1998 CERN Summer Student Lecture Programme

Particle Physics: The Standard Model

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The Ginzburg–Landau theory of superconductivity provides a phenomenological understanding of the Meissner effect: the observation that an external magnetic field does not penetrate the superconductor. Ginzburg and Landau introduce an "order parameter" ψ, such that $|\psi|^2$ is related to the density of superconducting electrons. In the absence of an impressed magnetic field, expand the free energy of the superconductor as

$$G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha\,|\psi|^2 + \beta\,|\psi|^4\,,$$

where α and β are phenomenological parameters.
(a) Minimize $G_{\text{super}}(0)$ with respect to the order parameter and discuss the circumstances under which spontaneous symmetry breaking occurs. Compute $\langle|\psi|^2\rangle_0$, the value at which $G_{\text{super}}(0)$ is minimized.
(b) In the presence of an external magnetic field $\mathbf{B}$, a gauge-invariant expression for the free energy is

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{2} + \frac{1}{2m^*}\psi^*(-i\nabla - e^*\mathbf{A})^2\psi.$$

(The effective charge e^* turns out to be $2e$, because $|\psi|^2$ represents the density of Cooper pairs.) Derive the field equations that follow from minimizing $G_{\text{super}}(\mathbf{B})$ with respect to ψ and $\mathbf{A}$. Show that in the weak-field approximation ($\nabla\psi \approx 0$, $\psi \approx \langle\psi\rangle_0$) the photon acquires a mass within the superconductor. [Reference: V. L. Ginzburg and L. D. Landau, *Zh. Eksp. Teor. Fiz.* **20,** 1064 (1950); English translation: see *Men of Physics: Landau,* Vol. II, edited by D. ter Haar, Pergamon, New York, 1965. For further information, see §21.6 of Weinberg's *The Quantum Theory of Fields,* vol. 2, "Modern Applications" (Cambridge University Press, Cambridge, 1996).]