Electroweak Precision Tests at GigaZ

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1. Introduction

2. Precision Tests of the SM

3. Precision Tests of the MSSM

4. Conclusions

1. Introduction

Definition:

LC at low energies: high luminosity
$\sim 10^9$ Z bosons/year
$\sim 10^6$ W pairs/year

} **GigaZ**

→ use the Z bosons to measure the left-right asymmetry:

$$A_{LR}\ (++,+-,-+,--)$$

use A_{LR} to determine the

effective leptonic mixing angle: $\delta \sin^2\theta_{\text{eff}} \approx 1\times 10^{-5}$

→ use the W bosons (at threshold) to measure the

W boson mass: $\delta M_W \approx 6$ MeV

Expected accuracies:

	LEP2/Tev.	LHC	LC	GigaZ
M_W	30 MeV	15 MeV	15 MeV	6 MeV
$\sin^2\theta_{\text{eff}}$	0.00017	0.00017	0.00017	0.00001
m_t	3 GeV	2 GeV	0.2 GeV	0.2 GeV
m_h	?	0.2 GeV	0.05 GeV	0.05 GeV

M_W determination at e^+e^- colliders

threshold measurement of

$$e^+e^- \to WW \to 4f(+\gamma)$$

Status of theoretical prediction for total cross section in the SM:

above threshold: $\pm 0.5\%$ with DPA
[A. Denner, S. Dittmaier, M. Roth, D. Wackeroth '00]

at threshold:
$\pm 2\%$ with leading log calculation $\to$ F

However: Only shape of XS in threshold region needed for M_W measurement
Larger error on absolute value tolerable

DPA not applicable in threshold region
$\Rightarrow$ full $\mathcal{O}(\alpha)$ calculation necessary

full $\mathcal{O}(\alpha)$ calculation in whole parameter space:
first results for simplest case: work in progress
[A. Vicini '01]

Sudakov logs:
$\sim 0.5\%$ at $\sqrt{s} = 500$ GeV
$\Rightarrow$ probably no problem in threshold region

Precision Observables (POs):

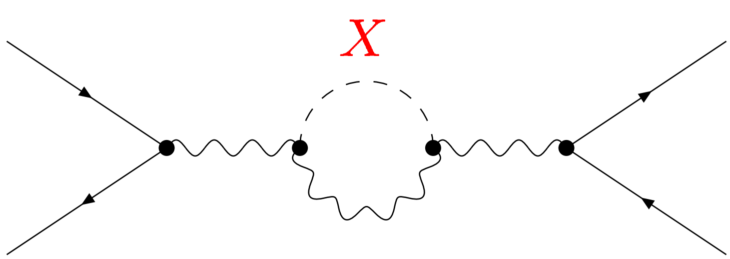
Comparison of electro-weak precision observables with theory:

EW Precision data:		Theory:
$\Delta\rho, \Delta r, M_W, \sin^2\theta_{\rm eff}$	$\leftrightarrow$	SM, MSSM , . . .

$\Downarrow$

Test of theory at quantum level:

Sensitivity to loop corrections



$\Downarrow$

Improve indirect constraints on unknown parameters: $m_h, m_{\tilde{t}}, \ldots$

Effects of "new physics"?

Theory: Precision observables M_W, $\sin^2\theta_{\rm eff}$:

1.) Theoretical prediction for M_W in terms of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\,\alpha}{\sqrt{2}\,G_\mu}(1 + \Delta r)$$

$\Updownarrow$

loop corrections

SM prediction for Δr, one-loop:

[*A. Sirlin '80*] [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\rm loop} = \quad \Delta\alpha \quad - \quad \frac{c_w^2}{s_w^2}\Delta\rho \quad + \quad \Delta r_{\rm rem}(m_h)$$

$$\sim \log\frac{M_Z}{m_f} \qquad \sim m_t^2$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

2.) Leptonic effective weak mixing angle:

$$\sin^2\theta_{\rm eff} = \frac{1}{4\,|Q_f|}\left(1 - \frac{\mathrm{Re}\,g_V^f}{\mathrm{Re}\,g_A^f}\right)$$

Higher order contributions are contained in

$$\Delta g_V^f, \quad \Delta g_A^f$$

Further available corrections in the SM:

- QCD corrections up to $\mathcal{O}(\alpha\alpha_s^2)$
  [*A. Djouadi, C. Verzegnassi '87*]
  [*F. Halzen, B. Kniehl '91*]
  [*K. Chetyrkin, J. Kühn, M. Steinhauser '95*]
  [*L. Avdeev, J. Fleischer, S. Mikhailov, O. Tarasov '95*]

- electroweak two-loop corrections

  – expansion in leading powers of m_t, M_H
  [*J. van der Bij, M. Veltman, '84*]
  [*J. van der Bij, F. Hoogeveen '87*]
  [*R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere '92*]
  [*J. Fleischer, O.V. Tarasov, F. Jegerlehner '93*]
  [*G. Degrassi, P. Gambino, A. Vicini '98*]
  [*G. Degrassi, P. Gambino, A. Sirlin '98*]

  – complete fermionic two-loop result for M_W
  [*A. Freitas, S.H., W. Hollik, W. Walter, G.W. '00*]

- leading electroweak three-loop terms
  $\mathcal{O}\left(G_F^3 m_t^6\right)$, $\mathcal{O}\left(G_F^2 \alpha_s m_t^4\right)$
  [*J. van der Bij, K. Chetyrkin, M. Faisst, G. Jikia, T. Seidensticker '00*]

Available corrections in the MSSM:

- full one-loop corrections to Δr
  [*P. Chankowski, A. Dabelstein, W. Hollik, W. Mösle, S. Pokorski, J. Rosiek '94*]
  [*D. Garcia, J. Solà '94*]

- Z-boson observables, one-loop
  [*D. Garcia, R. Jiménez, J. Solà '95*]
  [*D. Garcia, J. Solà '95*]
  [*A. Dabelstein, W. Hollik, W. Mösle '95*]
  [*P. Chankowski, S. Pokorski '96*]

- leading $\mathcal{O}(\alpha\alpha_s)$ corrections
  [*A. Djouadi, P. Gambino, S. H., W. Hollik, C. Jünger, G.W. '97*]
  [*S. H, W. Hollik, G.W. '98*]

- leading $\mathcal{O}\left(G_F^2 m_t^4\right)$ corrections ($M_{\mathrm{SUSY}} \to \infty$)
  [*S. H, G.W. '01*]

Interpretation in the SM:

Indirect determination of m_t from precision data:

$$m_t = 172^{+14}_{-11} \text{ GeV}$$

direct measurement:

$$m_t = 174.3 \pm 5.1 \text{ GeV}$$

One-loop corrections to precision observables:

$$\sim m_t^2$$
$$\sim \log m_h$$

$\Rightarrow$ Very high accuracy of measurements and theoretical predictions needed

Theoretical uncertainties:

– unknown higher-order corrections

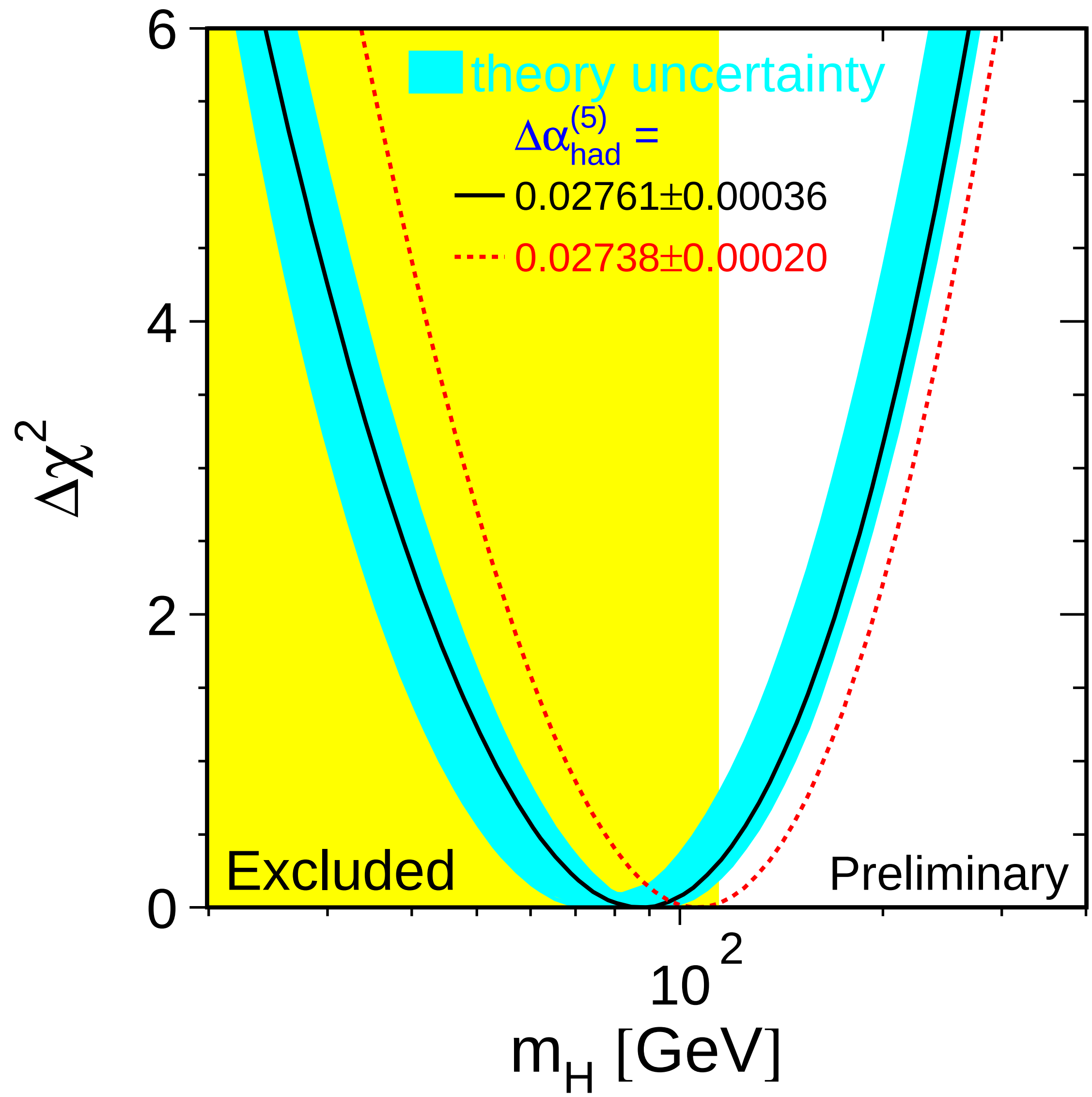
– exp. error of input parameters: m_t, $\Delta\alpha_{\text{had}}$, ...

