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Asymmetric Gluon Distributions and Hard Diffractive Electroproduction

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Abstract

Due to the momentum transfer $r \equiv p - p'$ from the initial proton to the final, the "asymmetric" matrix element $\langle p'|G \ldots G|p\rangle$ that appears in the pQCD description of hard diffractive electroproduction does not coincide with that defining the gluon distribution function $f_g(x)$. I outline a pQCD formalism based on a concept of a double distribution $F_g(x,y)$, which specifies the fractions xp, yr, $(1-y)r$ of the initial proton momentum p and the momentum transfer r, *resp.*, carried by the gluons. I discuss the one-loop evolution equation for the double distribution $F_g(x,y;\mu)$ and obtain the solution of this equation in a simplified situation when the quark-gluon mixing effects are neglected. For $r^2 = 0$, the momentum transfer r is proportional to p: $r = \zeta p$, and it is convenient to parameterize the matrix element $\langle p - r|G \ldots G|p\rangle$ by an asymmetric distribution function $\mathcal{F}_\zeta^g(X)$ depending on the total fractions $X \equiv x + y\zeta$ and $X - \zeta = x - (1-y)\zeta$ of the initial hadron momentum p carried by the gluons. I formulate evolution equations for $\mathcal{F}_\zeta^g(X)$, study some of their general properties and discuss the relationship between $\mathcal{F}_\zeta^g(X)$, $F_g(x,y)$ and $f_g(x)$.

1. Introduction. The process of hard diffractive exclusive electroproduction of vector mesons $\gamma^* + p \to V + p'$ is now an object of intensive theoretical studies [1–5]. It is considered to be one of the most effective ways to measure the gluon distribution function $f_g(x)$ in the low-x region. However, one can easily notice that due to the momentum transfer $r \equiv p - p'$ from the initial proton to the final, even for small $t \equiv r^2$, the matrix element of the gluonic operator in this case does not coincide with the usual forward matrix element defining the gluon distribution function $f_g(x)$. My goal in this letter is to develop a modified pQCD approach for the hard diffractive electroproduction, which takes into account the effects related to the momentum transfer. In this formalism, the basic function describing the gluon content of the "asymmetric" matrix element $\langle p - r| \ldots |p\rangle$ is the *double distribution* $F_g(x, y)$, which specifies the fractions xp, yr, $(1 - y)r$ of the initial proton momentum p and the momentum transfer r, *resp.*, carried by the gluons*. With respect to x, the function $F_g(x, y)$ looks like a distribution function while with respect to y it behaves like a distribution amplitude. Since the logarithmic scaling violation is an important feature of the gluon distribution function in the low-x region, I discuss the evolution equation for the double distribution $F_g(x, y; \mu)$. The relevant evolution kernel $R_{gg}(x, y; \xi, \eta)$ produces the GLAPD evolution kernel $P_{gg}(x/\xi)$ [6–8] when integrated over y, while integrating $R_{gg}(x, y; \xi, \eta)$ over x gives the expression coinciding with the evolution kernel $V_{gg}(y, \eta)$ for the gluon distribution amplitude. I construct the solution of the one-loop evolution equation for the double gluon distribution in a simplified situation when quark-gluon mixing effects are neglected. For $t = 0$ and vanishing hadron masses, the momentum transfer r is proportional to p: $r = \zeta p$ and, for this reason, it is convenient to parameterize the matrix element $\langle p - r|G \ldots G|p\rangle$ by the *asymmetric distribution function* $\mathcal{F}_\zeta^g(X)$ specifying the total fractions Xp, $(X - \zeta)p$ of the initial hadron momentum p carried by the gluons†. I formulate evolution equations for the asymmetric distribution functions $\mathcal{F}_\zeta^g(X)$ and discuss the relationship between this function, double gluon distribution $F_g(x, y)$ and the usual gluon distribution function $f_g(x)$.

2. Double distributions. The amplitude for the elastic electroproduction process $\gamma^* p \to p' V$ depends on the momentum p of the initial proton, the momentum transfer $r = p - p'$ and the momentum q of the produced vector meson. We will consider the limit in which one can neglect the squares of the meson $q^2 \equiv m_V^2$ and proton $p^2 \equiv m_p^2$ masses compared to the virtuality $-Q^2 \equiv (q - r)^2$ of the

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*Originally, the double distributions for the asymmetric matrix elements of quark operators were introduced in ref. [9] in application to virtual Compton scattering.

