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Applicability of Two Simplified Flood Routing Methods: Level-Pool and Muskingum-Cunge

by

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Abstract. Simplified flood routing models for unsteady flow simulation in reservoirs and rivers have advantages of relatively small computing requirements when compared to dynamic routing models based on the complete Saint-Venant equations of unsteady flow. The range of applicability as governed by accuracy is investigated for two simplified routing models, a level-pool reservoir routing model and a Muskingum-Cunge river routing model. The routing error for each simplified model is determined by systematic comparison with an accurate dynamic routing model (DAMBRK). Error properties of each simplified model are presented graphically as functions of dominant channel and flood hydrograph parameters.

Introduction

Within the National Weather Service (NWS) hydrology program for river and water resource forecasting services, sophisticated dynamic routing models such as DAMBRK, DWOPER, and FLDWAV (Fread, 1985; Chow et al., 1988), based on the complete one-dimensional Saint-Venant equations of unsteady flow, are being implemented, as resources are available, for particularly complex routing applications such as dam-break floods, major river systems subject to backwater effects, and tidal estuaries. This paper presents guidance for selecting which routing applications should be considered for the dynamic routing models and for two very simple routing models, i.e., (1) a level-pool reservoir routing model and (2) a diffusion-type Muskingum-Cunge river routing model. Suitability of the simplified models is assessed on the basis of the routing error as determined by the deviation of computed flows between the simplified models and the dynamic model (DAMBRK).

Level-Pool Reservoir Routing

Usually unsteady flow routing in reservoirs is approximated by a simple level-pool routing technique which is based on the principle of conservation of mass, i.e.,

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$$I(t) - Q(t) = dS/dt \qquad (1)$$

in which inflow (I) and outflow (Q) are functions of time (t), and the storage (S) is a function of the water-surface elevation (h) which changes with time (t). The reservoir is assumed always to have a horizontal water surface throughout its length, hence level-pool, and Q is assumed to be a function only of h(t). Eq. (1), an ordinary differential equation, can be solved by an iterative trapezoidal integration method. Using average values for I(t) and Q(t) over a Δt interval, and expressing dS/dt as the product of reservoir surface area (Sa, a known tabular function of h) and change of water-surface elevation (h) over the j^{th} time step (Δt^j), Eq. (1) becomes:

$$0.5(I^j+I^{j+1}) - 0.5(Q^j+Q^{j+1}) - 0.5(Sa^j+Sa^{j+1})(h^{j+1}-h^j)/\Delta t^j = 0 \qquad (2)$$

The inflows (I) at times j and j+1 are known from the specified inflow hydrograph, the outflow (Q) at time j can be computed from the known water-surface elevation (h^j) and an appropriate spillway discharge equation. The surface area (Sa^j) can be determined from the known value of h^j. The unknowns in the equation consist of h^{j+1}, Q^{j+1}, Sa^{j+1}; the latter two are known nonlinear functions of h^{j+1}. Hence, Eq. (2) can be solved for h^{j+1} by an iterative method such as Newton-Raphson, in which

$$h^{j+1}_{k+1} = h^{j+1}_k - f(h^{j+1}_k)/f'(h^{j+1}_k) \qquad (3)$$

and k is an iteration counter; $f(h_k^{j+1})$ is the left-hand side of Eq. (2) evaluated with the first estimate for h_k^{j+1} which for k=1 is either h^j (must be known at t=0) or a linear extrapolated estimate of h^{j+1}; and $f'(h_k^{j+1})$ is the derivative of Eq. (2) with respect to h^{j+1}. It can be approximated with a numerical derivative, i.e. $f'(h^j_k+1) \simeq [f(h^{j+1}_k+\varepsilon) - f(h^{j+1}_k+\varepsilon)] / [(h^{j+1}_k+\varepsilon) - (h^{j+1}_k-\varepsilon)]$ where ε is a small value, say 0.1 ft (0.03 m). One or two iterations usually solve Eq. (2) for h^{j+1}. Once h^{j+1} is obtained, Q^{j+1} is computed from the spillway discharge equation.

Accuracy of Reservoir Routing Models. The accuracy of level-pool routing models relative to the more accurate distributed dynamic routing models such as DAMBRK (Fread, 1985; Chow, et al., 1988) is shown in Fig. 1. The error (E_q, in percent) of the rising limb of the outflow hydrograph, as normalized by the peak outflow, is:

$$E_q = 100/Q_{D_p} \cdot [\sum_{i=1}^{N} (Q_{L_i} - Q_{D_i})^2/N]^{1/2} \qquad (4)$$

in which Q_{L_i} is the level-pool routed flow; Q_{D_i} is the dynamic routed flow, Q_{D_p} is the dynamic routed flow peak, and N is the number of computed discharges comprising the rising limb of the routed hydrograph. The error (E_q) increases as (1) reservoir mean depth (D_r) decreases, (2) reservoir length (L_r) increases, (3) time of rise (T_r) of inflow hydrograph decreases, and (4) inflow hydrograph volume decreases. These effects can be represented by three dimensionless parameters, σ_ℓ, σ_t, and σ_v; where, $\sigma_\ell = D_r/L_r$, $\sigma_t = L_r/[3600\ T_r\ (gD)^{1/2}]$ in which g is the gravity acceleration constant and T_r is the time (hrs) from beginning of rise until the peak of the hydrograph, and σ_v = hydrograph volume/reservoir volume. As shown in Fig. 1, E_q increases as σ_t increases and as σ_ℓ and σ_v decrease; also the influence of σ_v

