TOKAMAKS AND QUASI-AXISYMMETRIC SHAPING

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1. **Shaping primary determinant of fusion plasma equilibria.**
   Pressure and current profiles largely self-determined.

2. **Axisymmetric and quasi-axisymmetric tokamaks have same fundamental physics.**
   Most shaping freedom of tokamaks is non-axisymmetric, but need quasi-axisymmetry: (1) magnetic surfaces and (2) $B(\ell) = B(\ell + L)$.

3. **A tokamak DEMO probably requires non-axisymetric shaping.**
   An axisymmetric tokamak plasma is in a self-organized (self-determined) state with little freedom to design around problems.

Plasma Shaping is Primary Design Freedom in DEMO

A plasma equilibrium, $\vec{\nabla} p = \vec{j} \times \vec{B}$, is determined by the profiles of plasma pressure and current and the shape of the outermost plasma surface.

In DEMO, the density and temperature profiles are largely self-determined by fusion energy production and transport.

Bootstrap current also self-determined. In DEMO, $I_{boot} / I_{driven} > 4$.

Pressure and current are largely self-determined.
Shape is left as primary design freedom.

Importance of axisymmetric shaping appreciated and exploited.
(aspect ratio, ellipticity, triangularity, squareness)

Non-axisymmetric shaping has an order of magnitude more shaping parameters than axisymmetric shaping.

Definition of Quasi-Axisymmetry

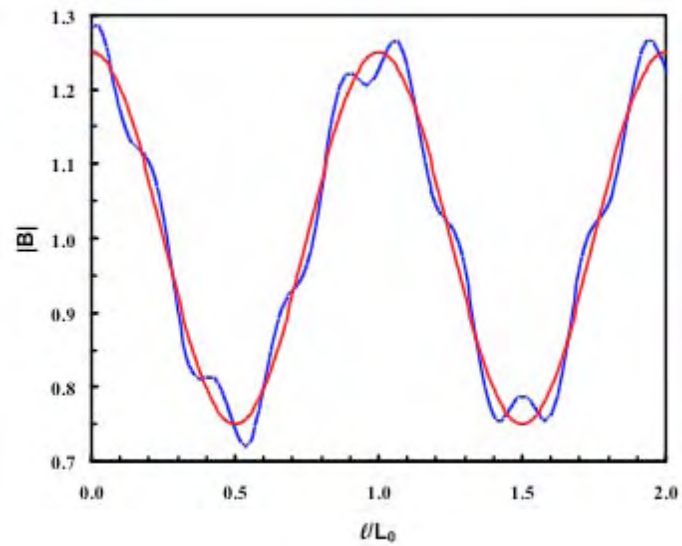
1. Good magnetic surfaces $\vec{B}\cdot\vec{\nabla}s(\vec{x})=0$.

   Ensures passing particle confinement.

2. $B(\ell)=B(\ell+L)$

   Ensures trapped particle confinement.

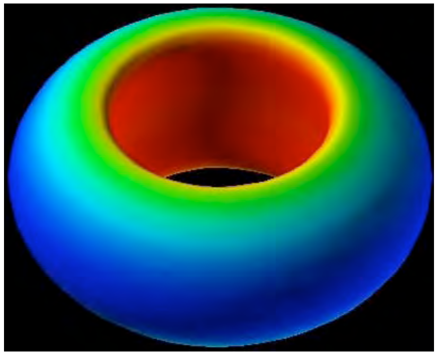
$$J=\oint \mathrm{v}_{\|} d\ell=\oint\sqrt{\tfrac{2}{m}\left\{E-\mu B(\ell)\right\}}\,d\ell$$



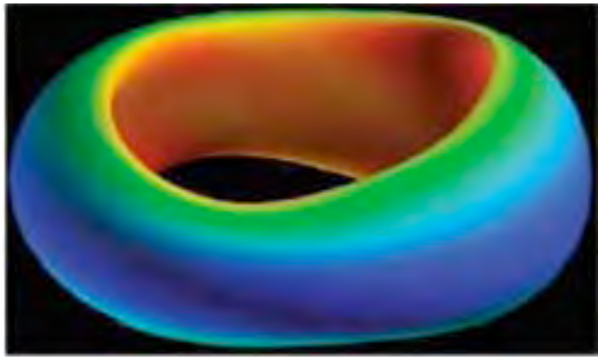
Red quasi-symmetric. Blue not quasi-symmetric.

Quasi-axisymmetry ensures particle confinement as in axisymmetry.

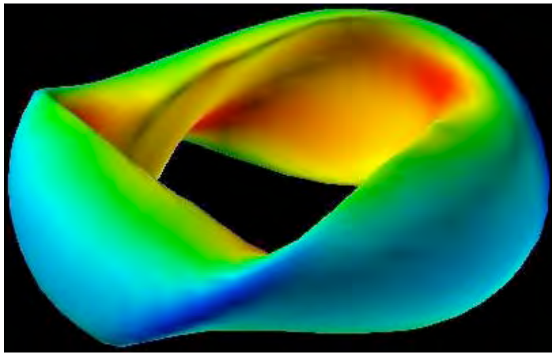
Shaping Freedom Retaining Tokamak-like Drifts



Tokamak $\iota_{vac}=0$



ARIES-RS but $\iota_{vac}/\iota=20\%$



NCSX $\iota_{vac}/\iota=75\%$

All three approximate quasi-axisymmetry, $B(\ell)=B(\ell+L)$.

Importance of Non-Axisymmetric Shaping

Experiments have demonstrated:

Non-axisymmetric shaping addresses major issues of fusion plasmas.

A number of examples will be discussed

A narrow focus on these issues obscures true importance:

Non-axisymmetric shaping is primary design freedom to obtain suitable equilibria for fusion.

Benefits of Weak Non-Axisymmetric Shaping, $\delta B/B \sim 10^{-3}$

Major area of tokamak innovations

1. **Error Field Control**

   $\delta B/B \sim 10^{-4}$ can cause tokamak disruptions.
   Simple coils mitigate effects but increase the error field.

2. **Resistive Wall Mode Stabilization (RWM)**

   Tokamaks with a strong bootstrap current and high beta are unstable to an external kink, the RWM.

3. **Edge Localized Mode (ELM) control**

   ELMs deposit unacceptable heat pulses on divertor plates.

For all three, currents in coils or walls produce a non-axisymmetric plasma equilibrium, which must be understood.

Benefits of Strong Non-Axisymmetric Shaping

Stellarator experiments show effectiveness.