†The asymmetric distribution functions are similar to, but not identical with the $t \to 0$ limit of the off-forward parton distributions introduced recently by X.Ji [10].

initial photon and the energy invariant $p \cdot q$. Thus, we set $p^2 = 0$ and $q^2 = 0$, and use q and p as the basic Sudakov light-cone 4-vectors. In the diffractive region, the invariant momentum transfer $r^2 \equiv t$ is small, and we actually will consider the limit $t = 0$. Note that in this case the on-shell condition $p'^2 \equiv (p + r)^2 = p^2$ results in the requirement $(p \cdot r) = 0$ which can be satisfied only if the two lightlike momenta p and r are proportional to each other: $r = \zeta p$, where $\zeta \equiv Q^2/2(p \cdot q)$ is the Bjorken variable which obeys $0 \leq \zeta \leq 1$.

The leading contribution in the large-Q^2, fixed-ζ limit is given by the diagrams shown in Fig.1. The long-distance dynamics is described there by the vector meson distribution amplitude $\varphi_V(\tau)$ and the asymmetric matrix element of the light-cone gluonic operator

$$\langle p - r \,|\, G_{\mu\alpha}^a(z_1) E_{ab}(z_1, z_2; A) G_{\nu\alpha}^b(z_2) \,|\, p\rangle \qquad (1)$$

in which the 4-vectors z_1, z_2 specifying the location of the two gluon vertices are separated by lightlike intervals $z_1^2 = 0$, $z_2^2 = 0$ from the virtual photon vertex. The factor $E_{ab}(z_1, z_2; A)$ is the usual P-exponential of the gluonic A-field along the straight line connecting z_1 and z_2, and the indices $a, b = 1, \ldots 8$ denote the gluon color.
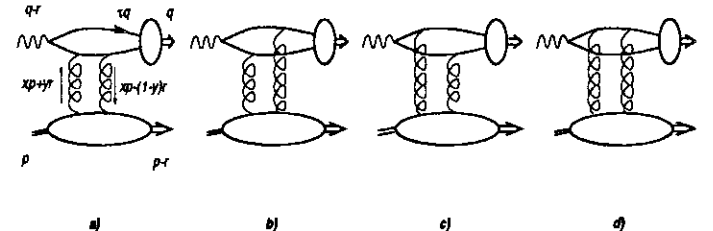

FIG. 1. Diagrams contributing to hard diffractive electroproduction of vector mesons.

The gluon momentum in this matrix element originates both from the initial hadron momentum p and from the momentum transfer r, so I write it as $xp + yr$ parameterizing the asymmetric matrix element of the light-cone gluon operator by the double distribution $F_g(x, y)$

$$\langle p - r \,|\, z_\mu z_\nu G_{\mu\alpha}^a(0) E_{ab}(0, z; A) G_{\nu\alpha}^b(z) \,|\, p\rangle|_{z^2=0} \qquad (2)$$

$$= \bar{u}(p - r)\hat{z}u(p)\,(z \cdot p) \int_0^1 \int_0^1 \left(e^{-ix(pz)-iy(rz)} + e^{ix(pz)-i\bar{y}(rz)} \right)$$

$$\times \theta(x + y \leq 1)\, F_g(x, y)\, dx\, dy.$$

Here and in the following I adhere to the convention $\bar{y} = 1 - y$, $\bar{x} = 1 - x$, *etc.*, for momentum fractions and use the notation $\gamma_\alpha z^\alpha \equiv \hat{z}$.

Due to the spectral properties $x \geq 0$, $y \geq 0$, $x + y \leq 1$ (this can be proved for any Feynman diagram using the approach of ref. [11]), both the initial active gluon and the spectators carry positive fractions of the initial hadron momentum p: $(x + \zeta y)$ for the gluon and $(\bar{x} - \zeta y) \geq y(1 - \zeta) \geq 0$ for the spectators. On the other hand, the fraction of the p-momentum carried by another gluon is given by $(x - \bar{y}\zeta)$ and it may take both positive and negative values.

The usual gluon distribution function $x f_g(x)$ corresponds to the limit $r = 0$. Hence, the double distribution $F_g(x, y)$ satisfies the reduction formula:

$$\int_0^{1-x} F_g(x, y)\, dy = x f_g(x). \tag{3}$$

3. Asymmetric distribution functions. Since $r = \zeta p$, the variable y appears in eq.(2) only in the combinations $x + y\zeta \equiv X$ and $x - \bar{y}\zeta \equiv X - \zeta$, where X and $(X - \zeta)$ are the total fractions of the initial hadron momentum p carried by the gluons. Introducing X as an independent variable, we can integrate the double distribution $F(X - y\zeta, y)$ over y to get

$$\mathcal{F}_\zeta(X) = \int_0^{\min\{X/\zeta,\, \bar{X}/\bar{\zeta}\}} F(X - y\zeta, y)\, dy, \tag{4}$$

