Electroweak Physics and Bulk Gauge Fields

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Electroweak Observables and Extra Dimensional Gauge Field Theories

- Introduction:

— Extra Dimensions

  Flat Space and Warped Extra Dimensions

— Bulk Gauge Fields and Localized Kinetic Terms in the Brane(s)

- The Flat Space Scenario

— Brane kinetic terms in 5d

Kaluza-Klein decomposition, Masses and Couplings

$\Longrightarrow$ One and Two opaque brane cases

$\Longrightarrow$ Phenomenological implications

- The Warped Extra Dimension Scenario

— Transparent and Opaque Branes

  KK decomposition, masses and couplings

  5d Propagator Analyses

— Effects of a Localized Higgs Field

— Phenomenology of the Ad5SM

  Fit to Electroweak Data

- Conclusions

Based on works done in collaboration with

Tim Tait and Carlos Wagner: Acta Phys. Polon. B33: 2355, 2002 (hep-ph/0207056)
and **Eduardo Ponton**: hep-ph/0212307
and **Antonio Delgado**: in preparation

Related works:

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Extra Dimensions

Strings indicate existence of 6–7 extra dimensions

- They may be there, we do not know at what scale
- They should be compact (small)

– if seen by SM particles $\longrightarrow$ they should be quite small: R $\leq 10^{-17} cm \approx 1/TeV$

– if not seen by SM particles, only by gravity $\longrightarrow$ they can be larger: R $\leq 1mm$

If gravity propagates in the extra dimensions $\Longrightarrow$
Newton's law modified: $M_{Pl}^2 = (M_{Pl}^{fund.})^{2+d} R^d$

$\Longrightarrow$ this lowers the fundamental Planck scale, dep. on the size and number of ED

- solution to hierarchy problem: but why R so large!

Possible solution to hierarchy problem $\rightarrow$ warped space:

Non-factorizable metric: $ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$

$\Longrightarrow$Solution to 5d Einstein eqs.

5d Planck mass relates to M_{Pl}:

$$M_{Pl}^2 = \frac{(M_{Pl}^{fund.})^3}{2k}(1 - e^{-2kL})$$

Assuming fundamental scales of the same order:
$M_{Pl} \sim M_{Pl}^{fund.} \sim v \sim k$ (k: warp factor)
$\Longrightarrow$ Physical Higgs v.e.v. suppressed by e^{-kL}

$$\tilde{v} = v\, e^{-kL} \simeq M_{Pl}\, e^{-kL}$$

$kL \approx 34 \Longrightarrow$ good solution to the hierachy problem

Gauge Fields in the Bulk

• Existence of TeV size or warped extra-dimensions, with gauge fields propagating in the bulk
$\Longrightarrow$ interesting theoretical possibility

• Fermions are easier to localize in 4d by the action of a scalar field; gauge fields propagate in the bulk in a natural way

• Models with gauge fields in extra dim. are not renormalizable $\Longrightarrow$ they must be regarded as an effective field theory up to a scale Λ close to the compactification scale
$\Lambda \sim R^{-1}$ (flat space)
$\Lambda \sim 10\, k\, e^{-kL}$ at IR brane, or $\Lambda \sim 10\, k$ at UV brane (warped extra dimensions)

• Possible gauge coupling unification at high scales

Localized Gauge Kinetic Term in the Brane

Consider a 5d space with fermion fields localized in 3-branes at y =0,L:

Loops of charged fields in the brane induce radiative corrections $\longrightarrow$ gauge kinetic terms in the brane

$$\mathcal{L} = \frac{-\sqrt{-g}}{4g_5^2}\left[\mathcal{F}^{MN}\mathcal{F}_{MN} + r_a\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}\delta(y) + r_b\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}\delta(y-L)\right]$$

$g \rightarrow$ determinant of the metric. $\sqrt{-g} = e^{-4ky}$

(k=0 flat space)

$M = 0, 1, 2, 3, 5;$ $\mu = 0, 1, 2, 3$

$g_5^{-2} \rightarrow$ dim. of mass,

local brane term coefficients: $r_i = g_5^2/g_i^2 \rightarrow$ dim. of length

and

$$\mathcal{F}^a_{MN} = \partial_M \mathcal{A}^a_N - \partial_N \mathcal{A}^a_M + f^{abc}\mathcal{A}^b_M \mathcal{A}^c_N$$

Brane terms are consistent with all the symmetries of the theory.

From general renormalizability arguments one expects such terms to appear at tree level.

• Most ED studies assumed *transparent branes* : dynamics governed by extra dim. kinetic term.

Brane Kinetic terms $\rightarrow$ novel, interesting phenomenology

Flat Space Scenario: Effects of Local Kinetic Terms

In the presence of a brane term, if $r_i \equiv g_5^2/g_i^2 \sim R$, the mixing effects between KK modes can drastically affect the KK decomposition

Considering the A_μ decomposition in term of $f^n(x_5)$'s

$$\mathcal{A}_\mu = \sum_n A_\mu^{(n)} f^n(x_5)$$

with $A_\mu(x_5) = A_\mu(-x_5)$ and in the un. gauge $A_5 = 0$ yields a diagonal kinetic term and a mass term, respectively

$$\int dx_5 \left[\frac{f^m f^n}{g_5^2} + \delta(x_5) \frac{f^m f^n}{g_1^2} + \delta(x_5 - \pi R) \frac{f^m f^n}{g_2^2} \right] = \delta_{nm}$$

$$\int dx_5 \frac{\partial_5 f^m \partial_5 f^n}{g_5^2} = m_n^2 \delta_{nm}$$

or

$$\int dx_5 f^n \left[\partial_5^2 + m_n^2 r_1 \delta(x_5) + m_n^2 r_2 \delta(x_5 - \pi R) + m_n^2 \right] f^n = 0$$

This is verified if

$$\left(\partial_5^2 + m_n^2 r_1 \delta(x_5) + m_n^2 r_2 \delta(x_5 - \pi R) + m_n^2 \right) f^n(x_5) = 0$$

where m_n are the masses to be determined.

The One Opaque Brane Solution

Solving the equation which determines the f^n 's

$$\left(\partial_5^2 + m_n^2 r_1 \delta(x_5) + m_n^2\right) f^n = 0$$

in the regions $x_5 < (>)\ 0$, imposing periodicity and continuity at x_5 =0

$$f^n(x_5 - 2\pi R) = f^n(x_5); \qquad f^n(0^+) = f^n(0^-)$$

$$\partial_5 f^n(0^+) - \partial_5 f^n(0^-) = -r_1 m_n^2 f^n(0)$$

The resulting wave functions have quantized KK mode masses, solutions of the transcendental eq.:

$$\frac{r_1\ m_n}{2} = -\tan[\pi\ R\ m_n]$$

The couplings can be determined from the resulting wave functions

$$f^n(x_5) = A_n \begin{cases} \cos[m_n x_5] + (m_n r_c/2)\sin[m_n x_5] & x_5 < 0 \\ \cos[m_n x_5] - (m_n r_c/2)\sin[m_n x_5] & x_5 \geq 0 \end{cases}$$