1. **Rotational transform, $\iota=1/q$, control**

   Axisymmetric tokamak magnetic configuration is determined by transport. ITER won't demonstrate configuration is maintainable.

2. **Plasma robustness**

   Non-axisymmetric fields can center plasma in chamber.
   Appears to prevent disruptions; reduces stability sensitivity.

3. **High plasma density**

   Greatly reduces drive for energetic-particle modes. Eases divertor problem.

4. **Confinement control**

   Good confinement observed. Less dependence on edge expected.

Importance of Rotational Transform Control

$$\iota = \iota_{vac} + \iota_{boot} + \iota_{drive} + \iota_{\Omega}$$

1. Magnetic configuration maintenance difficult in axisymmetry.

a. Steady state requires $I_{boot}/I_{driven} > 4$. ITER expects $I_{boot}/I_{driven} \sim 1$.

$$\text{Power requirement } P_{drive} > \frac{I_{drive}}{ec} m_e c^2 \gamma \nu_{ee}(\gamma).$$

b. Ohmic drive gives short pulses, $\tau_I \approx (0.2hrs)\left(\frac{a}{meters}\right)^2 \left(\frac{T_e}{10kev}\right)^{3/2}$.

Axisymmetry forces tokamaks to high *T* and low *n*.

2. ι_{boot}/ι measures self-organization of plasma.

Microturbulence determines profiles and bootstrap current. Magnetic configuration difficult to predict or control if $\iota - \iota_{boot} << \iota$.

3. ι_{vac} typically has reverse shear.

Stabilizing for neoclassical tearing modes and microturbulence.

Meaning of Robustness

In axisymmetry, no natural tendency exists to center plasma in the vacuum chamber.

Plasma location is a balance between vertical field and hoop stresses.

Stellarator fields form a cage centering the plasma.

Neil Pomphrey found little change in the location of the outer surface of the NCSX plasma if either the pressure or current were set to zero with fixed external field.

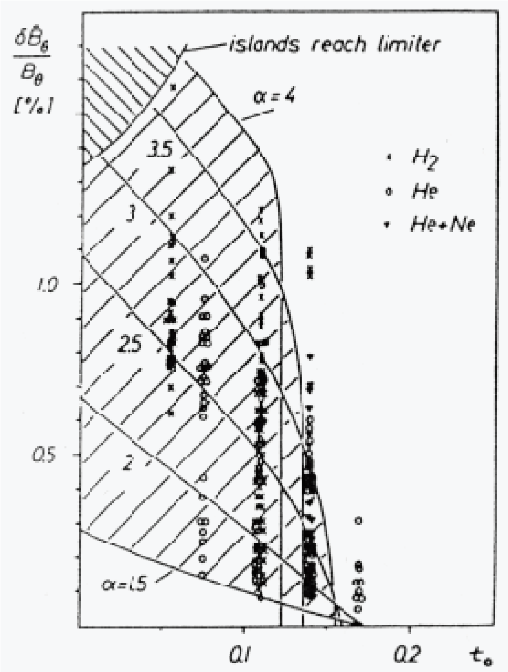
Implications of robust positional stability appear profound.

W7-AS experiments, PPCF 50, 053001 (2008):

"Stability studies do not show fast disruptive instabilities even close to operational limits but rather slow transitions to increased transport. Pressure driven MHD neither plays a role in the range of highest-β nor causes any limitation on the pressure gradient."

Robustness of Stellarator Equilibria
Against Disruptions

In tokamaks operational limits set by disruptions but not in stellarators. (Disruptions can unacceptably damage DEMO device.)



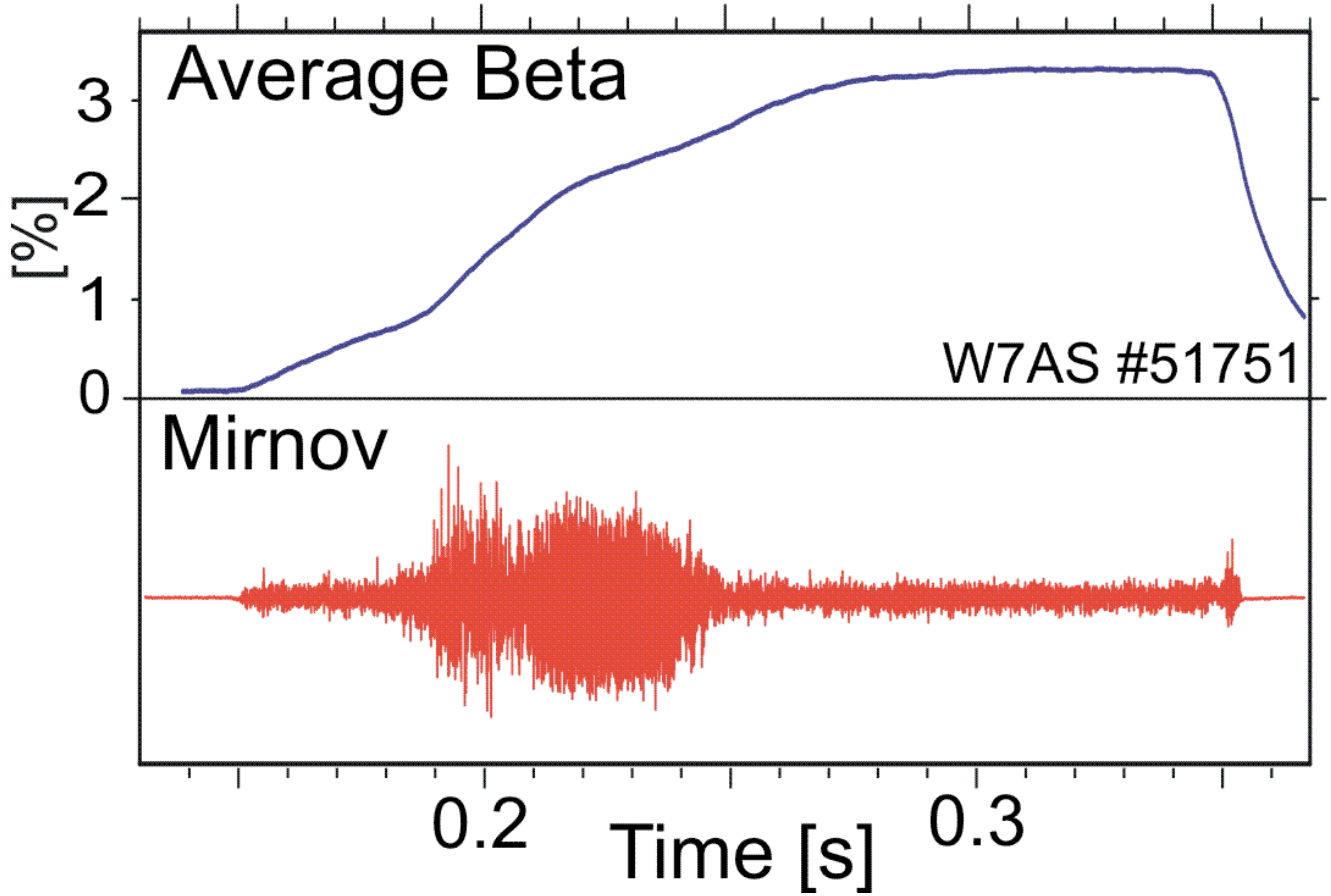
W7-A Team, Nuclear Fusion 20, 1093 (1980).