Todays theoretical uncertainty:

$$\delta M_W^{\text{theo}} \approx 6 \text{ MeV}$$
$$\delta \sin^2\theta_{\text{eff}}^{\text{theo}} \approx 0.00007$$

Global fit to all data in the SM:

[*LEPEWWG '01*]



What if SUSY will be discovered?

Assume: SUSY particles accessible at next generation of colliders

Exact SUSY: $m_f = m_{\tilde{f}}, \ldots$

$\Rightarrow$ SUSY must be broken: $M_{\text{SUSY}} \lesssim 1$ TeV

SUSY breaking mechanism not well understood; different models for SUSY breaking:

"Hidden sector": SUSY breaking	$\longrightarrow$	Visible sector: MSSM

"Gravity-mediated": mSUGRA

"Gauge-mediated": GMSB

"Anomaly-mediated": AMSB

. . .

Unconstrained MSSM:

no particular SUSY breaking mechanism assumed, parameterization of possible SUSY-breaking terms

$\Rightarrow$ 105 new parameters: masses, mixing angles, phases

Current situation:

– Good phenomenological description for universal breaking terms

– no preference for certain SUSY-breaking scenario

Possible future situation:

Measurements of SUSY observables

and improved precision for $M_W, \sin^2\theta_{\text{eff}}, \ldots$:

⇒ Determination of SUSY parameters

⇒ Patterns of SUSY breaking (new high scales)

Role of PO's:

Indirect information from precision observables

constraints on unaccessible scales

→ complementary to direct production processes

⇒ Sensitive test of the theory at the quantum level

Stringent direct test of SUSY:

Light Higgs boson h required

Tree level: $m_h < M_Z$

Yukawa couplings: $\frac{e\, m_t}{2 M_W s_w}, \frac{e\, m_t^2}{M_W s_w}, \ldots$

Dominant corrections to m_h from $t - \tilde{t}$-sector :
Leading one-loop term:

$$\Delta m_h^2 \sim \frac{m_t^4}{M_W^2} \log\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right)$$

Two-loop result (*FeynHiggs*):
[*S.H., W. Hollik, G.W. '99*]

$$\Rightarrow m_h \lesssim 135 \text{ GeV}$$

High-precision measurement of m_h:

$$\begin{aligned} \text{LHC}: &\quad \delta m_h \approx 0.2 \text{ GeV} \\ \text{LC}: &\quad \delta m_h \approx 0.05 \text{ GeV} \\ \text{Muon Collider}: &\quad \delta m_h \approx 0.1 \text{ MeV} \end{aligned}$$

$\Rightarrow m_h$ will be precision observable!

SM:

free parameter: m_h

$\rightarrow$ direct contribution via loop effects

$\Rightarrow$ Prediction for precision observables
M_W, $\sin^2\theta_{\text{eff}}$

MSSM:

free parameters: $M_{\text{SUSY}}, X_t, \ldots$

$\rightarrow$ direct contribution via loop effects

$\rightarrow$ indirect effect via contribution to m_h

$\Rightarrow$ Prediction for precision observables
M_W, $\sin^2\theta_{\text{eff}}$

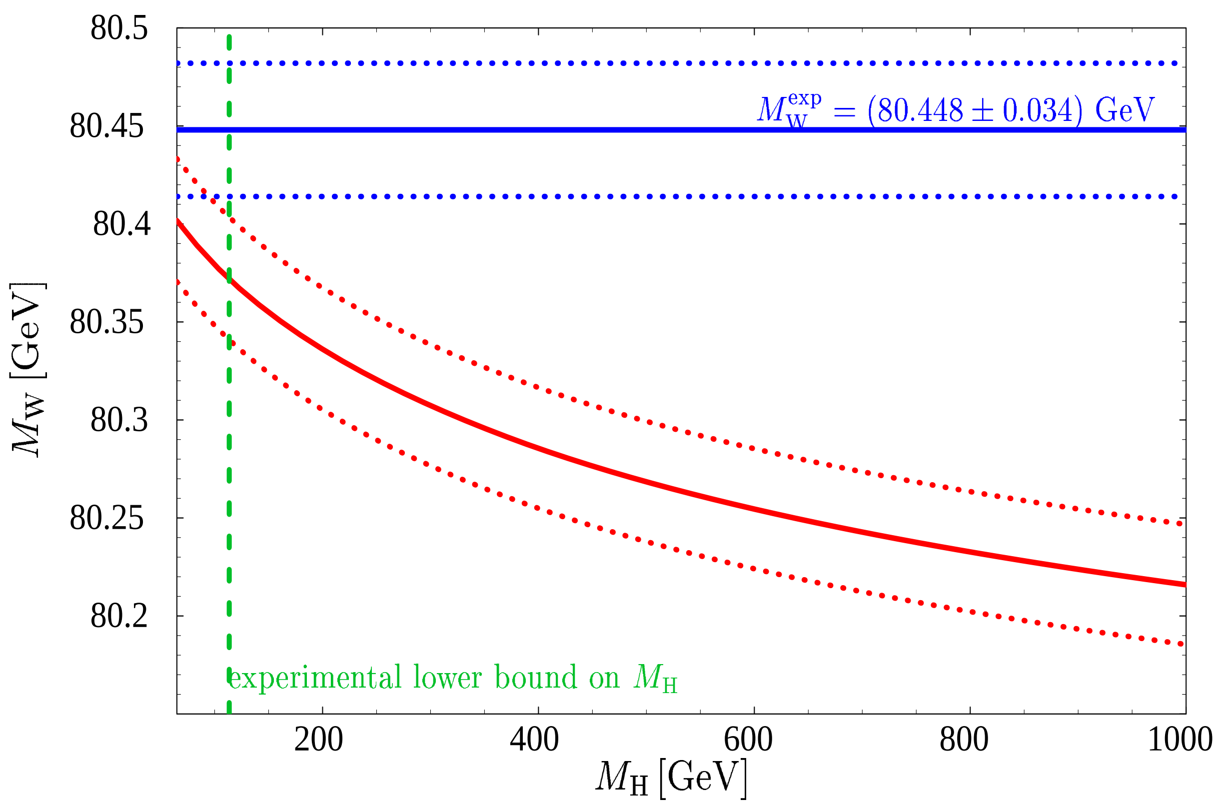
Prediction for precision observable m_h

2. Precision tests of the SM:

Present situation:

SM prediction for M_W vs. experimental result:

[*A. Freitas, S.H., W. Hollik, W. Walter, G.W. '00*]



$\Rightarrow$ no overlap at the 1 σ level

$\Rightarrow$ light Higgs preferred

Theoretical uncertainty dominated by uncertainty in input parameters: m_t, M_H, . . .

$\delta m_t = 5.1$ GeV $\Rightarrow$ $\delta M_W \approx 31$ MeV

Future theoretical uncertainties:

Parametric uncertainties from experimental errors of input parameters:

$$\begin{aligned}
\delta m_t &= \pm 130 \text{ MeV} & \Rightarrow \delta M_W &\approx \pm 1 \text{ MeV} \\
& & \delta \sin^2\theta_{\text{eff}} &\approx \pm 0.4 \times 10^{-5} \\
\delta m_h &= \pm 50 \text{ MeV} & & \\
\delta \alpha_s &= \pm 1 \times 10^{-3} & & \\
\delta M_Z &= \pm 2.1 \text{ MeV} & \Rightarrow \delta M_W &\approx \pm 2.5 \text{ MeV} \\
& & \delta \sin^2\theta_{\text{eff}} &\approx \pm 1.4 \times 10^{-5} \\
\delta(\Delta\alpha) &= \pm 5 \times 10^{-5} & \Rightarrow \delta M_W &\approx \pm 1 \text{ MeV} \\
& & \delta \sin^2\theta_{\text{eff}} &\approx \pm 1.8 \times 10^{-5}
\end{aligned}$$

$\Rightarrow$ Error of M_Z non-negligible!
(measurement of $M_W/M_Z \Rightarrow \delta M_W \approx \pm 1$ MeV)

Estimate for future theoretical uncertainties:

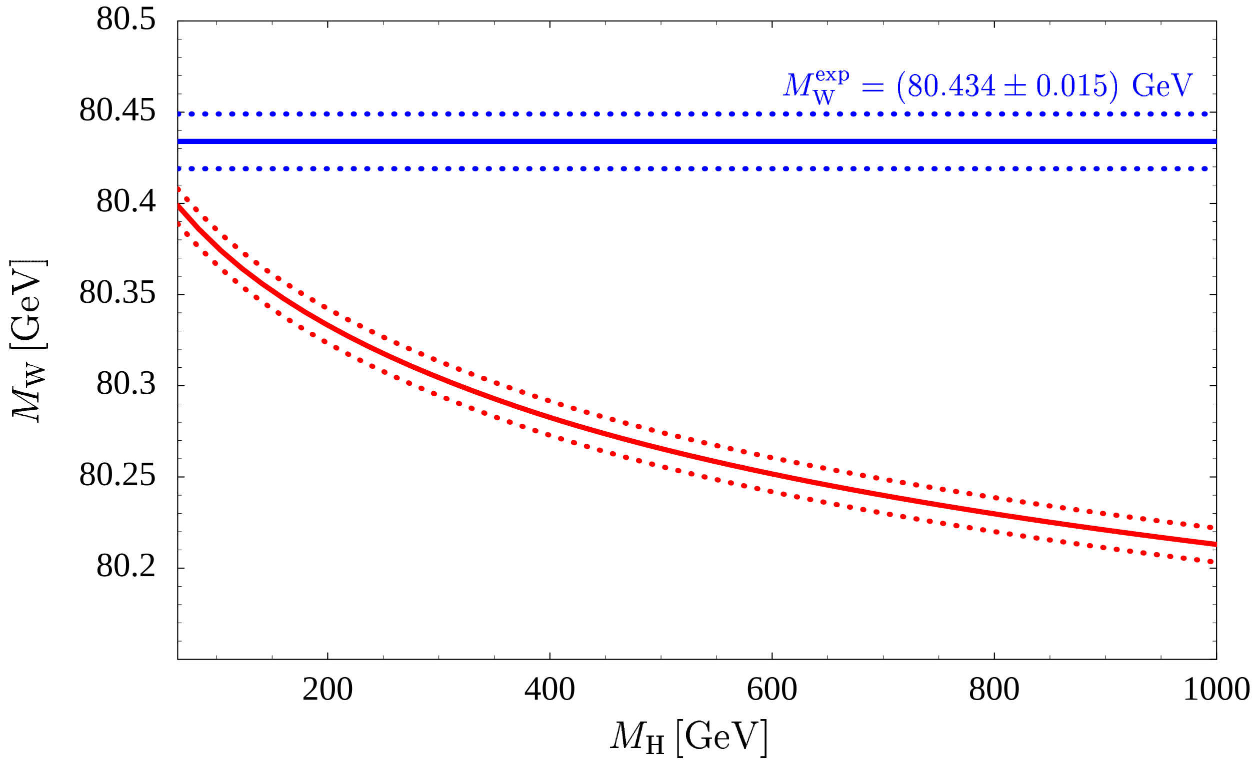
– unknown higher-order corrections
– $\delta(\Delta\alpha)$ uncertainty

$$\delta M_W^{\text{theo}} \approx \pm 3 \text{ MeV}, \quad \delta \sin^2\theta_{\text{eff}}^{\text{theo}} \approx \pm 3 \times 10^{-5}$$

