Saturation & BFKL dynamics in the HERA data at small x

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Plan of the talk:

- Introduction
- Dipole picture in DIS at small x
- Previous attempts with saturation models
- Physics at $x < 10^{-2}$, $Q^2 \lesssim$ 50GeV^2
- A CGC fit to the HERA F_2 data
- Summary

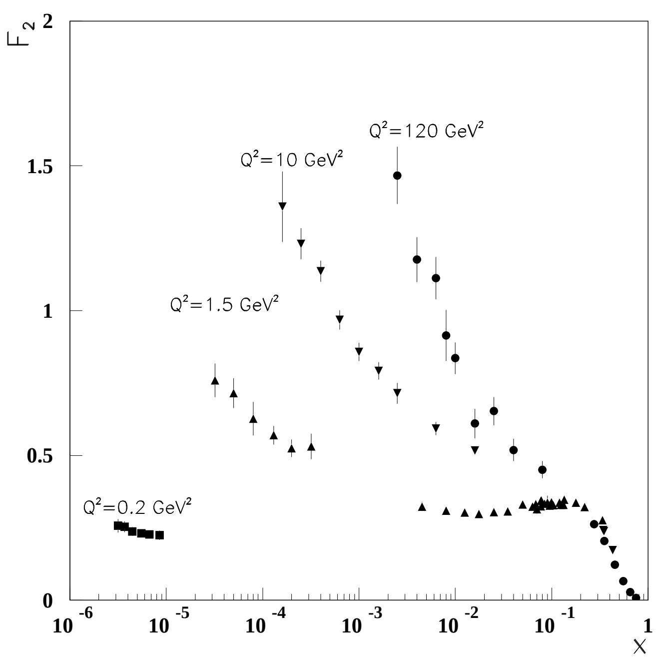
Introduction

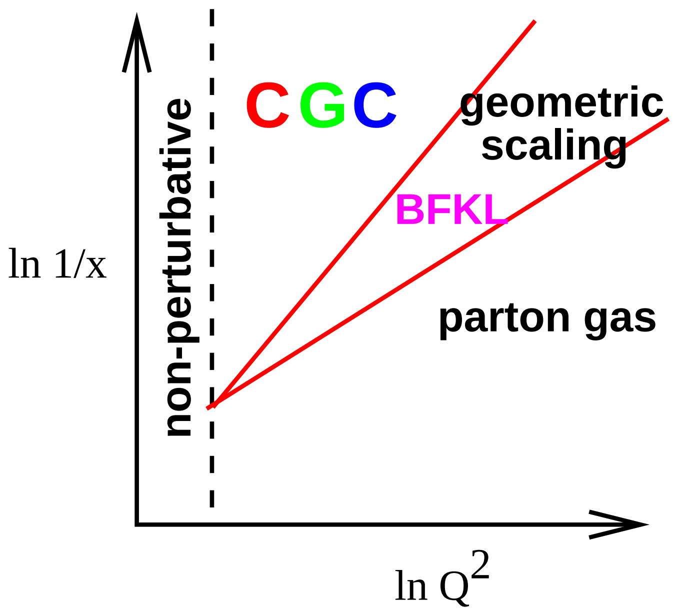
Deep Inelastic Scattering (DIS) at small x is the best place to see Color Glass Condensate.

- can directly probe the hadron wavefunction
- increase of gluon density at small x is observed in F_2
- can enter CGC regime with increasing energy
- relation to RHIC

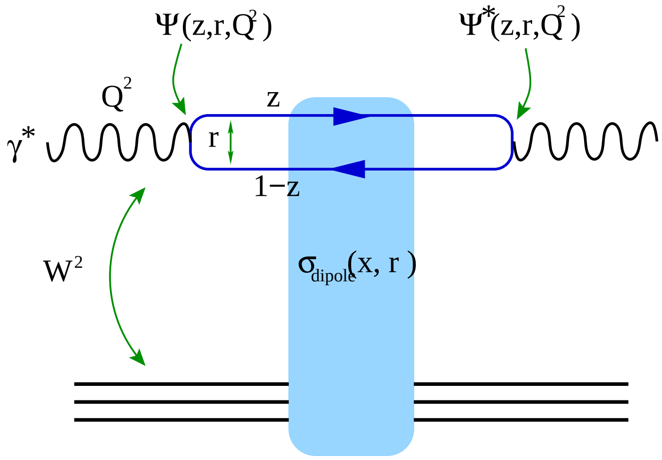
  Saturation scale $Q_s^2(x, A) \propto A^{1/3}(1/x)^{0.3}$