Early stellarators had large currents, but no disruptions for $\iota_{vac} > 0.15$.

Robustness of Stellarator Equilibria

Soft beta limits

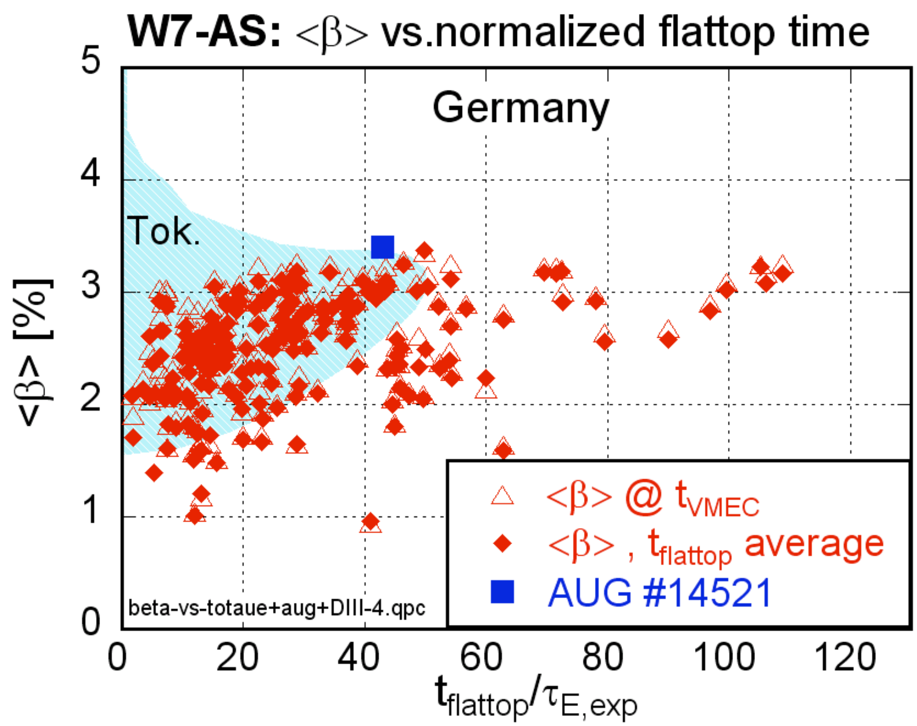
Stability limit can be so soft that an operational limit is not defined.

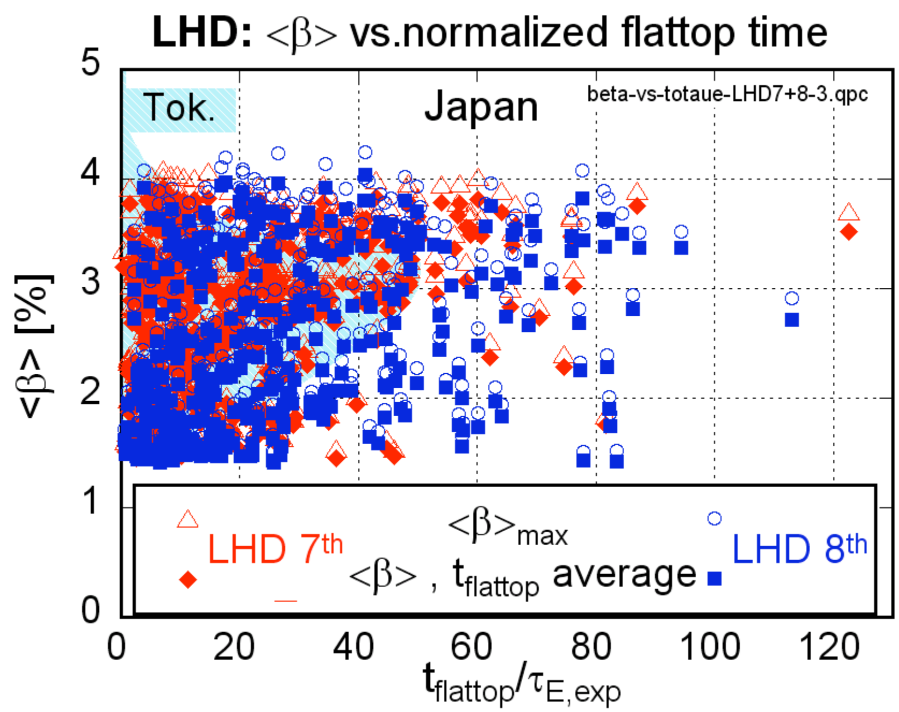


Robustness of Stellarator Equilibria

Maintenance of beta $\beta \equiv \left\langle \frac{2\mu_0 p}{B^2} \right\rangle$

Relevant β for DEMO is steady-state. In stellarators maximum and steady-state β similar. In tokamaks, steady-state β about 60% of its maximum *and for not as many confinement times*.





Higher betas achieved in short pulses in tokamaks and stellarators.

Importance of High Plasma Density

1. Low density implies a higher *T* for adequate fusion power density.

Makes the pressure of non-thermal α's high. $\left(p_\alpha \propto T_e^{3/2}\right)$

Gives strong drive for energetic particle instabilities.