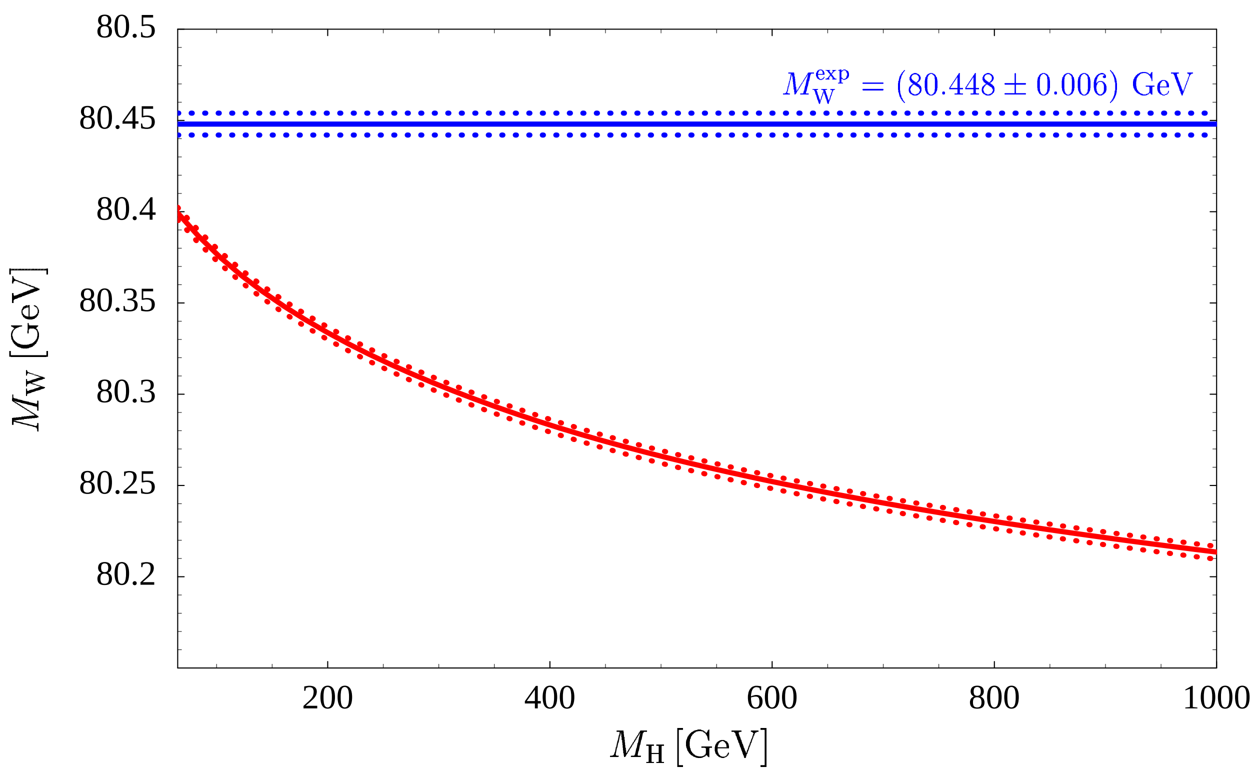
$M_W(m_h)$ in the SM:

prospective future accuracies:

LHC precision ($\delta M_W \approx 15$ MeV, $\delta m_t \approx 1.5$ GeV):



GigaZ precision ($\delta M_W \approx 6$ MeV, $\delta m_t \approx 0.15$ GeV):

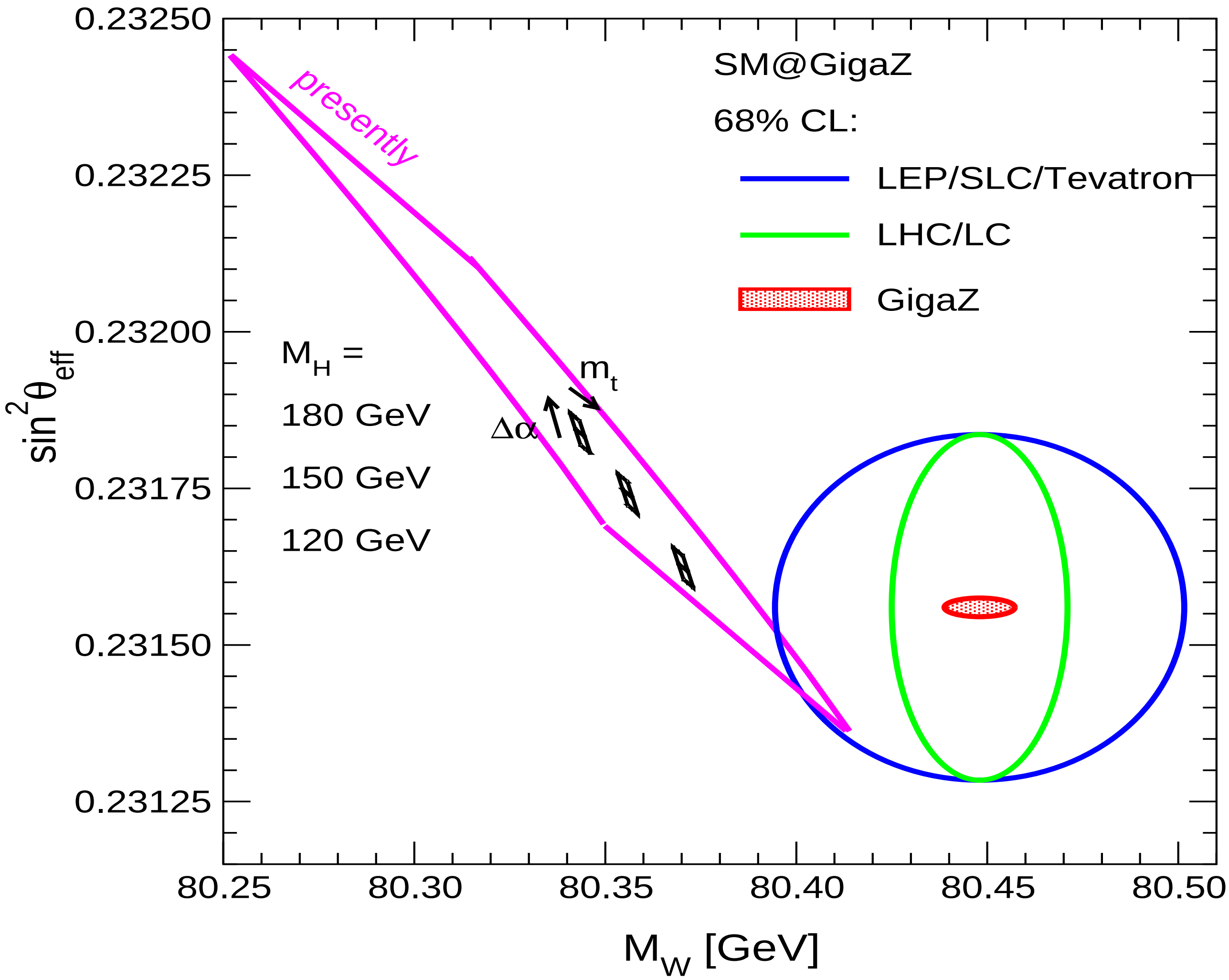


SM prediction for M_W and $\sin^2\theta_{\rm eff}$

vs. current and prospective accuracies at

LEP2/SLD/Tevatron , LHC/LC , GigaZ:

[J. Erler, S.H., W. Hollik, G.W. and P. Zerwas '00]