  DIS @ HERA $x \sim 10^{-4} \leftrightarrow$ AuAu @ RHIC $x \sim 10^{-2}$





Dipole picture in DIS at small x



F_2 structure function

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{\mathrm{EM}}} \sum_{T,L} \int_0^1 dz \int d^2 r_\perp \left|\Psi_{T,L}(z, r_\perp, Q^2)\right|^2 \sigma_{\mathrm{dipole}}(x, r_\perp)$$

where $\Psi_{T,L}$: LC wavefunc of γ^* (perturbatively calculable)

$\sigma_{\mathrm{dipole}}(x, r_\perp)$: Dipole-proton cross section

- $\propto r_\perp^2 xG(x, 1/r_\perp^2)$ at very short distance $r_\perp$,
- should be unitarized at large $r_\perp$ (up to log effects)

$$\sigma_{\mathrm{dipole}}(x, r_\perp) = 2 \int d^2 b \mathcal{N}(x, r_\perp, b_\perp) < \sigma_0$$

- Parametrizations of σ_{dipole} with saturation models work relatively well to describe the HERA data.

$\Rightarrow$ Aim: Provide a simple parametrization rooted in QCD

Previous attempts with saturation models

Golec-Biernat–Wüsthoff model [PRD59(99)014017,60(99)114023]

$$\sigma_{\rm dipole}(x, r_\perp) = \sigma_0 \left[1 - \exp\left\{-\frac{1}{4} r_\perp^2 Q_s^2(x)\right\}\right]$$

$$Q_s^2(x) = 1\,\text{GeV}^2 \left(\frac{x_0}{x}\right)^\lambda$$

- Saturation (unitarization): $\sigma_{\rm dipole} \to \sigma_0$ as $r_\perp \to \infty$
- Geometric scaling: $\sigma_{\rm dipole}(x, r_\perp) = \sigma_{\rm dipole}(r_\perp^2 Q_s^2(x))$

$\Rightarrow$ Good fit to old HERA data $(F_2, F_2^{\rm Diff})$ for $x < 10^{-2}$

[H1: NPB470(96)3,497(97)3, ZEUS:Z.P.C72(96)399,EPJ.C7(99)609]

3 parameters: $\sigma_0 = 23\text{mb}, x_0 = 3 \times 10^{-4}, \lambda \simeq 0.3$

But, doesn't work well for new HERA data at high Q^2

[H1: EPJ.C21(01)33, ZEUS:PLB487(00)53,EPJ.C21(01)443]