with A_n det. from proper normalization

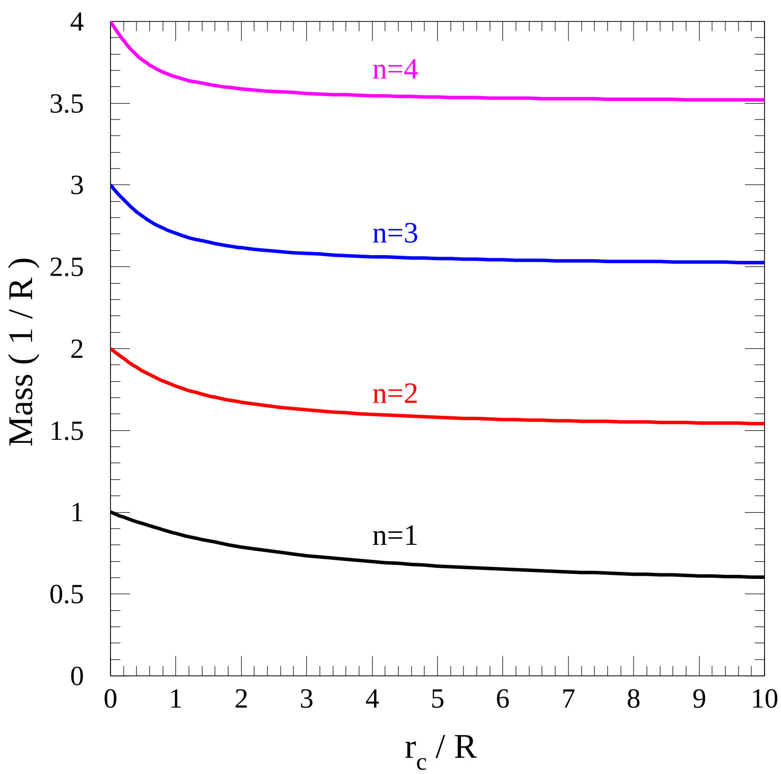
$$\frac{1}{g_5^2}\int_0^{2\pi R} |f^n|^2 (1 + r_1\delta(x_5)) dx_5 = 1$$

• Mass Spectrum somewhat lighter than in the non-localized case, but rather similar.

Apart from the zero mode, the mass eq. implies

$$\frac{n'}{R} \geq m_n \geq \frac{n'}{R} - \frac{1}{2R} \quad \text{with} \quad n' \geq 1$$

Lightest KK mode has $m_1 \geq 1/2R$.



$r_1 = 0 \Longrightarrow$ no brane term $\longrightarrow m^n_{KK} \equiv m_n = n/R$

$r_1 \to \infty \Longrightarrow$ no bulk term

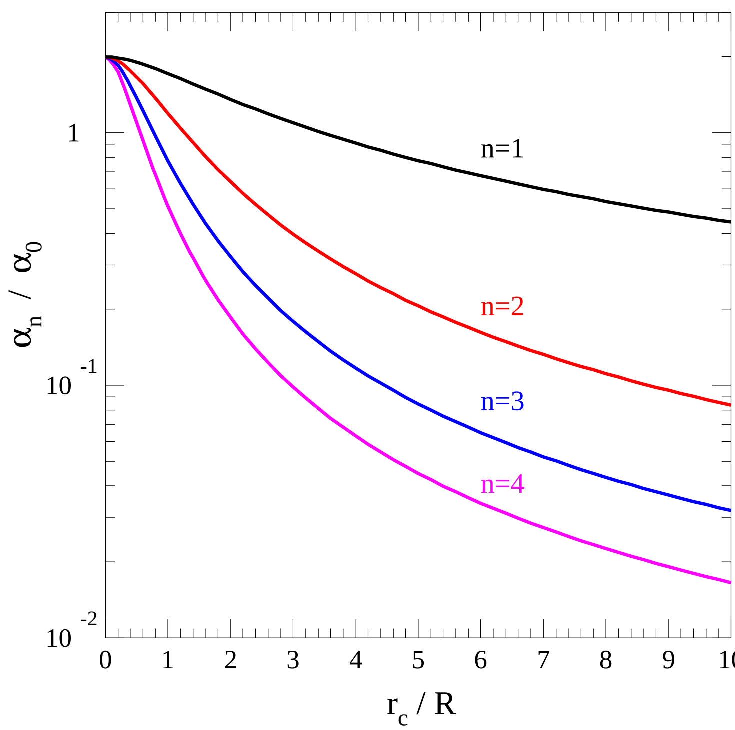
• Behaviour of Couplings quite interesting!

The n-mode and zero mode gauge field couplings to brane matter:

$$g_1^n = f^n(0) \qquad\qquad g_1^0 = f^0(0) = \left(\frac{2\pi R}{g_5^2} + \frac{1}{g_1^2}\right)^{-1/2}$$

– If $r_1 > \mathrm{R} \Longrightarrow$ only the first KK mode couples relatively strongly. The others decouple fast!!

– If $r_1 \leq \mathrm{R} \Longrightarrow$ a few KK modes couple strongly while the others, with $m_n > 1/r_1$, rapidly decouple



All Couplings weaker than in the r_1 =0 case.

Recall: r_1 =0 $\longrightarrow$ the couplings of the KK modes to the fermions are universal, apart from the zero mode:

$$g_1^{(0)} = g_5/(2\pi R)^{1/2} \qquad g_1^{(n)} = \sqrt{2}\ g_1^{(0)}$$

The Two Opaque Brane Scenario

Consider two branes: at $x_5 = 0$ and $x_5 = \pi R$

The interactions of the gauge fields in one brane are affected by the strength of the local coupling in the other

Using similar techniques than those for the one brane case, matching solutions across the branes,
the quantized KK masses are:

$$\frac{(r_1 + r_2)\, m_n}{2\,(1 - m_n^2 r_1 r_2/4)} = -\tan[\pi\, R\, m_n]$$

If $r_1, \;\; r_2 \gg R \rightarrow$ small mass solution compare with 1/R:

$$m_1^2 = \frac{2}{\pi R}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

- In the symmetric case, $r_1 = r_2 = r$
  $m_1^2 = 2/\pi R r$ which $\rightarrow 0$ for large r.

The rest of the KK mode masses approach their canonical values $m_{n+1} = n/R$ for large r

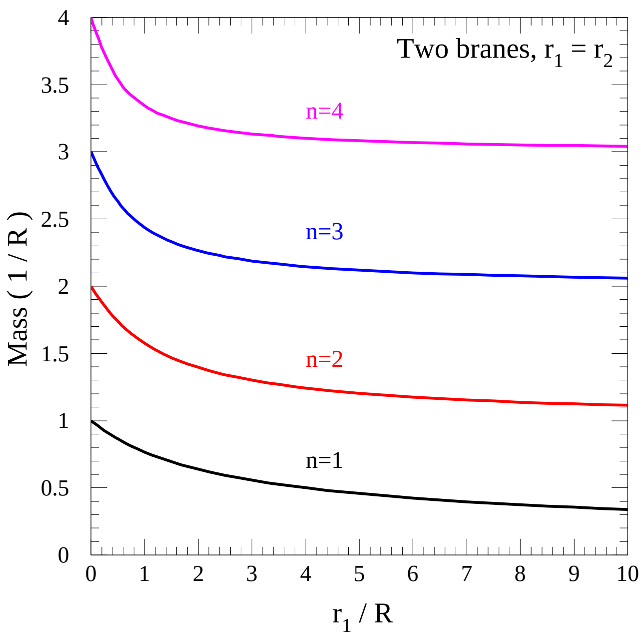
– Radical modifications to the spectrum:

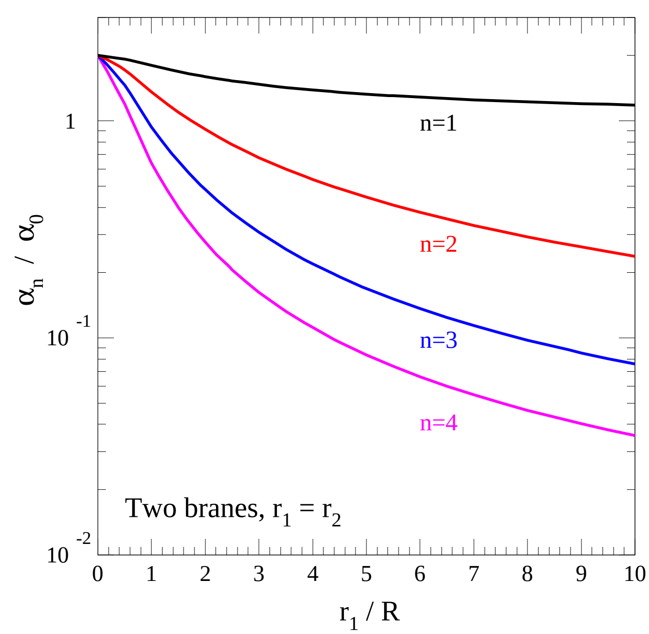
While for one brane KK masses of order n/R, with $n \geq 1$ rapidly approach $(n - 1/2)/R$, as $r_1 \gg 1$;
in the two brane case they approach $(n-1)/R$ for increasing r_i.