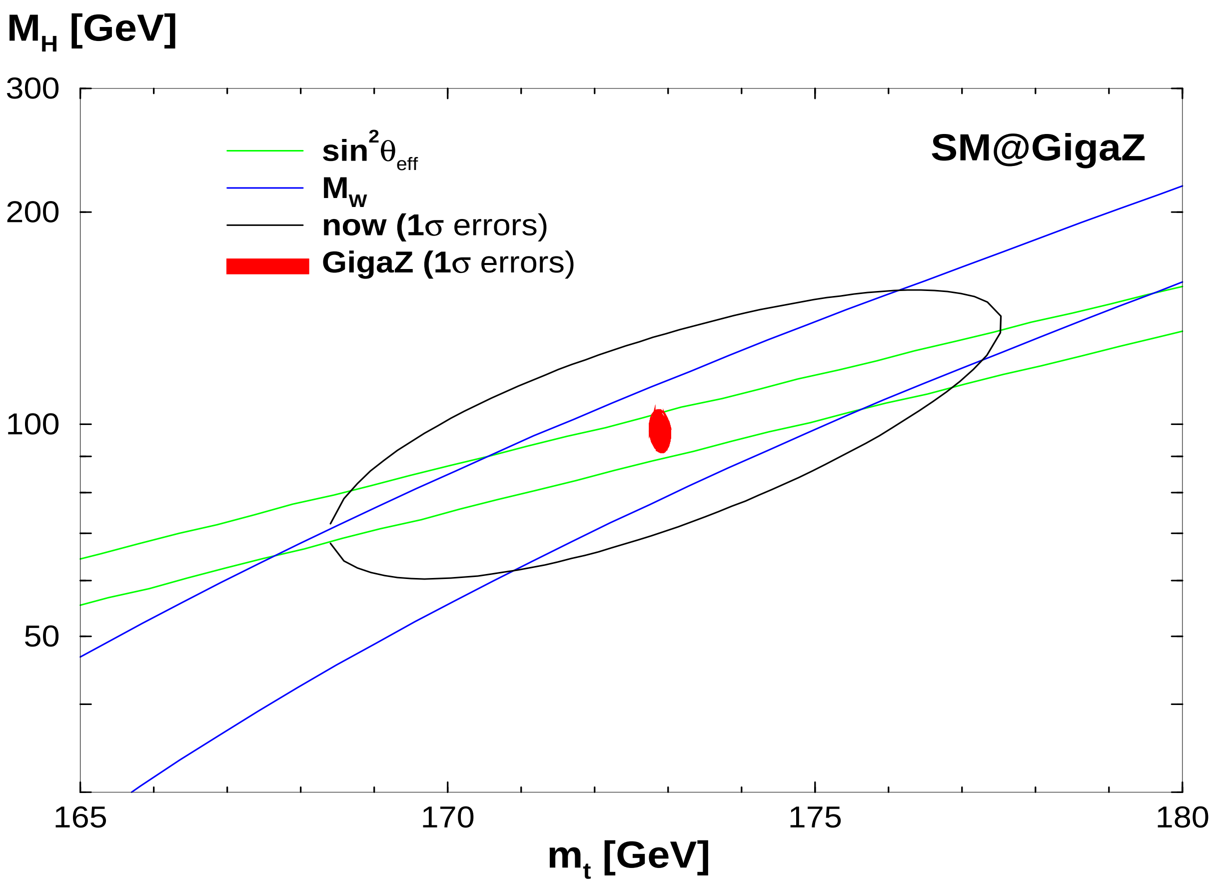


$\Rightarrow$ test of the SM with very high precision

high sensitivity to new physics scales

Indirect determination of m_t, M_H at GigaZ:

[J. Erler, S.H., W. Hollik, G.W. and P. Zerwas '00]



Expected precisions of indirect M_H-determinations:

[J. Erler, S.H., W. Hollik, G.W. and P. Zerwas '00]

	M_W	$\sin^2\theta_{eff}$	all
now	200 %	62 %	60 %
Tevatron Run IIA	77 %	46 %	41 %
Tevatron Run IIB	39 %	28 %	26 %
LHC	28 %	24 %	21 %
LC	18 %	20 %	15 %
GigaZ	12 %	7 %	7 %

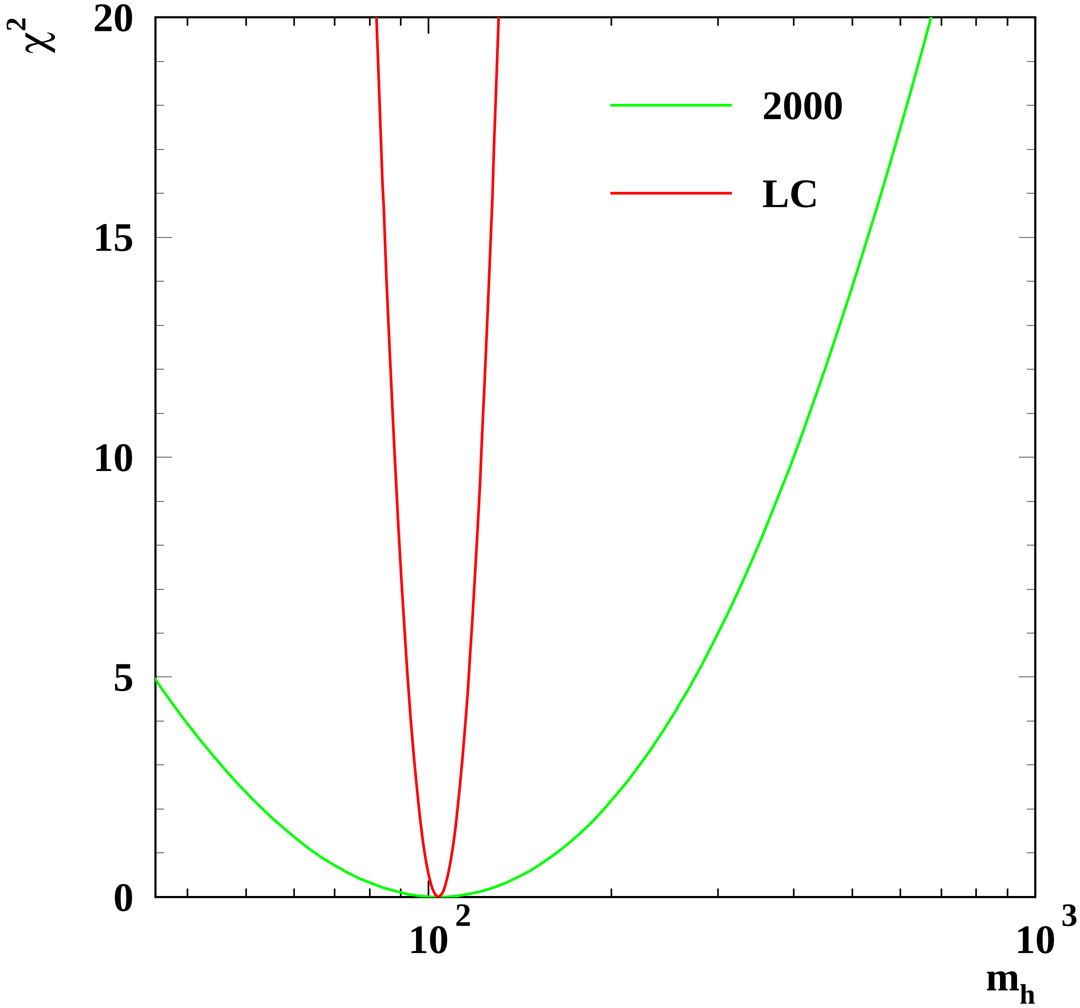
New "Blue band" plot:

Fit for the SM Higgs boson mass

Today → GigaZ

(neglected theoretical uncertainty)

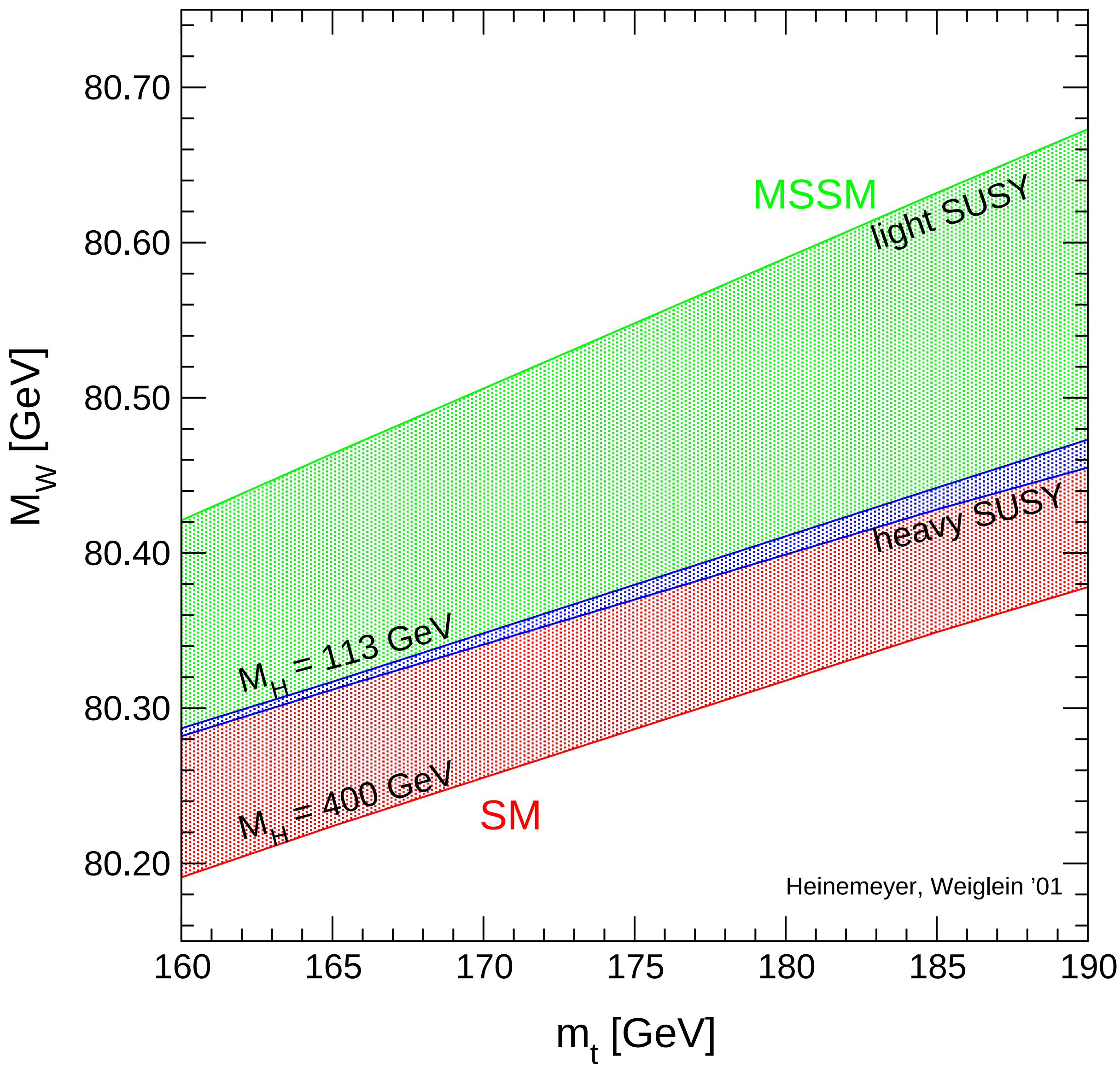
[*R. Hawkings and K. Mönig '00*]