Improvement of the GBW model

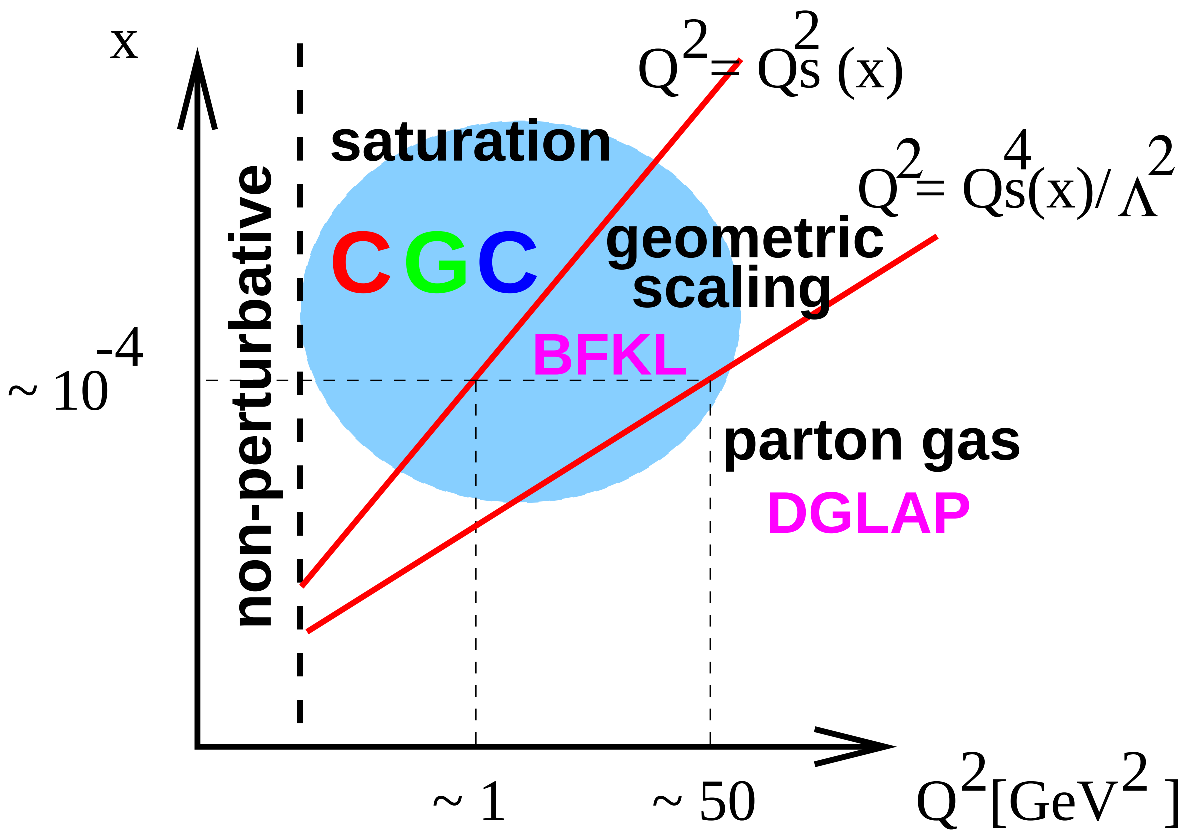
- by adding DGLAP evolution at high Q^2 (small $r_\perp$)

  Bartels, Golec-Biernat, Kowalski, PRD66(02)014001
- by further adding impact parameter (b) dependence

  Kowalski& Teaney, PRD68(03)114005

Physics at $x < 0.01$, $Q^2 \lesssim 50\text{GeV}^2$

But, BFKL, rather than DGLAP, should be
the right physics in the transition regime!
We focus on the region with not too large Q^2.



CGC (saturation) for $Q^2 < Q_s^2(x)$
BFKL dynamics in $Q_s^2 < Q^2 < Q_s^4/\Lambda^2 \sim 50\text{GeV}^2$

Physics at $x < 0.01$, $Q^2 \lesssim 50\text{GeV}^2$

Saturation for $Q^2 < Q_s^2(x)$ [Levin-Tuchin, Iancu-McLerran]

A solution to BK and JIMWLK equations:
Scattering amplitude $\mathcal{N}$: $\sigma_{\text{dipole}} = 2\pi R^2 \mathcal{N}(x, r_\perp)$

$$\mathcal{N}(x, r_\perp) \simeq 1 - \exp\left\{-\frac{1}{2c}\ln^2 r_\perp^2 Q_s^2(x)\right\}$$

coherent multiple scatterings ($\neq$ Glauber with indep. scat.)