increases as σ_ℓ decreases. An analysis of Fig. 1 indicates that E_q exceeds 10% for (a) most reservoirs subjected to rapidly rising unsteady flows in which T_r is less than 1 hr such as dam-break floods or intermittent turbine releases, and (b) very long reservoirs (L_r > 50 mi) subjected to flash floods with T_r less than 18 hrs.

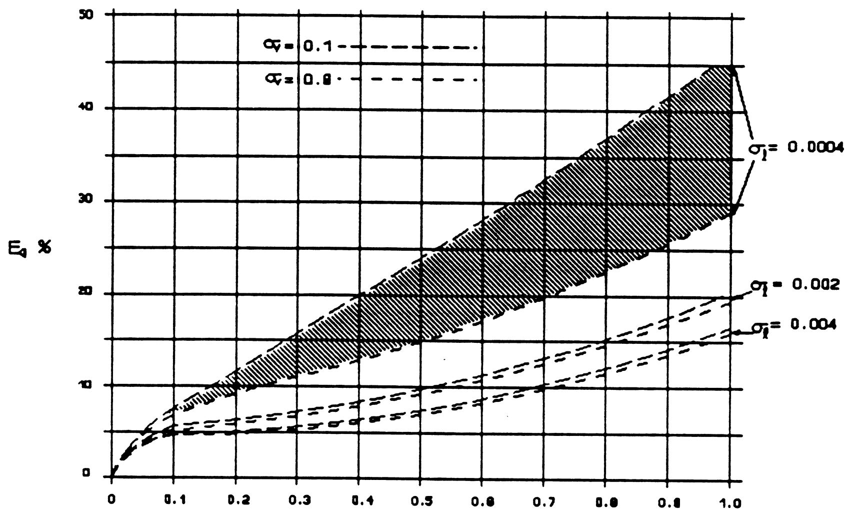

Fig. 1 Level-pool routing error (E_q) as a function of σ_t, σ_ℓ and σ_v (dimensionless parameters)

Muskingum-Cunge Method

The popular Muskingum method, described in standard reference books, e.g. Chow, et al., 1988, can be modified by computing the routing coefficients in a particular way as shown by Cunge (1969); this changes the kinematic-based Muskingum method to one based on the diffusion analogy which is capable of predicting hydrograph attenuation. This modified Muskingum method (known as the Muskingum-Cunge method) is most effectively used as a distributed flow routing technique. The recursive equation applicable to each Δx_i subreach for each Δt^j time step is:

$$Q_{i+1}^{j+1} = C_1\, Q_i^{j+1} + C_2\, Q_i^j + C_3\, Q_{i+1}^j + C_4 \qquad (5)$$

The coefficients C_1, C_2, and C_3 are positive values whose sum must equal unity, and the last term (C_4) accounts for the effect of lateral inflow ($\bar{q}_i$) along the Δx_i subreach:

$$C_1 = (\Delta t - 2KX)/[2K(1-X) + \Delta t] \qquad (6)$$

$$C_2 = (\Delta t + 2KX)/[2K(1-X) + \Delta t] \qquad (7)$$

$$C_3 = [2K(1-X) - \Delta t]/[2K(1-X) + \Delta t] \qquad (8)$$

$$C_4 = \bar{q}_i\, \Delta x\, \Delta t/[2K(1-X) + \Delta t] \qquad (9)$$

in which K is a storage constant and X is a weighting factor. It can be shown (Cunge, 1969) that Eq. (5) is a finite-difference form of the classical kinematic wave equation; however, if X is expressed as a particular function of the flow properties, Eq. (5) is able to account for wave attenuation but not for reverse (negative) flows or backwater effects. In this method, K and X are computed as follows:

$$K = \Delta x / \bar{c} \tag{10}$$

$$X = 0.5 \left[1 - \bar{Q}/(\bar{c}\, \bar{B}\, S\, \Delta x)\right] \tag{11}$$

in which $\bar{c}$ is the kinematic wave celerity, $\bar{Q}$ is discharge, $\bar{B}$ is cross-sectional top-width associated with $\bar{Q}$, and S is the energy slope approximated by the water surface slope as computed from the backwater solution of the initial steady flow condition to properly approximate the energy slope for channels with irregular and even adverse channel bottom slopes. The bar (-) indicates the variable is averaged over Δx and Δt. The coefficients, defined by Eqs. (6-9), are functions of Δx and Δt (the independent parameters), and K and X, which are functions of Q (the dependent variable) and its corresponding water-surface elevation (h) that may be obtained from a steady, uniform flow formula such as the Manning equation. Using a nonlinear solution procedure, the coefficients (K and X) are computed from the known flow properties, i.e., Q_i^j, Q_i^{j+1}, Q_{i+1}^j, h_i^j, h_i^{j+1}, h_{i+1}^j, and estimated values of the unknown flow (Q_{i+1}^{j+1}) and its h value. The estimated values are determined by extrapolation from previously computed values. The solution procedure is iterative and converges when computed and estimated values of h agree within a suitably small tolerance, 0.01 ft (0.003 m).