The two Brane scenario is fundamentally different from the one brane case:

- lightest KK mode may acquire masses much smaller than the usual KK masses of order 1/R
- can couple strongly to the branes when $r_i \gg 1$

The lightest KK mode coupling approaches the zero mode coupling for large r/R





On the other hand, the two brane case also has the same fast decoupling behavior of the heavy KK modes:

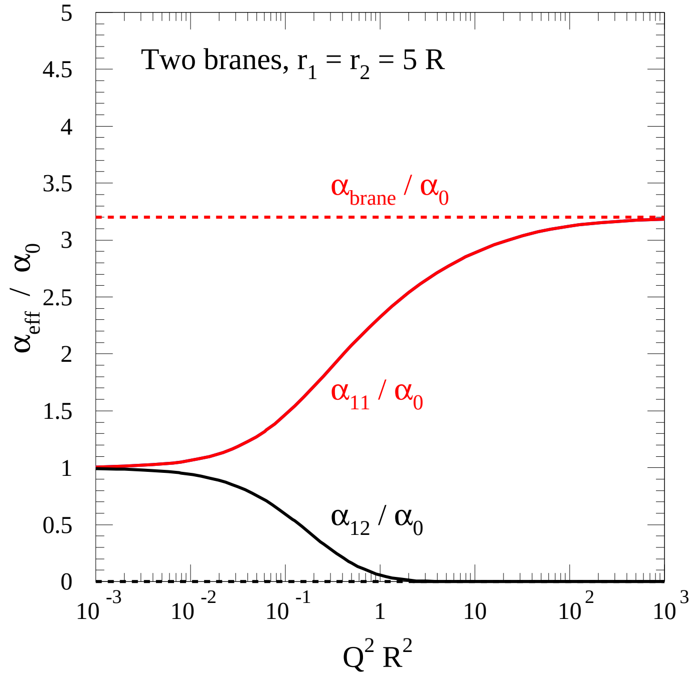
- The effective coupling in the UV regime is controlled by the local brane coupling $\Longrightarrow$ the theory does not enter a strong coupling regime!

Effective Coupling Between Brane Fields

The net force between fields either on the same brane or on different branes is a sum over all of the KK modes

$$\frac{1}{4\pi}\sum_{n\geq 0}\frac{g_n^i g_n^j}{Q^2+m_n^2}\rightarrow\frac{\alpha_{ij}(Q^2)}{Q^2}$$

Q^2: momentum transfer; g_n^i: coupling of nth KK mode to i-th brane; $\alpha_{ij}(Q^2)$: effective intra and inter-brane coupling



• at high Q^2, the fields see only their own brane, with its local brane coupling

• This implies that the coupling between two fields on different branes tends to zero at large Q^2

• at low Q^2, the fields see only the zero mode gauge field.

• Precision measurements studies of the 5dSM in flat space $\Longrightarrow$ the lightest KK masses of order a few TeV.

• Localized kinetic terms for the gauge bosons can relax the constraint on the compactification scale
$\rightarrow$ due to reduced coupling strength of KK modes with brane matter fields.

Warped Extra Dimensions:

$\rightarrow$ interesting alternative
But, if gauge fields in the bulk, precision measurements put stringent lower bound, about 27 TeV, on the mass of the lightest KK excitations

By analogy to flat space:
can localized kinetic terms lower this bound and make the lightest KK modes accesible via direct searches at future colliders?

Warped Extra dimensions: The Transparent Brane Case

From the action of the general 5d Lagangrian for $r_i = 0$; with the decomposition

$$A^\lambda(x_\mu, y \equiv x_5) = \sum_n f_n(y) A_n^\lambda(x^\mu)$$

plus requiring that A_n^λ obey the 4d eq. of motion for a free massive gauge field requires:

$$\left[\partial_y^2 - 2k\partial_y + e^{2ky} m_n^2\right] f_n(y) = 0$$

The solutions to this equation are Bessel functions

$$f_n(y) = \mathcal{N}_m e^{k|y|} \left\{ J_1\left(\frac{m_n}{k} e^{k|y|}\right) + b_n Y_1\left(\frac{m_n}{k} e^{k|y|}\right) \right\}$$

$\mathcal{N}_m$ is an over-all normalization factor, determined by requiring each KK mode to have canonically normalized kinetic terms

$$\frac{1}{g_5^2} \int_0^L dy\, f_n(y) f_m(y) = \delta_{nm}$$

$$\frac{1}{g_5^2} \int_0^L dy\, e^{-2ky}\, f_n'(y) f_m'(y) = m_n^2 \delta_{nm}$$

Imposing the conditions of continuity of the derivative of the wave functions at the two 3d branes:
y=0 $\rightarrow$ UV brane and y=L $\rightarrow$ IR brane.

$$b_n^0 = -\frac{J_0\left(\frac{m_n}{k}\right)}{Y_0\left(\frac{m_n}{k}\right)} \qquad b_n^L = -\frac{J_0\left(\frac{m_n}{k}e^{kL}\right)}{Y_0\left(\frac{m_n}{k}e^{kL}\right)}$$

Equating $b^0 = b^L \Longrightarrow$ trascendental equation for the quantized masses m_n

for $m_n/k \ll 1$, Solutions approx. given by zeros of
$J_0\left(\frac{m_n}{k}e^{kL}\right) \equiv x_n$
$\Longrightarrow m_n = ke^{-kL}x_n$ with $x_n - x_{n-1} \rightarrow \pi$

Couplings of KK modes to matter in the branes:

For large values of kL, KK modes couple universally (up to a sign) to fermions in the IR brane (y=L),
except the zero mode
$g_n/g^0 = \sqrt{2kL}$

KK mode couplings to fermions in the UV brane:
vary with the KK mode number and are suppressed with respect to n=0 mode.

Effects of Local Kinetic Terms

Allowing for opacity in both IR and UV branes, the orthonormality cond. for the KK mode decomposition become

$$\frac{1}{g_5^2}\int_0^L dy\,[1+2\,r_{UV}\,\delta(y)+2\,r_{IR}\delta(y-L)]\,f_n(y)f_m(y)=\delta_{nm}$$

$$\frac{1}{g_5^2}\int_0^L dy\,e^{-2ky}\;f_n'(y)f_m'(y)=m_n^2\delta_{nm}$$

Conditions simultaneously solved if $f_n(y)$ satisfies

$$\left[\partial_y^2-2k\partial_y+e^{2ky}m_n^2(1+2\,r_{UV}\,\delta(y)+2\,r_{IR}\delta(y-L))\right]f_n(y)=0$$

(same eq. obtained from imposing that gauge fields are on the mass shell).

- The bulk solution for KK modes remains the same after introducing brane opacity

- boundary conditions at y=0 and y=L should reflect discontinuity in the derivatives

New solutions have same form but different coefficients b's:

$$b^0 = -\frac{J_0\left(\frac{m_n}{k}\right) + m_n\, r_{UV}\, J_1\left(\frac{m_n}{k}\right)}{Y_0\left(\frac{m_n}{k}\right) + m_n\, r_{UV}\, Y_1\left(\frac{m_n}{k}\right)}$$

$$b^L = -\frac{J_0\left(\frac{m_n}{k}e^{kL}\right) - m_n\, r_{IR}\, e^{kL} J_1\left(\frac{m_n}{k}e^{kL}\right)}{Y_0\left(\frac{m_n}{k}e^{kL}\right) - m_n\, r_{IR}\, e^{kL} Y_1\left(\frac{m_n}{k}e^{kL}\right)}$$

$\Longrightarrow$ different admixture of J_1 and Y_1 in the sol.