3. Precision tests of the MSSM

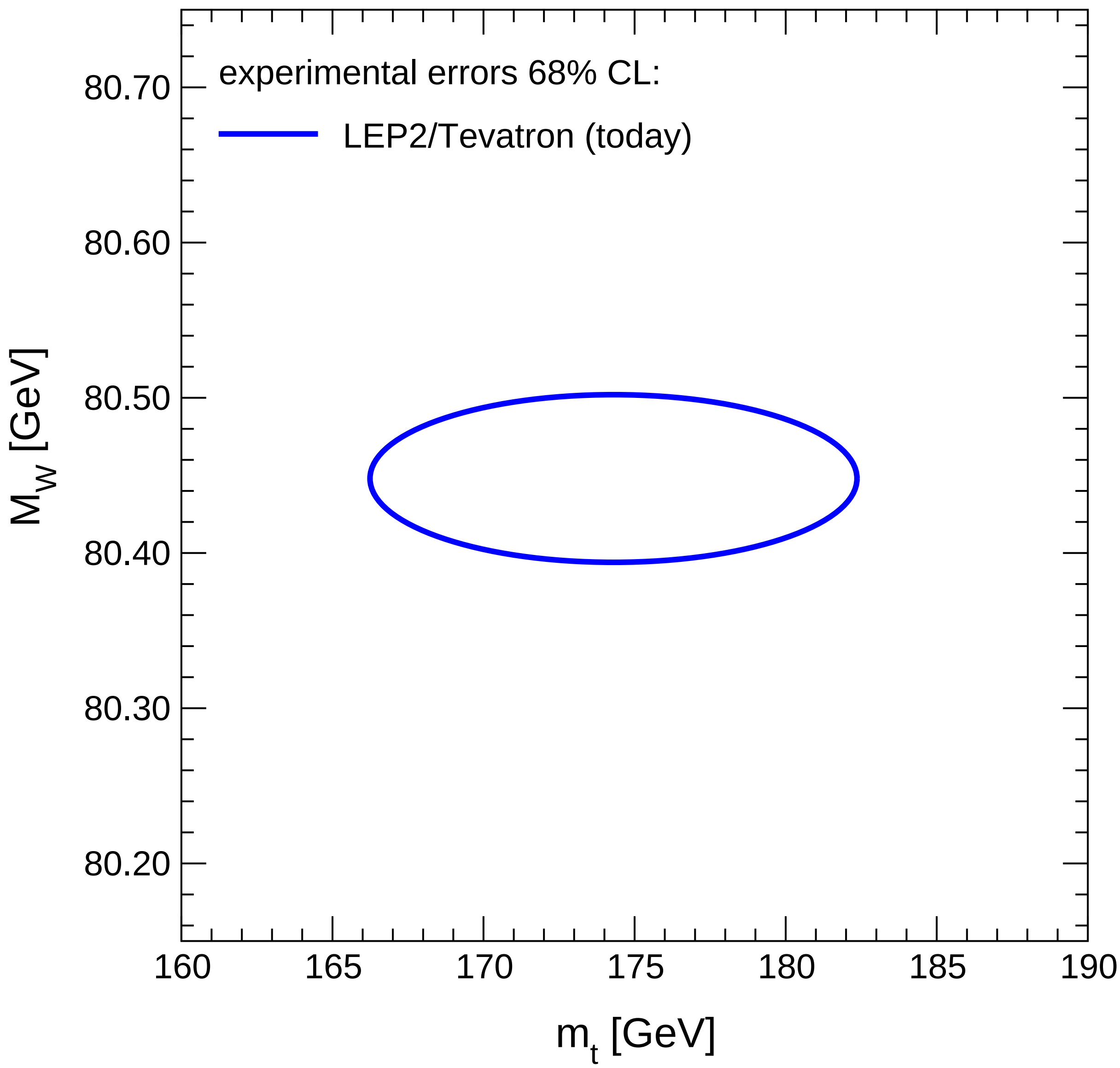
Prediction for M_W in the SM and the MSSM :

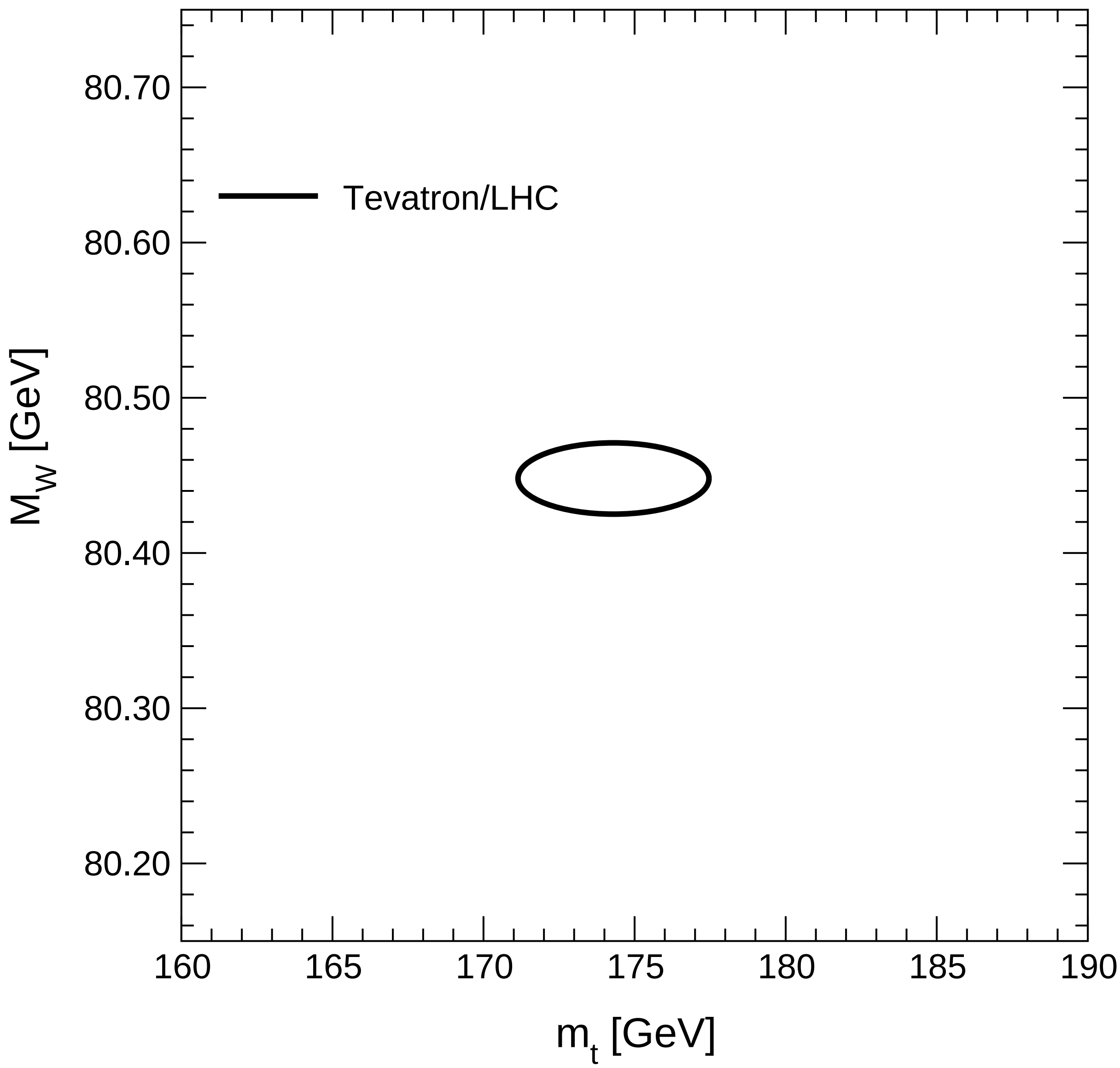
[*S.H., G.W. '01*]

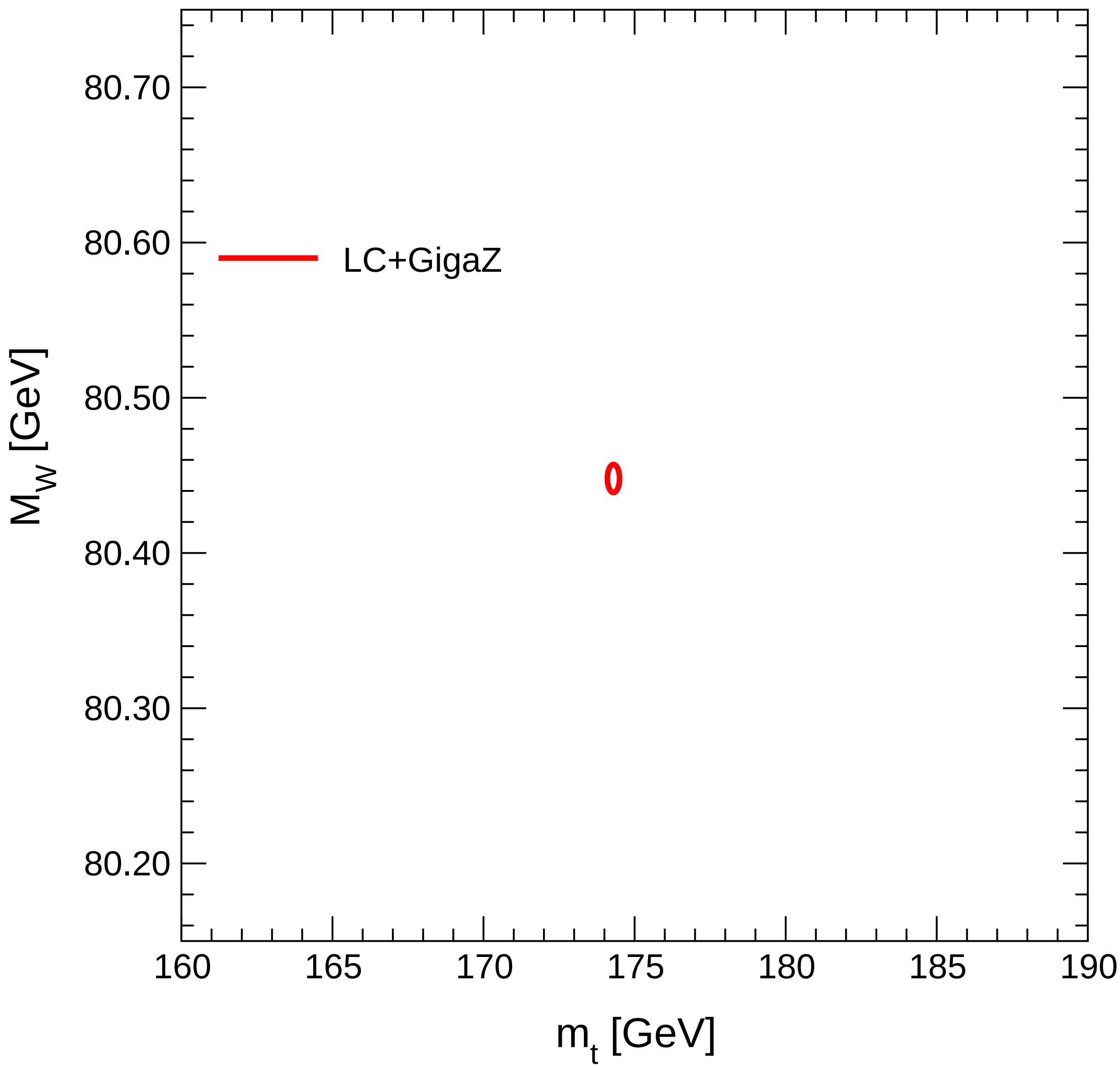


SM uncertainty: unknown Higgs mass

MSSM uncertainty: unknown masses of SUSY particles


experimental errors 68% CL:
LEP2/Tevatron (today)
80.70
80.60
80.50
80.40
80.30
80.20
M_W [GeV]
160
165
170
175
180
185
190
m_t [GeV]