BFKL dynamics in $Q_s^2 < Q^2 < Q_s^4/\Lambda^2 \sim 50\text{GeV}^2$

[Iancu-KI-McLerran, Mueller-Triantafyllopoulos]

saddle point approx. + saturation boundary

$$\mathcal{N}(x, r_\perp) \simeq \underbrace{\left(r_\perp^2 Q_s^2\right)^\gamma}_{\text{geometric scaling}} \underbrace{\exp\left\{-\frac{1}{2\kappa\lambda Y}\left(\ln\frac{1}{r_\perp^2 Q_s^2}\right)^2\right\}}_{\text{diffusion, scaling violation}}$$

- $\gamma \simeq 0.63 < 1$: anomalous dimension
- $\kappa = \chi''(\gamma)/\chi'(\gamma) \simeq 9.9$: diffusion coefficient
- λ: saturation exponent $Q_s^2(Y) \propto e^{\lambda Y}$ ($Y = \ln 1/x$)

Diffusion effectively increases the anomalous dimension.
Data very sensitive to the precise value of λ

A CGC fit to the HERA F_2 data

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$\sigma_{\rm dipole}(x, \mathbf{r}) = 2\pi R^2 \mathcal{N}(rQ_s, Y)$, $Q_s^2(x) = (x_0/x)^\lambda \text{GeV}^2$,

$$\mathcal{N}(rQ_s, Y) = \mathcal{N}_0 \left(\frac{r^2 Q_s^2}{4}\right)^{\gamma + \frac{\ln(2/rQ_s)}{\kappa\lambda Y}} \quad \text{for} \quad rQ_s \le 2$$

$$\mathcal{N}(rQ_s, Y) = 1 - e^{-a \ln^2(b\, rQ_s)} \quad \text{for} \quad rQ_s > 2$$

$\gamma = 0.63$, $\kappa = 9.9$ (fixed by LO-BFKL $\simeq$ resummed NLO)

a, b fixed by continuity at $rQ_s = 2$ where $\mathcal{N} = \mathcal{N}_0$.

3 parameters R, x_0 and λ (The same as in GBW. $\sigma_0 = 2\pi R^2$)

- R, x_0 : non-perturbative.
- λ : saturation exponent. Perturbatively computable, but left free to account for theoretical uncertainty. Resummed NLO: $\lambda \sim 0.3\pm$ 15 % [Triantafyllopoulos]

Fit to the new data within

$$\boxed{x \le 10^{-2} \text{ and } 0.045 \le Q^2 \le 45\text{GeV}^2}$$

A CGC fit to the HERA F_2 data

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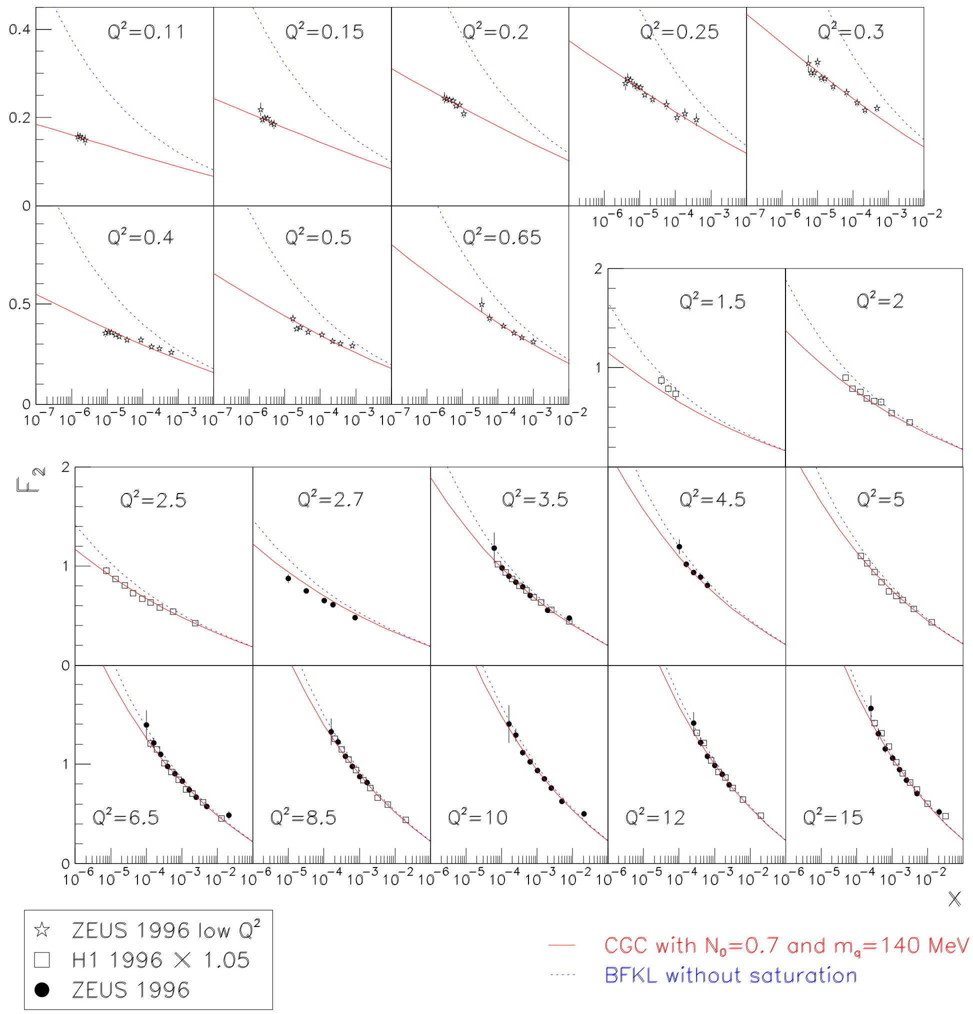

