TOWARDS AN ELECTROMAGNETIC PRESSURE STANDARD: DIELECTRIC PERMITTIVITY OF HELIUM AND ARGON MEASURED WITH QUASI-SPHERICAL MICROWAVE RESONATORS AND CROSS CAPACITORS

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Abstract

To develop electrically based pressure standards, we measured the dielectric permittivity of helium $\varepsilon_{\mathrm{He}}(p,T)$ and argon $\varepsilon_{\mathrm{Ar}}(p,T)$ using two independent techniques (quasi-spherical microwave cavities and cross capacitors). At 7 MPa, the microwave results for helium differ from NIST's pressure standards by $(18\pm37)\times10^{-6}$.

Introduction

For electrically based pressure standards to compete with conventional pressure standards, which at 7 MPa have relative uncertainties of 17×10^{-6} [1], we must measure $\varepsilon_{\mathrm{He}}(p,T)$ with part-per-billion precision. This is a demanding requirement for which we have developed specialized apparatus: quasi-spherical microwave cavities and cross capacitors. We determined the dielectric permittivity of helium $\varepsilon_{\mathrm{He}}(p,T)$ and argon $\varepsilon_{\mathrm{Ar}}(p,T)$ from a quasi-spherical microwave cavity (2.5 cm radius) and the expression

$$\varepsilon\mu_{\mathrm{r}} = \left[(f_0+g_0)/(f_{\mathrm{p}}+g_{\mathrm{p}})\right]^2\left(1+2\kappa_{\mathrm{T}}p/3\right) \quad . \qquad (1)$$

Independently, we determined $\varepsilon_{\mathrm{He}}(p,T)$ and $\varepsilon_{\mathrm{Ar}}(p,T)$ from multi-ring toroidal cross capacitor measurements (10 pF) and the expression

$$\varepsilon(p,T) = (C_{\mathrm{p}}/C_0)(1+\kappa_{\mathrm{T}}p/3) \quad . \qquad (2)$$

Here, f_0 and g_0 are the frequency and half-width of a microwave resonance in vacuum; f_{p} and g_{p} are the frequency and half-width of the same microwave mode at pressure p; C_0 and C_{p} are the capacitance of the cross capacitor in vacuum and at pressure p; μ_{r} is the relative magnetic permeability, and κ_{T} is the isothermal compressibility of the microwave cavity or of the cross capacitor. (κ_{T} accounts for the deformation of the apparatus under hydrostatic pressure.) The data range from 273 K to 295 K and up to 7 MPa.

At present, the microwave results are more accurate than the capacitance results [2]. In a quasi-spherical cavity, the degenerate microwave modes of a spherical cavity are split into separate, easily measured components, while retaining the high quality factors typical of spherical cavity resonators. Using copper-plated steel cavities, we obtained *Q*s of 25,000 and thus measured frequencies to a few parts in 10^9. The stability of cross-capacitors makes them more suitable for high accuracy measurements of $\varepsilon_{\mathrm{He}}(p,T)$ than conventional coaxial capacitors [3]. We used a commercial bridge to measure capacitance and obtained capacitance ratios with an uncertainty 100 parts in 10^9. Within this comparatively large uncertainty, the capacitance results and the microwave results are consistent.

Helium used as a Pressure Standard

The pressure is obtained from electromagnetic measurements of $\varepsilon_{\mathrm{He}}$ by combining the dielectric virial equation for helium

$$\frac{\varepsilon_{\mathrm{He}}-1}{\varepsilon_{\mathrm{He}}+2} = \rho A_{\varepsilon,\mathrm{He}}\left(1+b\rho+c\rho^2+\dots\right) \quad , \qquad (3)$$

with the density virial equation of state for helium

$$p = RT\rho\left(1+B_\rho\,\rho+C_\rho\,\rho^2+\dots\right) \quad , \qquad (4)$$

to obtain

$$p_{\mathrm{electro}} = \frac{RT}{A_\varepsilon}\frac{\varepsilon-1}{\varepsilon+2}\left(1+\frac{B-b}{A_\varepsilon}\frac{\varepsilon-1}{\varepsilon+2}+\dots\right) \quad . \qquad (5)$$

where first principles calculations determine the polarizability $A_{\varepsilon,\mathrm{He}}$ [4-6], permeability μ_{r} [7], and the dielectric and density virial coefficients b [8] and B_ρ [9]. We used measurements from the literature to determine the third dielectric and density virial coefficients c [10] and C_ρ [11]. In Eq. (4), R is the molar gas constant [12] and T is the thermodynamic temperature [13].