2. High *T* and low *n* at edge makes divertor problem difficult.

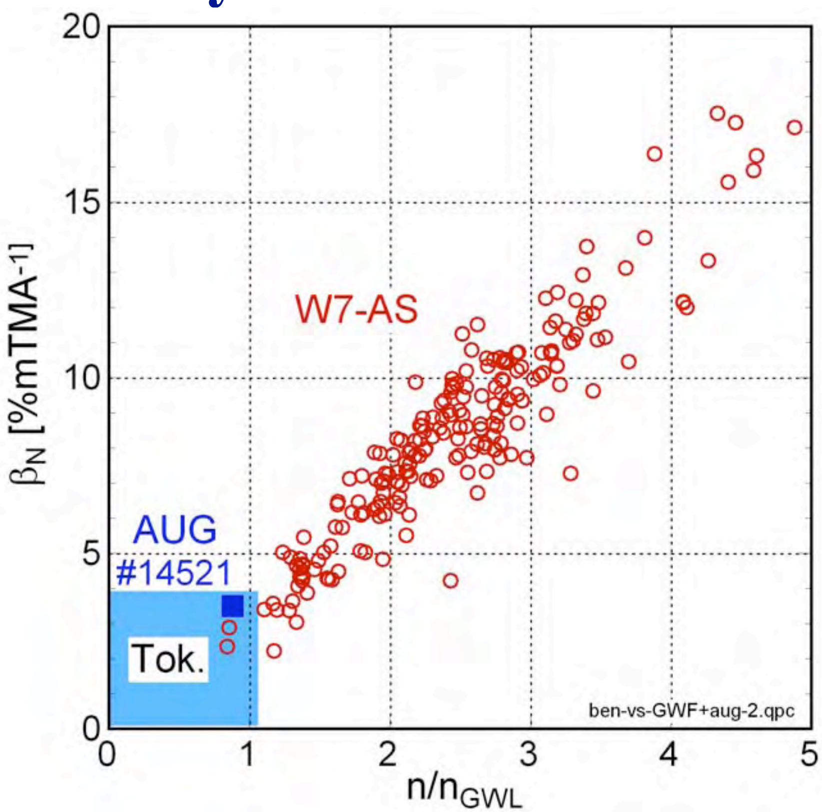
W7-AS and LHD have stable detached divertor plasmas, which unload power through radiation.

Power in radiation easier to handle than in particles hitting divertor.

Cause of Low Operating Density of Tokamaks

1. Efficiency of current drive $\propto 1/n$. $\quad Power > \dfrac{I_{driven}}{ec} m_e c^2 \gamma \nu_{ee}(\gamma).$
2. Greenwald density limit ($n < n_{GWL} \equiv I_{equiv} / \pi a^2$) to avoid disruptions.

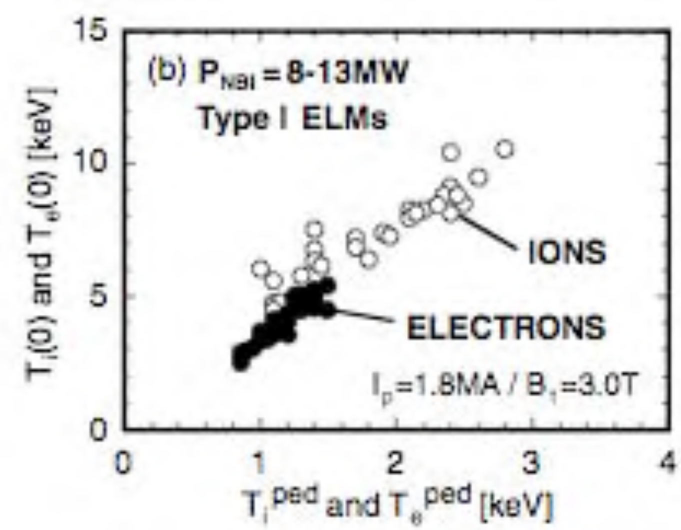
Neither density limit restricts stellarators.



$$\beta_N \equiv \langle \beta \rangle / (I_{equiv} / aB).$$

Importance of Confinement Control

1. Axisymmetric tokamak confinement determined by edge.



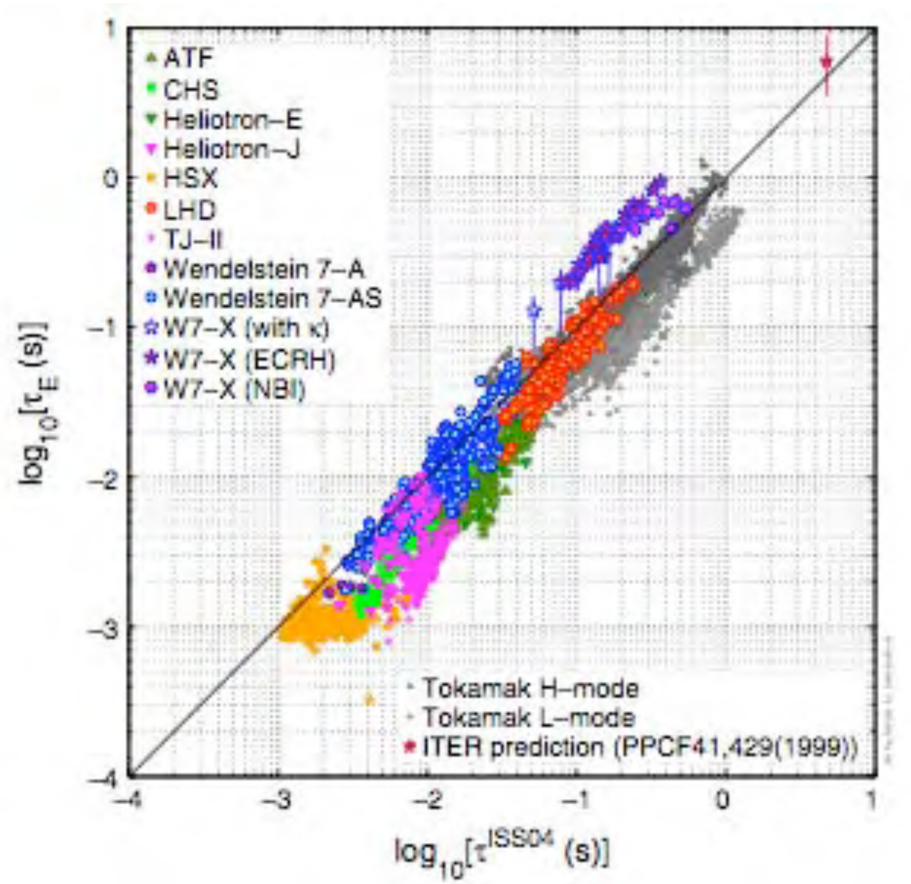
JT-60 ELMy H-mode {NF 42, 76 (2002)}

2. Engineering not plasma physics determines required χ_{eff}.

Plasma size: minimum set by 1.5m of blanket and shields.
maximum set by wall loading and total power.