where $\bar{\zeta} \equiv 1 - \zeta$. Since $\zeta \leq 1$ and $x + y \leq 1$, the variable X satisfies a natural constraint $0 \leq X \leq 1$.
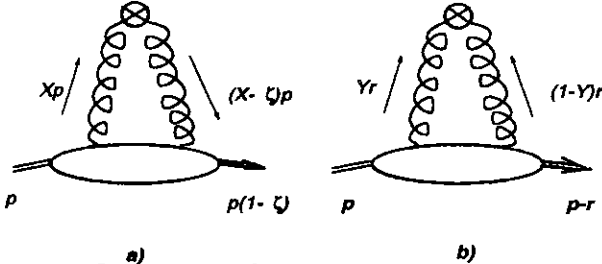

FIG. 2. Momentum flow corresponding to the asymmetric gluon distribution function in two regimes: *a)* $X \geq \zeta$ and *b)* $X \equiv Y\zeta \leq \zeta$.

In the region $X > \zeta$, where the initial gluon momentum Xp is larger than the momentum transfer $r = \zeta p$, the function $\mathcal{F}_\zeta^g(X)$ can be treated as a generalization of the usual distribution function $x f_g(x)$ for the asymmetric case when the final hadron momentum p' differs by ζp from the initial momentum p (but remains collinear to it). In this case, $\mathcal{F}_\zeta^g(X)$ describes a gluon going out of the hadron with a positive fraction Xp of the original hadron momentum and then coming back into the hadron with a changed (but still positive) fraction $(X - \zeta)p$. The Bjorken ratio ζ serves here as an external parameter specifying the momentum asymmetry of the matrix element. Hence, one deals now with a family of asymmetric distribution functions $\mathcal{F}_\zeta^g(X)$ whose shape changes when ζ is changed. The basic distinction between the double distributions $F(x, y)$ and the asymmetric distribution functions $\mathcal{F}_\zeta(X)$ is that the former are universal functions which do not depend on the momentum asymmetry parameter ζ, while the latter are explicitly labelled by it. When $\zeta \to 0$, the limiting curve for $\mathcal{F}_\zeta(X)$ reproduces the gluon distribution function:

$$\mathcal{F}_{\zeta=0}^g(X) = X f_g(X). \tag{5}$$

Note, that if $X < \zeta$, the "returning" gluon has a negative fraction $(X - \zeta)$ of the light-cone momentum p. Hence, it is more appropriate to treat it as a gluon going out of the hadron and propagating together with the original one. Writing X as $X = Y\zeta$, we immediately obtain that the gluons carry now positive fractions $Y\zeta p \equiv Yr$ and, respectively, $(1 - Y)r \equiv \bar{Y}r$ of the momentum transfer r. Hence, in the region $X = Y\zeta < \zeta$, the asymmetric distribution function looks like a distribution amplitude $\Psi_\zeta(Y)$ for a two-gluon state with the total momentum $r = \zeta p$:

$$\Psi_\zeta(Y) = \int_0^Y F((Y - y)\zeta, y)\, dy. \tag{6}$$

The asymmetric distribution function can be also defined directly through the matrix element

$$\langle p' \,|\, z_\mu z_\nu G_{\mu\alpha}^a(0) E_{ab}(0, z; A) G_{\nu\alpha}^b(z) \,|\, p\rangle|_{z^2=0} \tag{7}$$

$$= \bar{u}(p')\hat{z}u(p)\,(z \cdot p) \int_0^1 \left(e^{-iX(pz)} + e^{i(X-\zeta)(pz)} \right) \mathcal{F}_\zeta^g(X)\, dX.$$

To re-obtain the relation between $\mathcal{F}_\zeta(X)$ and the double distribution function $F_g(x, y)$, one should combine this definition with eq.(2).

4. Leading contribution. The parameterization (7) can be used as a starting point for constructing a QCD parton-type formalism. The only problem is that our gauge-invariant definition of the gluon distribution is in terms of the field strength tensor $G_{\mu\nu}$ while the usual Feynman rules involve the vector potential A_μ. A possible way out is to utilize the light-cone gauge $q^\mu A_\mu(z; q) = 0$ in which A_μ can be expressed in terms of $G_{\mu\nu}$

$$A_\mu(z; q) = q^\nu \int_0^\infty G_{\mu\nu}(z + \sigma q)\, d\sigma, \tag{8}$$