Again imposing condition of equality of b's determines the numerical solutions to the quantized masses

Normalization condition determines $\mathcal{N}_n$:

$$\frac{1}{g_5^2}\left[r_{UV} f_n^2(0) + r_{IR} f_n^2(L) + \int_0^L dy f_n^2(y)\right] = 1$$

and hence the coupling of KK modes to brane fields localized at $y \Longrightarrow g_n = f_n(y)$

• Zero mode solution: $m_n = 0$ and constant $f_0(y) \rightarrow$ constant coupling to all charged brane fields for any y, as required by gauge inv.

$$g_0 = f_0(y) = \frac{g_5}{\sqrt{L + r_{IR} + r_{UV}}}$$

Interaction Lagrangian

To examine the couplings of the KK tower to various types of fields, either confined to branes or living in the bulk, consider some representative interaction terms in the 5d theory,

$$\mathcal{L} = \int_0^L dy \left\{ \delta(y - y_\psi) \;\; \left[\overline{\psi} \mathcal{A}_\mu \gamma^\mu \psi\right] \right.$$
$$+ \frac{1}{g_5^2} \left(1 + 2\, r_{UV} \delta(y) + 2\, r_{IR} \delta(y - L)\right) \left[2(\partial_\mu \mathcal{A}_\nu^a - \partial_\nu \mathcal{A}_\mu^a) f^{abc} \mathcal{A}_b^\mu \mathcal{A}_c^\nu\right]$$
$$\left. + \frac{1}{g_5^2} \left(1 + 2\, r_{UV} \delta(y) + 2\, r_{IR} \delta(y - L)\right) \left[f^{abc} f^{ade} \mathcal{A}_b^\mu \mathcal{A}_c^\nu \mathcal{A}_\mu^d \mathcal{A}_\nu^e\right] \right\}$$

Inserting the KK mode decomposition above it follows:

- gauge boson-fermion coupling

$g_n = f_n(y_\psi)$

- interactions among the bulk gauge fields for a non-Abelian theory

for the A_n-A_m-A_l and A_n-A_m-A_l-A_k modes

$$g_{nml} = \frac{1}{g_5^2} \int_0^L dy \left(1 + 2\, r_{UV} \delta(y) + 2\, r_{IR} \delta(y - L)\right) f_n(y)\, f_m(y)\, f_l(y)$$
$$g_{nmlk} = \frac{1}{g_5^2} \int_0^L dy \left(1 + 2\, r_{UV} \delta(y) + 2\, r_{IR} \delta(y - L)\right) f_n(y)\, f_m(y)\, f_l(y)\, f_k(y)$$

Zero mode has constant couplings $g_0 \delta_{mn}$ (3-point) and $g_0^2 \delta_{mn}$ (4-point) with all other KK mode pairs
$\Longrightarrow$ couples universally both to bulk and brane fields

Opaque IR Brane

Analysis based on 5d Propagator

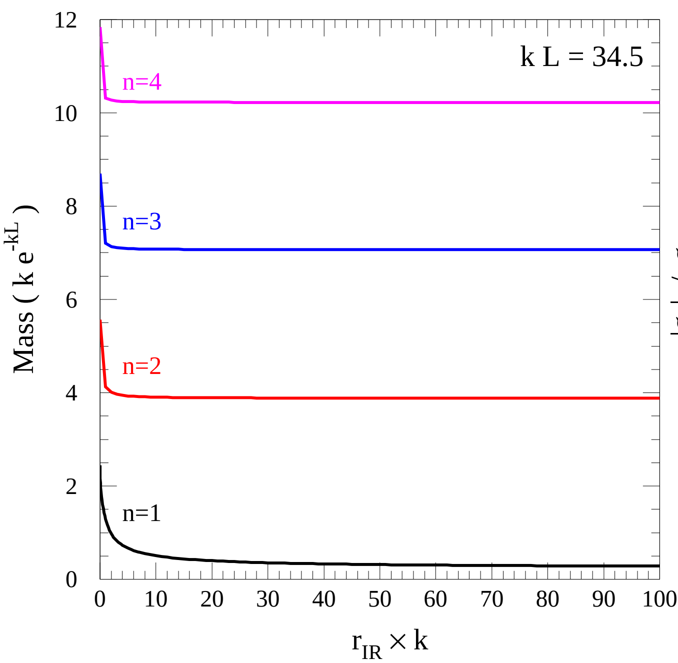
For $p \ll k\, e^{-kL}$, $\qquad G_p(L,L) \sim -g_5^2 / \left[p^2 (L + r_{IR})\right]$

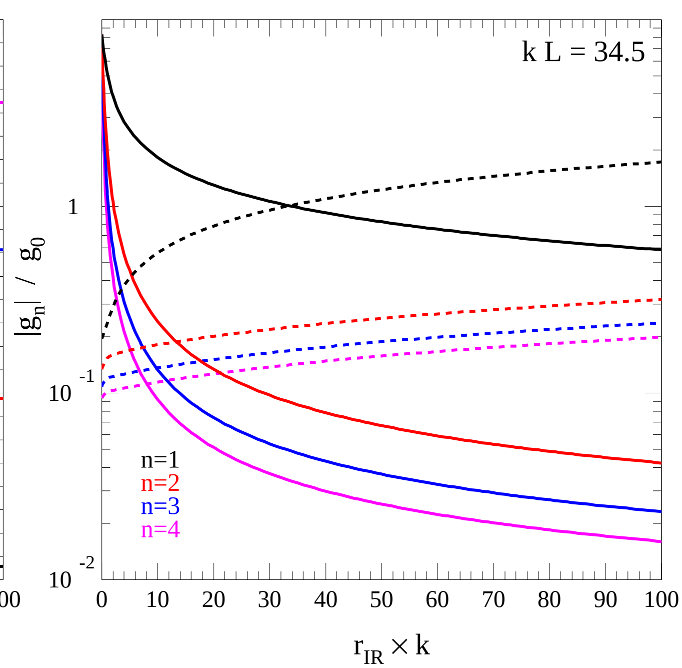
$\Longrightarrow$ zero mode with $g_0 = g_5 / \sqrt{L + r_{IR}}$.

For $p \gg k\, e^{-kL}$, $\qquad G_p(L,L) \simeq -g_5^2 e^{kL} / \left[p(1 + p\, r_{IR}\, e^{kL})\right]$

if $k\, r_{IR} \gtrsim 1 \Longrightarrow$ 4d behavior $G_p(L,L) \simeq -g_5^2 / \left(p^2\, r_{IR}\right)$

- For sufficiently large $k r_{IR}$ there is a mode with mass of order $k\, e^{-kL}$ that couples to the brane with strength $g_1 = \sqrt{\frac{L}{r_{IR}}}\; g_0$ while the other modes decouple.