Tevatron/LHC
80.70
80.60
80.50
80.40
80.30
80.20
M_W [GeV]
160
165
170
175
180
185
190
m_t [GeV]

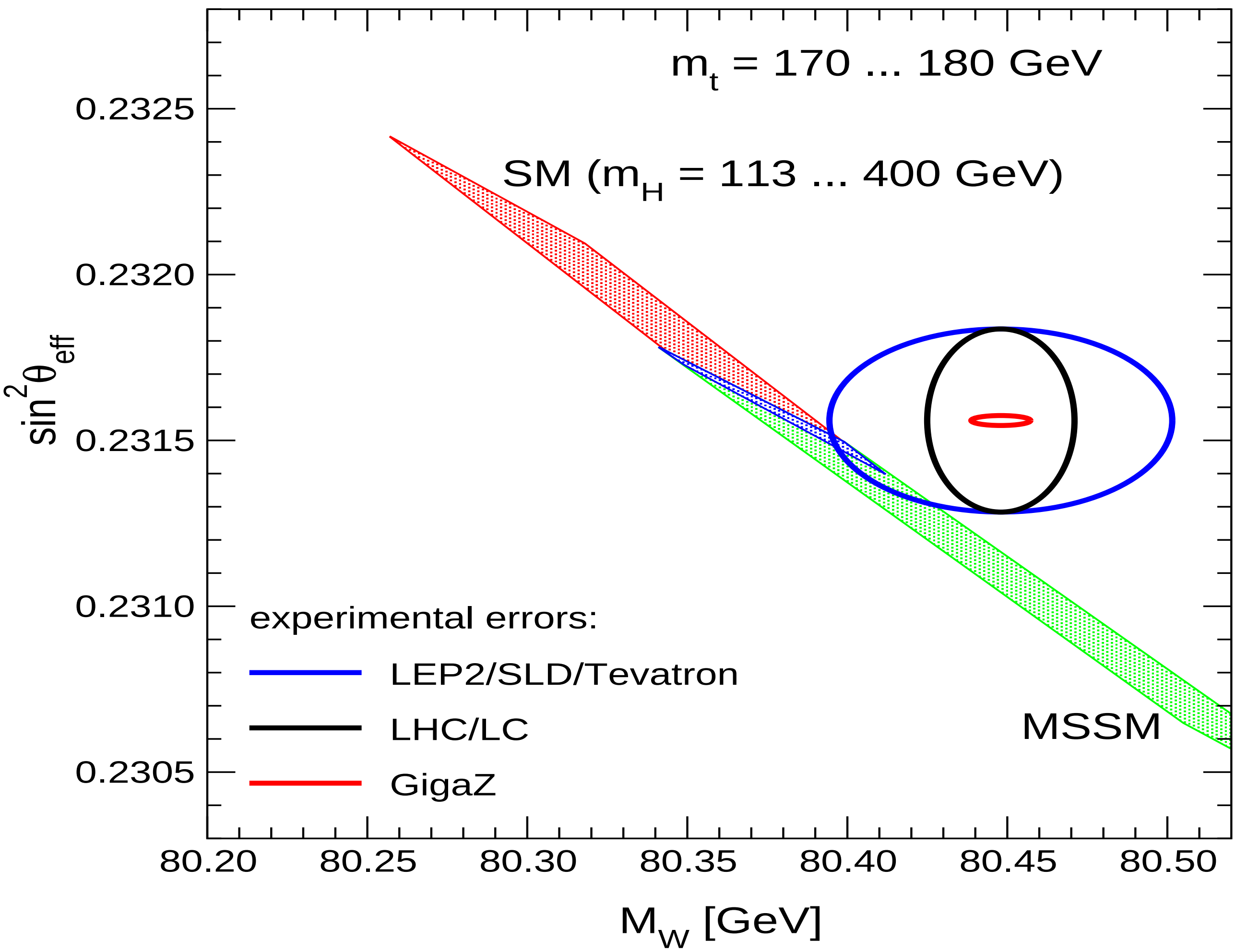

LC+GigaZ
80.70
80.60
80.50
80.40
80.30
80.20
160
165
170
175
180
185
190
M_W [GeV]
m_t [GeV]

Prediction for M_W, $\sin^2\theta_{\text{eff}}$ in the SM and MSSM

vs. prospective accuracies at

LEP2/SLD/Tevatron , LHC/LC, GigaZ:

[*S.H., G.W. '01*]



⇒ large improvement of experimental accuracy:

LEP2/SLD/Tevatron → LHC/LC → GigaZ

⇒ Very sensitive test of theory

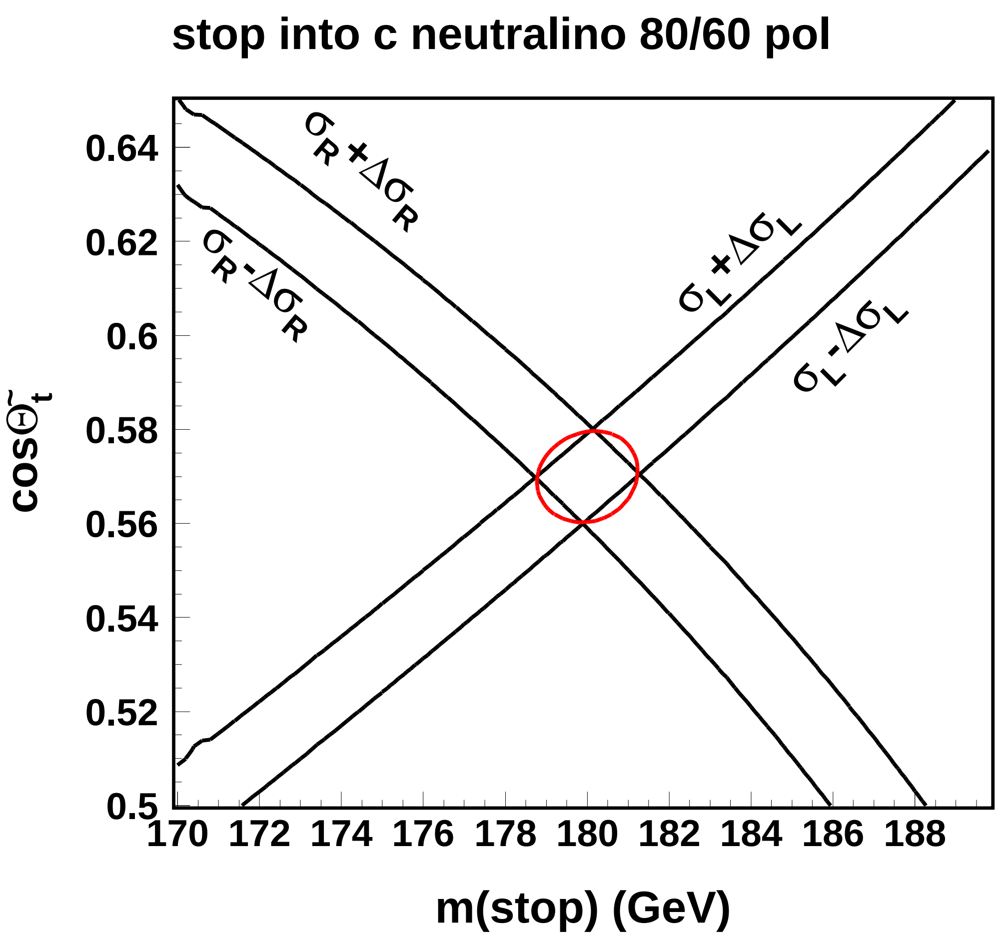
The Higgs mass as a precision observable (Scenario I:)

$\rightarrow$ Combination of direct and indirect information on $\tilde{t}$ sector parameters:

Direct information:

$e^+e^- \rightarrow \tilde{t}_1\tilde{t}_1$ at high luminosity LC

(80% pol. e^- beam, 60% pol. e^+ beam, $\sqrt{s} = 500$ GeV, $\mathcal{L} = 500$ fb^{-1})
[*R. Keränen, H. Nowak, A. Sopczak '00*]