Confinement in Stellarators

Empirical confinement scaling as in ELMy H-mode tokamaks.



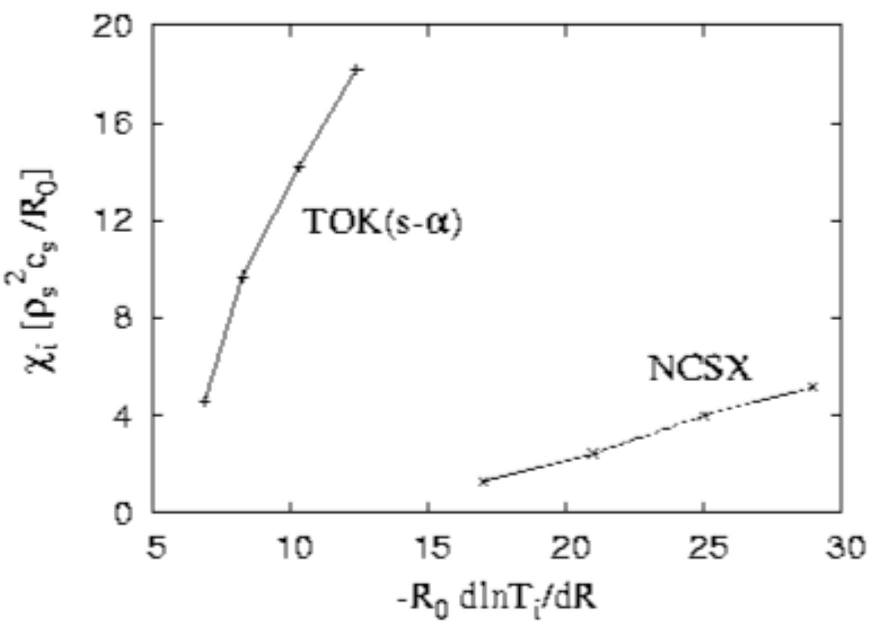
Confinement Control Using Non-axisymmetry

The strong variation in the plasma cross section along the magnetic field lines implies a large local magnetic shear: ~R/N_p not ~qR as in axisymmetry.

Local shear converts $k_\perp \sim 1/\rho_i$ into $k_{||}$, which causes Landau damping.

Gene code (Jenko and Xanthopoulos) found ITG microturbulence in NCSX when compared to the cyclone base case tokamak:

1. Less affected by zonal flows
2. Less stiff (χ_i rises more slowly when critical gradient exceeded.)



Stiffness of χ_i of Major Importance

Transport stiff if χ_i rises rapidly above a critical gradient, $\dfrac{d\ln(T_i)}{dr} = \dfrac{1}{L_c}$.

If very stiff, temperature gradient is just above critical gradient for any heat flux.

Edge T_a and central T_0 related by $\dfrac{T_0}{T_a} = \exp\left(\dfrac{a}{L_c}\right)$ as seen on JT-60.

High edge temperature makes problems of edge more difficult.

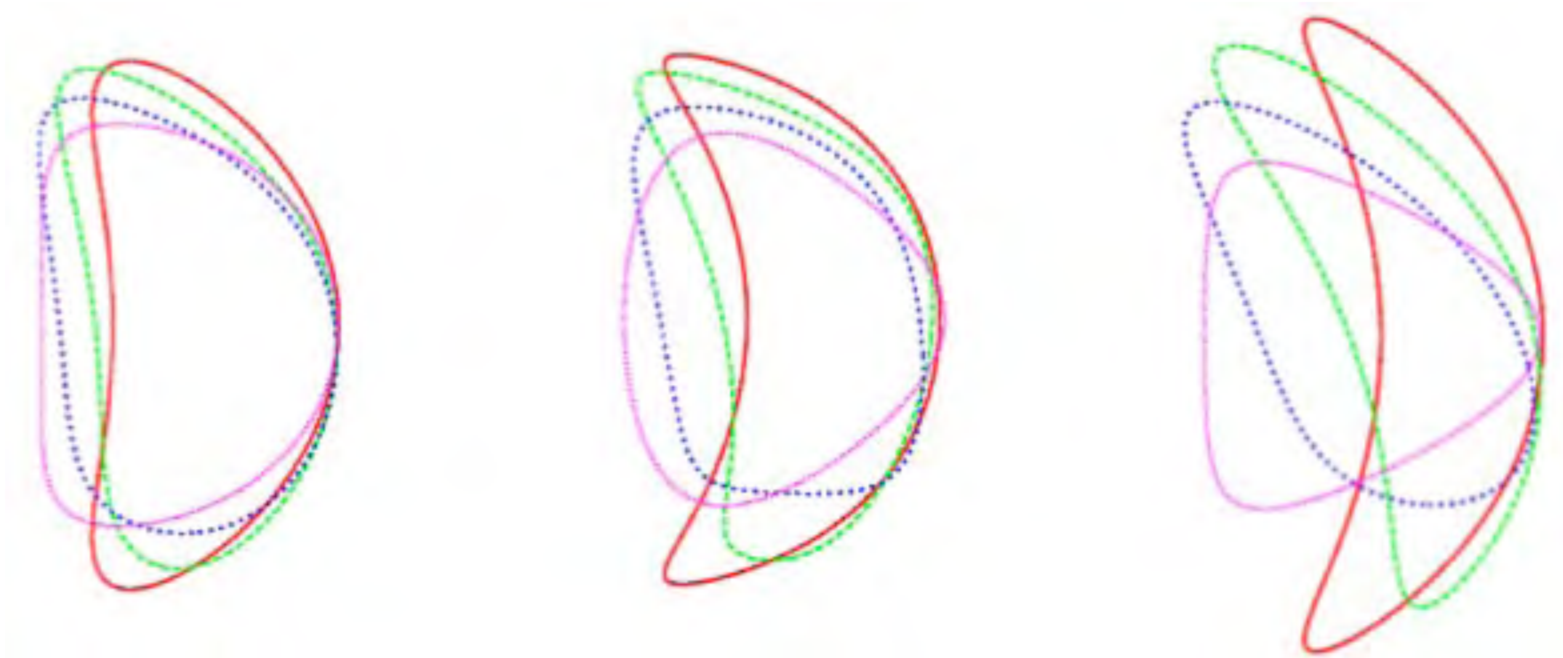
Sensitivity to plasma edge rules out some solutions.