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so that the definition (7) can be applied directly. This gives

$$\langle p' \mid A_\mu^a(z_1; q) A_\nu^a(z_2; q) \mid p \rangle|_{z_1^2=0, z_2^2=0} = \frac{\bar{u}(p')\hat{q}u(p)}{(q \cdot p)} \left(g_{\mu\nu} - \frac{p_\mu q_\nu + p_\nu q_\mu}{(p \cdot q)} \right) \quad (9)$$

$$\times \int_0^1 \left(e^{-iX(pz_1)+i(X-\zeta)(pz_2)} + e^{i(X-\zeta)(pz_1)-iX(pz_2)} \right) \frac{\mathcal{F}_\zeta^g(X)}{X(X-\zeta+i\epsilon)} dX.$$

In ref. [5], the amplitude of hard diffractive electroproduction was calculated for the longitudinal polarization of both the virtual photon ($\epsilon_{\gamma^*}^\mu = (q^\mu + \zeta p^\mu)/Q$) and produced meson ($\epsilon_V^\mu = q^\mu/m_V$). In this case, we obtain

$$T_{LL}(p,q,r) = \frac{\sqrt{1-\zeta}}{Qm_V} \int_0^1 \frac{\varphi_\rho(\tau)}{\tau\bar{\tau}} d\tau \int_0^1 \frac{\mathcal{F}_\zeta^g(X)dX}{X(X-\zeta+i\epsilon)}, \quad (10)$$

where $\sqrt{1-\zeta}$ comes from $\bar{u}(p') = \sqrt{1-\zeta}\,\bar{u}(p)$ and $\varphi_V(\tau)$ is the distribution amplitude of the longitudinal vector meson. The amplitude has the imaginary part due to the factor $1/(X-\zeta+i\epsilon)$:

$$\frac{1}{\pi}\operatorname{Im}T_{LL}(\zeta) = \frac{\sqrt{1-\zeta}\,\mathcal{F}_\zeta^g(\zeta)}{\zeta Qm_V} \int_0^1 \frac{\varphi_V(\tau)}{\tau\bar{\tau}} d\tau. \quad (11)$$

In ref. [5], the gluonic matrix element was approximated by the gluon distribution function $f_g(x)$. To get our result from that of ref. [5], one should substitute there $f_g(\zeta)$ by $\sqrt{1-\zeta}\,\mathcal{F}_\zeta^g(\zeta)/\zeta$.

Though the asymmetric distribution function $\mathcal{F}_\zeta^g(X)$ coincides with $Xf_g(X)$ in the limit $\zeta = 0$, these two functions differ in the general case when $\zeta \neq 0$. Furthermore, the imaginary part appears for $X = \zeta$, i.e., in a highly asymmetric configuration in which the second gluon carries a vanishing fraction of the original hadron momentum. Hence, one cannot exclude the possibility that $\mathcal{F}_\zeta^g(\zeta)$ visibly differs from the function $\zeta f_g(\zeta)$ which corresponds to a symmetric configuration in which the final gluon has the momentum equal to that of the initial one.

To get a feeling about the interrelationship between $\mathcal{F}_\zeta^g(X)$ and $Xf_g(X)$, let us consider a toy model $F^{mod}(x,y) = A(n+1)(1-x-y)^n$ for the gluon double distribution. Then $xf^{mod}(x) = A(1-x)^{n+1}$ while

$$\mathcal{F}_\zeta^{mod}(X) = \frac{A}{1-\zeta}\left[\left((1-X)^{n+1} - (1-X/\zeta)^{n+1}\right) \theta(X < \zeta) \right.$$

$$\left. + (1-X)^{n+1}\theta(X > \zeta) \right]. \quad (12)$$

Hence, $\zeta f^{mod}(\zeta) = A(1-\zeta)^{n+1}$ while $\mathcal{F}_\zeta^{mod}(\zeta) = A(1-\zeta)^n$, i.e., the two functions are rather close to each other for small ζ. This model also reveals a characteristic feature of the asymmetric distribution function $\mathcal{F}_\zeta^{mod}(X)$: the parameter ζ specifying the momentum asymmetry of the gluonic matrix element serves as a boundary between two regions $X < \zeta$ and $X > \zeta$ in which $\mathcal{F}_\zeta^g(X)$ is given by different analytic expressions. An important property of $\mathcal{F}_\zeta^{mod}(X)$ is that it rapidly varies in the region $X \lesssim \zeta$ and vanishes for $X = 0$. Since $X = 0$ can be arranged only when both x- and y-parameters of the double distribution $F(x,y)$ are zero, the fact that $\mathcal{F}_\zeta^{mod}(0) = 0$ is quite general. Note, however, that the limiting curve $\mathcal{F}_{\zeta=0}^{mod}(X) = (1-X)^{n+1}$ does not vanish for $X = 0$, i.e., the limits $\zeta \to 0$ and $X \to 0$ do not commute.