F_2 as a function of x in bins of $Q^2 \leq 15\text{GeV}^2$.

red line: CGC with $\mathcal{N}_0 = 0.7$ and $m_q = 140\text{MeV}$

blue line: BFKL without saturation

$R = 0.641\text{fm}$, $x_0 = 0.267 \times 10^{-4}$, $\lambda = 0.253$

with $\chi^2/dof = 0.81$ (156 data points)

A CGC fit to the HERA F_2 data

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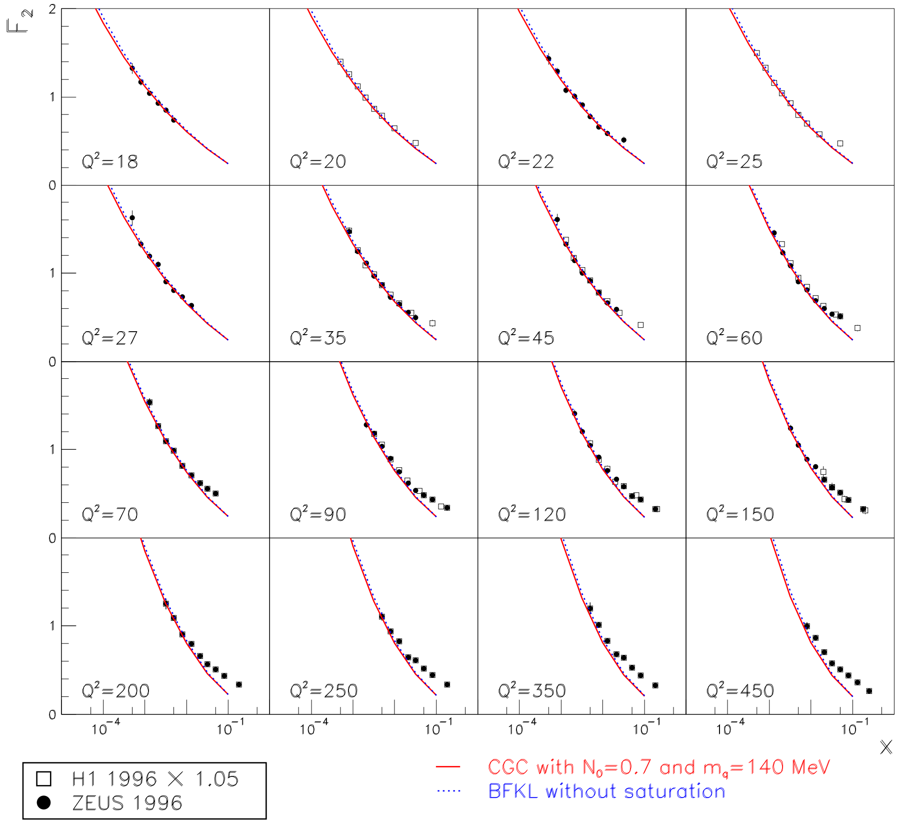


Larger Q^2: (The fit is extrapolated in bins $Q^2 \geq 60\text{GeV}^2$.)