Detached radiative divertor impossible

Power in radiation easier to handle than in particles hitting divertor.

Shaping Required to Address Tokamak Issues

Quasi-axisymmetric, $<\beta>=4\%$, $A=4$, three field periods



$\iota_{vac}=0.045$
$\iota_{vac}/\iota_{edge}=18\%$
Vertical stability

$\iota_{vac}=0.09$
$\iota_{vac}/\iota_{edge}=37\%$
No current drive
Holds plasma from walls

$\iota_{vac}=0.27$
$\iota_{vac}/\iota_{edge}=60\%$
No RWM

Long-Poe Ku

Technical Challenge

Construction delays of W7-X and cancellation of NCSX have focused attention on the technical challenge of non-axisymmetric shaping.

Many stellarators (including the largest LHD) have been built without major problems or delays. Jeff Harris has a list of 28 since 1970.

Theory and design can ease the technical challenge:

1. Less restrictive tolerances.
2. More efficiently produced magnetic fields.
3. Different aspect ratios.
4. Hybrid coil systems: helical, modular, and saddle coils.
ferritic and superconducting blocks.

If an axisymmetric DEMO can be built, so can a DEMO with some level of non-axisymmetric shaping.

Distributions of External Magnetic Field

Coils required to support non-axisymmetric plasma are a central issue.

Between coils and plasma, the field produced by the coils satisfies

$$\vec{\nabla} \times \vec{B}_{ext} = 0 \text{ and } \vec{\nabla} \cdot \vec{B}_{ext} = 0, \text{ so } \vec{B}_{ext} = \vec{\nabla}\phi, \text{ where } \nabla^2\phi = 0.$$

A magnetic field distribution $\phi_j(\vec{x})$ is uniquely defined by:

1. $\nabla^2\phi_j = 0$
2. no infinities in plasma volume
3. $b_j(\theta,\varphi) \equiv \left(\vec{n} \cdot \vec{\nabla}\phi_j\right)_{\substack{plasma\\surface}}$

General external field is $$\vec{B}_{ext} = \frac{\mu_0 G_0}{2\pi}\vec{\nabla}\varphi + \sum_j c_j \vec{\nabla}\phi_j(\vec{x})$$

Each c_j is proportional to the current that produces that distribution.

Distributions can be ordered by efficiency. As $|\vec{x}| \to \infty$, each distribution satisfies $\phi_j(\vec{x}) \propto \exp\left(k_j|\vec{x}|\right)$. Most efficient distribution has smallest k_j.

Eased Technical Challenge

I. Less restrictive tolerances by use of trim coils

Error field mitigation in stellarators can be studied using CAS3D code.

Strategy

1. External magnetic field distributions can be ordered by the degradation they produce in the quality of the plasma: $\left(\delta\vec{B}_{ext}\cdot\hat{n}\right)_1$, $\left(\delta\vec{B}_{ext}\cdot\hat{n}\right)_2$, etc.

2. Design a trim coil set that can null $\left(\delta\vec{B}_{ext}\cdot\hat{n}\right)_1$ for expected machine accuracy without driving $\left(\delta\vec{B}_{ext}\cdot\hat{n}\right)_2$ too strongly.

3. Fundamental limitation is how many error field distributions $\left(\delta\vec{B}_{ext}\cdot\hat{n}\right)_j$ can be simultaneously controlled.

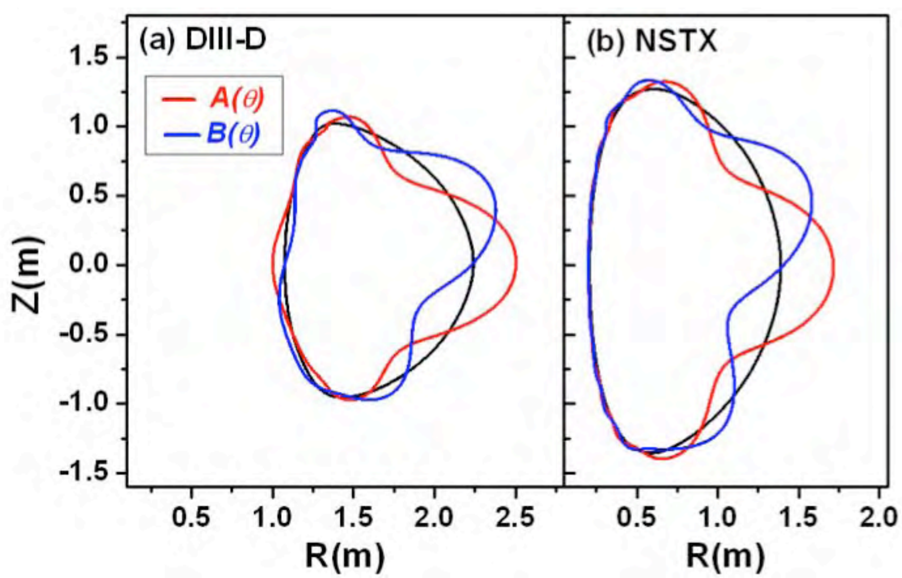
Error field mitigation (or correction) does not mean error field elimination or even error field reduction.

Tokamak example of error field mitigation strategy

Errors with $\delta B/B=10^{-4}$ can cause disruptions so mitigation is required.

Drive for magnetic islands at q=1,2,3 surfaces studied using IPEC code.

J-K Park et al, PRL 99, 195003 (2007)



$\delta\vec{B}_{ext}\cdot\hat{n} = A(\theta)\cos\varphi + B(\theta)\sin\varphi$ on plasma surface of most important error field.

Tokamak islands tend to be about ten times as sensitive to the first external perturbation $\left(\delta\vec{B}_{ext}\cdot\hat{n}\right)_1$ as to the second.

Eased Technical Challenge

II. More efficiently produced magnetic fields

Coils must: 1. Provide net toroidal magnetic flux.

Provided by toroidal field coils.

2. Ensure $\vec{B}\cdot\hat{n} = \vec{B}_{pl}\cdot\hat{n} + \vec{B}_{ext}\cdot\hat{n} = 0$ on plasma.

In a tokamak, provided by poloidal field coils.

$$\vec{B}_{ext} = \frac{\mu_0 G_0}{2\pi}\vec{\nabla}\varphi + \sum_j c_j \vec{\nabla}\phi_j(\vec{x}) \quad \text{where} \quad \phi_j(\vec{x}\to\infty) \propto \exp\left(k_j|\vec{x}|\right)$$

Plasmas and coils require common design so only efficiently produced distributions are required. (small k_j)

Efficiency limits axisymmetric shaping to 4 parameters (*aspect ratio, ellipticity, triangularity, and squareness*). Same limit gives 10 times as many non-axisymmetric parameters.