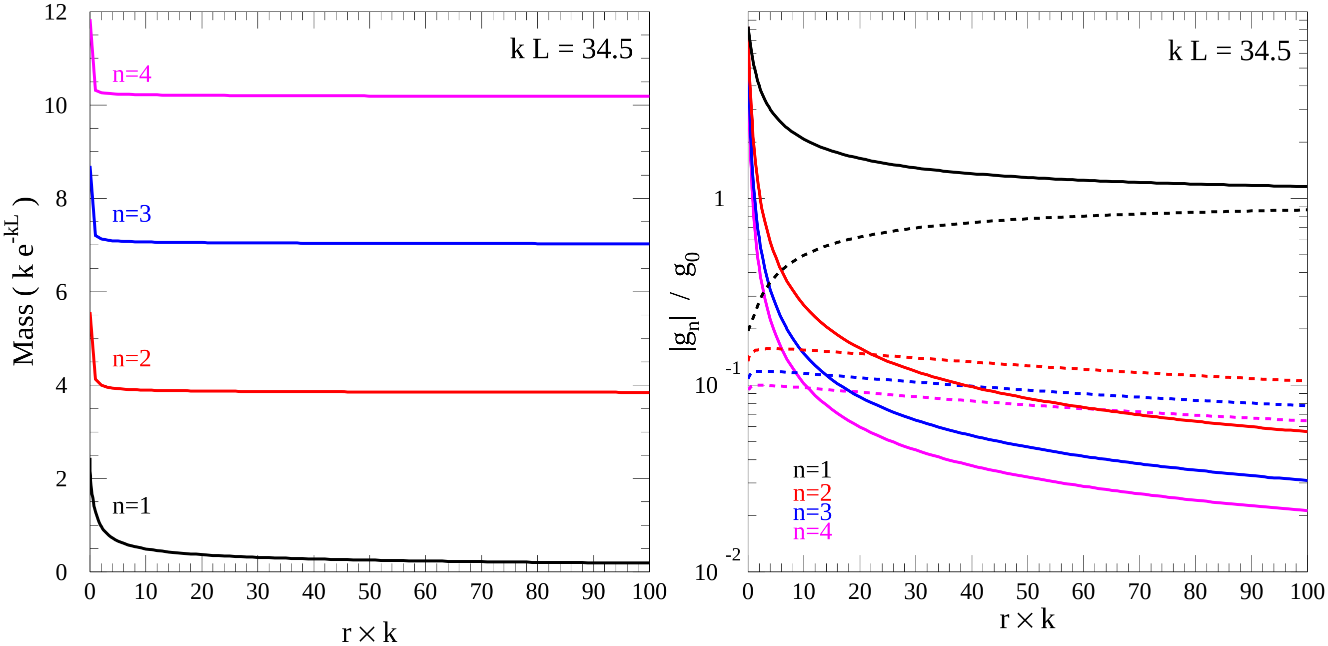


- For $r_{IR} \gg L \Longrightarrow g_1 \ll g_0$: Only the zero mode couples to IR brane matter with $\Longrightarrow g_0^2 \approx g_{IR}^2 = g_5^2 / r_{IR}$

Opaque IR and UV Branes

$r_{IR}, r_{UV} \to \infty \longrightarrow$ an observer on either brane must be insensitive to the extra dimensions + other brane.
$\Longrightarrow$ physics at each brane determined by the local coupling

$\Longrightarrow$ two massless modes should appear, one l.c. of them couples to each brane with local brane coupling strength



$\underline{r_{IR} = r_{UV} = r}$:

• large r: first KK-mode mass $\to 0$ and its coupling becomes equal (and opposite in sign for UV brane fields) to the zero mode one

• $r \to \infty$: bulk propagation switches off $\equiv$ two brane gauge theories which do not interact with each other. The higher modes decouple from both branes.

Localized Higgs Effect

To address the hierarchy problem, the Higgs responsible for EWSB must be localized on the IR brane

In the low energy effective theory after canonical normalization of the Higgs kinetic term

$$-\int d^4x \left\{ \eta^{\mu\nu} (D_\mu H)^\dagger D_\nu H + \lambda \left(|H|^2 - \frac{1}{2} v^2 e^{-2kL} \right)^2 \right\}$$

yielding a localized gauge boson mass prop. to the localized v.e.v., $\tilde{v} = e^{-kL} v$

Effects on the gauge field propagation induced by a Higgs v.e.v. on the IR brane

- Conditions for a diagonal KK decomposition:

$$\frac{1}{g_5^2} \int_0^L dy f_n(y) f_m(y) \left[1 + 2\, r_{UV} \delta(y) + 2\, r_{IR} \delta(y - L) \right] = \delta_{nm}$$

$$\frac{1}{g_5^2} \int_0^L dy e^{-2ky} \left\{ f_n'(y) f_m'(y) + 2\, g_5^2 v^2 \delta(y - L)\ f_n(y) f_m(y) \right\} = m_n^2 \delta_{nm}$$

- Localized mass only affects the b. c. at y=L

$$b^L = - \frac{J_0 \left(\frac{m_n}{k} e^{kL} \right) - \left[m_n\, r_{IR} - g_5^2 \tilde{v}^2 / m_n \right]\ e^{kL} J_1 \left(\frac{m_n}{k} e^{kL} \right)}{Y_0 \left(\frac{m_n}{k} e^{kL} \right) - \left[m_n\, r_{IR} - g_5^2 \tilde{v}^2 / m_n \right]\ e^{kL} Y_1 \left(\frac{m_n}{k} e^{kL} \right)}$$

KK masses (for $r_{UV} = 0$)
$\rightarrow$ determined by $b^0 = b^L$ with $b^0 = -J_0 \left(\frac{m_n}{k} \right) / Y_0 \left(\frac{m_n}{k} \right)$

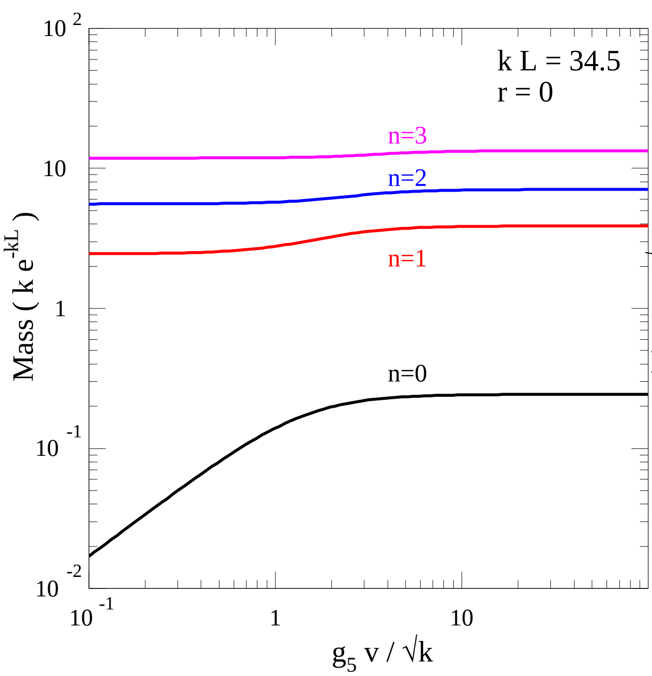
Transparent IR Brane

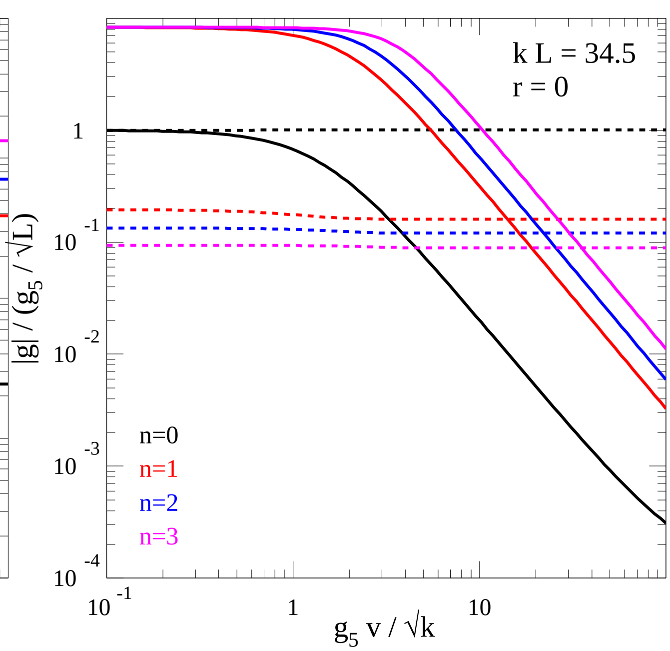
The presence of the v.e.v. tries to induce a repulsion of the gauge field from the brane location

For $v \ll k \Longrightarrow$ zero mode approx. flat with mass $g_5\, \tilde{v}/\sqrt{L}$.