For this reason, if ζ is small, the substitution of $\mathcal{F}_\zeta(X)$ by $Xf_g(X)$ may be a good approximation for all X-values except for the region $X \lesssim \zeta$. However, the imaginary part is given by the value of $\mathcal{F}_\zeta(X)$ at the point $X = \zeta$ belonging to this region and it is not clear a priori how close are the functions $\mathcal{F}_\zeta(\zeta)$ and $\zeta f_g(\zeta)$.

5. Evolution of the double distribution. On the light-cone, the matrix elements have ultraviolet divergences, which are removed by subtraction prescription characterized by a scale μ: $F_g(x,y) \to F_g(x,y;\mu)$. Under renormalization, the gluonic operator

$$\mathcal{O}_g(uz,vz) = z_\mu z_\nu G_{\mu\alpha}^a(uz) E_{ab}(uz,vz;A) G_{\nu\alpha}^b(vz) \quad (13)$$

mixes with the flavor-singlet quark operator

$$\mathcal{O}_Q(uz,vz) = \frac{i}{2} \sum_q (\bar{\psi}_q(uz)\hat{z}E(uz,vz;A)\psi_q(vz) - \bar{\psi}_q(vz)\hat{z}E(vz,uz;A)\psi_q(uz)). \quad (14)$$

For simplicity, we will ignore here the quark-gluon mixing and analyze below the evolution of the double gluon distribution $\tilde{F}_g(x,y;\mu)$ corresponding to the "quenched approximation". In this case

$$\mu\frac{d}{d\mu}\tilde{F}_g(x,y;\mu) = \int_0^1 d\xi \int_0^1 R_{gg}(x,y;\xi,\eta;g(\mu))\tilde{F}_g(\xi,\eta;\mu)d\eta. \quad (15)$$

The easiest way to get explicit expressions for $R_{ab}(x,y;\xi,\eta;g)$ is to use the Balitsky-Braun evolution equation [12] for the light-cone operators[‡]

[‡]Instead of the original kernels $K_{ab}(u,v)$ from ref. [12] we prefer to use the kernels $B_{ab}(u,v) = -K_{ab}(\bar{u},v)$ which have the symmetry property $B_{ab}(u,v) = B_{ab}(v,u)$.

$$\mu \frac{d}{d\mu} \mathcal{O}_a(0, z) = \int_0^1 \int_0^1 \sum_b B_{ab}(u, v) \mathcal{O}_b(uz, \bar{v}z) \theta(u + v \le 1) \, du \, dv, \qquad (16)$$

where $a, b = g, Q$ and [12]

$$B_{gg}(u, v) = \frac{\alpha_s}{\pi} N_c \left(4(1 + 3uv - u - v) + \frac{\beta_0}{2N_c} \delta(u)\delta(v) \right.$$
$$\left. + \delta(u)\left[\frac{\bar{v}^2}{v} - \delta(v)\int_0^1 \frac{dz}{z} \right] + \delta(v)\left[\frac{\bar{u}^2}{u} - \delta(u)\int_0^1 \frac{dz}{z} \right] \right). \qquad (17)$$

Here, $\beta_0 = 11 - \frac{2}{3}N_f$ is the lowest coefficient of the QCD β-function.

Our kernel $R_{gg}(x, y; \xi, \eta; g)$ is related to the $B_{gg}(u, v)$-kernel by

$$R_{gg}(x, y; \xi, \eta; g) = \frac{1}{\xi} B_{gg}(y - \eta x/\xi, \bar{y} - \bar{\eta} x/\xi). \qquad (18)$$

This gives

$$R_{gg}(x, y; \xi, \eta; g) = \qquad (19)$$
$$\frac{\alpha_s}{\pi} N_c \frac{1}{\xi} \left\{ 4[x/\xi + 3(y - \eta x/\xi)(\bar{y} - \bar{\eta} x/\xi)] \theta(0 \le x/\xi \le \min\{y/\eta, \bar{y}/\bar{\eta}\}) \right.$$
$$+ \frac{\theta(0 \le x/\xi \le 1)(x/\xi)^2}{(1 - x/\xi)} \left[\frac{1}{\eta}\delta(x/\xi - y/\eta) + \frac{1}{\bar{\eta}}\delta(x/\xi - \bar{y}/\bar{\eta}) \right]$$
$$\left. + \delta(1 - x/\xi)\delta(y - \eta)\left[\frac{\beta_0}{2N_c} - \int_0^1 \frac{dz}{z} \right] \right\}.$$