Strategy for Common Design of Plasma and Coils

(closely related to error field mitigation)

Important plasma properties can be calculated. For example:

1. Driving field for a magnetic island at $\iota = n/m$ rational surface.
   Easily assessed with perturbed equilibrium code CAS3D.
2. Deviation of action $\delta J = J - J_0(\psi_t)$. $\qquad J \equiv \oint v_{\|} d\ell$
3. Least stable δW of an ideal MHD stability analysis.

Given a set of J_p plasma properties: $\qquad$ Typically $J_p \sim 15$

1. Carolin Nührenberg's CAS3D code can find the J_p external magnetic field distributions $\phi_j(\vec{x})$ that control these properties.

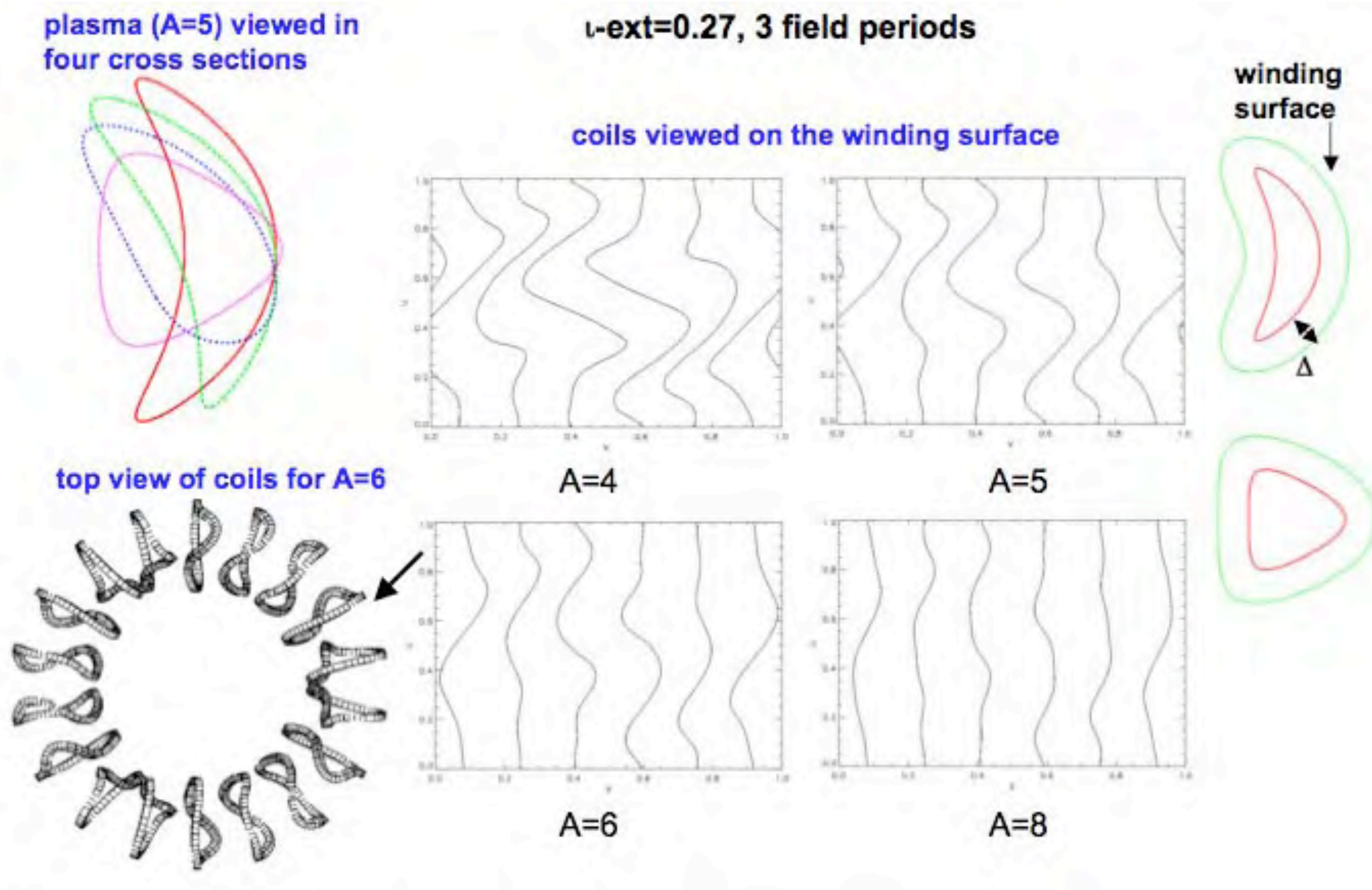
   Irrelevant whether distributions not in this set of J_p are driven.

2. Peter Merkel's codes allow one to find the J_p external field distributions that most efficiently drive the controlling distributions.

Eased Technical Challenge

III. Different Aspect Ratios (L-P Ku)

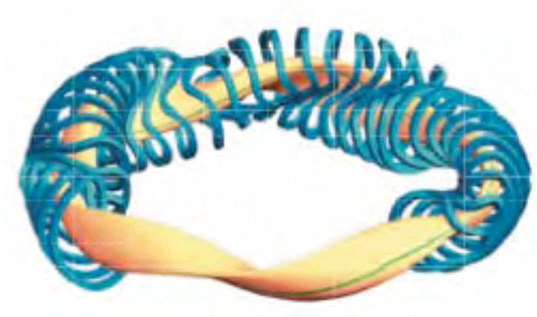
Stable at $<\beta>=4\%$, quasi-axisymmetric, $\Delta/a=0.6$, three periods

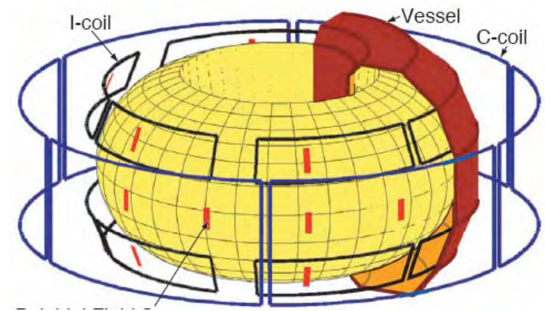


Eased Technical Challenge

IV. Hybrid coil systems (helical, modular, saddle)


Helical


Modular



Saddle

Coils systems that consist of more than one type could give simpler designs.