For $v \gg k$

- Zero mode no longer flat with mass more and more insensitive to v as it bends away from the brane.
- KK mode gauge couplings with $m_n/ke^{-kL} > g_5 v/\sqrt{k}$, and zero mode coupling, tend to small values.
- The ratio of the KK mode couplings to the zero mode tends to a constant larger than $\sqrt{2\,k\,L}$ $\quad$ ($v = 0$ value)



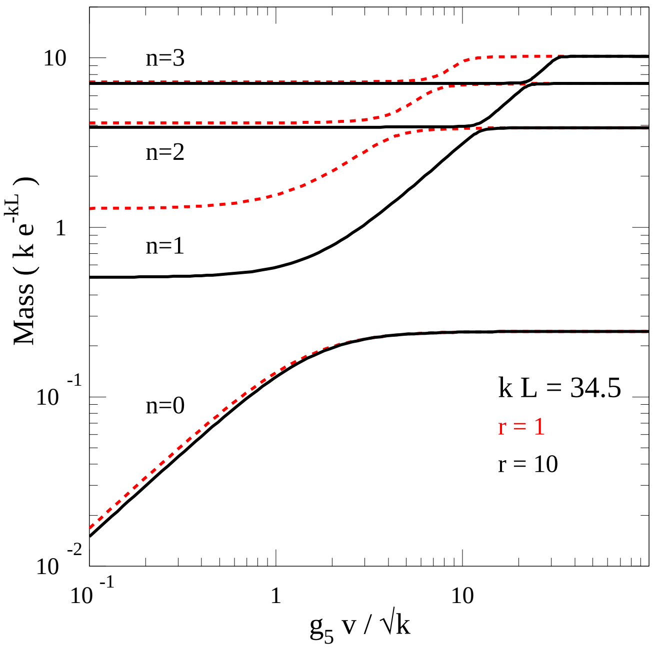


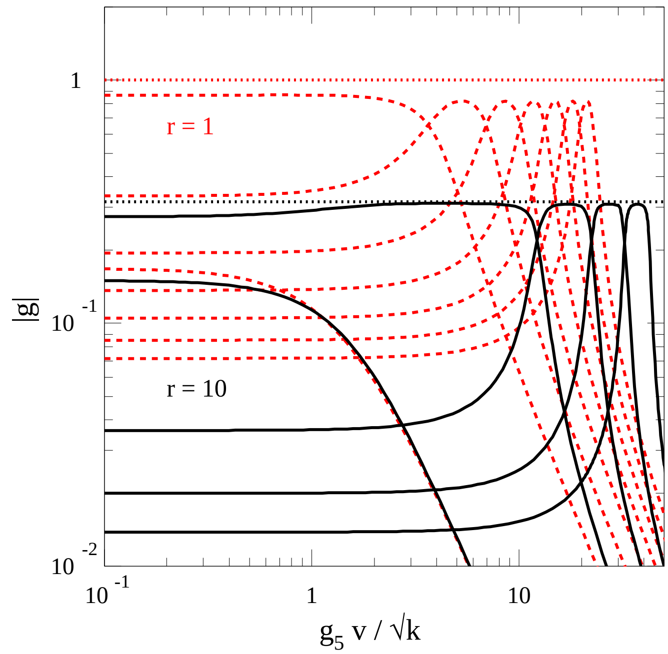
Higgs Field in Opaque IR Brane

From the propagator with endpoints at the IR brane

$$G_p(L,L) \sim -g_5^2/\left(p^2\, r_{IR} + g_5^2 \tilde{v}^2\right) \qquad p \gg k\, e^{-kL} \quad (r_{IR} k > 1)$$

$$G_p(L,L) \sim -g_5^2/\left[p^2\,(L + r_{IR}) + g_5^2 \tilde{v}^2\right] \qquad p \ll k\, e^{-kL}$$





• For $v \ll k$ and $p \ll k\, e^{-kL}$, $\rightarrow$ single state with $g_0 \simeq g_5/\sqrt{L + r_{IR}}$ and $m_0 \simeq g_0 \tilde{v}$, up to correc. of order $g_5^2 v^2/k \Longrightarrow$ zero mode grows linearly with $\tilde{v}$

• If $r_{IR} \gg L$, both propagators describe a single 4d state with mass $g_{IR}\, \tilde{v}$ and coupling g_{IR}.
All other modes decouple.

• For $v/k \gtrsim 1 \rightarrow$ zero mode mass becomes insensitive to v; other modes successively have masses prop. to v. and couplings $g_{IR} = g_5/\sqrt{r_{IR}}$

Electroweak Theory

Higgs and all fermions confined to the IR brane
Gauge fields propagate in the bulk.

After EWSB,

$$\mathcal{L}^5_{EW} = \sqrt{-g}\left\{-\frac{1}{4g_5^2}\mathcal{W}_{MN}\mathcal{W}^{MN}\left(1+2r_2\delta(y-L)\right)-\frac{1}{4{g'_5}^2}\mathcal{B}_{MN}\mathcal{B}^{MN}\left(1+2r_1\delta(y-L)\right) -v^2\delta(y-L)\left[\mathcal{W}^1_M\mathcal{W}^M_1+\mathcal{W}^2_M\mathcal{W}^M_2+\left(\mathcal{W}^3_M-\mathcal{B}_M\right)\left(\mathcal{W}^M_3-\mathcal{B}^M\right)\right]\right\}$$

Introducing the gauge rotations to diagonalize the masses in terms of the bulk couplings,
$s \equiv g'_5/\sqrt{g_5^2+{g'_5}^2}$.

For $r_1 = r_2$, and going to the basis
$\mathcal{W}^3_\mu = c^2\mathcal{Z}_\mu + \mathcal{A}_\mu \quad \mathcal{B}_\mu = -s^2\mathcal{Z}_\mu + \mathcal{A}_\mu$
$\Longrightarrow$photon and Z boson KK towers decouple.

$$\mathcal{L}^5_{EW} = \sqrt{-g}\left\{-\frac{s^2}{2e_5^2}\mathcal{W}^+_{MN}\mathcal{W}^{MN}_-\left[1+2r\delta(y-L)\right]-\frac{1}{4e_5^2}\mathcal{F}_{MN}\mathcal{F}^{MN}\left[1+2r\delta(y-L)\right] -\frac{s^2c^2}{4e_5^2}\mathcal{Z}_{MN}\mathcal{Z}^{MN}\left[1+2r\delta(y-L)\right]-2v^2\delta(y-L)\left(\mathcal{W}^+_M\mathcal{W}^M_-+\frac{1}{2}\mathcal{Z}_M\mathcal{Z}^M\right)\right\}$$

with $1/e_5^2 = 1/g_5^2 + 1/{g'_5}^2 \to$ 5d photon coupling.

• the zero modes will be associated with the weak gauge bosons observed in experiments.