As usual, the divergent integral provides the regularization for the singularities of the kernel for $x = \xi$ (or $y = \eta$). It is easy to verify that the kernel $R_{gg}(x, y; \xi, \eta; g)$ has the property that $x + y \le 1$ if $\xi + \eta \le 1$. Using the expression for $R_{gg}(x, y; \xi, \eta; g)$ and the explicit form of the $P_{gg}(x/\xi)$ kernel [7,8], one can check the reduction formula

$$\int_0^{1-x} R_{gg}(x, y; \xi, \eta; g) dy = \frac{1}{\xi} P_{gg}(x/\xi). \qquad (20)$$

Integrating $R_{gg}(x, y; \xi, \eta; g)$ over x one should get the evolution kernel for the gluon distribution amplitude

$$\int_0^{1-y} R_{gg}(x, y; \xi, \eta; g) dx = V_{gg}(y, \eta; g). \qquad (21)$$

To solve the evolution equation, we apply first the standard trick used to solve the GLAP equation: integrate $x^n R_{gg}(x, y; \xi, \eta; g)$ over x. Using the property $R_{gg}(x, y; \xi, \eta; g) = R_{gg}(x/\xi, y; 1, \eta; g)/\xi$, we get

$$\mu \frac{d}{d\mu} \tilde{F}_g^{(n)}(y; \mu) = \int_0^1 R_{gg}^{(n)}(y, \eta; g) \tilde{F}_g^{(n)}(\eta; \mu) d\eta, \qquad (22)$$

where $\tilde{F}_g^{(n)}(y; \mu)$ is the nth x-moment of $\tilde{F}_g(x, y; \mu)$

$$\tilde{F}_g^{(n)}(y; \mu) = \int_0^1 x^n \tilde{F}_g(x, y; \mu) dx \qquad (23)$$

and the kernel $R_{gg}^{(n)}(y, \eta; g)$ is given by

$$R_{gg}^{(n)}(y, \eta; g) = \frac{\alpha_s}{\pi} N_c \left\{ \left(\frac{y}{\eta} \right)^{n+2} \left(\frac{4}{n+2} + \frac{12}{n+1}\bar{y}\eta \right. \right.$$
$$\left. - \frac{12}{n+2}(y\bar{\eta} + \eta\bar{y}) + \frac{12}{n+3}y\bar{\eta} + \frac{1}{\eta - y} \right) \theta(y \le \eta)$$
$$\left. + \delta(y - \eta)\left[\frac{\beta_0}{2N_c} - \int_0^1 \frac{dz}{z} \right] + \{y \to \bar{y}, \eta \to \bar{\eta}\} \right\}. \qquad (24)$$

It is straightforward to establish that $R_{gg}^{(n)}(y, \eta; g)$ has the property

$$R_{gg}^{(n)}(y, \eta; g) w_n(\eta) = R_{gg}^{(n)}(\eta, y; g) w_n(y),$$

where $w_n(y) = (y\bar{y})^{n+2}$. Hence, the eigenfunctions of $R_{gg}^{(n)}(y, \eta; g)$ are orthogonal with the weight $w_n(y) = (y\bar{y})^{n+2}$, i.e., they are proportional to the Gegenbauer polynomials $C_k^{n+5/2}(y-\bar{y})$ (cf. [13,14] and refs. [15,16] where the general algorithm was originally applied to the evolution of gluonic distribution amplitudes). Now, we can write the general solution of the evolution equation

$$\tilde{F}_g^{(n)}(y; \mu) = (y\bar{y})^{n+2} \sum_{k=0}^\infty A_{nk} C_k^{n+5/2}(y - \bar{y}) \left[\log(\mu/\Lambda) \right]^{-\gamma_k^{(n)}/\beta_0}, \qquad (25)$$

where the anomalous dimensions $\gamma_k^{(n)}$ are given by the eigenvalues of the kernel $R_{gg}^{(n)}(y, \eta; g)$:

$$\gamma_k^{(n)} = 2N_c \left[-\frac{1}{(k+n)(k+n+1)} - \frac{1}{(k+n+2)(k+n+3)} + \sum_{j=1}^{k+n+1} \frac{1}{j} \right] - \frac{1}{2}\beta_0. \qquad (26)$$