Error Properties of Muskingum-Cunge Routing. In order to assess the magnitude of errors associated with the nonlinear Muskingum-Cunge routing algorithm, a number of routing applications having a range of hydrographs and channel bottom slopes were simulated with the Muskingum-Cunge method as well as with a highly accurate implicit dynamic routing algorithm within the DAMBRK model (Fread, 1985; Chow, et al., 1988). The Muskingum-Cunge algorithm's peak discharges and corresponding water-surface elevations were compared with those computed by the dynamic routing algorithm. The difference between the two was taken as the magnitude of the error associated with the Muskingum-Cunge algorithm. This error (ε) was defined as $\varepsilon = (\varepsilon_Q^2 + \varepsilon_h^2)^{1/2}$, where ε_Q, and ε_h are the peak error in the discharge and depth, normalized about each peak. The results of this empirical error analysis via comparative routings through 10-, 20-, 50-, and 100-mile channel reaches (L) are shown in Fig. 2. The lines representing a constant value of ε are plotted against the dominant hydrograph property, T_r (the time of rise in hrs) along the vertical axis, and the energy slope, S (which is approximated by the average channel bottom slope, ft/ft). The Muskingum-Cunge algorithm is shown for 10% ε curves for the various L routing reaches. The shaded area below each curve represents all conditions of S and T_r that cause the error (ε) to exceed 10% while the unshaded areas are ε values less than 10%. The family of curves is represented by the following expression:

$$T_{r_{min}} = 0.0024\ S^{-1.03}\ (L/20)^{0.13\ S^{-0.18}} \tag{12}$$

These curves show that as S increases, there is a gradual nonlinear decrease in the minimum T_r value that can be accommodated by the algorithm for a given ε value. In general, Fig. 2 indicates that the Muskingum-Cunge algorithm incurs errors less than 10% when applied to rapidly rising hydrographs ($T_r > 1$ hr) for channels with

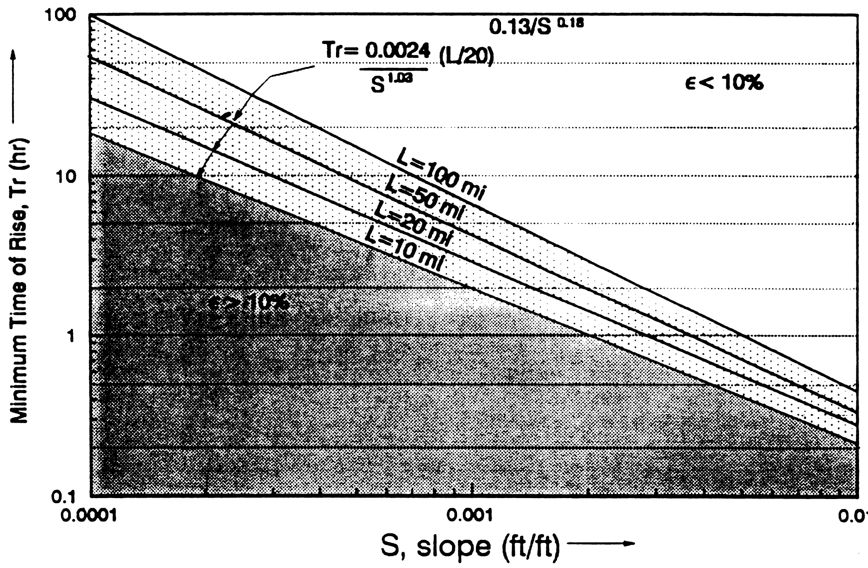

Fig. 2 Minimum allowable time of rise (Tr) which restricts errors (ϵ) in Muskingum-Cunge routing applications to less than 10%

S > 0.002 ft/ft (10 ft/mi). Also, the minimum allowable T_r gradually increases to 20-100 hrs (depending on the reach length) as the slope decreases to 0.0001 (0.5 ft/mi).

Conclusions

The level-pool reservoir routing model is shown empirically to incur errors exceeding 10% for (a) most reservoirs subjected to rapidly rising unsteady flows in which the time of rise of the hydrograph is less than 1.0 hour such as dam-break floods or intermittent turbine releases and (b) very long reservoirs (length > 50 mi) subjected to flash floods with times of rise less than 18 hours. The Muskingum-Cunge river routing model's error properties, excluding those due to neglecting backwater effects, increase with (a) decrease in the time of rise of the hydrograph, (b) decrease in the channel bottom slope, and (c) increase in the length of the routing reach. In general, the Muskingum-Cunge method incurs errors exceeding 10% when applied to hydrographs having a time of rise (hrs) smaller than that given by Eq. (12).

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