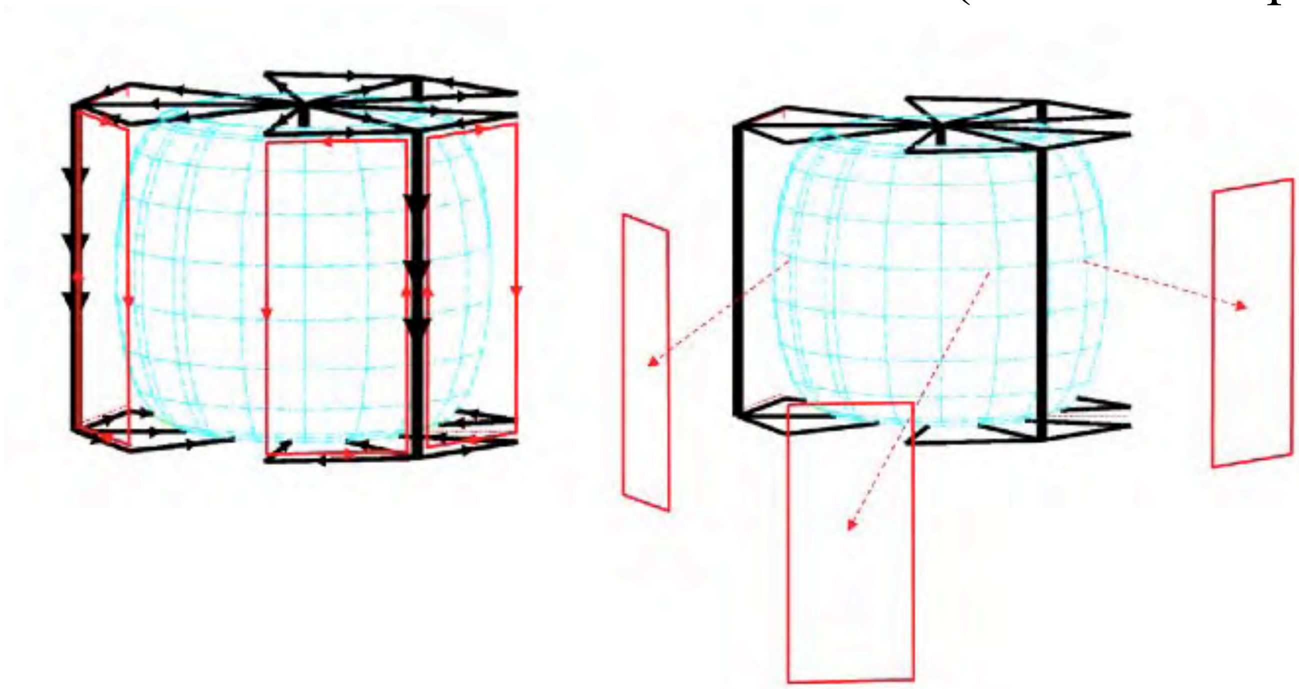
Blocks of ferritic materials and bulk superconductors can also be used.

Eased Technical Challenge

Tokamak Example of Hybrid Coil System

Tokamak toroidal field coils can be concentrated in a few columns:

To reduce ripple, plugs in access ports contain:
(1) saddle coils, (2) ferritic blocks, (3) bulk superconductors.
(Boozer-Pomphrey).



Easier Maintenance Access to Plasma Chamber.

Two Views of Tokamak Research

1. As a faith-based initiative:

Demonstration that a weakly controlled plasma indeed self-organizes into an attractive state for fusion. Usual vision *(focused on largest machine)*

No need for "for theorists who possess complicated minds and access to supercomputers." Physics Today, December 2008, page 14.

2. Based on intelligent design:

Understanding of what design features reduce the risk of DEMO and optimizes its attractiveness for fusion power.

Need to understand available design space—mostly non-axisymmetric.

Central Points on Theoretical Research

1. If a set of plasma properties can be calculated, the external magnetic field can be found that optimizes those properties.

2. For each calculable plasma property, a specific external magnetic field distribution that controls that property can be determined.

3. Coil designs can use the most efficient external magnetic field distributions that span a space equal to the number of plasma properties to be controlled. (about 15 properties)

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Simulation codes that handle non-axisymmetry required even to know what construction accuracy is required.

Tokamak fusion systems must be close to quasi-axisymmetry, so codes that efficiently calculate perturbations to quasi-axisymmetry are needed.

Central Points on Engineering Design

1. Coil designs should be focused on the small set of the most efficient magnetic field distributions required for physics control.

Independent circuits for magnetic field distributions that control plasma properties ensure important device flexibility.

Use of efficiently produced field allows coils to be far away:

Reduces effects of field errors.

Allows simplifications in first wall shape.

2. Different types of coils and blocks of passive superconducting and ferritic material may simplify device construction and maintenance.

Central Points on Experimental Research

A program that extends the design space of tokamaks would:

1. ensure leadership in the world fusion program.
2. allow optimal use of the data from ITER for fusion power.

A credible experiment is required:

1. must have low collisionality and high beta to be credible.
2. such an experiment costs about 40M$/yr to operate.
3. could be constructed with annual costs less than operating budget.

About 8 operating tokamaks in the world are larger programs, but a new device is needed to study non-axisymetric shaping with $\delta B/B>10^{-3}$.

Would require about 15% of the U.S. non-ITER fusion budget—whether such an experiment is built in the U.S. is a priority not a budget issue.

Closing Questions

1. What level and type of non-axisymmetric shaping is required to:
    a. maintain the magnetic configuration?
    b. avoid disruptions?
    c. remove restrictive limits on the plasma density?
    d. allow large ratios of the central to the edge temperature?

2. What are the engineering constraints on non-axisymmetry?

3. Do alternative solutions exist to those provided by quasi-axisymmetric shaping, or should tokamak research proceed by just keeping the faith?

A fusion system cannot be perfectly axisymmetric, so non-axisymmetric fields must be controlled, but:

a. at what level? b. of what type? c. for what purpose?

See:

Stellarators and the path from ITER to DEMO, A. Boozer, PPCF **50** 124005 (2008).
Uses of non-axisymmetric shaping in magnetic fusion, A. Boozer, submit. Phys. Fluids.