Phenomenology of the AdS$_5$SM

Considering the gauge field decomposition

$$\mathcal{A}^\lambda(X) = \sum_n f_A^n(y) A_n^\lambda(x^\mu), \quad \mathcal{Z}^\lambda(X) = \sum_n f_Z^n(y) Z_n^\lambda(x^\mu),$$
$$\mathcal{W}^{\pm\lambda}(X) = \sum_n f_W^n(y) W_n^{\pm\lambda}(x^\mu),$$

the effective 4D Lagrangian for the zero modes reads:

$$\mathcal{L} = -\tfrac{1}{2} W_{\mu\nu}^+ W_-^{\mu\nu} - \tfrac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - m_W^2 W_\mu^+ W_-^\mu$$
$$- \tfrac{m_Z^2}{2} Z_\mu Z^\mu + \tfrac{1}{\sqrt{2}} f_W \left(\overline{\psi} \gamma^\mu T_+ \psi W_\mu^+ + \overline{\psi} \gamma^\mu T_- \psi W_\mu^- \right)$$
$$+ f_Z \overline{\psi} \gamma^\mu (T_3 - s^2 Q) \psi Z_\mu + f_A \overline{\psi} \gamma^\mu Q \psi A_\mu \ ,$$

• The quantities m_W and m_Z are determined numerically for a given choice of the parameters of the model:
e_5, s, v, r, L, and k
$k \simeq$ 5d Planck scale sets the over-all dimensionful scale

$$m_Z = \frac{e\tilde{v}}{s\,c} (1 - \eta\,\epsilon + \cdots) \qquad m_W = \frac{e\tilde{v}}{s} \left(1 - c^2 \eta\,\epsilon + \cdots\right)$$

where

$$\eta\,\epsilon = \frac{2k^2L^2 - 2kL + 1}{8k(L + r_{IR})} \frac{e^2 v^2}{s^2 c^2 k^2} \ .$$

• f_W, f_Z and f_A → zero mode wave functions at $y = L$,

$$f_A = e_5/\sqrt{L+r} \equiv e$$
$$f_Z = \frac{\sqrt{g_5'^2 + g_5^2}}{\sqrt{L+r}} (1 - 2\eta\epsilon) \equiv (e/sc) \hat{f}_Z = \sqrt{g^2 + g'^2}\, \hat{f}_Z$$
$$f_W = \frac{g_5}{\sqrt{L+r}} (1 - 2c^2\eta\epsilon) \equiv (e/s) \hat{f}_W = g \hat{f}_W$$

The photon experiences no symmetry-breaking
$\rightarrow$ zero mode wave function is flat

Use precisely measured quantities: α_Z, M_Z, G_μ to determine SM electroweak parameters e, $\sin^2\theta_W \equiv s_0$, and $\tilde{v}$.

Determine e_5, s and v from data and leave r, L as free parameters in the fit

- $\alpha_Z^{-1} = 128.92(3)$. yields $e = e_5/\sqrt{L+r}$
- W boson at zero momentum transfer with entire KK tower effects:
  $1/\tilde{v}^2 = 4\sqrt{2}G_\mu \equiv f_W^2/m_W^2 + \sum_{n\neq 0} f_{W_n}^2/m_{W_n}^2$
  with $G_\mu = 1.16639(1) \times 10^{-5}$ GeV^{-2} yields $\tilde{v} \simeq 123$ GeV.
- Having e_5 and v as a function of r, L $\rightarrow$ adjust s so that the Z zero mode mass, $m_Z \equiv M_Z = \frac{e\tilde{v}}{s_0\, c_0} = 91.1875(21)$

This defines s as a function of s_0

$$s = s_0\left(1 - \frac{c_0^2}{c_0^2 - s_0^2}\eta\,\epsilon + \cdots\right)$$

Fit to precision observables $\Longrightarrow$ determine r and L region consistent with data.

Parametrization in terms of Oblique Corrections

Once G_μ, M_Z and α_Z are fixed, all corrections to the zero mode tree-level Lagrangian may be absorbed into corrections to the gauge bosons wave functions, and the W-mass.

$\Longrightarrow$ always possible when fermion couplings corrections are flavor independent $\Longrightarrow$ rescaling fermion interactions to unity redefining the gauge boson fields.

Therefore, although the corrections are at tree-level, they can be parametrized as a function of the same oblique parameters as the one-loop corrections

Tree-level 5d contributions to S, T and U

$$\begin{aligned}
\overline{S} &= \frac{4s^2c^2}{\alpha}\left(1-\frac{1}{\hat{f}_Z^2}\right) = -16 s_0^2 c_0^2 \frac{\eta\epsilon}{\alpha} \\
\overline{T} &= \frac{1}{\alpha}\left(\frac{m_W^2}{c^2 m_Z^2 \hat{f}_W^2} - \frac{1}{\hat{f}_Z^2}\right) - 2 s_0^2 \frac{\eta\epsilon}{\alpha} \\
\overline{U} &= \frac{4s^2}{\alpha}\left[1-\frac{1}{\hat{f}_W^2} - c^2\left(1-\frac{1}{\hat{f}_Z^2}\right)\right] = \mathcal{O}(\epsilon^2)
\end{aligned}$$

Non-Oblique Corrections

Due to non-oblique corrections associated with heavy KK modes → define effective S, T and U parameters to describe the precision electroweak observables.

• Z-pole precision observables properly described introducing an effective parameter, $T_{\rm eff}$, which includes the non-oblique corrections to the weak mixing angle,

$$s^2 - s_0^2 = \frac{\alpha}{c^2 - s^2}\left(\frac{1}{4}S - s^2 c^2 T_{\rm eff}\right) , \qquad (1)$$

where $T_{\rm eff} = T + \Delta T$

$$\Delta T = -\frac{1}{\alpha}\frac{\delta G_\mu}{G_\mu} = -\frac{2c_0^2}{\alpha}\eta\,\epsilon + \mathcal{O}(\epsilon^2)$$

δG_μ → non-oblique contribution from KK mode exchange.
This definition of $T_{\rm eff}$ recovers the relation between s and s_0 derived from M_z

• The non-oblique corrections to G_μ affect also the expression of m_W/m_Z, but with a relative coefficient between the oblique and non-oblique corrections encoded in $T_{\rm eff}$ different from the one in s^2.

To properly parameterize m_W^2/m_Z^2 in terms of effective parameters $S_{\rm eff} = S$, $T_{\rm eff}$ and $U_{\rm eff}$

$$\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha c^2}{c^2 - s^2}\left(-\frac{1}{2}S_{\rm eff} + c^2 T_{\rm eff} + \frac{c^2 - s^2}{4s^2}U_{\rm eff}\right) ,$$

one can introduce $U_{\rm eff} = U - 4s^2 \Delta T$.

The above parameterization serves to describe all Z-pole observables as well as the W mass.

The tree-level five-dimensional contributions, including the non-oblique corrections to G_μ give:

$$\overline{S}_{\rm eff} = -\frac{16 s_0^2 c_0^2}{\alpha} \eta\,\epsilon + \ldots \approx -366\,\eta\,\epsilon\ , \quad (2)$$

$$\overline{T}_{\rm eff} = -\frac{2}{\alpha} \eta\,\epsilon + \ldots \approx -258\,\eta\,\epsilon\ , \quad (3)$$

$$\overline{U}_{\rm eff} = \frac{8 s_0^2 c_0^2}{\alpha} \eta\,\epsilon + \ldots \approx 183\,\eta\,\epsilon\ , \quad (4)$$

We see that $\overline{S}_{\rm eff}$ and $\overline{T}_{\rm eff}$ are negative while $\overline{U}_{\rm eff}$ is positive (and not small).