Uncertainty

$u(\varepsilon_{\mathrm{He}}-1)$: We determined κ_{T} of the maraging steels used to make the resonator and the cross capacitor by combining our resonance ultrasound spectroscopy data [14] with published thermophysical property data. At 7 MPa, the uncertainty $u(\kappa_{\mathrm{T}})$ contributed 7.6×10^{-6} to the relative standard uncertainty $u_{\mathrm{r}}(\varepsilon_{\mathrm{He}}-1)$ for the microwave cavity and 3×10^{-6} to $u_{\mathrm{r}}(\varepsilon_{\mathrm{He}}-1)$ for the capacitor. Inconsistencies among microwave modes and hysteresis of the microwave cavity combined with $u(\kappa_{\mathrm{T}})$ give the total relative standard uncertainty $u_{\mathrm{r}}(\varepsilon_{\mathrm{He}}-1) = 9.6\times10^{-6}$. For the capacitor the bridge uncertainty dominated the uncertainty in $(\varepsilon_{\mathrm{He}}-1)$ and was more than twice as large as for the microwave cavity based measurements.

$u_{\mathrm{r}}(\rho)$: Near 7 MPa, the total relative standard uncertainty of the density obtained from Eqs. (1) and (3) is 10.2×10^{-6}.

The contribution from $u(b)$ is 3.3×10^{-6}. The contribution to $u_r(\rho)$ from the molar polarizability $A_{\varepsilon,\mathrm{H}\varepsilon}$ is $< 1\times10^{-6}$. [4-6]

$u_r(p)$: Near 7 MPa, the largest contributions to the relative standard uncertainty in pressures $u_r(p)$ deduced from Eq. (4) comes from $\rho u(B_\rho)$ and $\rho^2 u(C_\rho)$ and is 20×10^{-6} and 23×10^{-6} respectively. Smaller uncertainties come from the gas constant ($u_r(R) = 1.7\times10^{-6}$ [9]), the temperature of the stirred bath ($u(T) = 1.5$ mK, $u_r(T) = 5\times10^{-6}$), and the uncertainty of the thermodynamic temperature near 273 K, which we estimate as 0.9 mK ($u_r(T_K) = 3\times10^{-6}$) from Fig. 13 of [13]. The total standard uncertainty is displayed as the shaded region in Fig. 1. Near 7 MPa the total relative uncertainty in pressure is 33×10^{-6}. The minimum in the relative pressure uncertainty occurs at about 3.5 MPa and is 18×10^{-6}.

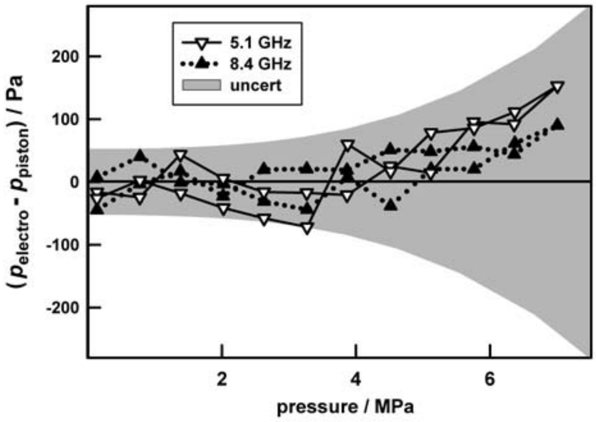

Figure 1. Comparison of pressure determined using the microwave resonator and the theory for $\varepsilon(p)$ at 273 K to a NIST pressure transfer standard (piston gauge). The shaded region indicates the total uncertainty of the present measurements.

Conventional Standards: At NIST, pressure standards near 7 MPa are piston gauges. They are calibrated against a mercury manometer at low pressures (0.02 to 0.36 MPa) and extrapolated to higher pressures. Figure 1 displays the difference between pressures obtained from the microwave measurements described above and a transfer piston gauge based on NIST's pressure standards. Near 7 MPa, the relative standard uncertainty, $u_r(p)$ is 17×10^{-6} for this gauge [1].

Comparison: We compared pressures obtained from $\varepsilon_{\mathrm{He}}$ to a piston gauge traceable to NIST's pressure standards; the difference was (123±233) Pa at 7 MPa, or, fractionally, $(18\pm37)\times10^{-6}$. At 3.5 MPa the fractional difference was $(0\pm25)\times10^{-6}$.

Argon used as Secondary Standard

We also measured the ratio $(\varepsilon_{\mathrm{Ar}}-1)/(\varepsilon_{\mathrm{He}}-1) \approx 8$ under the same conditions that we measured $(\varepsilon_{\mathrm{He}}-1)$; namely 273 K to 295 K and up to 7 MPa. Argon will become a convenient secondary pressure standard because the pressure dependence of its dielectric constant is 8 times larger than that of helium. When theory provides more accurate values of B_ρ and C_ρ for helium, we will be able to reduce the uncertainty of the argon secondary standard without repeating the difficult measurements of $\varepsilon_{\mathrm{He}}(p,T)$. When a more accurate capacitance bridge becomes available, we shall repeat our comparison of the capacitor and the resonator with smaller uncertainties.

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