- $\lambda \sim 0.25$ is in agreement with the resummed NLO BFKL result
- Scaling violation is essential (pure scaling fit doesn't work well)
- BFKL w/o saturation cannot reproduce the low Q^2 data
- Good agreement even for $Q^2 \ll 1$ GeV^2 (quark-hadron duality)
- Deviation at high Q^2: no DGLAP, no valence quarks

Summary

- Presented a new analytic parametrization of the dipole cross section which is rooted in QCD.

  BFKL dynamics near saturation
  - geometric scaling as leading effect
  - its violation by the diffusion term at high Q^2

  CGC, saturation
  - correct asymptotic behaviour (from BK or JIMWLK eqs.)

- This works very well in describing HERA $F_2(x, Q^2)$ data within $x \leq 10^{-2}$ and $0.045 \leq Q^2 \leq 45\text{GeV}^2$.

  3 parameter fit (R, x_0 and λ) gives $\chi^2/dof \sim 0.8$.

- Our analysis suggests that the CGC may have been seen in the HERA DIS data at small x.

The CGC fit

$\mathcal{N}_0$/model	0.5	0.6	0.7	0.8	0.9	GBW
χ^2	146.43	129.88	123.63	125.61	133.73	243.87
χ^2/d.o.f	0.96	0.85	0.81	0.82	0.87	1.59
x_0 ($\times 10^{-4}$)	0.669	0.435	0.267	0.171	0.108	4.45
λ	0.252	0.254	0.253	0.252	0.250	0.286
R (fm)	0.692	0.660	0.641	0.627	0.618	0.585

Table 1: The CGC fits for different values of $\mathcal{N}_0$ and 3 quark flavors with mass $m_q = 140$ MeV. Also shown is the fit obtained by using the GBW model.

