Fringe Fields in COSY Infinity

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- **Avenues for fringe field specification in COSY:**
  - on-axis multipole strengths defined by 6-parameter Enge functions
  - symplectic scaling
  - multipole or field data from magnet design codes and/or measurements

- Most important aspects of fringe field representation and implementation are available in **COSY's User Guide and Reference Manual, pages 30-40,** available on the web at

http://cosy.nscl.msu.edu//cosymanu/

Practical aspects

- The command **FR <mode>;** specifies the mode of fringe field map computation:
  - by default, fringe fields neglected $\Leftrightarrow$ **FR 0;**
  - if **<mode>=1,1.9,2,3** fringe fields are taken into account, and the meaning of the modes are the following:
    - mode **1** is a low precision version of mode **3**
    - mode **1.9** is a "linearly corrected" version of mode **2**
    - mode **2** is the symplectic scaling of fringe fields (fast, symplectic, very accurate close to the reference map)
    - mode **3** is numerical integration through the fringe field (most accurate and time consuming, integrator has adjustable precision)
- Flexibility: modes allow computation of fringe fields of:
  - every element in the lattice
  - only certain multipole orders
  - entrance and/or exit
  - magnetic and/or electric
  - only certain elements
  - stand alone fringe field maps for later reuse (modes **-1, -2**)
- Related commands:
  - **FC, FC2, FD, FD2, FP, ESET, WSM**
  - used to specify the Enge coefficients, defaults, accuracy of numerical integration, new reference files for symplectic scaling

Practical aspects (cont.)

- The command **GE**
  - specifies the zeroth and first order y dependence of the scalar potential (electric and/or magnetic) in analytic form (as a DA vector) at support points
- The command **MF**
  - applicable in case of midplane symmetry
  - takes as input measured field data in the midplane
- The command **MGE**
  - arbitrary combination of normal and skew multipole data at support points along straight axis (from measurement or magnet design codes)

- All **3 methods** use the same numerical integrator that is also used for **FR 3**

Technical details

- Enge function

$$F(s) = \frac{1}{1 + \exp \sum_{i=0}^{n} a_i \left(\frac{s}{D}\right)^i}$$

  where s is the arclength, D the full aperture of the element, and a_i are the Enge coefficients, with property that $F(s) \to 1$ for large positive s, and $\to 0$ for large negative s at element entrance; $s = 0$ at effective field boundary; other way around at exit

  - COSY uses a 6 parameter Enge function to specify on-axis multipoles for the **FR** modes (see **FC** and **FD**)

- Symplectic scaling:
  - compute and store fringe field map depending on parameters
  - reload and relate it through scaling transformations to the actual case - done automatically

- Multipole/Field data at support points:
  - Gaussian interpolation in 1D (MGE) and 2D (MF)

$$B_y(x,s) = \sum_{i_x} \sum_{i_s} \frac{BY(i_x, i_s)}{\pi N^2} \exp\left[-\frac{(x - x(i_x))^2}{(\Delta x)^2 N^2} - \frac{(s - s(i_s))^2}{(\Delta s)^2 N^2} \right]$$

  - assures differentiability, locality, and smoothing of data

- Scalar potential for **GE**
  - spline interpolation; preserves derivatives at support points

Fields are always Maxwellian

- In summary, data for fringe field map computation may be specified in one of the following formats:
  - analytically:
    - Enge function falloff of on-axis multipoles
    - scalar potential at support points
    - multipole decomposition of magnet design code generated magnet models
  - numerically, from measured data
    - field components in one plane (curved reference orbit)
    - on-axis multipoles (straight reference orbit)

- It does not matter which method one utilizes: internally COSY makes sure that the **fields always satisfy Maxwell's equations** up to the order of calculation, i.e.

$$\Delta\phi = 0$$

  - COSY first builds the scalar potential consistent with vanishing Laplacian, and then obtains fields as partial derivatives of the potential
  - no numerical differentiation involved $\Rightarrow$ high order derivatives needed for map computation are correct