They coincide with the standard anomalous dimensions $\gamma_{gg}(N)$ [17,18]: $\gamma_k^{(n)} = \gamma_{gg}(n+k+1)$. Note, that γ_0^0 is formally given by the negative infinity, while all other anomalous dimensions are finite and non-negative. Hence, in the formal $\mu \to \infty$ limit we have $\tilde{F}_n(y, \mu \to \infty) = 0$ for all $n \geq 1$. This means that

$$\tilde{F}_g(x, y; \mu \to \infty) \sim \delta(x)(y\bar{y})^2,$$

i.e., in each of its variables the limiting function $\tilde{F}_g(x, y; \mu \to \infty)$ acquires the characteristic asymptotic form dictated by the nature of the variable: $\delta(x)$ is specific for the distribution functions [17,18], while the $(y\bar{y})^2$-form is the asymptotic shape for the lowest-twist gluonic distribution amplitudes [15,16]. It is easy to see that if the double distribution has the asymptotic form $F^{as}(x, y) = 120\, C\, \delta(x)\, y^2(1-y)^2$, then $x f^{as}(x) = C\, \delta(x)$ while $\mathcal{F}_\zeta^{as}(X) = 120\, C\, X^2(1 - X/\zeta)^2/\zeta$. Note that in this case $\mathcal{F}_\zeta^{as}(\zeta) = 0$, i.e., the function which determines the magnitude of the imaginary part of T_{LL} vanishes.

6. Evolution equations for asymmetric distribution functions. Introducing the asymmetric distribution function $\mathcal{F}_\zeta^Q(X)$ for the flavor-singlet quark combination (14)

$$\langle p - r \,|\, \mathcal{O}_Q(0, z) \,|\, p\rangle|_{z^2=0}$$
$$= i\bar{u}(p-r)\hat{z}u(p)\int_0^1 \left(e^{-iX(pz)} - e^{i(X-\zeta)(pz)} \right) \mathcal{F}_\zeta^Q(X)\, dX \qquad (27)$$

and using eq.(16), we obtain a set of coupled evolution equations for $\mathcal{F}_\zeta^g(X)$ and $\mathcal{F}_\zeta^Q(X)$:

$$\mu\frac{d}{d\mu}\mathcal{F}_\zeta^a(X; \mu) = \int_0^1 \sum_b W_\zeta^{ab}(X, Z; g)\, \mathcal{F}_\zeta^b(Z; \mu)\, dZ, \qquad (28)$$

where a and b denote g or Q. To relate the functions $W_\zeta^{ab}(X, Z; g)$ to the Balitsky-Braun evolution kernels [12]

$$B_{QQ}(u, v) = \frac{\alpha_s}{\pi}C_F\left(1 + \delta(u)[\bar{v}/v]_+ + \delta(v)[\bar{u}/u]_+ - \frac{1}{2}\delta(u)\delta(v)\right), \qquad (29)$$

$$B_{gQ}(u, v) = \frac{\alpha_s}{\pi}C_F\left(2 + \delta(u)\delta(v)\right) \quad, \quad B_{Qg}(u, v) = \frac{\alpha_s}{\pi}N_f\left(1 + 4uv - u - v\right) \qquad (30)$$

($B_{gg}(u, v)$ was displayed earlier by eq.(17)), it is convenient to introduce first the auxiliary kernels $M_\zeta^{ab}(X, Z; g)$:

$$M_\zeta^{ab}(X, Z) = \int_0^1\int_0^1 B_{ab}(u, v)\,\delta(X - \bar{u}Z + v(Z - \zeta))\,\theta(u + v \leq 1)\, du\, dv. \qquad (31)$$

In terms of these kernels, we have

$$W_\zeta^{gg}(X, Z) = M_\zeta^{gg}(X, Z) \quad, \quad W_\zeta^{QQ}(X, Z) = M_\zeta^{QQ}(X, Z), \qquad (32)$$

$$W_\zeta^{gQ}(X, Z) = \int_X^1 M_\zeta^{gQ}(\tilde{X}, Z)\, d\tilde{X} \quad, \quad W_\zeta^{Qg}(X, Z) = \frac{d}{dX} M_\zeta^{Qg}(X, Z). \qquad (33)$$

Note that the expressions for the kernels describing the quark-gluon mixing are slightly more involved than those for the diagonal ones. The reason is that the definition (7) of the gluon distribution has an extra $(z \cdot p)$ factor which generates a derivative acting on $\mathcal{F}_\zeta^g(X)$.