The CGC fit

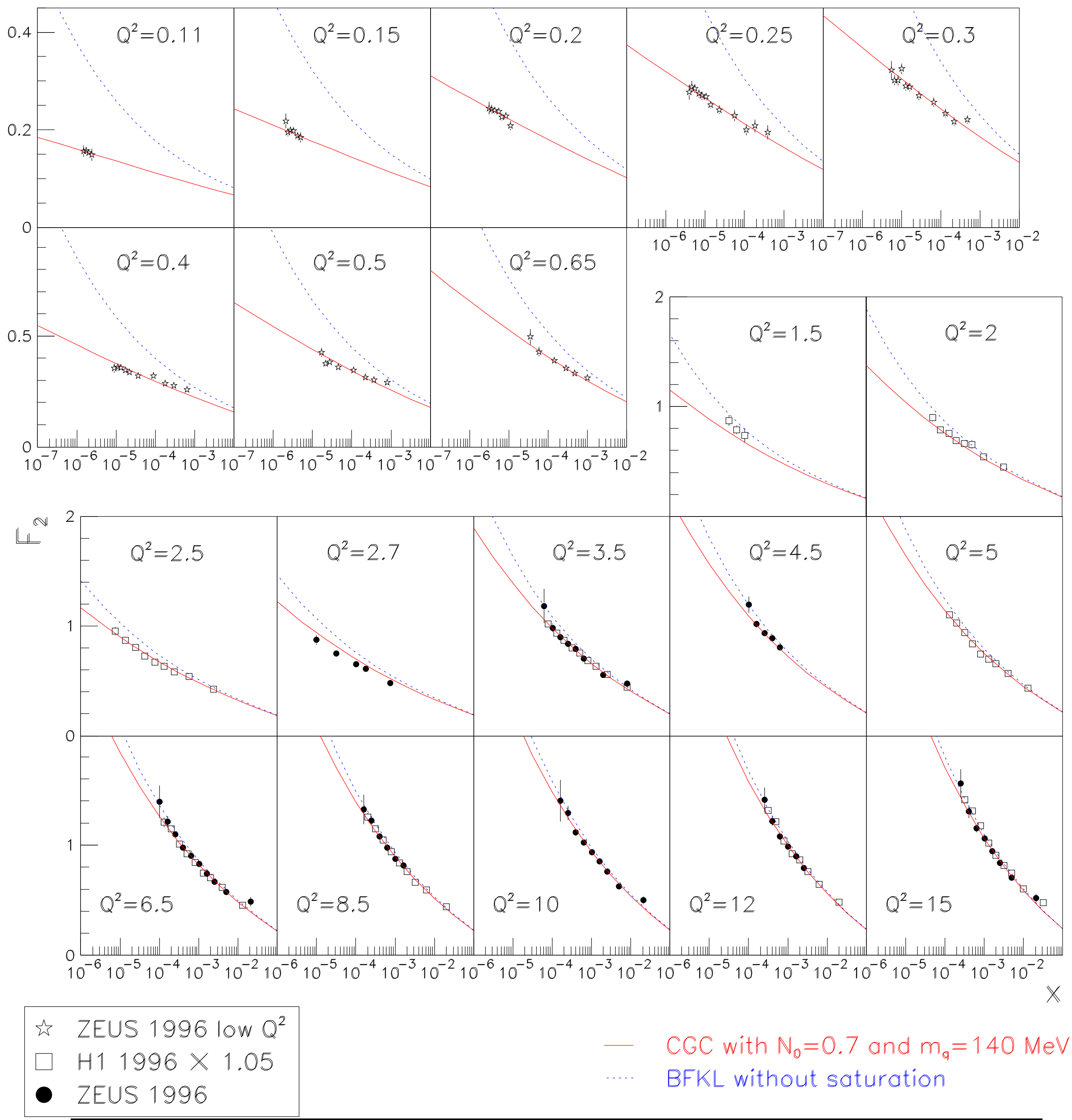
	$m_q = 50$ MeV			$m_q = 10$ MeV		
$\mathcal{N}_0$/model	0.5	0.7	0.9	0.5	0.7	0.9
χ^2	148.02	108.52	108.76	149.27	107.64	106.49
χ^2/d.o.f	0.97	0.71	0.71	0.98	0.70	0.70
x_0 ($\times 10^{-4}$)	2.77	0.898	0.333	3.32	1.06	0.382
λ	0.290	0.281	0.274	0.295	0.285	0.276
R (fm)	0.604	0.574	0.561	0.593	0.566	0.554

Table 2: The CGC fits for three values of $\mathcal{N}_0$ and quark masses $m_q = 50$ MeV (left) and $m_q = 10$ MeV (right).

$\mathcal{N}_0$	χ^2	χ^2/d.o.f	x_0 ($\times 10^{-4}$)	λ	R (fm)	γ
0.7	215.70	1.42	3.79	0.313	0.572	0.845

