	1.0
170	1.0			
	-0.03	1.0		
	0.40	0.16	1.0	
	0.23	0.07	0.41	1.0
175	1.0			
	0.16	1.0		
	0.42	0.29	1.0	
	0.26	0.19	0.36	1.0

Table 1: Statistical correlation between methods at varying masses.

ME	ν_η	P_z	ν_ϕ
1.0			
0.06	1.0		
0.33	0.18	1.0	
0.15	0.17	0.26	1.0

Table 2: Statistical correlation between methods

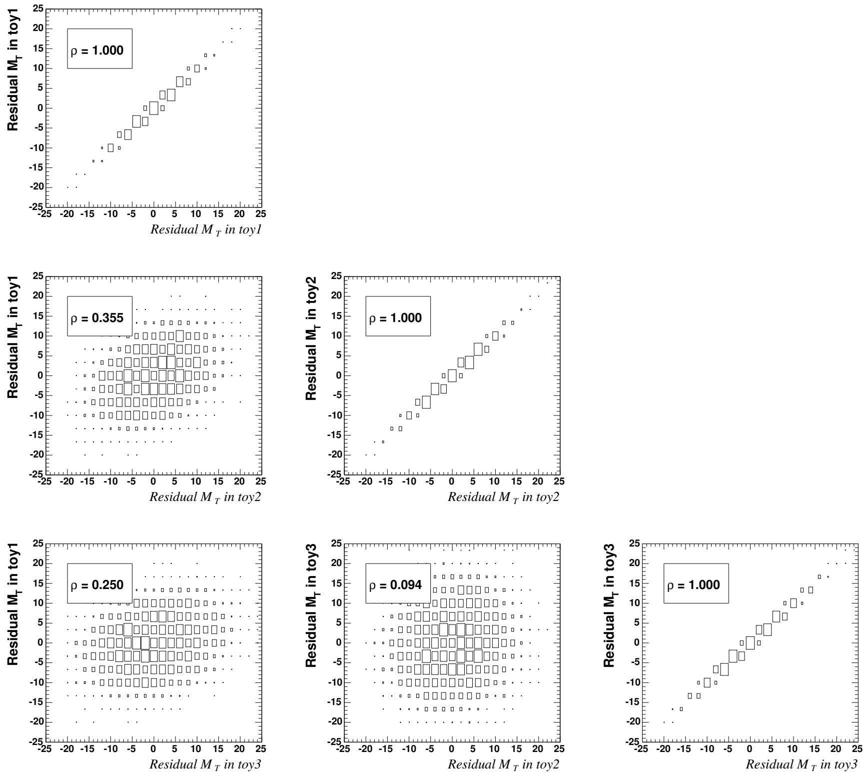

Figure 7: Correlation in mass residual in high-statistics toy pseudo-experiments with $\rho_{12} = 0.35, \rho_{23} = 0.10, \rho_{13} = 0.25$.

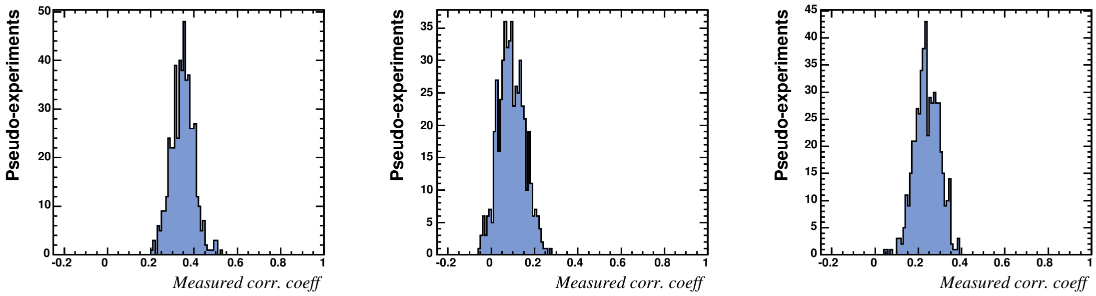

Figure 8: Correlation in mass residual in low-statistics toy pseudo-experiments with $\rho_{12} = 0.35, \rho_{23} = 0.10, \rho_{13} = 0.25$. The width of the distributions indicate that the statistical error of ρ is ≈ 0.05.