The full $S_{\rm eff}$, $T_{\rm eff}$, and $U_{\rm eff}$ is given as the sum of the extra dimensional contributions, and the Higgs contributions

$$S_H \simeq \frac{1}{12\pi} \log\left(\frac{m_h^2}{m_{ref}^2}\right) \quad (5)$$

$$T_H \simeq -\frac{3}{16\pi c_0^2} \log\left(\frac{m_h^2}{m_{ref}^2}\right) \quad (6)$$

$$U_H \simeq 0 \quad (7)$$

Comparison with Data

We consider two different fits to the precision electroweak data:

- based on the analysis of the whole available SLD/LEP and Tevatron data

$$\begin{aligned} S &= 0.00 \pm 0.11 \\ T &= -0.03 \pm 0.13 \\ U &= 0.27 \pm 0.14 \ . \end{aligned} \tag{8}$$

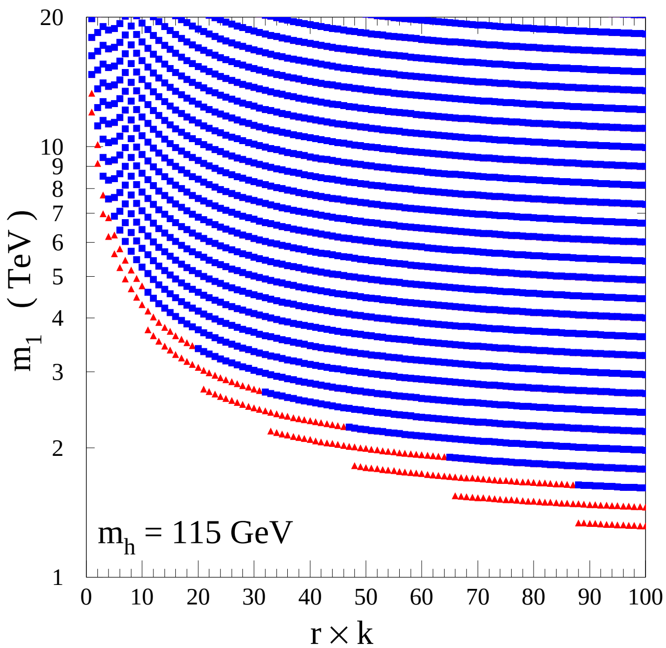
- ignoring the b-forward backward asymmetry, A^b_{FB},

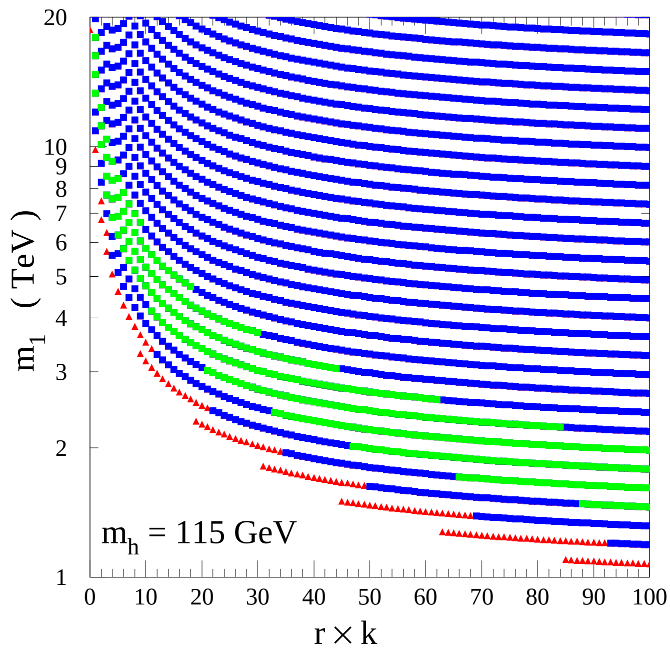
$$\begin{aligned} S &= -0.14 \pm 0.12 \\ T &= -0.08 \pm 0.13 \\ U &= 0.20 \pm 0.14 \ , \end{aligned} \tag{9}$$

Note: weak mixing angle extracted from lepton asymmetries differs by more than 3 σ from its value from the hadron asymmetries.

m_H from the EW fit close to its experimental limit only by virtue of A^b_{FB} (measured A^b_{FB}, about 2.6 away from SM prediction). Removing the hadron asymm. from the fit $\rightarrow$ considerably lower values of m_H.

Fit to precision Data

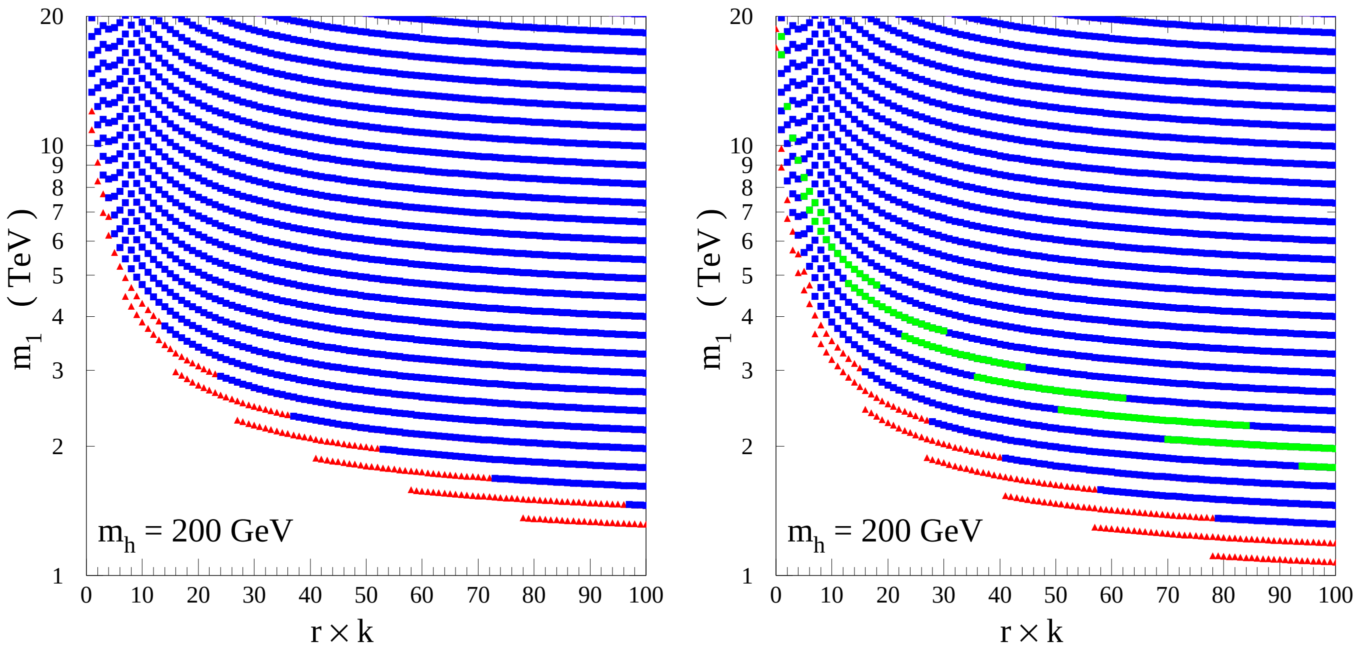




The regions of 1σ (green), 2σ (blue) and 3σ (red) agreement with precision electroweak observables, in the plane $r \times k$ and the first KK photon mass for all precision data in the fit (left) and only leptonic determinations of $\sin^2\theta_W$ (right).

- For $r \to 0 \Longrightarrow m_1 \gtrsim 20$ TeV.
- As r increases $\Longrightarrow$ a first KK photon with fermionic couplings of the order of the zero mode couplings, and a mass of a few TeV may appear
- For $r \times k \sim 20$ and $m_1 \sim 3.5$ TeV $\Longrightarrow$ fit to the entire data set as good as in the SM, and the fit to the data without A^b_{FB} consistent within 1σ (improvement over SM fit!)
- Varying $m_h = 200$ GeV $\to$ small variation

Fit to precision Data



The regions of 1σ (greens), 2σ (blue) and 3σ (red) agreement with precision electroweak observables, in the plane $r \times k$ and the first KK photon mass for all precision data in the fit (left) and only leptonic determinations of $\sin^2\theta_W$ (right).

Conclusions

– Extra Dimensions with Bulk Gauge Fields open many interesting possibilities, including grand unification

– Gauge field kinetic terms in the brane $\rightarrow$ drastic consequences for the KK mode spectrum and couplings:

• Decoupling of the heavier KK gauge bosons from the matter field in the brane

• The effective coupling in the Ultraviolet regime is controlled by the local brane coupling.

– Warped extra dimensions with gauge brane kinetic terms

• allow for lightest KK gauge bosons of a few TeV.

• If fermion fields also propagate in the bulk and one allows for moderate UV and IR brane kinetic terms

Gauge Coupling Unification possible

with lightest KK mode of order a few TeV.