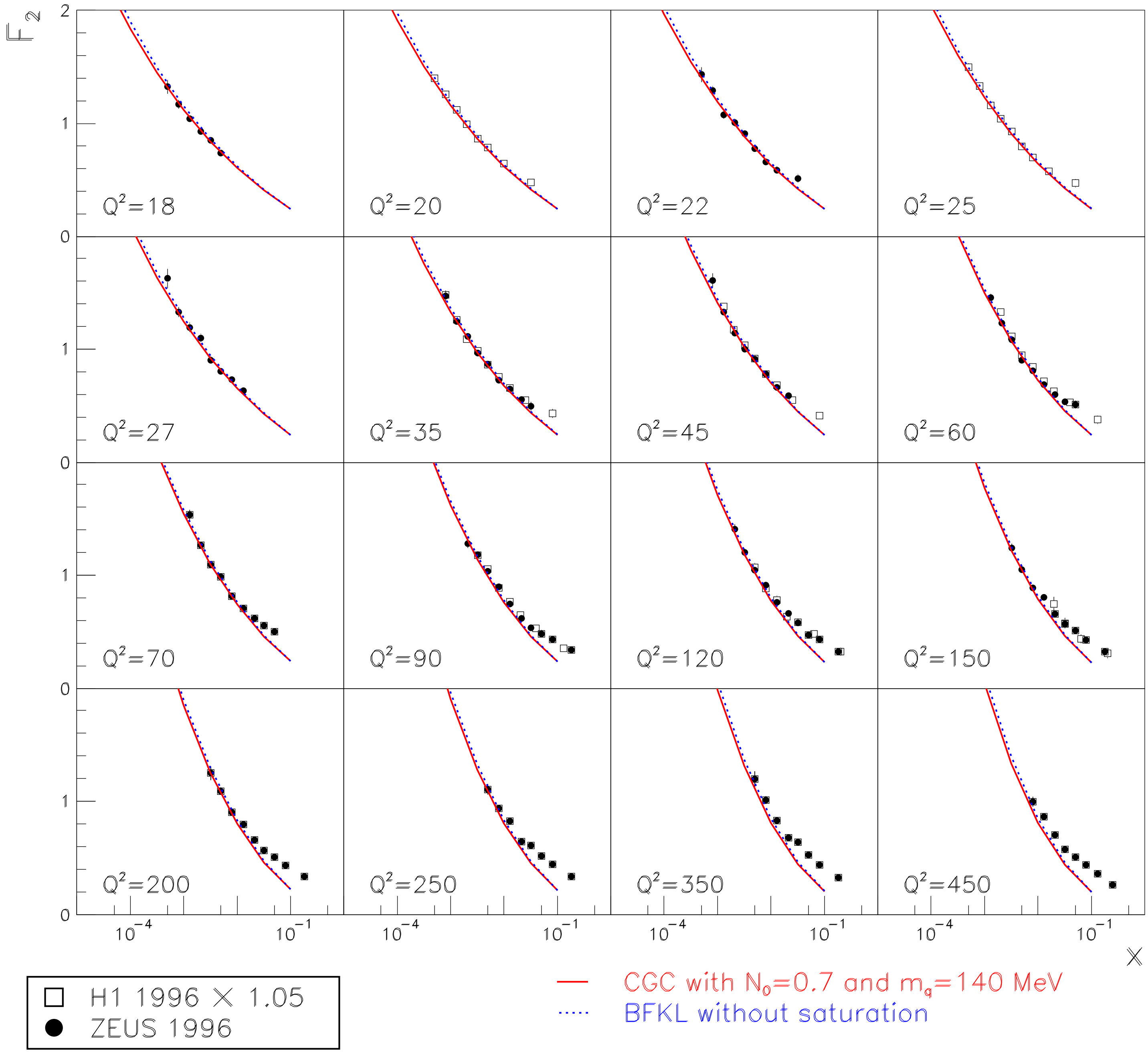
Table 3: A 4 parameter fit for geometric scaling + saturation.

The CGC fit



Q^2=0.11
Q^2=0.15
Q^2=0.2
Q^2=0.25
Q^2=0.3
Q^2=0.4
Q^2=0.5
Q^2=0.65
Q^2=1.5
Q^2=2
Q^2=2.5
Q^2=2.7
Q^2=3.5
Q^2=4.5
Q^2=5
Q^2=6.5
Q^2=8.5
Q^2=10
Q^2=12
Q^2=15
F_2
x
☆ ZEUS 1996 low Q^2
□ H1 1996 × 1.05
● ZEUS 1996
CGC with N_0=0.7 and m_q=140 MeV
BFKL without saturation

The CGC fit



F_2
Q²=18
Q²=20
Q²=22
Q²=25
Q²=27
Q²=35
Q²=45
Q²=60
Q²=70
Q²=90
Q²=120
Q²=150
Q²=200
Q²=250
Q²=350
Q²=450
x
H1 1996 × 1.05
ZEUS 1996
CGC with N₀=0.7 and m_q=140 MeV
BFKL without saturation