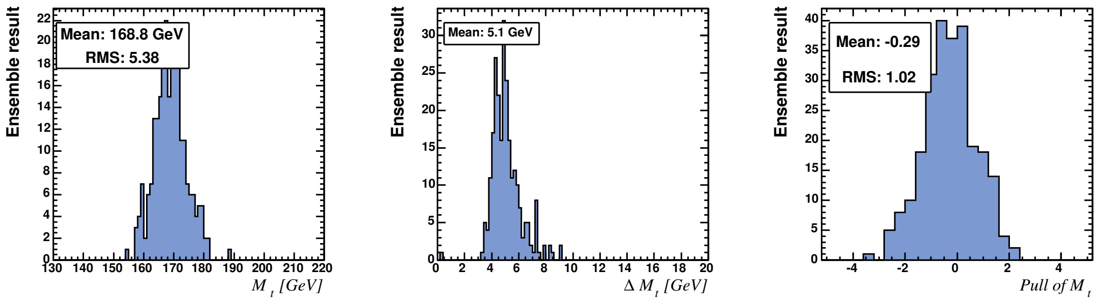

Figure 9: Measured combined mass, statistical error and pull distribution for the four-measurement combination in pseudo-experiments with $M_t = 165, 170, 175$ GeV. Results from simulated samples at $M_t = 165$ have been shifted by $+5$ GeV, and results from $M_t = 175$ have been shifted by -5 GeV.

Source	ME	ν_η	P_z	ν_ϕ	Correlation
iJES					1.0
aJES					1.0
bJES	0.8	0.6	0.6		1.0
cJES	2.2	1.0	2.4		1.0
dJES	0.8	0.6	0.5		1.0
rJES	1.1	2.3	2.1		1.0
Signal	1.5	1.1	0.8		1.0
MC	0.8	0.9	0.6		1.0
UN/MI					1.0
BG	1.6	3.0	1.5		1.0
Fit	0.6	1.3	0.9		0.0
Total	3.6	4.4	3.8		

Table 3: Correlations between systematic uncertainties.

3 Systematic Correlation

We break down the systematic uncertainty in each measurement using the categories defined to aid combination of CDF and DZero's mass measurements[8]. We assume that each category is either 0% or 100% correlated. Table 3 gives the size of the uncertainty in each channel for each measurement, and indicates the assumption regarding the correlations.

Method	Total	Corr	Matrix		Weight	Pull
ME	1.0				42%	-0.67
ν_η	0.26	1.0			31%	0.41
P_z	0.46	0.33	1.0		16%	0.32
ν_ϕ	0.30	0.29	0.38	1.0	12%	0.19

Table 4: Total correlation matrix, weight and pull of each method in the combined Run2 result.

Source	Magnitude
iJES	0.0
aJES	0.0
bJES	0.7
cJES	1.9
dJES	0.7
rJES	1.7
Signal	1.3
MC	0.8
UN/MI	0.0
BG	1.9
Fit	0.5
Total	3.7

Table 5: Final systematic errors for the Run2 result by category.

4 Run2 Results

The combined result for Run2 is

$$M_t = 168.2 \pm 4.5_{stat} \pm 3.7_{syst}.$$

The weights and pulls of each measurement can be found in Table 4. The systematic error is broken down by category in Table 4. The four measurements are shown with the combined result in Figure 10.

If the measured statistical correlations are varied coherently up or down by $\Delta\rho = 0.05$, the combined result changes by less than 25 MeV, and the statistical error by approximately 100 MeV.

Method	Total	Corr	Matrix			Weight	Pull
ME	1.0					36%	-0.60
ν_η	0.26	1.0				27%	0.42
P_z	0.46	0.33	1.0			13%	0.33
ν_ϕ	0.30	0.29	0.38	1.0		10%	0.20
Run1	0.17	0.15	0.16	0.15	1.0	14%	-0.07

Table 6: Total correlation matrix, weight and pull of each method in the combined Run1+Run2 result.

5 Run1+Run2 Results

We combine the four Run2 measurements with the CDF Run1 dilepton mass measurement [9]. The statistical uncertainty is uncorrelated, and we follow the breakdown of the systematic error and uncertainty from [8]. Specifically, we assume 100% correlation between Run1 and Run2 in b-jet energy scale (bJES), fragmentation, out-of-cone corrections (cJES), calorimeter response, relative corrections and multiple interactions (rJES), signal modelling, background modelling, and generator uncertainty; we assume 0% correlation between Run1 and Run2 in calibration samples (dJES) and in method if fit.

The combined result for Run2 is

$$M_t = 168.1 \pm 4.1_{stat} \pm 3.8_{syst}.$$

The weights and pulls of each measurement can be found in Table 5. The systematic error is broken down by category in Table 5. The four measurements are shown with the combined result in Figure 10.