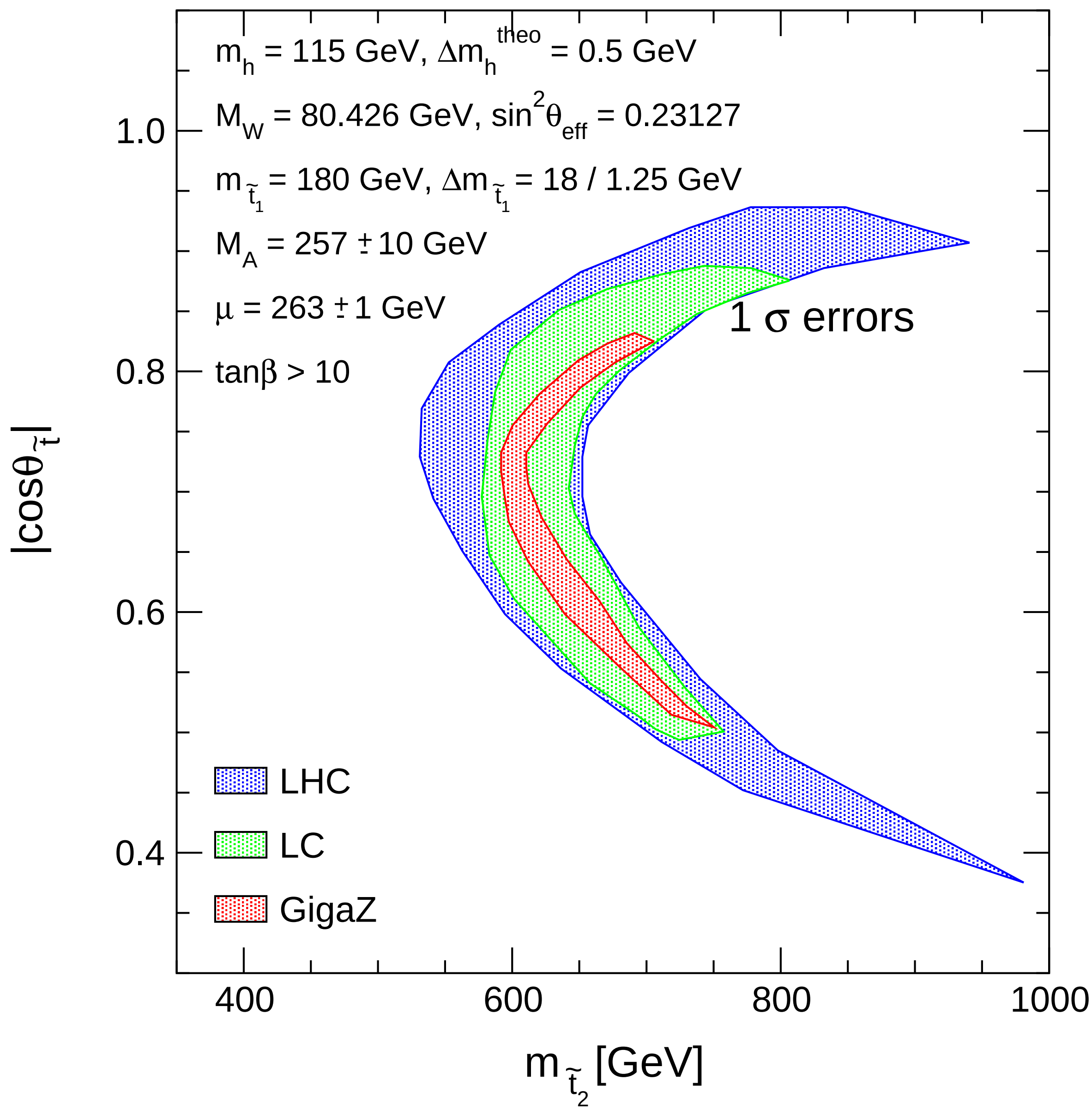


$\Rightarrow$ precise direct determination of $m_{\tilde{t}_1}$ and $\theta_{\tilde{t}}$

Indirect information:

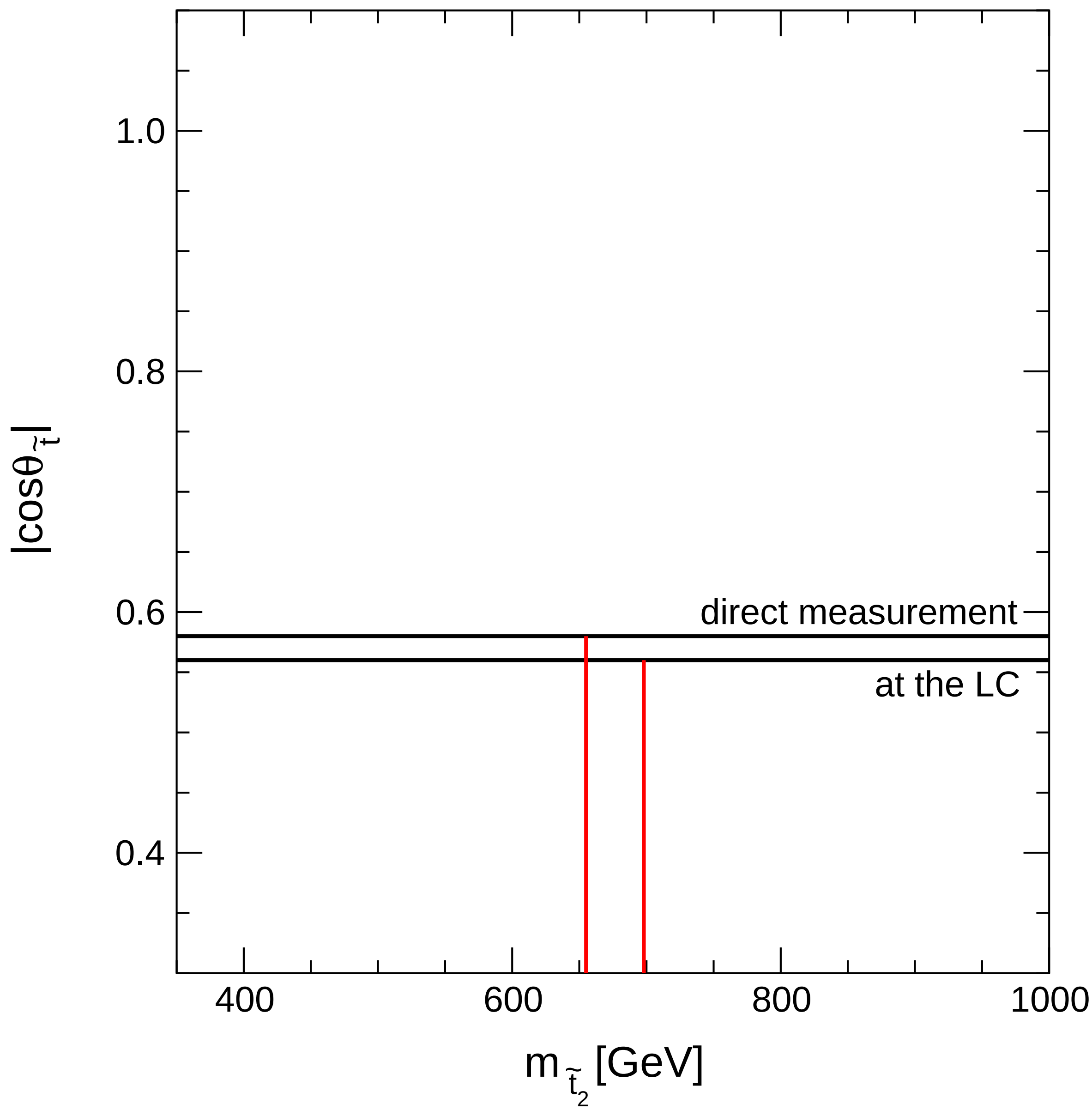
Constraints on $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$ from precision observables $m_h, M_W, \sin^2\theta_{\text{eff}}$ at LHC , LC , GigaZ :
[S.H., G.W. '00]

Allowed region in $m_{\tilde{t}_2}$-$\cos\theta_{\tilde{t}}$ plane:
($M_A = 257 \pm 10$ GeV, $\tan\beta > 10$):



Complementary of direct ↔ indirect information

⇒ Indirect determ. of $m_{\tilde{t}_2}$ with high precision



1.0
0.8
0.6
0.4
$|\cos\theta_{\tilde{t}}|$
direct measurement
at the LC
400
600
800
1000
$m_{\tilde{t}_2}$ [GeV]

Scenario II:

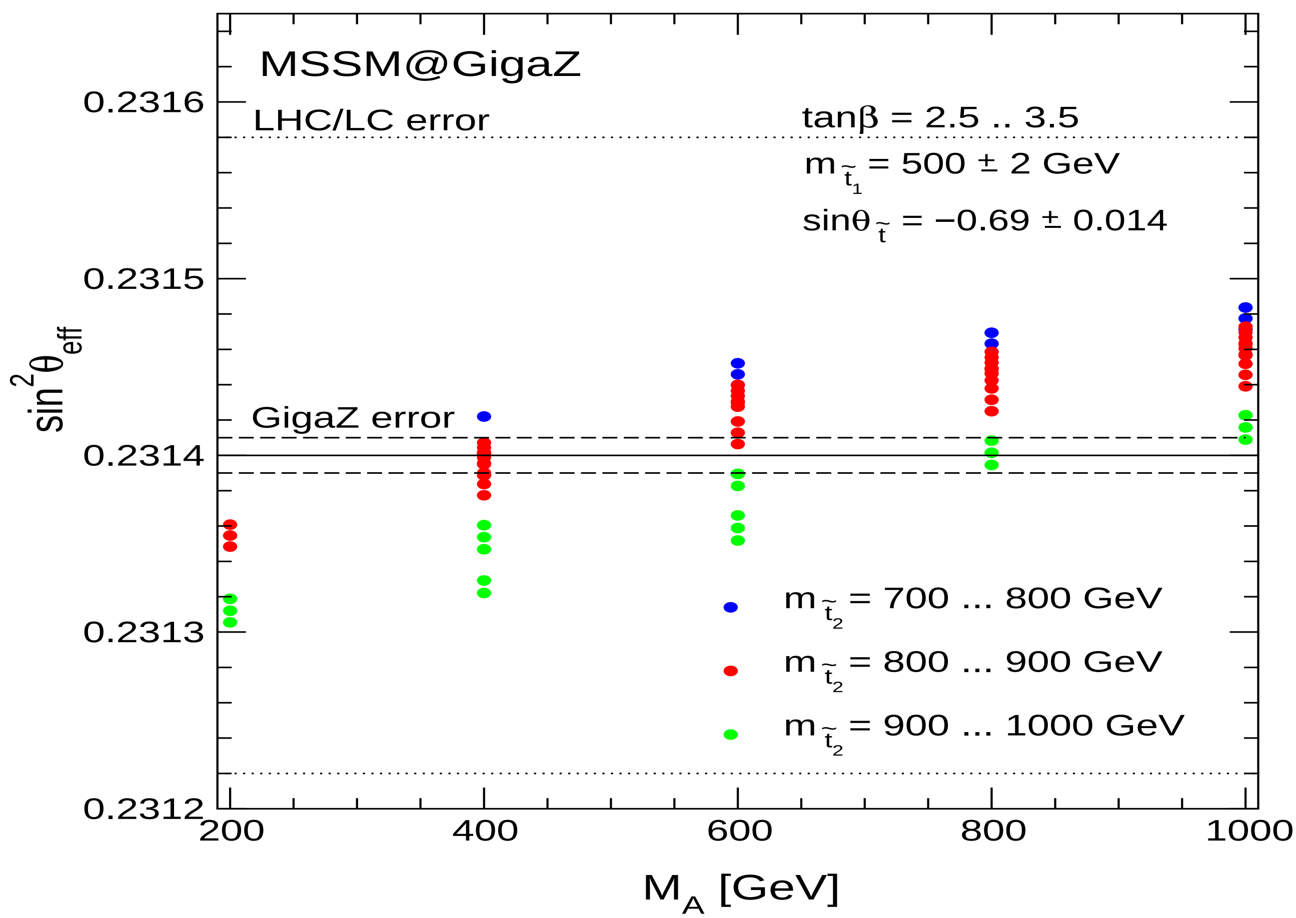
[*J. Erler, S.H., W. Hollik, G.W. and P. Zerwas '00*]

no experimental information on $m_{\tilde{t}_2}$, M_A

$\sin^2\theta_{\text{eff}} = 0.23140 \pm 0.00001$

$m_{\tilde{t}_1} = 500 \pm 2$ GeV, $\sin\theta_{\tilde{t}} = -0.69 \pm 2\%$, $\tan\beta = 3 \pm 0.5$