Integrating the delta-function in eq.(31), one obtains four different types of the θ-functions, each of which corresponds to a specific evolution regime for the asymmetric distribution functions. In particular, for $W_\zeta^{gg}(X, Z; g)$ we obtain

$$W_\zeta^{gg}(X, Z; g) = \frac{1}{Z}\int_0^1 dv\, B_{gg}\left([1 - X/Z - v(1 - \zeta/Z)], v\right) \qquad (34)$$

$$\times\left\{ \theta(Z \geq X \geq \zeta)\,\theta\left(0 \leq v \leq \frac{1 - X/Z}{1 - \zeta/Z}\right) + \theta(Z \geq \zeta \geq X)\,\theta(0 \leq v \leq X/\zeta) \right.$$

$$+ \theta(X \leq \zeta)\,\theta(Z \leq \zeta)\,[\,\theta(X \leq Z)\,\theta(0 \leq v \leq X/\zeta)$$

$$\left. + \theta(X \geq Z)\,\theta\left(\frac{X/Z - 1}{\zeta/Z - 1} \leq v \leq X/\zeta\right)\,]\right\}.$$

For the first two terms in this sum, the original fraction Z is bigger than the momentum asymmetry parameter ζ. In this region of Z's, the resulting fraction X cannot be increased by the evolution: in both terms we have $X \leq Z$. Such a situation is typical for the evolution of distribution functions.

The last two terms in eq.(34) correspond to the region where the original fraction Z is smaller than ζ. In this case, the evolution can either decrease the fraction ($X \leq Z$ for the first term in the square brackets) or increase it ($X \geq Z$ in the second one). However, if the initial fraction Z is less than ζ, the evolved fraction X cannot be larger than ζ or, what is the same, the parameter $Y \equiv X/\zeta$ specifying the fraction Yr of the momentum transfer r carried by this gluon cannot exceed 1. This property is a characteristic feature of the evolution of distribution amplitudes.

Qualitatively, the evolution of the asymmetric distribution functions proceeds in the following way. Due to the GLAP-type evolution, the momenta of the partons decrease, and distributions become peaked in the regions of smaller and

smaller X. However, when the parton momentum degrades to values smaller than the momentum transfer $r = \zeta p$, the further evolution is like that for a distribution amplitude: it tends to make the distribution symmetric with respect to the central point $X = \zeta/2$ of the $(0, \zeta)$ segment. In two extreme cases, when $\zeta = 0$ or $\zeta = 1$, the evolution is more trivial. For $\zeta = 0$, $\mathcal{F}_\zeta(X)$ reduces to a usual distribution function governed by the GLAP evolution equation, while for $\zeta = 1$ we always have $Z \le \zeta$ for the initial fraction and the function $\mathcal{F}_\zeta(X)$ experiences a purely Brodsky-Lepage evolution. In other words, $W^{ab}_{\zeta=0}(X, Z; g) = P_{ab}(X/Z, g)/Z$ and $W^{ab}_{\zeta=1}(X, Z; g) = V_{ab}(X, Z; g)$ [§].

7. *Conclusions.* In this letter, I demonstrated that one can describe the asymmetric matrix element $\langle \bar{\zeta}p | G \ldots G | p \rangle$ either by the universal double distribution $F_g(x, y)$ or by the asymmetric distribution function $\mathcal{F}_\zeta(X)$ which explicitly depends on the momentum asymmetry parameter ζ and specifies the total fractions X and $X - \zeta$ of the original hadron momentum p carried by the gluons. Using $\mathcal{F}_\zeta(X)$ gives a formalism that looks very similar to the standard QCD parton approach, in which the gluon content of the hadron is described by the gluon distribution function $f_g(x)$. Moreover, $\mathcal{F}_\zeta(X)$ coincides with $X f_g(X)$ in the $\zeta \to 0$ limit, and this fact suggests an approximation $\mathcal{F}_\zeta(X) \approx X f_g(X)$ for small ζ. One should realize, however, that the electroproduction amplitude is dominated by the imaginary part whose magnitude is determined by $\mathcal{F}_\zeta(\zeta)$. Since the function $\mathcal{F}_\zeta(X)$ rapidly varies in the region $X \lesssim \zeta$ and vanishes for $X = 0$ (which is not the case with $X f_g(X)$), the relation $\mathcal{F}_\zeta(\zeta) = \zeta f_g(\zeta)$ may be strongly violated. This pessimistic expectation is not supported by a toy model for the double distribution $F^{mod}(x, y) = A(1 - x - y)^n$, in which $\zeta f^{mod}_g(\zeta)$ differs from $\mathcal{F}^{mod}_\zeta(\zeta)$ by an extra factor $(1 - \zeta)$ only, the latter being close to 1 for small ζ. However, the structure of the double distribution $F_g(x, y)$ in general case may be more involved, and a detailed analysis of the interrelationship between $\mathcal{F}_\zeta(\zeta)$ and $\zeta f_g(\zeta)$ is an interesting problem for future studies.

[§]Originally, this observation was made by X.Ji and P.Hoodbhoy [19] in application to evolution of the $t = 0$ limit of the off-forward parton distributions of ref. [10].

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