Source	Magnitude
iJES	0.0
aJES	0.0
bJES	0.7
cJES	2.0
dJES	0.6
rJES	1.9
Signal	1.5
MC	0.8
UN/MI	0.0
BG	1.7
Fit	0.5
Total	3.8

Table 7: Final systematic errors for the Run1+Run2 by category.

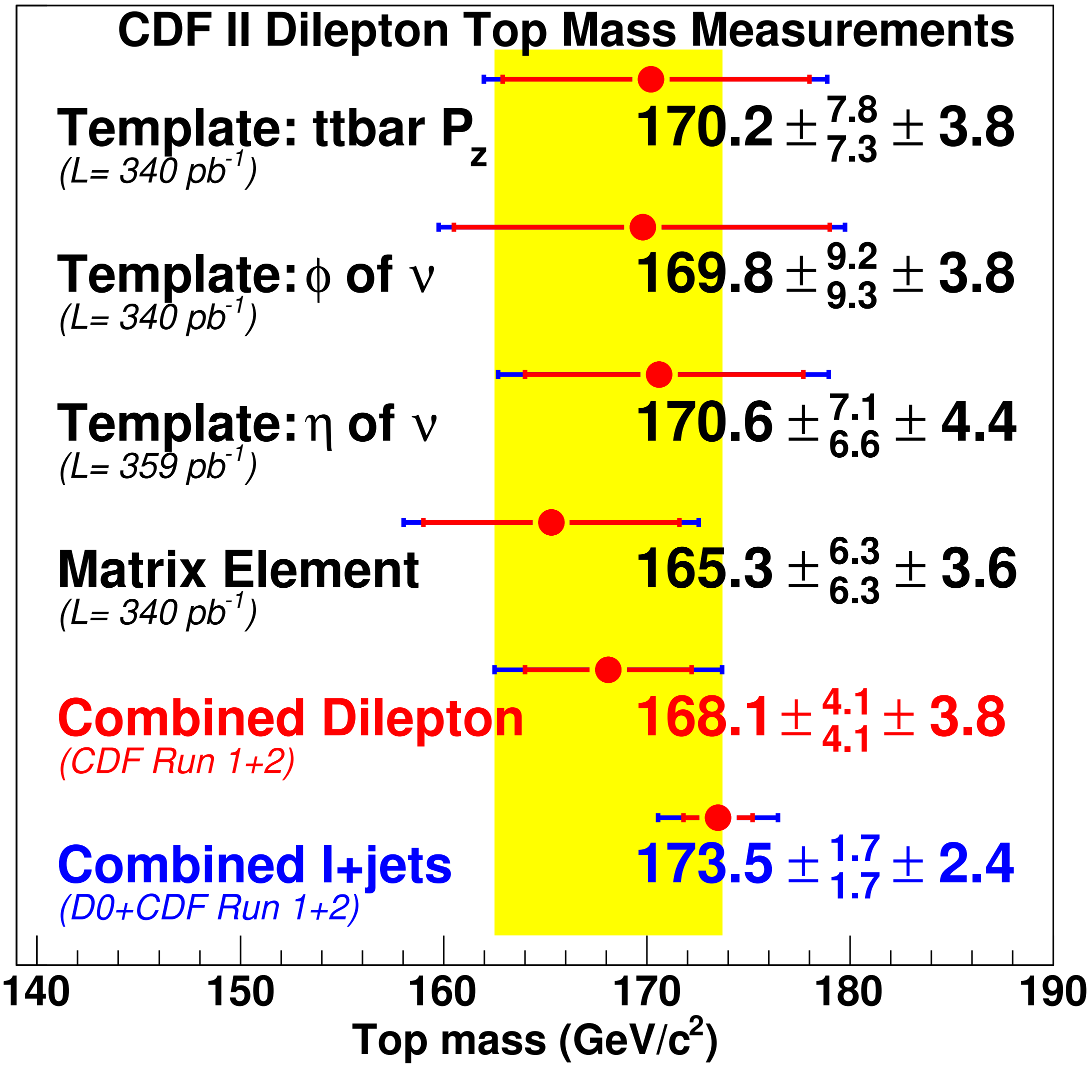

Figure 10: The four individual measurements, the combined Run1+Run2 dilepton result and the combined Run1+Run2 CDF+DZero l+jets result [8].

References

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[9] F. Abe *et. al.*, The CDF Collaboration, Phys. Rev. Lett. **82**, 271 (1999)