Constraints for M_A only from $\sin^2\theta_{\text{eff}}$:



⇒ Logarithmic M_A dependence

⇒ no constraints on M_A from LHC/LC measurement

Now: use all PO's

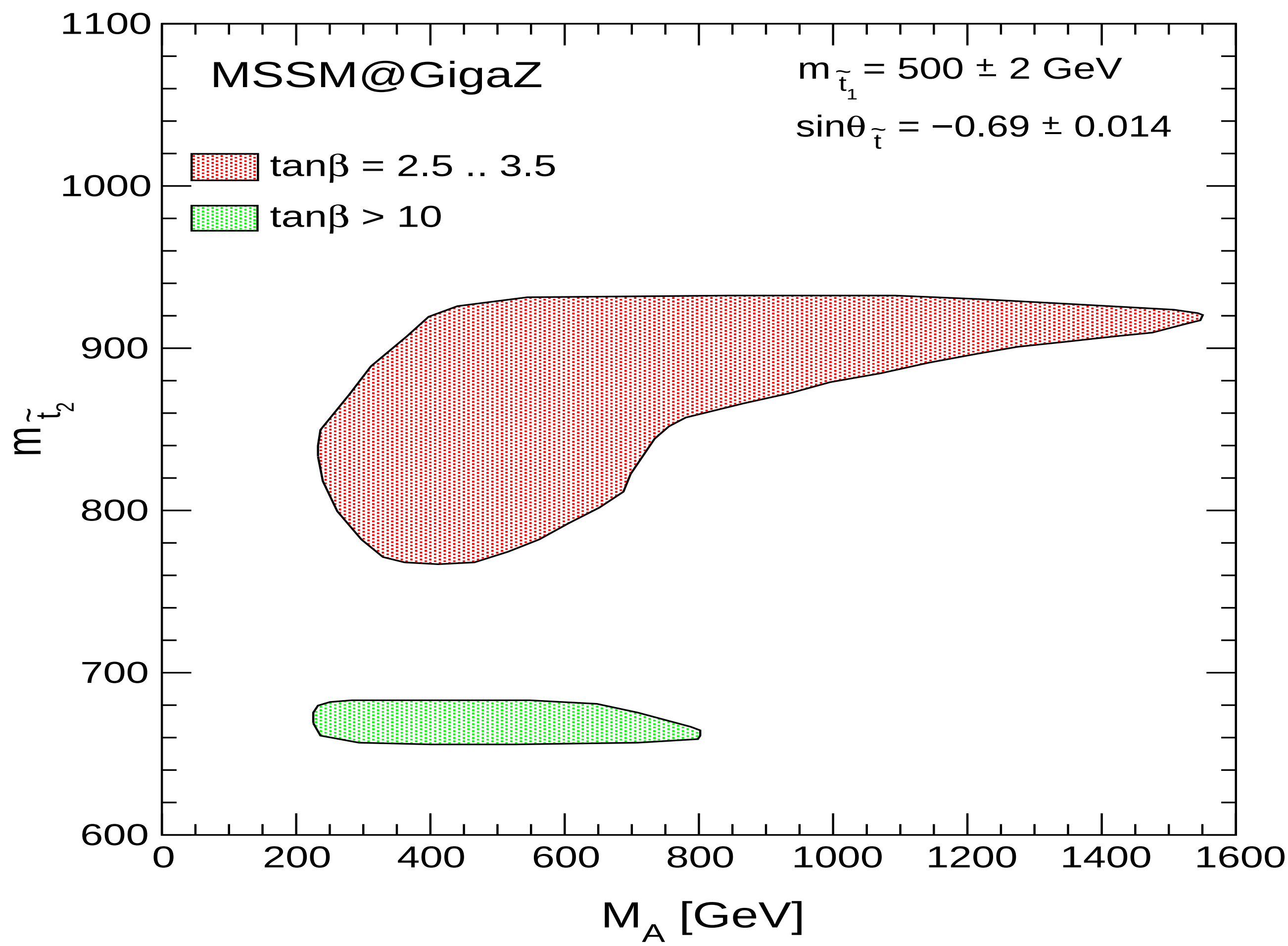
$\to M_W, \sin^2\theta_{\text{eff}}, m_h$:

$M_W = 80.4 \pm 0.006$ GeV, $\sin^2\theta_{\text{eff}} = 0.23140 \pm 0.00001$
$m_h = 115 \pm 0.05$ GeV, $\delta m_h^{\text{theo}} = \pm 0.5$ GeV

$2.5 < \tan\beta < 3.5$ or $\tan\beta \geq 10$
(from measurements in gaugino sector)

$m_{\tilde{t}_1} = 500 \pm 2$ GeV, $\sin\theta_{\tilde{t}} = -0.69 \pm 2\%$, $A_b = A_t \pm 10\%$,
$\mu = -200 \pm 1$ GeV, $M_2 = 400 \pm 2$ GeV, $m_{\tilde{g}} = 500 \pm 10$ GeV

Allowed region in M_A-$m_{\tilde{t}_2}$ plane:



$\Rightarrow$ Constraints on $m_{\tilde{t}_2}$, upper bound on M_A

Scenario III:

[J. Erler, S.H., W. Hollik, G.W. and P. Zerwas '00]

no experimental information on $\tan\beta$, M_A

$m_{\tilde{t}_2} = 520 \pm 1$ GeV or $m_{\tilde{t}_2} = 640 \pm 10$ GeV

$M_W = 80.4 \pm 0.006$ GeV, $\sin^2\theta_{\rm eff} = 0.23138 \pm 0.00001$

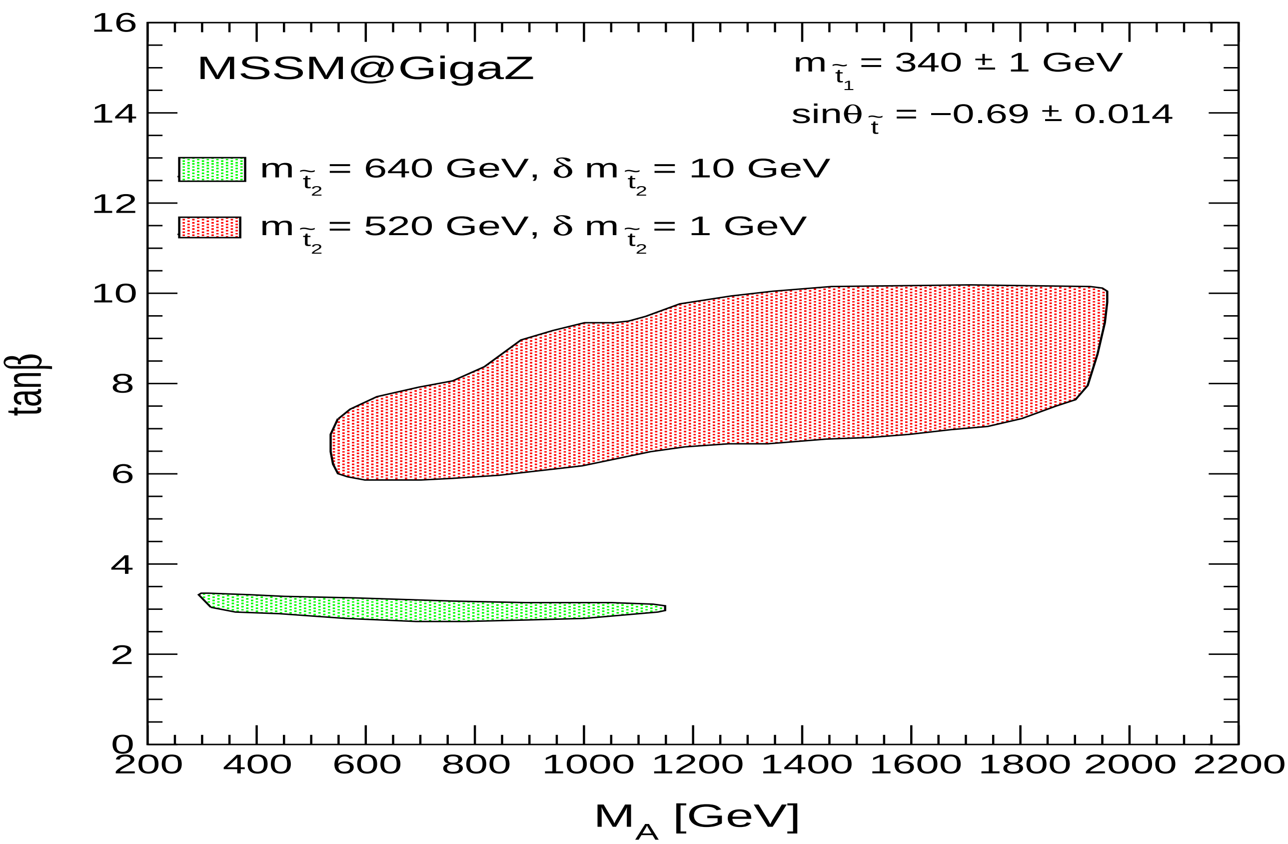
$m_h = 110 \pm 0.05$ GeV, $\delta m_h^{\rm theo} = \pm 0.5$ GeV

$m_{\tilde{t}_1} = 340 \pm 1$ GeV, $\sin\theta_{\tilde{t}} = -0.69 \pm 2\%$,

$A_b = -640 \pm 60$ GeV, $\mu = 316 \pm 1$ GeV,

$M_2 = 152 \pm 2$ GeV, $m_{\tilde{g}} = 496 \pm 10$ GeV

Allowed region in M_A-$\tan\beta$ plane:



$\Rightarrow$ Constraints on $\tan\beta$, upper bound on M_A

4. Conclusions

- **GigaZ:** $\rightarrow$ $\delta \sin^2\theta_{\text{eff}} = 0.00001$
  $\delta M_W = 6$ MeV,

  Combined with high precision of LC for
  m_h , m_t, $m_{\tilde{t}}$, . . .

  $\Rightarrow$ Highly sensitive test of SM and MSSM

- SM:
  Indirect determination of M_H:

  $$\delta M_H / M_H \approx 7\%$$

- MSSM:
  m_h is precision observable
  - Combination of direct and indirect constraints in the $\tilde{t}$ sector:
    determination of $m_{\tilde{t}_2}$
  - Constraints on $\tan\beta$
  - upper bound on M_A possible

- High sensitivity to scales beyond SM and MSSM