Negative Specific Heat of the MIT Bag with Surface Tension

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The unusual thermostatic properties of the heavy hadronic resonances with exponential mass spectrum, Hagedorns, were recently discussed in details in [1, 2]. Since in the framework of MIT Bag Model [3] these resonances can be regarded as the finite bags, it would be interesting to study the effect of the surface energy on thermodynamic properties of Hagedrorns. The inclusion of surface tension can be accounted for by writing the pressure of a spherical bag as

$$p = \frac{\sigma}{3} T^4 - B - a_s(T)\, V^{-\frac{1}{3}} = \frac{\sigma}{3} T^4 - B - \frac{a_s(T)}{\alpha\, R}\,, \quad (1)$$

where $a_s(T)$ is the temperature dependent surface energy coefficient, R is the bag radius and $\alpha \equiv \left[\frac{4\pi}{3}\right]^{\frac{1}{3}}$. Using the thermodynamic identities for the free energy F and entropy S

$$p = -\left(\frac{\partial F}{\partial V}\right)_T\,, \quad \text{and} \quad S = -\left(\frac{\partial F}{\partial T}\right)_V\,, \quad (2)$$

one can find all thermodynamic functions as follows

$$F \;=\; -\left[\frac{\sigma}{3} T^4 - B\right] V + \frac{3}{2}\, a_s(T)\; V^{\frac{2}{3}}\,, \quad (3)$$

$$S \;=\; \frac{4\,\sigma}{3} T^3 V - \frac{3}{2}\, \frac{d a_s(T)}{d\;\, T}\; V^{\frac{2}{3}}\,, \quad (4)$$

$$\varepsilon V \;=\; \left[\sigma T^4 + B\right] V + \frac{3}{2}\left[a_s(T) - \frac{d a_s(T)}{d\;\, T}\right]\; V^{\frac{2}{3}}\,. \quad (5)$$

In evaluating the expression (3) the integration constant was fixed (an arbitrary function of T) to zero because for the bag of zero volume the free energy should vanish.

While the magnitude of $a_s(T) > 0$ is needed the consequences of this surface term are surprising. In Eq. (1) the surface term appears as an <u>additional</u> pressure to the bag pressure. Therefore, setting the total pressure to zero $p = 0$, we obtain for the bag temperature

$$T = T(R) = \left[\frac{3}{\sigma}\left(B + \frac{a_s(T)}{\alpha\, R}\right)\right]^{\frac{1}{4}}\,. \quad (6)$$

When R is large we recover the previous bag temperature and the associated physics. When R becomes small, however, the bag temperature increases! In other words, the bag has negative heat capacity!

In the standard bag model the heat capacity is infinite: no matter how much energy is fed to the bag, its temperature remains constant [1, 2]. The only effect is to make the bag larger. This is completely consistent with what we observe in isobaric phase transitions in ordinary matter. Here the isobaric condition is produced by the bag constant, and the phase transition is from hadronic to partonic phase.

Surface effects, however, lead to an apparently non-thermodynamic behavior: the more we feed the energy to the bag, the lower its temperature becomes. In other words, the bag's heat capacity is negative. Indeed, from Eqs. (2) and (4) one can find the heat capacity at constant pressure C_p and at constant volume C_V:

$$C_p \equiv T\left(\frac{\partial S}{\partial T}\right)_p = C_V - \frac{3\, T V^{\frac{4}{3}}}{a_s(T)}\left[\frac{4\sigma}{3} T^3 - \frac{1}{V^{\frac{1}{3}}}\, \frac{d\; a_s}{d\;T}\right]^2, \quad (7)$$

$$C_V \equiv T\left(\frac{\partial S}{\partial T}\right)_V = 4\sigma\, T^3\, V - \frac{3}{2}\, T\, V^{\frac{2}{3}}\, \frac{d^2\; a_s}{d\;T^2}\,. \quad (8)$$

In evaluating the expression for C_p we used an explicit form of the derivative

$$\left(\frac{\partial V}{\partial T}\right)_p = -\frac{3\, V^{\frac{4}{3}}}{a_s(T)}\left[\frac{4\sigma}{3} T^3 - \frac{1}{V^{\frac{1}{3}}}\, \frac{d\; a_s}{d\;T}\right]. \quad (9)$$

From Eqs (7) and (8) it is clearly seen that for any $T > 0$ there is a volume of the bag $V_p(T)$ above which the heat capacity C_p, that corresponds to the bag equilibrium in vacuum, becomes negative. This leads to what is called a convex intruder in the entropy or an unusual behavior of its second derivative:

$$\left(\frac{\partial^2 S}{\partial E^2}\right)_{p=0} \;=\; -\;\frac{1}{T^2\, C_p}\,, \quad (10)$$

which becomes positive for $V > V_p(T)$.

In the literature on this subject it is argued [4] that all small systems (comparable in size with the range of the prevailing force) should show this effect. We, however, would like to stress that, in contrary to the small systems, a convex intruder in the bag model exists not for small systems, but for large ones and it does not disappear in thermodynamic limit, when the bag volume becomes infinite. Also the convex intruder in small systems [4] corresponds to a continuous change of the sign of the second derivative $\left(\frac{\partial^2 S}{\partial E^2}\right)$, whereas for the bag model it has a simple pole at $V = V_p(T)$.

Therefore, it might be interesting to verify through their decay products that heavier resonances have such peculiar properties, in particular, that they have a smaller temperature (but never lower than $T(R = \infty)$).

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